E C O N O M I C S B U L L E T I N

Expected utility: a defense

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Abstract

In recent papers Matthew Rabin and Richard H. Thaler have argued that expected utility theory generates implausible predictions about individuals' attitudes toward small vs. large risks. Specifically, these authors argued that expected utility theory, plus the assertion that individuals reject small risks that are actuarially unfavorable, implies that agents should reject large risks which in fact they would accept. In this paper we question the presumption that the small risks are in fact rejected: they have risk−return characteristics that are superior to those of the daily returns on common stocks, which individuals generally find acceptable.

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Matthew Rabin [2] and Rabin and Richard H. Thaler [3] argued that expected utility theory generates implausible predictions about individuals' attitudes toward large vs. small risks.[1] For example, they showed that if an agent rejects a 50-50 gamble between losing $10 and winning $11, he will also reject a 50-50 gamble between losing $100 and winning an infinite amount of money. They asserted that, from introspection, the small gamble would be rejected and the large gamble accepted. The conclusion is that expected utility must be wrong.

Is it so obvious that the small risky prospect just described will be rejected? Consider an agent who is just indifferent between accepting and rejecting the small gamble at all levels of wealth. That agent has negative exponential utility $U(x) = -e^{-\alpha x}$, with α equal to the value that solves $-e^{-\alpha x} = (-e^{-\alpha(x+11)} - e^{-\alpha(x-10)})/2$, which is the same for all x. Now imagine that the gamble is repeated 365 times—once each day for a year. An agent with negative exponential utility will be indifferent between making the bet once, not at all, or every day, since past outcomes do not affect his risk aversion. Since the expected payoff of the gamble played once is 0.5, the expected payoff of 365 independent repetitions is $365 * 0.5 = 182.5$. Similarly, the standard deviation of the gamble played once is 10.5, so its standard deviation played 365 times is $10.5 * \sqrt{365} = 200.6$. We have here an investment with a risk-return tradeoff that is considerably better than we see on the stock market.

Every day individuals maintain positions in risky portfolios that look less attractive than Rabin-Thaler's win-11/lose-10 gamble.[2] One concludes that, in terms of what they actually do, individuals accept Rabin-Thaler's gamble. This poses an interesting research question: why do individuals reject in questionnaires and experiments risky prospects that are more attractive than real-world prospects that they accept every day? Whatever the explanation for this discrepancy between what people do and what they say they do, there are no implications whatsoever here for the validity or lack thereof of expected utility, at least insofar as the goal is to model what agents actually do.[3]

[1] Richard Watt [5], Ariel Rubinstein [4] and Ignacio Palacios-Huerta, Roberto Serrano and Oscar Volij [1] have also written criticisms of Rabin-Thaler. The last of these makes essentially the same point as here.

[2] This argument invokes intuition from security markets, in which agents can optimize over the number of shares held, in analyzing Rabin-Thaler's example, in which agents' choice is whether to accept or reject a random prospect of a given size. There is no reason to think that this difference invalidates our conclusion that individuals in fact accept random prospects that according to Rabin-Thaler's introspection, they say they reject.

[3] It may be objected that this example is unrealistic because of its dependence on the negative exponential utility function. As seen above, that utility function is appropriate if we are to analyze an agent who is indifferent between accepting and rejecting a random prospect of given absolute magnitude at all levels of wealth.

The example can be recast so that the random prospect is proportional to the agent's wealth and the agent's utility function displays contant relative risk aversion rather than constant absolute risk aversion. In that case a small gamble consisting of a 0.35% gain vs. a 0.30% loss corresponds to a large gamble with expectation of 9.6% and standard deviation of 6.8%. This example is qualitatively

References

[1] Ignacio Palacios-Huerta, Roberto Serrano, and Oscar Volij. Rejecting small gambles under expected utility. Brown University, 2002.

[2] Matthew Rabin. Risk aversion and expected utility theory: A calibration theorem. *Econometrica*, 68:1281–1292, 2000.

[3] Matthew Rabin and Richard H. Thaler. Response. *Journal of Economic Perspectives*, 16:229–230, 2002.

[4] Ariel Rubinstein. Comments on the risk and time preference in economics. Reproduced, Princeton University, undated.

[5] Richard Watt. Defending expected utility theory. *Journal of Economic Perspectives*, 16:227–229, 2002.

similar to that given in the text. Details are available from the author.