E C O N O M I C S   B U L L E T I N

Econometrics of the Forward Premium Puzzle

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Abstract

This paper compares the "level" regression of the future spot rate on the current forward rate, which yields a slope coefficient close to unity, to the forward premium puzzle, i.e., a regression of the change in the spot exchange rate on the forward premium, which paradoxically yields a slope coefficient that is frequently negative. We argue that the striking difference between these two otherwise equivalent regressions follows from the existence of a bias together with the non-stationarity of underlying variables. In addition, we contend that non-rationality may potentially explain the existence of the bias that generates the forward premium puzzle.

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1 Introduction

The Forward Premium Puzzle is an empirical paradox in the foreign exchange market that continues to pose a challenge to international economists. An ordinary least squares (OLS) regression of the future change in the log of the spot exchange rate on the forward premium (the log of the forward exchange rate minus the log of the spot exchange rate) is expected to yield a coefficient of unity under risk neutrality and rational expectations. Instead, regression estimates of this "forward premium" specification yield a coefficient that is significantly less than unity and frequently negative. Much of the burgeoning literature attempting to solve the puzzle has focused on explanations involving a risk premium in the forward exchange market, with mixed findings[1].

A second specification involving spot and forward exchange rates, referred herein as the "level" specification, was pursued early in this literature – an OLS regression of the log of the future spot exchange rate on the log of the current forward exchange rate[2]. Although not without econometric concerns, this regression typically yields a coefficient close to unity, a finding which seems consistent with rational expectations. Comparing estimates from these two equivalent specifications suggests a related puzzle – how can a small and often insignificant deviation of the coefficient from unity in the level specification become so greatly magnified that it causes a sign reversal in the forward premium specification?

The simplest approach to evaluate the forecasting ability of the forward exchange rate would seem to be the level form. However, the variables in the level form (the future spot and current forward exchange rates) are non-stationary I(1), which implies that regressing one of them on the other may lead to inconsistency given the well-known unit root problem[3]. The forward premium form involves stationary I(0) variables (the future change in the spot exchange rate and the forward premium), so the resulting regression coefficient is consistent, which explains the literature's almost universal reliance on this specification. More recently, Evans and Lewis (1993) demonstrate that the variables in the level specification, the future spot and the current forward exchange rates, are cointegrated, implying that the level regression is in fact super consistent[4]. If so, the level form indeed yields legitimate estimates and one need not focus only on the traditional forward premium specification. Therefore, there seems a contradiction in the implications of the two results - the "level" estimate suggests that the forward rate is an accurate predictor of the future spot exchange rate, while the "forward premium" estimate suggests otherwise.

To explore this apparent contradiction, we focus on a non-rational explanation for the bias in the two specifications, both because of the empirical challenge discovered by Fama (1984) for a risk premium approach as well as the theoretical and empirical support for non-rationality in Chakraborty (2008) and Chakraborty and Evans (2008). The theoretical analysis leads to stark empirical predictions, which are then tested using data on spot and forward exchange rates between the US dollar and four other major currencies. The general conclusion is that the dramatic

[1] For discussion about the forward premium puzzle, see Froot and Thaler (1990) and Obstfeld and Rogoff (1997, pp. 588-91). For surveys of the research see Lewis (1995) and Engel (1996).

[2] For some early papers, see Cornell (1977), Levich (1979), Frankel (1980) and McCallum (1994); for a recent discussion, see Zivot (2000).

[3] For an early statement, see Granger and Newbold (1974). For a full treatment of the unit root problem, see Hamilton (1994, 557-562).

[4] See Engle and Granger (1987) and Hamilton (1994, 571-629) for general development on cointegration and super consistency. For related applications of cointegration to spot and forward exchange rates, see Hakkio and Rush (1989), Hai, Mark, and Wu (1997), and Zivot (2000).

difference in the coefficient deviation from unity and possible sign reversal shifting from the level to the forward premium specification can be explained by the variance-covariance properties of the relevant I(0) and I(1) variables in the two specifications, i.e., the fact that the variables are stationary in the forward premium form and non-stationary in the level form.

The next section develops, for both specifications, the theoretical decomposition of the coefficients as variances and covariances of the relevant variables, section 3 presents estimation results, and section 4 concludes.

2 Level and Forward Premium Models

The "level" specification of the relationship between the forward exchange rate f_t and the future spot exchange rate s_{t+1} , where both exchange rates are defined as the dollar price of foreign exchange and expressed in logarithms, is the following:

$$s_{t+1} = \delta + \gamma f_t + \psi_{t+1} \tag{1}$$

where δ is the intercept, γ is the slope coefficient, and ψ is a random error term.

Table 1: Estimates from the "Level" regression equation $s_{t+1} = \delta + \gamma f_t + \psi_{t+1}$ using *Monthly* and *Quarterly* data on four exchange rates.

Monthly Data				
Currency	AUD	CAD	GBP	JPY
No. of obs	201	201	201	201
$\hat{\gamma}$	0.982	0.998	0.949*	0.958*
	(0.014)	(0.012)	(0.024)	(0.019)
$\bar{R}^2$	0.96	0.97	0.88	0.93
Quarterly Data				
Currency	AUD	CAD	GBP	JPY
No. of obs	67	67	67	67
$\hat{\gamma}$	0.934	0.989	0.841*	0.859*
	(0.046)	(0.036)	(0.073)	(0.058)
$\bar{R}^2$	0.86	0.92	0.66	0.77

Note: * and ** represent 5% and 1% levels of significance for $H_0 : \gamma = 1$, respectively. *Standard Errors* are in parentheses.

The key null hypothesis is that the slope coefficient γ is unity under rational expectations and risk neutrality. The results from this regression using recent data on four exchange rates US dollar price of Australian dollar (AUD), Canadian dollar (CAD), British pound (GBP) and Japanese yen (JPY), are presented in Table 1[5]. As the results show, the estimate $\hat{\gamma}$ is very close to unity in both quarterly and monthly data over the same range, and in case of AUD and CAD are insignificantly different from unity. For GBP and JPY, although the deviations from unity are significant, the magnitudes are very close to unity. This suggests only a small deviation from the null hypothesis.

Table 2: Estimates from the "Forward Premium" regression equation $\Delta s_{t+1} = \alpha + \beta(f_t - s_t) + \mu_{t+1}$ using *Monthly* and *Quarterly* data on four exchange rates.

Monthly Data				
Currency	AUD	CAD	GBP	JPY
No. of obs	201	201	201	201
$\hat{\beta}$	-0.53^*	-0.26	0.85	-1.52^*
	(0.67)	(0.71)	(0.93)	(1.07)
$\bar{R}^2$	-0.01	-0.01	-0.01	0.01
Quarterly Data				
Currency	AUD	CAD	GBP	JPY
No. of obs	67	67	67	67
$\hat{\beta}$	-1.23^*	-0.60^*	0.45	-1.94^*
	(1.02)	(0.78)	(1.06)	(1.18)
$\bar{R}^2$	0.01	-0.01	-0.01	0.02

Note: * and ** represent 5% and 1% levels of significance for $H_0 : \beta = 1$, respectively. *Standard Errors* are in parentheses.

The traditional "forward premium" specification is

$$\Delta s_{t+1} = \alpha + \beta(f_t - s_t) + \mu_{t+1} \tag{2}$$

where α is the intercept, β is the slope coefficient, and μ is a random error term. The null hypothesis in this form is that the slope coefficient β is unity. The results from this regression using the

[5] A detailed description of the data and sources and more formal empirical analyses are given in the empirical section.

same data described above are presented in Table 2. As the results show, the estimate $\hat{\beta}$ is significantly less than unity and negative in the majority of the cases, replicating the forward premium puzzle. The only exception is GBP, where $\hat{\beta}$ is positive but less than unity. Therefore, the bias is uniformly downward, suggesting a large deviation from the null hypothesis.

2.1 General Model

Suppose first that agents are risk averse. In this case, the forward rate is their expected value of the future spot rate minus a premium they are willing to forego in order to eliminate foreign exchange risk. Thus,

$$E_t[s_{t+1}] = f_t + RP_t \tag{3}$$

where $E_t[s_{t+1}]$ is the expected value in period t of the spot rate in period $t+1$, and RP_t is the risk premium in period t. Next, define e_{t+1} as the forecast error such that

$$s_{t+1} = E_t[s_{t+1}] + e_{t+1} \tag{4}$$

If agents are not rational, they make systematic forecast errors and e_{t+1} is correlated with variables in period t. Otherwise, e_{t+1} is uncorrelated with any information in period t.

Combining eq. (3) and eq. (4) we obtain

$$s_{t+1} = f_t + RP_t + e_{t+1} \tag{5}$$

Next, subtract s_t from both sides of eq. (5):

$$\Delta s_{t+1} = (f_t - s_t) + RP_t + e_{t+1} \tag{6}$$

Now, the OLS estimator of γ in eq. (1) is $\hat{\gamma}$:

$$\hat{\gamma} = \frac{Cov(s_{t+1}, f_t)}{V(f_t)} \tag{7}$$

where V and Cov are the sample variance and covariances, respectively. Similarly, the OLS estimate of β in eq. (2) is $\hat{\beta}$:

$$\hat{\beta} = \frac{Cov(\Delta s_{t+1}, f_t - s_t)}{V(f_t - s_t)} \tag{8}$$

Combining eq. (7) and eq. (5) yields

$$\hat{\gamma} = 1 + \frac{Cov(f_t, RP_t)}{V(f_t)} + \frac{Cov(f_t, e_{t+1})}{V(f_t)} \tag{9}$$

Similarly, combining eq. (8) and eq. (6) yields

$$\hat{\beta} = 1 + \frac{Cov(f_t - s_t, RP_t)}{V(f_t - s_t)} + \frac{Cov(f_t - s_t, e_{t+1})}{V(f_t - s_t)} \tag{10}$$

Thus, in general, risk aversion and/or non-rationality offer plausible explanations why OLS estimates of γ and β may differ from unity. To see this, consider the special case of risk neutrality and rational expectations. With risk neutrality, $RP_t = 0$, and the second terms on the right-hand sides of eqs. (9) and (10) become zero. In addition, if agents possess rational expectations, then the forecast error e_{t+1} is uncorrelated with the information set in period t (including $f_t - s_t$), which implies that the third terms on the right-hand sides of eqs. (9) and (10) are also zero. Thus, with risk neutrality and rational expectations, eqs. (9) and (10) collapse to $\hat{\gamma} = \hat{\beta} = 1$[6].

Next, consider the conditions required to generate the forward premium puzzle, i.e., $\hat{\beta}$ less than unity and often negative. For $\hat{\beta}$ to be less than unity, it follows from eq. (10) that

$$\frac{Cov(f_t - s_t, RP_t)}{V(f_t - s_t)} + \frac{Cov(f_t - s_t, e_{t+1})}{V(f_t - s_t)} < 0 \tag{11}$$

which implies that at least one of the two terms on the left-hand side of eq. (11) must be negative, and their sum must be negative. For $\hat{\beta}$ to be negative, it follows from eq. (10) that

$$\frac{Cov(f_t - s_t, RP_t)}{V(f_t - s_t)} + \frac{Cov(f_t - s_t, e_{t+1})}{V(f_t - s_t)} < -1 \tag{12}$$

Which of the two terms dominates in generating a downward bias? To address this question we rely on the famous result of Fama (1984). According to Fama's decomposition, for the risk premium term to dominate, the variance and covariance need to have certain properties that are not supported by data[7]. In our paper we take Fama's results as given and focus on a non-rationality approach.

2.2 Non-rationality

Assume that agents are risk neutral, i.e., $Cov(f_t - s_t, RP_t) = 0$, but that expectations are not rational, i.e., $Cov(f_t - s_t, e_{t+1}) \neq 0$. Thus, the forecast error in the next period is correlated with information in this period, and agents make systematic errors in prediction of the spot exchange rate. In order for $\hat{\beta} < 1$, it follows from eq. (10) in this case that $Cov(f_t - s_t, e_{t+1}) < 0$. Since it is not possible a priori to predict the sign of this covariance, non-rationality can potentially explain the puzzle of $\hat{\beta} < 1$ if $Cov(f_t - s_t, e_{t+1})$ is negative.

This also has implications for the "level" regression. In order for there to be a downward bias in $\hat{\gamma}$ from unity, $Cov(f_t, e_{t+1}) < 0$ must hold. Alternatively, if $\hat{\gamma}$ is insignificantly different from unity or very close to unity, then $Cov(f_t, e_{t+1})$ must be very close to zero. In this case, non-rational expectations cannot explain the results and we must look for alternative explanations for the bias in $\hat{\beta}$. However, it may so happen that $Cov(f_t, e_{t+1})$ is negative yet for some other reason the deviation is minimal in $\hat{\gamma}$. In that case, non-rational expectations would retain its potential as an explanation, and the reason behind the minimal deviation in $\hat{\gamma}$ from unity needs to be explored. In this paper, we show that this argument is indeed the case - the reason the bias in $\hat{\gamma}$ is small can be found in the non-stationary properties of f_t.

[6]Eq. (10) is not new. It is a variant of the derivations found in Frankel et. al. (1987) and Frankel et. al. (1989).

[7]Our data also do not support these properties required for a risk premium explanation. We omit these results for brevity.

2.3 Comparing the Level to the Forward Premium Specification

Suppose that agents are risk-neutral, and that non-rationality is the only source of bias in eqs. (9) and (10). Thus, from eq. (9) the bias in $\hat{\gamma}$ in the level specification is $\frac{Cov(f_t,e_{t+1})}{V(f_t)}$, and from eq. (10) the bias in $\hat{\beta}$ in the forward premium specification is $\frac{Cov(f_t-s_t,e_{t+1})}{V(f_t-s_t)}$. Evidence discussed in the introduction suggests that, paradoxically, the bias in the level specification is minimal, yet the bias in the forward premium specification is strongly negative, causing a frequent sign reversal in the coefficient estimate $\hat{\beta}$. A plausible resolution to this paradox can be found by exploring the stationary-nonstationary properties of the relevant variables in the two bias terms. Our econometric argument relies on the following propositions and corollary, with proofs in the appendix.

Proposition 1 *If a_t is a univariate stationary variable following an AR(1) process and b_t is a univariate non-stationary variable following a random walk, then for given initial observations a_0 and b_0 the conditional covariance $Cov(a_t,b_t|a_0,b_0)$ is non-stationary in finite sample size but converges to a finite value as the sample size $t \to \infty$, i.e. is asymptotically stationary.*

Corollary 1 *If a_t is a univariate stationary variable following an AR(1) process and b_t is a univariate non-stationary variable following random walk, then for given initial observations a_0 and b_0 the ratio of the conditional covariance to conditional variance $\frac{Cov(a_t,b_t|a_0,b_0)}{V(b_t|b_0)}$ is a decreasing function of sample size t in finite samples and* $\lim_{t\to\infty}\frac{Cov(a_t,b_t|a_0,b_0)}{V(b_t|b_0)} = 0$.

Proposition 2 *If a_t and b_t are two univariate stationary variables following AR(1) processes, then for given initial observations a_0 and b_0 the change in the ratio of the conditional covariance to conditional variance $\frac{Cov(a_t,b_t|a_0,b_0)}{V(b_t|b_0)}$ with changing t is ambiguous, but it is asymptotically stationary.*

First, consider the bias term in the level specification, $\frac{Cov(f_t,e_{t+1})}{V(f_t)}$. Assume that the forward exchange rate f_t is a non-stationary variable and the forecast error e_{t+1} is stationary, conjectures supported by empirical evidence presented below. Also, assume that the first observation in any relevant series is considered as fixed or given. Given these statistical properties of f_t and e_{t+1}, according to *Corollary 1* the bias term in the level specification $\frac{Cov(f_t,e_{t+1})}{V(f_t)}$ is likely to be relatively "small" for samples of at least moderate size, although from *Proposition 1* $Cov(f_t,e_{t+1})$, which is the true source of the bias (possibly due to non-rationality), may remain significantly different from zero[8]. Furthermore, *Corollary 1* also suggests that $\frac{Cov(f_t,e_{t+1})}{V(f_t)}$ should decline in absolute value moving from smaller to larger samples in general. Therefore, the $\frac{Cov(f_t,e_{t+1})}{V(f_t)}$ term should decline moving from quarterly data to monthly data for a fixed number of years as the number of observations increases, although $Cov(f_t,e_{t+1})$ may not change significantly. This implies that,

[8]This conclusion is based on the assumption that f_t follows random walk and e_{t+1} follows a stationary AR(1) process.

although the bias exists, it does not appear in the level form regression of moderately large sample size because of the non-stationarity property of f_t, and therefore $\hat{\gamma}$ remains very close to unity. These implications of the level model are tested below.

Next, consider the bias term in the forward premium specification: $\frac{Cov(f_t - s_t, e_{t+1})}{V(f_t - s_t)}$. Since estimates of $\hat{\beta}$ are significantly less than unity and often negative, this bias term is expected to be relatively "large" in magnitude and negative, and in the majority of cases we should find that $\frac{Cov(f_t - s_t, e_{t+1})}{V(f_t - s_t)} < -1$. Since, the variables $f_t - s_t$ and e_{t+1} are stationary, according to *Proposition 2* the behavior of $\frac{Cov(f_t - s_t, e_{t+1})}{V(f_t - s_t)}$ with increasing sample size is ambiguous. Thus, the bias term in the forward premium form $\frac{Cov(f_t - s_t, e_{t+1})}{V(f_t - s_t)}$ may have any finite magnitude and is not systematically related to sample size, implications also tested below. However, theoretically the sign of the covariance term is ambiguous without placing restrictions on the source of the non-rationality.

Therefore, under non-rationality, the bias exists even in the "level" specification, but $\hat{\gamma}$ remains close to unity given the non-stationary properties of f_t and s_t. However, the deviation in $\hat{\beta}$ from unity in the forward premium specification is "large" given the stationary properties of its variables. Thus, this analysis offers a potential explanation for the apparent puzzle of little or no bias in the level specification of a regression between the spot and forward exchange rates, yet a dramatic negative bias with a frequent sign reversal in the traditional forward premium specification.

3 Empirical Evidence

3.1 Data

The data are monthly and quarterly series on four exchange rates – the US dollar prices of the Australian dollar (AUD), Canadian dollar (CAD), British pound (GBP) and Japanese Yen (JPY). Quarterly data are for the period 1988-Q4 to 2005-Q3. Monthly data are for the period 1988:12 to 2005:9. The spot exchange rate, one-month forward rate and three-months forward rate data are from *Bloomberg*. All raw exchange rate data are closing mid-prices for which the value-date is the last business day of the month/quarter. The future spot rate for a given period is constructed by observing the spot rate for which the value-date is the last business day one month/quarter ahead. Thus, end-points are adjusted properly. Logarithmic transformation is made on each series.

3.2 Non-stationarity of s_{t+1} and f_t

The first step is to show that s_{t+1} and f_t are non-stationary, so that the legitimacy of the "level" regression requires cointegration. Also, *Proposition 1* and *Corollary 1* could be applied only if f_t is non-stationary. Therefore, the Augmented Dickey-Fuller unit root test is applied on all the s_{t+1} and f_t from the data. The results are presented in Table 3. As the results show (except for the 3-month forward rate in GBP from quarterly data), future spot and current forward rates are non-stationary as the null hypothesis of a unit root could not be rejected even at the 10% level. Using 3-month forward rate series in GBP with quarterly data, one could not reject the same null at 5% level. Therefore, s_{t+1} and f_t are indeed non-stationary.

Table: 3. ADF unit root test results on current *Forward Rate* (f_t) and *Future Spot Rate* (s_{t+1}) from *Monthly* and *Quarterly* data on four exchange rates.

Monthly Data

Currency	Variable	No.of Obs	ADF Stat	10%Critical Value
AUD	f_t	201	-1.76	-2.57
	s_{t+1}	200	-1.94	-2.57
CAD	f_t	201	-0.97	-2.57
	s_{t+1}	200	-1.02	-2.57
GBP	f_t	201	-2.53	-2.57
	s_{t+1}	200	-2.43	-2.57
JPY	f_t	201	-1.94	-2.57
	s_{t+1}	200	-2.01	-2.57

Quarterly Data

Currency	Variable	No. of Obs	Stat	10% Critical Value
AUD	f_t	67	-1.80	-2.59
	s_{t+1}	66	-1.66	-2.59
CAD	f_t	67	-0.97	-2.59
	s_{t+1}	66	-0.97	-2.59
GBP	f_t	67	-2.63	-2.59
	s_{t+1}	66	-2.37	-2.59
JPY	f_t	67	-2.00	-2.59
	s_{t+1}	66	-2.12	-2.59

3.3 Cointegration of the Level Specification

Valid OLS estimation of the level specification requires cointegration between the future spot and current forward exchange rates. With cointegration, the regression estimates will be super consistent. To test this cointegration requirement, a Vector Error Correction model is estimated using the four exchange rates from both monthly and quarterly data in our sample. Engel and Granger (1987) describe the error correction model. Our analysis assumes a cointegrating relationship between s_{t+1} and f_t.

Cointegration is tested using two alternative VAR specifications - a VAR(1) given by

$$\Delta x_t = \Pi(s_t - \Phi f_{t-1}) + \Lambda_1 \Delta x_{t-1} + \zeta_t \tag{13}$$

and a VAR(2) given by

$$\Delta x_t = \Pi(s_t - \Phi f_{t-1}) + \Lambda_1 \Delta x_{t-1} + \Lambda_2 \Delta x_{t-2} + \zeta_t \tag{14}$$

Table: 4. Vector Error Correction estimates involving future spot rate (s_{t+1}) and current forward rate (f_t) from four exchange rates with VAR(1) [$\Delta x_t = \Pi(s_t - \Phi f_{t-1}) + \Lambda_1 \Delta x_{t-1} + \zeta_t$] and VAR(2) [$\Delta x_t = \Pi(s_t - \Phi f_{t-1}) + \Lambda_1 \Delta x_{t-1} + \Lambda_2 \Delta x_{t-2} + \zeta_t$] specifications.

Currency	AUD		CAD		GBP		JPY	
Monthly Data	*VAR*(1)	*VAR*(2)	*VAR*(1)	*VAR*(2)	*VAR*(1)	*VAR*(2)	*VAR*(1)	*VAR*(2)
No. of obs	199	198	199	198	199	198	199	198
$\hat{\Pi}_s$	1.71*	1.46	1.22	1.67	0.08	1.21	1.65	−0.23
	(0.67)	(0.75)	(0.87)	(0.94)	(1.02)	(1.11)	(1.12)	(1.09)
$\hat{\Pi}_f$	0.32**	0.23**	0.21**	0.13*	0.33**	0.19**	0.26**	0.13**
	(0.05)	(0.06)	(0.04)	(0.05)	(0.05)	(0.05)	(0.04)	(0.03)
$\hat{\Phi}$	−1.009	−1.009	−1.009	−1.01	−1.01	−1.015	−0.99	−0.99
	(0.003)	(0.004)	(0.003)	(0.001)	(0.004)	(0.006)	(0.003)	(0.004)
Quarterly Data	*VAR*(1)	*VAR*(2)	*VAR*(1)	*VAR*(2)	*VAR*(1)	*VAR*(2)	*VAR*(1)	*VAR*(2)
No. of obs	65	64	65	64	65	64	65	64
$\hat{\Pi}_s$	1.97	−0.38	1.37	1.85*	1.17*	0.46*	−0.32	−0.89*
	(0.67)	(0.31)	(1.01)	(0.93)	(0.46)	(0.20)	(0.80)	(0.39)
$\hat{\Pi}_f$	−0.04	0.04**	0.12	0.02	−0.01	−0.01	0.11**	0.05*
	(0.03)	(0.01)	(0.08)	(0.07)	(0.02)	(0.01)	(0.03)	(0.02)
$\hat{\Phi}$	−1.07	−0.87	−1.03	−1.05	−1.20	−1.47	−0.92	−0.82
	(0.02)	(0.04)	(0.02)	(0.02)	(0.06)	(0.15)	(0.02)	(0.03)

Note: * and ** represent 5% and 1% levels of significance for $H_0: \Pi_s = 0$ and $\Pi_f = 0$, respectively. *Standard Errors* are in parentheses.

where, $x_t = \begin{pmatrix} s_{t+1} \\ f_t \end{pmatrix}$, $\Pi = \begin{pmatrix} \hat{\Pi}_s \\ \hat{\Pi}_f \end{pmatrix}$, Φ is a scalar and the cointegrating coefficient on f_t when the coefficient on s_{t+1} is normalized to 1, $\Lambda_i = \begin{pmatrix} \Lambda_i^{ss} & \Lambda_i^{sf} \\ \Lambda_i^{fs} & \Lambda_i^{ff} \end{pmatrix}$ $(i = 1,2)$ is a 4 x 4 matrix of coefficients, and $\zeta_t = \begin{pmatrix} \zeta_{st} \\ \zeta_{ft} \end{pmatrix}$ represents regression error terms. $(s_t - \Phi f_{t-1})$ is the cointegrating relationship or the error correction term. Legitimate OLS regression of s_{t+1} on f_t requires $\Phi = 1$. Also, if there is cointegration then both or at least one of the coefficient estimates $\hat{\Pi}_s$ and $\hat{\Pi}_f$ must be significant.

Test results are presented in Table 4. As Table 4 shows, most of the coefficient estimates $\hat{\Pi}_f$ are significant even at the 1% level in the monthly data and in some variables of the quarterly data. Except for one case, either one or both of $\hat{\Pi}_s$ and $\hat{\Pi}_f$ are significant at least at the 5% level. Thus, the estimates in Table 4 clearly indicate that the non-stationarity of the variables in the level specification leads to super-consistency in OLS estimation due to cointegration. As a consequence, a side-by-side comparison of both the level and forward premium coefficient estimates is feasible in order to determine why they are so dramatically different. The other observation is that the cointegrating coefficient estimates $\hat{\Phi}$ are very close to 1 in almost all the cases[9]. This suggests the possibility that, in the long run, f_t predicts s_{t+1}. This also suggests that the forecast error $(e_{t+1} = s_{t+1} - f_t)$ is stationary. Therefore, combining the results from the previous subsection, *Proposition 1* and *Corollary 1* could be applied to e_{t+1} and f_t.

3.4 Direct Estimates of the Bias

Finally, we explore a more direct method of testing the theoretical predictions in Section 2. Table 5 presents, for the level model, estimates of $Cov(f_t, e_{t+1})$ and $V(f_t)$. Table 6 presents analogous estimates for the forward premium model. The level form variances and covariances are normalized by dividing by the variance of the corresponding spot rate s_t, and those from the forward premium form are normalized by dividing by the variance of the corresponding forecast error (e_{t+1})[10].

The evidence is consistent with that presented in Tables 1 and 2. The forward rate has a much larger variance in Table 5 compared to any other variance or covariance estimates in Tables 5 and 6, as predicted given its non-stationarity. Also, the covariance between the forward rate and the forecast error in Table 5 is very small, as predicted by theory developed in Section 2. Thus, the $\frac{Cov(f_t, e_{t+1})}{V(f_t)}$ is small in magnitude, but this does not necessarily imply $Cov(f_t, e_{t+1}) = 0$ (i.e. expectations are rational)[11]. Also, moving from quarterly to monthly data, $Cov(f_t, e_{t+1})$ diminishes and $V(f_t)$ increases in magnitude, possibly due to the much larger number of observations as predicted by *Corollary 1*. The numerator and denominator terms for the bias $\frac{Cov(f_t - s_t, e_{t+1})}{V(f_t - s_t)}$ in the

[9] The significance level of the test for $H_0 : \Phi = 1$ could not be determined from the given standard errors, as Engel and Granger (1987) show that standard t-tests are biased for this regression.

[10] This is because f_t is non-stationary and hence the normalization would be more appropriate if another non-stationary variable (in this case s_t) is used, while $f_t - s_t$ is stationary and hence normalization using a stationary variable (in this case e_{t+1}) is suitable.

[11] In some cases in Table 5 the ratio of $Cov(f_t, e_{t+1})$ to $V(f_t)$ is slightly different from $\frac{Cov(f_t, e_{t+1})}{V(f_t)}$ and the ratio of $Cov(f_t - s_t, e_{t+1})$ to $V(f_t - s_t)$ is different from $\frac{Cov(f_t - s_t, e_{t+1})}{V(f_t - s_t)}$. This is due to rounding.

forward premium specification in Table 6 are also small and are roughly of the same order of magnitude. Consequently, the bias in $\hat{\beta}$ is sufficiently large and negative so that the coefficient becomes negative, creating the forward premium puzzle.

Table 5: "Level" specification: Variance and covariance terms for the forward rate (f_t) and forecast error (e_{t+1}) using *Monthly* and *Quarterly* data on four exchange rates.

Currency	AUD	CAD	GBP	JPY
Monthly Data				
No. of obs	201	201	201	201
$Cov(f_t, e_{t+1})$	-0.018	-0.002	-0.050	-0.043
$V(f_t)$	0.985	0.982	0.980	1.01
$\frac{Cov(f_t, e_{t+1})}{V(f_t)}$	-0.018	-0.002	-0.051	-0.042
Quarterly Data				
No. of obs	67	67	67	67
$Cov(f_t, e_{t+1})$	-0.064	-0.011	-0.148	-0.146
$V(f_t)$	0.965	0.951	0.933	1.039
$\frac{Cov(f_t, e_{t+1})}{V(f_t)}$	-0.067	-0.011	-0.159	-0.141

Note: The variances $V(f_t)$ and covariances $Cov(f_t, e_{t+1})$ are normalized by dividing by the variance of the corresponding spot exchange rate $V(s_t)$.

In sum, the empirical evidence in Tables 3 through 6 is strongly consistent with the theoretical analysis relating the presence of a modest bias in the slope coefficient in the level specification to a substantial bias and sign reversal in the slope coefficient in the forward premium specification.

Table 6: "Forward Premium" specification: Variance and covariance terms for the forward premium ($f_t - s_t$) and forecast error (e_{t+1}) using *Monthly* and *Quarterly* data on four exchange rates.

Currency	AUD	CAD	GBP	JPY
Monthly Data				
No. of obs	201	201	201	201
$Cov(f_t - s_t, e_{t+1})$	-0.017	-0.012	-0.001	-0.011
$V(f_t - s_t)$	0.011	0.010	0.006	0.004
$\frac{Cov(f_t - s_t, e_{t+1})}{V(f_t)}$	-1.53	-1.26	-0.15	-2.53
Quarterly Data				
No. of obs	67	67	67	67
$Cov(f_t - s_t, e_{t+1})$	-0.031	-0.038	-0.008	-0.030
$V(f_t - s_t)$	0.014	0.024	0.014	0.011
$\frac{Cov(f_t - s_t, e_{t+1})}{V(f_t)}$	-2.23	-1.60	-0.55	-2.94

Note: The variances $V(f_t - s_t)$ and covariances $Cov(f_t - s_t, e_{t+1})$ are normalized by dividing by the variance of the corresponding forecast error $V(e_{t+1})$.

4 Conclusion

This paper explores the econometrics behind the forward premium puzzle from a novel perspective. By appealing to non-rationality and the stationarity-nonstationarity properties of the relevant variables, we can explain why there is a small deviation from unity in the coefficient of a regression of the future spot exchange rate on the current forward exchange rate (the level specification), and yet the bias in the traditional forward premium specification is large enough to frequently yield a negative regression coefficient, i.e., the forward premium puzzle. We thus argue that the relationship between spot and forward exchange rates can be better understood by examining their link using both the level and forward premium specifications jointly rather than focusing solely on the traditional forward premium specification.

In this paper, we make no conjecture about the source of non-rationality that may generate the negative covariance between the forecast error and the forward premium. One potential source of non-rationality that is consistent with the model and evidence presented herein is recursive least squares learning, as developed in Chakraborty (2008) and Chakraborty and Evans (2008). In conclusion, we suggest that non-rationality be considered for future research into the forward premium puzzle.

Appendix

Proof of Proposition 1: Suppose, a_t follows a univariate AR(1) process $a_t = ca_{t-1} + u_{at}$ where, c is a constant with $0 < c < 1$ and $u_{at} \sim iid(0, \sigma_a^2)$ is a stationary process. Similarly, b_t follows a univariate non-stationary process $b_t = b_{t-1} + u_{bt}$ where, $u_{bt} \sim iid(0, \sigma_b^2)$ is a stationary process. Also suppose, $E(u_{at}, u_{bt}) = \sigma_{ab}$. The given initial observations are a_0 and b_0.

The conditional means of a_t and b_t (conditional on a_0 and b_0) are given by $E(a_t|a_0)$ and $E(b_t|b_0)$.

Now, $E(a_t|a_0) = E[(ca_{t-1} + u_{at})|a_0] = E[\{c(ca_{t-2} + u_{at-1}) + u_{at}\}|a_0]$
$= E[c^t a_0 + \sum_{s=0}^{t-1} c^s u_{at-s}] = c^t a_0 + \sum_{s=0}^{t-1} c^s E(u_{at-s}) = c^t a_0$.
(Since, $E(u_{ax}) = 0$, for $x = 1, ..., t$).
Similarly, $E(b_t|b_0) = b_0$.
Thus, conditional covariance between a_t and b_t is given by
$Cov[(a_t, b_t)|a_0, b_0] = E[(a_t b_t)|a_0, b_0] - E(a_t|a_0)E(b_t|b_0)$
$= E[(ca_{t-1} + u_{at})(b_{t-1} + u_{bt})|a_0, b_0] - c^t a_0 b_0$
$= E[(ca_{t-1}b_{t-1} + u_{at}b_{t-1} + u_{bt}a_{t-1} + u_{at}u_{bt})|a_0, b_0] - c^t a_0 b_0$
$= E(ca_{t-1}b_{t-1}|a_0, b_0) + E(u_{at}b_{t-1}|a_0, b_0) + E(u_{bt}a_{t-1}|a_0, b_0) + E(u_{at}u_{bt}|a_0, b_0) - c^t a_0 b_0$
$= E(ca_{t-1}b_{t-1}|a_0, b_0) + \sigma_{ab} - c^t a_0 b_0$.
(Since, $E(u_{at}b_{t-1}|a_0, b_0) = E(u_{bt}a_{t-1}|a_0, b_0) = 0$, and
$E(u_{at}u_{bt}|a_0, b_0) = E(u_{at}u_{bt}) = \sigma_{ab}$).
By recursive substitution we get
$Cov[(a_t, b_t)|a_0, b_0] = c^t E(a_0 b_0) + (1 + c + c^2 + ... + c^{(t-1)})\sigma_{ab} - c^t a_0 b_0$
$= c^t a_0 b_0 + \sigma_{ab}\frac{(1-c^t)}{(1-c)} - c^t a_0 b_0$
$= \sigma_{ab}\frac{(1-c^t)}{(1-c)}$

Therefore, $Cov[(a_t, b_t)|a_0, b_0]$ is nonstationary in finite sample but converges to $\frac{\sigma_{ab}}{(1-c)}$ as $t \longrightarrow \infty$ and hence, $c^t \longrightarrow 0$. (The magnitude converges but the sign depends on the sign of σ_{ab}).

Thus, $Cov[(a_t, b_t)|a_0, b_0]$ is asymptotically stationary.■

Proof of Corollary 1: From the above proof
$Cov[(a_t, b_t)|a_0, b_0] = \sigma_{ab}\frac{(1-c^t)}{(1-c)}$
Also, $V(b_t|b_0) = E(b_t^2|b_0) - (E(b_t|b_0))^2$
$= E(b_t^2|b_0) - b_0^2$
$= E[(b_{t-1} + u_{bt})^2|b_0] - b_0^2$
$= E[(b_{t-1}^2 + 2b_{t-1}u_{bt} + u_{bt}^2)|b_0] - b_0^2$
$= E(b_{t-1}^2|b_0) + 2E(b_{t-1}u_{bt}|b_0) + E(u_{bt}^2|b_0) - b_0^2$
$= E(b_{t-1}^2|b_0) + E(u_{bt}^2|b_0) - b_0^2$ (Since, $E(b_{t-1}u_{bt}|b_0) = E(b_{t-1}u_{bt}) = 0$)
By recursive substitution we get
$= E(b_0^2|b_0) + \sum_{s=0}^{t} E(u_{bt}^2|b_0) - b_0^2$
$= b_0^2 + t\sigma_b^2 - b_0^2$

$= t\sigma_b^2$

Thus, $V(b_t|b_0)$ is clearly non-stationary and explosive.

Therefore, $\frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)} = \frac{\sigma_{ab}\frac{(1-c^t)}{(1-c)}}{t\sigma_b^2}$ and as $t \to \infty$ and hence, $c^t \to 0 \Rightarrow \frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)} \to 0$.

Thus, in large sample $\frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)}$ is negligible. But even in finite sample the magnitude of $\frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)}$ is decreasing in t. In other words $\Delta|\frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)}|$ is negative for positive t.

Proof by *Mathematical Induction*:

Suppose, $\Delta|\frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)}| < 0$ for $t = T$.

Thus, $|\frac{Cov[(a_T,b_T)|a_0,b_0]}{V(b_T|b_0)}| - |\frac{Cov[(a_{T-1},b_{T-1})|a_0,b_0]}{V(b_{T-1}|b_0)}| < 0$.

$\Rightarrow |\frac{\sigma_{ab}\frac{(1-c^T)}{(1-c)}}{T\sigma_b^2}| - |\frac{\sigma_{ab}\frac{(1-c^{T-1})}{(1-c)}}{(T-1)\sigma_b^2}| < 0$

$\Rightarrow |\frac{\sigma_{ab}}{\sigma_b^2(1-c)}|[\frac{(1-c^T)}{T} - \frac{(1-c^{T-1})}{T-1}] < 0$

$\Rightarrow [\frac{(1-c^T)}{T} - \frac{(1-c^{T-1})}{T-1}] < 0$

$\Rightarrow \frac{(1-c^T)}{T} < \frac{(1-c^{T-1})}{T-1}$

$\Rightarrow Tc^{T-1} - Tc^T < 1 - c^T$

$\Rightarrow c(Tc^{T-1} - Tc^T) < Tc^{T-1} - Tc^T < 1 - c^T$, (Since, $0 < c < 1$)

$\Rightarrow Tc^T - Tc^{T+1} < 1 - c^T$

$\Rightarrow T - Tc^{T+1} < 1 - c^T - Tc^T + T$

$\Rightarrow \frac{(1-c^{T+1})}{T+1} < \frac{(1-c^T)}{T}$

$\Rightarrow |\frac{Cov[(a_{T+1},b_{T+1})|a_0,b_0]}{V(b_{T+1}|b_0)}| - |\frac{Cov[(a_T,b_T)|a_0,b_0]}{V(b_T|b_0)}| < 0$

$\Rightarrow \Delta|\frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)}| < 0$ for $t = T+1$.

Hence, if $\Delta|\frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)}| < 0$ for $t = T$ then it is also true for $t = T+1$.

Now, $\Delta|\frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)}| < 0$ for $t = 2$, (Since, $\frac{(1-c^2)}{2} < (1-c)$ for $0 < c < 1$) [We don't consider $t = 1$ as $\Delta|\frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)}|$ is not defined for $t = 1$].

$\Rightarrow \Delta|\frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)}| < 0$ for $t = 3, 4, 5.....$

$\Rightarrow \frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)}$ is decreasing in t.$\blacksquare$

Proof of Proposition 2: Suppose, a_t follows a univariate AR(1) process $a_t = ca_{t-1} + u_{at}$ where, c is a constant with $0 < c < 1$ and $u_{at} \sim iid(0, \sigma_a^2)$ is a stationary process. Similarly, $b_t = db_{t-1} + u_{bt}$ where, d is a constant with $0 < d < 1$ and $u_{bt} \sim iid(0, \sigma_b^2)$ is a stationary process. Also suppose, $E(u_{at}, u_{bt}) = \sigma_{ab}$.

In this case following the previous methodology it could be shown that

$$\frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)} = \frac{\sigma_{ab}\frac{(1-c^t)}{(1-c)}}{\sigma_b^2\frac{(1-d^t)}{(1-d)}}$$

We cannot conclude anything unambiguously about how $\frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)}$ behaves with increasing t as that would depend upon the magnitudes of σ_{ab}, σ_b^2, c and d.

Also, $\lim_{t\to\infty} \frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)} = \frac{\sigma_{ab}(1-d)}{\sigma_b^2(1-c)}$. Thus, $\frac{Cov[(a_t,b_t)|a_0,b_0]}{V(b_t|b_0)}$ is asymptotically stationary.$\blacksquare$

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