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A Dynamic Measure of the Effective Tax Rate

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Abstract

This article shows how the existing forward-looking measures of the effective tax rate may be biased when firms operate in a dynamic context. Using option pricing techniques we thus propose a measure of the effective tax rate which embodies future business changes.

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1 Introduction

Business projects can usually be altered during their lifetime. Since firms are aware that they can defer, expand, contract or abandon a project, their interest on real options is growing up[1]. Despite of this, the existing literature usually disregards future strategy changes when measuring a forward-looking effective tax rate (ETR)[2].

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[1]Graham and Harvey (2001) show that about 26% of the companies surveyed always or almost always incorporate real options when evaluating a project. Furhermore, McDonald (2000) argues that even when firms use standard techniques, it is possible that they apply ad hoc rules of thumb which proxy for optimal timing behaviour.

[2]See e.g. Devereux (2003), Devereux and Hubbard (2003), and Sϕrensen (2003).

The aim of this article is therefore to show how the existing measures of the ETR are biased when firms operate in a dynamic context. Using option pricing techniques we show that volatility may have an asymmetric effect on the ETR.

The paper is structured as follows. Section 2 shows how the ETR should account for timing. Section 3 introduces a continuous-time model and discusses a dynamic measure of the ETR. Section 4 summarizes the findings and identifies some topics for further research.

2 The ETR in a dynamic context

The ETR is usually measured as the scaled wedge between the pre- and post-tax net present value of a project, say $NPV(t) - NPV^T(t)$, *given the investment time* t. Dividing the wedge by $NPV(t)$ we thus obtain:

$$ETR_S(t) = \frac{NPV(t) - NPV^T(t)}{NPV(t)}. \tag{1}$$

Though $ETR_S(t)$ is a forward-looking measure, as it accounts for future profitability, it is inherently static. In fact, it is based on the implicit assumption that firms' strategies do not change over time. On the contrary, evidence shows that managerial flexibility is a fairly important input for business activities. In this case, the standard NPV approach fails to yield a reliable valuation of future projects. The intuition behind this point is straightforward. Suppose that, at time t, the firm can delay investment until time T. If the firm invests immediately, it will enjoy the profit stream between time t and time T. If it waits until time T, it has the possibility of enjoying better market conditions. Thus, investing at time t implies the exercise of the option to delay and entails paying an opportunity cost for the flexibility lost in the firm's strategy[3]. This implies that the NPV must exceed the summation of the investment cost and of the opportunity cost. Otherwise the loss in flexibility would not be compensated. As will be shown, in this case, the ETR_S is a biased measure.

In a dynamic context, the firm's problem is one of choosing the optimal exercise timing for its business option[4], i.e.

$$V(t) = \max_t E\left\{NPV(t)\, e^{-rt}\right\}. \tag{2}$$

[3]For further details see Trigeorgis (1996).

[4]Without any loss in generality, we assume that the firm owns one option. Actually,

Similarly, the post-tax value of the project can be expressed as

$$V^T(t) = \max_t E\left\{\left[NPV^T(t)\right]e^{-rt}\right\}. \quad (3)$$

As shown in Panteghini (2002), the standard NPV rule yields the same ranking as the real-option one only if:

1. *delaying the decision is impossible or the initial time t is the optimal timing for any project;*
2. *taxation is neutral.*

The first condition is trivial. When the value of flexibility is nil, t is the optimal timing and the standard NPV rule is correct. The second condition can be explained as follows. Suppose we have an increase in the tax rate. Its effect is twofold. On the one hand, the present value of future discounted profits is reduced: this induces the firm to delay investment. On the other hand, the option value decreases. As the opportunity cost drops, investment is stimulated. Neutrality means that these two contrasting effects perfectly offset each other.

When neither of the above conditions holds, the solutions of (2) and (3) are different. This implies that distortive taxation has a different impact on the net present value and on the firm's option, thereby affecting the intertemporal decision. To show the importance of the intertemporal effect, we thus propose the following measure of the ETR:

$$ETR_D(t) = \frac{\max_t E\{NPV(t)e^{-rt}\} - \max_t E\left\{NPV^T(t)e^{-rt}\right\}}{\max_t E\{NPV(t)e^{-rt}\}}, \quad (4)$$

As can be seen $ETR_D(t)$, takes into account the intertemporal decision. It is straightforward to show that when the value of the option is nil, (4) collapses to (1). Therefore, $ETR_S(t)$ can be regarded as a special case of $ETR_D(t)$.

It is worth pointing out that the above formula can be used to compute both average and marginal taxation. In this latter case, in fact, it is sufficient to set economic rents equal to zero[5].

firms may hold more than one option, e.g. regarding new locational choices, R&D investments, new marketing programs or the definition of the type and quality of the goods produced.

[5]For further details see Devereux (2003).

3 An application: the basic investment timing problem

To have a feeling of how $ETR_D(t)$ is affected by dynamic strategies, we present a basic investment timing problem with risky payoff. Using a continuous-time framework, we study the behavior of a risk-neutral representative firm, who must decide whether and when to undertake an investment project. We assume that risk is fully diversifiable, the risk-free interest rate r is fixed, and that the project's payoff follows a geometric Brownian motion

$$d\Pi(t) = \sigma\Pi(t)dz,$$

where σ is the variance parameter and z is a Wiener process. The firm starts to earn the payoff once a non-depreciable sunk cost, say I, is paid. In the absence of taxation the NPV is therefore

$$NPV\left(\Pi\left(t\right)\right) = \frac{\Pi\left(t\right)}{r} - I. \tag{5}$$

Let us next introduce taxation. Without any loss of generality, we define the tax base as the firm's current income, net of an imputation rate ρ[6]. Thus, given the tax rate τ, current tax payments are $T\left(t\right) = \tau\left[\Pi(t) - \rho I\right]$, and the post-tax income is

$$\Pi^T(t) = (1-\tau)\Pi(t) + \rho\tau I. \tag{6}$$

Given (6), the post-tax NPV is

$$NPV^T\left(\Pi\left(t\right)\right) = \frac{(1-\tau)\Pi\left(t\right)}{r} - \left(1 - \frac{\rho\tau}{r}\right)I. \tag{7}$$

The optimal investment timing T can be associated to a profit level $\overline{\Pi}$. This entails that whenever current payoff reaches $\bar{\Pi}$, the firm invests[7]. Thus the firm's problem (3) can be rewritten as[8]

$$\Psi\left(\Pi(t), \beta_1\right) = \max_{\bar{\Pi}>0}\left[\left(\frac{\Pi(t)}{\bar{\Pi}}\right)^{\beta_1} NPV^T(\bar{\Pi})\right], \tag{8}$$

[6]This assumption is in line with Boadway and Bruce (1984).

[7]For further details see Dixit and Pindyck (1994, Ch.6).

[8]A full derivation can be found in Panteghini (2002).

where $\beta_1 > 1$ and $NPV^T(\bar{\Pi})$ is the firm's static project value when $\Pi(t) = \bar{\Pi}$. Hereafter, for simplicity we will omit the time variable t.

Using (7) and solving the problem (8) yields the trigger point above which investment is profitable

$$\bar{\Pi} \equiv \left[\frac{1 - \frac{\rho}{r}\tau}{1-\tau}\right]\widetilde{\Pi}, \tag{9}$$

where $\widetilde{\Pi} \equiv \frac{\beta_1}{\beta_1 - 1} rI > rI$ is the laissez-faire trigger point.

The tax treatment of the investment cost is crucial for the sign of the difference $\left(\bar{\Pi} - \widetilde{\Pi}\right)$. Easy computations show that $\left(\bar{\Pi} - \widetilde{\Pi}\right) \propto (r - \rho)$. When therefore $\rho < r$, the inequality $\bar{\Pi} > \widetilde{\Pi}$ holds, namely the firm's propensity to invest is reduced by taxation. This can be regarded as the underinvestment case. When, conversely, $\rho > r$ overinvestment takes place. When, finally, $\rho = r$, the tax system is neutral, i.e. $\bar{\Pi} = \widetilde{\Pi}$.

Since underinvestment is the most common case, in the following discussion we will assume that $\rho < r$[9]. Depending on the current level of payoff we can find three cases. Firstly, when $\widetilde{\Pi} < \bar{\Pi} < \Pi$, investing immediately is the optimal choice irrespective of taxation. According to Panteghini's (2002) findings, timing does not affect the investment decision and ETR_D collapses to ETR_S. This entails that volatility does not matter and that ETR_S is unbiased.

In the following two cases, instead, a bias emerges. When $\widetilde{\Pi} < \Pi < \bar{\Pi}$, investing immediately is the better choice in the absence of taxation, whereas delaying is preferable under taxation. This implies that, in the absence of taxation, the standard NPV rule holds, whereas, under taxation, the intertemporal effect matters. Thus substituting (5) and (8) into (4) yields

$$ETR_D = 1 - \frac{\Psi\left(\overline{\Pi}, \beta_1\right)}{\frac{\Pi}{r} - I} \text{ when } \widetilde{\Pi} < \Pi < \bar{\Pi}. \tag{10}$$

If Π lies between $\widetilde{\Pi}$ and $\bar{\Pi}$, both current (Π) and expected profitability ($\bar{\Pi}$) affect ETR_D. Moreover, volatility affects ETR_D. By applying the Envelope Theorem, it is easy to ascertain that $\frac{dETR_D}{d\sigma} \propto -\frac{\partial\Psi\left(\overline{\Pi},\beta_1\right)}{\partial\beta_1}\frac{d\beta_1}{d\sigma} < 0$.

When $\Pi < \widetilde{\Pi} < \bar{\Pi}$, immediate investment is non-profitable in either case. Substituting (7) and (8) into (4) yields

[9]Under the assumption that $\rho > r$, the sign of results would be reverted.

$$ETR_D = 1 - (1-\tau)\left(\frac{1-\tau}{1-\frac{\rho}{r}\tau}\right)^{\beta_1 - 1} \text{ when } \Pi < \widetilde{\Pi} < \bar{\Pi}. \tag{11}$$

It is straightforward to show that $\frac{\partial ETR}{\partial \sigma} \propto (\rho - r)$.

As shown by (10) and (11), an increase in σ reduces ETR_D. This entails that the higher the standard deviation the greater is the bias caused by the use of ETR_S. The intuition behind this result is straightforward. On the one hand, an increase in the tax rate reduces both the present value of future discounted profits and the option value. On the other hand, an increase in volatility makes the timing option more valuable, thereby raising the opportunity cost of investing immediately. Therefore, the distortive effect of taxation on the option value is partially compensated by higher volatility. This reduces the effective impact of taxation on investment decisions, namely the 'true' ETR turns to be lower.

4 Concluding remarks

In this paper we have proven that the standard measure of the ETR may be biased, when managerial flexibility is a worthy input. In particular, we have shown that if taxation is distortive and profitability is low enough, then volatility affects the ETR. If, instead, taxation is neutral and/or profitability is high enough the ETR is unaffected. This asymmetry implies that market conditions may deeply affect the measurement of effective taxation.

This article should be considered as the starting point for further research. In particular, the introduction of a more general model, dealing with different kinds of options and stochastic processes, and the definition of algorithms able to proxy for the effect of volatility are left to future work.

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