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# The term structure of interest rates with nonlinear adjustment: Evidence from a unit root test in the nonlinear STAR framework

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## *Abstract*

This paper investigates the term structure of interest rates in Japan using the unit root test in a nonlinear STAR framework. The results provide strong evidence against the unit root of the yield spread between long−term and short−term interest rates, compared with standard unit root tests assuming only linear adjustment. This finding shows that the term structure of interest rates is stable with nonlinear adjustment.

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## 1. Introduction

The expectation hypothesis regarding the term structure of interest rates is one of the most significant issues in monetary economics, with a large number of studies having investigated the topic (e.g., for a survey, Campbell, 1995). The rigorous acceptance of the hypothesis involves a stationary yield spread defined as the difference between continuous long-term and short-term interest rates or the cointegration relationship with cointegrating vector $(1, -1)'$ between it (e.g., Campbell and Shiller, 1987). For that reason, representative unit root and cointegration tests including those derived by Dickey and Fuller (1979) and Engle and Granger (1987) have been employed. Since these tests assume only linear adjustment, many investigators analyzing the expectation hypothesis of the term structure have, for the sake of simplicity, neglected the inherent nonlinear adjustment caused by market friction including transaction costs.

However, recently, some authors have emphasized such nonlinear adjustment. For example, as pointed out by Anderson (1997), while investors do not attempt to adjust their portfolios within the range of transaction costs, they do make adjustments beyond the range of transaction costs through the selling and buying of bonds. This means that the term structure adjustment is nonlinear. In addition, Balke and Fomby (1997) showed that the power of cointegration tests assuming linear adjustment fell under a nonlinear process. These facts imply that neglecting the inherent nonlinear adjustment leads the expectation hypothesis of the term structure of interest rates to produce misspecified results.

The purpose of this paper is to investigate the term structure of interest rates in Japan using the unit root test in the nonlinear smooth transition autoregressive (STAR) framework, as proposed by Kapetanios, Shin, and Snell (2003). Their STAR approach tests for a unit root against a nonlinear stationary process based on the STAR process. The use of this test can clearly and directly further the discussion of whether the term structure of interest rates is stable with nonlinear adjustment: If the STAR approach could provide clear evidence of a nonlinear stationary yield spread based on the STAR process, and if standard unit root tests allowing for only linear adjustment could not reject the unit root of the yield spread, the results would show that the yield spread, i.e., the long-run equilibrium relationship between long-term and short-term interest rates, is stable with nonlinear adjustment.

The paper proceeds as follows. In section 2, we describe the term structure of interest rates along with the unit root test introduced by Kapetanios *et al.* (2003). In section 3, the data and our findings are presented. Section 4 provides a summary of the paper.

## 2. The term structure of interest rates and the unit root test in the nonlinear exponential smooth transition autoregressive (ESTAR) process

The expectation hypothesis expressed by Hall, Anderson, and Granger (1992) is as follows:

$$R_{k,t} = \frac{1}{k}\left[\sum_{j=1}^{k} E_t\big[R_{1,t+j-1}\big]\right] + L_{k,t}, \tag{1}$$

where $R_{k,t}$ is the time $t$ continuously compounded yield (log-compounded yield) to maturity of the $k$ period pure discount bond, $E_t$ is the expectation operator based on available information, and $L_{k,t}$ denotes the liquidity premium of the $k$ period. Equation (1) denotes that the long-term interest rate can be represented as a weighted average of the present and expected future short-term interest rates. To accept equation (1) accurately, two interest rates in different periods involve a cointegrating vector $(1,-1)'$. That is, a stationary yield spread is required:

$$S_{k,t} = R_{k,t} - R_{1,t} = \frac{1}{k}\sum_{i=1}^{k-1}\sum_{j=1}^{j=i} E_t \Delta R_{1,t+j} + L_{k,t}. \tag{2}$$

Given equation (2), if $R_{k,t}$ and $R_{1,t}$ have a unit root, we can test for cointegration using unit root tests including those derived by Dickey and Fuller (1979) because equation (2) has the pre-specified cointegrating vector $(1,-1)'$.

However, representative unit root tests assuming only linear adjustment is misspecified if the adjustment process is nonlinear. The term structure of interest rates has the inherent nonlinear adjustment caused by market frictions including transaction costs. In order to take into account such nonlinear adjustment, this paper employs the unit root test in the nonlinear STAR framework developed by Kapetanios *et al.* (2003). The STAR process expresses an unstable behavior near equilibrium but has quick convergence for large deviations from equilibrium. As pointed out by Anderson (1997), the STAR model is more reasonable in the case where different investors have different transaction costs [1].

To begin with, as the test assumption regarding the unit root, let $\{x_t\}_1^T$ denote an observable process which can be decomposed as $x_t = \mu + \delta t + y_t$, where $\mu$ and $\delta$ are constant and trend parameters, respectively. Kapetanios *et al.* (2003) proposed the following univariate STAR model:

$$y_t = \beta y_{t-1} + \gamma y_{t-1}\Theta(\theta; y_{t-d}) + \epsilon_t, t = 1, ..., T, \tag{3}$$

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[1] A three-regime threshold model also exists which follows a unit root process in the middle regime but triggers a mean-reverting process in the outer regime. This threshold model implies that all investors have the same threshold or transaction costs (see Anderson, 1997).

where $\epsilon_t \sim$ i.i.d.$(0,\ \sigma^2)$, and $\beta$ and $\gamma$ are unknown parameters. The transition function of the exponential form is as follows:

$$\Theta(\theta; y_{t-d}) = 1 - \exp(-\theta y_{t-d}^2), \tag{4}$$

where it is assumed that $\theta \geq 0$, and $d \geq 1$ is the delay parameter. The value of the exponential transition function is from 0 to 1 ($\Theta : R \rightarrow [0, 1]$),

$$\Theta(0) = 0; \lim_{x \rightarrow \pm\infty} \Theta(x) = 1.$$

From (3) and (4), an ESTAR model is derived,

$$y_t = \beta y_{t-1} + \gamma y_{t-1}[1 - \exp(-\theta y_{t-d}^2)] + \epsilon_t. \tag{5}$$

We transform (5) as

$$\Delta y_t = \phi y_{t-1} + \gamma y_{t-1}[1 - \exp(-\theta y_{t-d}^2)] + \epsilon_t, \tag{6}$$

where $\phi = \beta - 1$.

A null hypothesis with a unit root implies that $\phi = 0$ and $\theta = 0$. $\gamma$ is not identified under the null hypothesis. Under the alternative hypothesis, $y_t$ is a nonlinear stationary process ($\phi = 0, \theta > 0, -2 < \gamma < 0$). In addition, we set $d = 1$ as a delay parameter. Therefore, (6) is the following form imposed by $\phi = 0$ and $d = 1$:

$$\Delta y_t = \gamma y_{t-1}[1 - \exp(-\theta y_{t-1}^2)] + \epsilon_t. \tag{7}$$

Although the null hypothesis $\theta = 0$ must be tested against the alternative hypothesis $\theta > 0$, this hypothesis cannot be tested, since $\gamma$ is not identified under the null hypothesis. Therefore, by using a first-order Taylor series approximation, instead of (7) we test the hypothesis as follows:

$$\Delta y_t = \delta y_{t-1}^3 + error. \tag{8}$$

When the errors in (7) are serially correlated, (7) and (8) result in the following regression with $p$-order augmentation:

$$\Delta y_t = \sum_{j=1}^{p} \rho_j \Delta y_{t-j} + \gamma y_{t-1}[1 - \exp(-\theta y_{t-1}^2)] + \epsilon_t, \tag{9}$$

$$\Delta y_t = \sum_{j=1}^{p} \rho_j \Delta y_{t-j} + \delta y_{t-1}^3 + error. \tag{10}$$

The $t$ statistic of the auxiliary regression in (8) and (10) for $\delta = 0$ against $\delta < 0$ is the following:

$$t_{NL} = \hat{\delta}/\text{s.e.}(\hat{\delta}), \tag{11}$$

where $\hat{\delta}$ is the OLS estimate of $\delta$ and s.e.$(\hat{\delta})$ is the standard error of $\hat{\delta}$. We can test a unit root against a nonlinear stationary process by using the above procedure.

## 3. Data and Empirical results

In the present paper, we employ the government bond as a long-term interest rate, and the lending rate as a short-term interest rate. The monthly data obtained from the International Monetary Fund's International Financial Statistics CD-ROM consists of 439 periods between 1966:10-2003:4. Table 1 presents the results of unit root tests of two interest rates in logarithms. ADF and GLS denote Dickey and Fuller (1979) and Ng and Perron (2001) tests, respectively. STAR is the unit root test by Kapetanios *et al.* (2003). Since, as emphasized by Ng and Perron (1995), unit root tests are sensitive to lag length, we determine lag length using two lag criterions: the Akaike Information Criterion and *t-sig* introduced by Ng and Perron (1995) [2]. These tests include only a constant because interest rates and the theory of the term structure do not support a trend. As shown in Table 1, all of the tests do not reject the unit root of interest rates. Therefore, the results show that interest rates have a unit root process, thus justifying the unit root tests of the yield spread (cointegration tests for pre-specified cointegrating vector).

Table 1: Unit root tests for two interest rates[3]

| | ADF | | GLS | | STAR | |
|---|---|---|---|---|---|---|
| | LR | SR | LR | SR | LR | SR |
| AIC | 0.86(8) | -0.11(2) | 1.50(8) | 0.55(2) | -0.27(8) | 0.09(2) |
| *t-sig* | 0.74(10) | -0.25(7) | 1.49(7) | 0.40(7) | -0.39(10) | -0.01(7) |

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[2] For GLS, we use the MAIC (Modified Akaike Information Criterion) proposed by Ng and Perron (2001). MAIC has good size and power, compared with BIC (Baysian Information Criterion) and AIC. For *t-sig*, we set the maximum lag=12. *t-sig* selects the lag order $k$ via top-down testing. To begin with, we estimate the equation with the maximum lag (here, the maximum lag $k_{max}$=12). We use the lag order if the $t$-statistic of the parameter of the maximum lag is significant. If the $t$-statistic is not significant, we estimate the equation with the lag=$k_{max} - 1$. That is, when the $t$ statistic of the parameter of the lag=$k_{max} - q$ is significant at a conventional level, we employ the lag order.

[3] Parentheses show lag length.

Table 2 shows the results of the unit root tests of the yield spread. The results of ADF and GLS tests fail to reject a unit root even at a 10% significance level. In contrast, the test by Kapetanios *et al.* (2003) provides strong evidence against the unit root of the yield spread at a 1% significance level, even when different lag criterions are employed. This finding asserts that the long-run equilibrium relationship between long-term and short-term interest rates is stable with nonlinear adjustment. Given the unit root of interest rates and a nonlinear stationary yield spread, we estimate the nonlinear effect $\theta$ imposing $\gamma = -1$ on equation (9) [4]. From Table 3, note that while each interest rate does not have $\theta$ at a significant level, the yield spread has a significant $\theta$. This estimation also shows that the long-run equilibrium relationship between interest rates is stable with nonlinear adjustment.

Table 2: Unit root tests for the yield spread [5]

| | ADF | GLS | STAR |
|---|---|---|---|
| AIC | -1.51(8) | -1.41(8) | -3.54***(8) |
| *t-sig* | -1.58(7) | -1.12(11) | -3.58***(7) |

Table 3: Nonlinear effect[6]

| | LR | | SR | | Yield Spread | |
|---|---|---|---|---|---|---|
| | $\theta$ | $t_\theta$ | $\theta$ | $t_\theta$ | $\theta$ | $t_\theta$ |
| AIC | 0.001 | 0.278 | -0.001 | -0.091 | 0.125 | 3.317 |
| *t-sig* | 0.001 | 0.401 | 0.001 | 0.003 | 0.126 | 3.357 |

## 4. Summray

This paper has investigated the term structure of interest rates in Japan using the unit root test developed by Kapetanios *et al.* (2003). While representative tests assuming only linear adjustment fail to reject the unit root of the yield spread between long-term and short-term interest rates, the test in the STAR framework allowing for nonlinear adjustment provides clear evidence that the yield spread is a nonlinear stationary process. These results show that the long-run equilibrium relationship between long-term and short-term interest rates is stable with nonlinear adjustment.

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[4] Since joint estimation of $\gamma$ and $\theta$ incurs sever identification problems and makes the convergence of nonlinear algorithms difficult, we impose $\gamma = -1$, similar to Kapetanios *et al.* (2003).

[5] LR and SR denote long-term and short-term interest rates in logarithms, respectively. Parentheses show lag length. (***) Significant at a 1% level. (**) Significant at a 5% level. (*) Significant at a 10% level. Critical values are obtained from Dickey and Fuller (1979), Ng and Perron (2001), and Kapetanios *et al.* (2003).

[6] $t_\theta$ shows the $t$-statistic of $\theta$.

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