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Neutrality Properties of Firm Taxation under Default Risk

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This note discusses the neutrality conditions of a Firm Tax. In particular, it proves that the neutrality result found by Bond and Devereux (1995) holds under different default conditions.

1 Introduction

In a pioneering article Boadway and Bruce (1984) proposed a neutral corporate tax design, whose base is equal to current earnings, less depreciation and the opportunity cost of finance. According to this scheme, named Firm Tax (FT), the opportunity cost of finance is given by the nominal interest rate on default-free bonds multiplied by the tax-written-down value of the firm's depreciable assets. Bond and Devereux (1995) (hereafter BD) demonstrated that, in the absence of credit market imperfections, the FT is neutral under income, capital and default risk.

It is worth noting that BD do not explicitly introduce any default condition. As we know, instead, there exist at least two different default conditions. First of all, default may be triggered when the firms' asset value falls to the debt's value. In this case debt is approximated with a positive net-covenant and can be termed *protected debt* (see Leland, 1994). Alternatively,

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shareholders may have the opportunity to inject equity capital in order to meet the firm's debt obligations. As long as they issue new capital and pay the interest rate they can exploit future recoveries in the firm's profitability. In this case shareholders can decide *when* to default[1]. Following Leland (1994), debt will then be named *unprotected debt*[2]. Since BD treat default as an exogenous event, out of shareholders' control, their result is implicitly based on the assumption of protected debt. In this note we will prove that neutrality holds even under unprotected debt financing.

2 The model

In this section we introduce a continuous-time model describing the investment decision by a representative firm. At time t, the firm invests if the project's expected Net Present Value (NPV_t) is positive.

By assumption risk is fully diversifiable, the risk-free interest rate r is fixed and the firm is risk-neutral. Moreover, we assume that the firm's EBIT (Earnings Before Interests and Taxes) follows a geometric Brownian motion

$$\frac{d\Pi_t}{\Pi_t} = \sigma dz, \tag{1}$$

where σ is the variance parameter[3]. Moreover, the firm's project entails the payment of a non-depreciable investment of given size[4], say I.

Following Leland (1994) we assume that the firm pays a coupon C to the lender, and that the credit market is perfectly competitive. Thus, given C,

[1] The decision to default is thus equivalent to the exercise of a put option.

[2] Notice that both protected and unprotected debt are realistic financial instruments. Leland (1994) argues that minimum net-worth requirements, implied by protected debt, are common in short-term debt financing, whereas they are fairly rare in long-term debt instruments. Moreover, Graham and Harvey (2001) find that firms issue short-term debt to time market interest rates. However more than 63% of the firms surveyed state that debt maturity is aimed at matching with assets' lifetime. This entails that firms use a mix of short- and long-term debt istruments, which may be subject to different default conditions.

[3] The quality of results would not change if we introduced a drift in the Geometric brownian motion.

[4] Notice that the introduction of depreciation would not affect the qualitative nature of results.

the market value of debt is computed by applying a non-arbitrage condition[5]. In the event of default, the firm is expropriated by the lender.

Let us next analyze the two default conditions. For simplicity, hereafter, we will omit the time variable.

Protected debt When debt is protected, the threshold value of Π below which default takes place is exogenous. To this end, we define $\overline{\Pi}$ as the firm's EBIT which leads to zero net cash flow, i.e. $(1-\tau)\overline{\Pi} - C + \tau r I = 0$. When, therefore, Π falls down to $\overline{\Pi}$, the firm is expropriated and, therefore, the value of equity falls to zero, namely

$$E\left(\overline{\Pi}, C\right) = 0. \tag{2}$$

Unprotected debt When debt is unprotected shareholders can choose when to default. This entails that the threshold point below which default takes place, say $\widehat{\Pi}$, is optimally chosen. Following Leland (1994), $\widehat{\Pi}$ is computed by applying a Value Matching Condition (VMC) and a Smooth Pasting Condition (SPC). The former condition requires that, when Π reaches $\widehat{\Pi}$, the value of equity falls to zero, i.e.

$$E\left(\widehat{\Pi}, C\right) = 0; \tag{3}$$

the latter condition requires that the marginal value of equity is nil, i.e.

$$\frac{\partial E\left(\widehat{\Pi}, C\right)}{\partial \Pi} = 0. \tag{4}$$

As will be shown, $\widehat{\Pi}$ is affected by taxation and, thus, potentially distortive. Despite this fact, we will prove that neutrality holds even under unprotected debt financing.

3 The neutrality result

Under the FT, the base is given by the difference between Π and a tax allowance, equal to the risk-free interest rate r times the book value of the

[5]Notice that this is equivalent to setting the value of debt and then computing the relevant interest rate. For simplicity we also assume that debt cannot be renegotiated.

asset I. Therefore, tax payments are equal to $T = \tau(\Pi - rI)$ and the firm's net cash flow is

$$Y = \Pi - C - \tau(\Pi - rI) = (1 - \tau)\Pi - C + \tau rI \tag{5}$$

Following BD, tax charges are assumed to be independent of the ownership of the firm. This entails that, after expropriation, the lender is subject to the same tax treatment as shareholders.

Let us next analyze the effects of taxation on the project's Net Present Value ($NPV(\Pi, C)$). Defining $E(\Pi, C)$ and $D(\Pi, C)$ as the value of equity and debt, respectively, yields

$$NPV(\Pi, C) = E(\Pi, C) + D(\Pi, C) - I. \tag{6}$$

Defining NPV_{LF} as the *laissez-faire* Net Present Value, we can prove that:

Proposition 1 *Given the tax rate* τ*, the FT is neutral, i.e.*

$$NPV(\Pi, C) = (1 - \tau) \cdot NPV_{LF}, \tag{7}$$

irrespective of the default condition applied.

Proof. See the Appendix. ■

The intuition behind Proposition 1 is straightforward. In the Appendix, we show that $\overline{\Pi} > \widehat{\Pi}$. This entails that, under unprotected debt financing, default does not take place when the net cash flow is nil. When Π lies between $\widehat{\Pi}$ and $\overline{\Pi}$, shareholders face a negative cash flow. However, they prefer to inject equity capital in order to exploit future recoveries in the firm's profitability. The existence of such a put option entails that, *coeteris paribus*, the value of equity is greater under unprotected debt financing. On the other hand, for any C, the value of unprotected debt is less than that of protected debt. This is due to the fact that the shareholders' ability to delay default reduces the value of the firm in the event of default[6]. We can thus argue that any switch from protected to unprotected debt financing entails both an increase in the value of equity and a decrease in the value of debt. As proven in Proposition 1, however, these two effects neutralise each other. This implies that $NPV(\Pi, C)$ is $(1 - \tau)$ times NPV_{LF} irrespective of the characteristics of debt.

[6] Recall that, when expropriation takes place, the firm's expected EBIT is lower (i.e. $\widehat{\Pi} < \overline{\Pi}$).

4 Appendix: Proof of Proposition 1

Let us compute the value of debt. Before default, the lender receives C. After default, the lender becomes shareholder and the value of debt turns to be equity. Using dynamic programming one can write debt as

$$D(\Pi, C) = \begin{cases} Y dt + e^{-rdt}\xi\left[D(\Pi + d\Pi, C)\right] & if \quad \Pi \in [0, \widetilde{\Pi}), \\ C dt + e^{-rdt}\xi\left[D(\Pi + d\Pi, C)\right] & if \quad \Pi \in (\widetilde{\Pi}, \infty). \end{cases} \tag{8}$$

where $\xi[.]$ is the expectation operator and $\widetilde{\Pi} = \overline{\Pi}, \widehat{\Pi}$. Expanding (8) and using Itô's Lemma, one obtains its closed-form solution

$$D(\Pi, C) = \begin{cases} \frac{(1-\tau)\Pi}{r} + \tau I + \sum_{j=1}^{2} B_j \Pi^{\beta_j} & if \quad \Pi \in [0, \widetilde{\Pi}), \\ \frac{C}{r} + \sum_{j=1}^{2} D_j \Pi^{\beta_j} & if \quad \Pi \in (\widetilde{\Pi}, \infty), \end{cases} \tag{9}$$

where β_1 and β_2 are, respectively, the positive and negative roots of the characteristic equation[7] $\psi(\beta) \equiv \frac{\sigma^2}{2}\beta(\beta - 1) - r = 0$.

In the absence of any financial bubbles, we have $B_1 = D_1 = 0$[8]. If, moreover, the lender's claim is null when $\Pi = 0$, the boundary condition $D(0, C) = 0$ holds as well. This implies that $B_2 = 0$, irrespective of the quality of debt. To compute D_2, let us stitch together the two branches of function (9) at point $\Pi = \widetilde{\Pi}$. Using (9) and solving for D_2 one easily obtains

$$D_2 = \left[\frac{(1-\tau)\widetilde{\Pi} - C}{r} + \tau I\right]\widetilde{\Pi}^{-\beta_2},$$

which yields

$$D(\Pi, C) = \begin{cases} \frac{(1-\tau)\Pi}{r} + \tau I & if \quad \Pi \in [0, \widetilde{\Pi}), \\ \frac{C}{r} + \left[\frac{(1-\tau)\widetilde{\Pi} - C}{r} + \tau I\right]\left(\frac{\Pi}{\widetilde{\Pi}}\right)^{\beta_2} & if \quad \Pi \in (\widetilde{\Pi}, \infty). \end{cases} \tag{10}$$

As can be seen, the value of D_2 depends on the default condition applied (namely on the trigger point $\widetilde{\Pi}$).

[7] $\beta_{1,2} = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$, with $\beta_1 > 1$ and $\beta_2 < 0$.

[8] For details on this condition see Dixit and Pindyck (1994).

Using dynamic programming we can write the value of equity as

$$E(\Pi, C) = \begin{cases} 0 & if \quad \Pi \in [0, \widetilde{\Pi}), \\ \\ Y dt + e^{-rdt}\xi\left[E(\Pi + d\Pi, C)\right] & if \quad \Pi \in (\widetilde{\Pi}, \infty). \end{cases} \tag{11}$$

Substituting (5) into (11), expanding and using Itô's Lemma yields

$$rE(\Pi, C) = [(1-\tau)\Pi - C + \tau r I] + \frac{\sigma^2}{2}\Pi^2 E_{\Pi\Pi}(\Pi, C). \tag{12}$$

Solving (12) one can rewrite (11) as

$$E(\Pi, C) = \begin{cases} 0 & if \quad \Pi \in [0, \widetilde{\Pi}), \\ \\ \frac{(1-\tau)\Pi - C + \tau r I}{r} + \sum_{i=1}^{2} A_i \Pi^{\beta_i} & if \quad \Pi \in (\widetilde{\Pi}, \infty). \end{cases} \tag{13}$$

4.1 Protected debt

Let us first compute the value of equity under protected debt financing. Under the assumption that no financial bubbles exist, A_1 is nil. To compute A_2, substitute (13) into (2). It is straightforward to show that A_2 is nil and that

$$E(\Pi, C) = \begin{cases} 0 & if \quad \Pi \in [0, \overline{\Pi}) \\ \\ \frac{(1-\tau)\Pi - C + \tau r I}{r} & if \quad \Pi \in (\overline{\Pi}, \infty). \end{cases} \tag{14}$$

4.2 Unprotected debt

Let us next turn to the unprotected-debt case. Substituting (13) into (3) and (4), one obtains a two-equation set where $\widehat{\Pi}$ and A_2 are the unknowns. Rearranging yields $\widehat{\Pi} = \frac{\beta_2}{\beta_2 - 1}\frac{C - \tau r I}{1-\tau} < \overline{\Pi}$, and $A_2 = -\frac{(1-\tau)}{r}\frac{1}{\beta_2}\widehat{\Pi}^{1-\beta_2} > 0$. The value of equity is

$$E(\Pi, C) = \begin{cases} 0 & if \quad \Pi \in [0, \widehat{\Pi}) \\ \\ \frac{(1-\tau)\Pi - C + \tau r I}{r} - \frac{(1-\tau)}{r}\frac{1}{\beta_2}\widehat{\Pi}\left(\frac{\Pi}{\widehat{\Pi}}\right)^{\beta_2} & if \quad \Pi \in (\widehat{\Pi}, \infty). \end{cases} \tag{15}$$

Comparing (14) with (15), therefore, one can see that the value of equity is higher under unprotected debt financing. Using (10), (14), and (15), one finally shows that condition (7) holds under both protected and unprotected debt financing. This concludes the proof.■

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