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Power Law Scaling in the World Income Distribution

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Abstract

We show that over the period 1960−1997, the range comprised between the 30th and the 85th percentiles of the world income distribution expressed in terms of GDP per capita invariably scales down as a Pareto distribution. Furthermore, the time path of the power law exponent displays a negatively sloped trend. Our findings suggest that the cross−country average growth process appears to be scale invariant but for countries in the tails of the world income distribution, and that the relative volatility of smaller countries' growth processes have increased over time.

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1. Introduction

In the literature sprung up in recent years on the dynamics of the world distribution of per capita GDPs across countries, two empirical results have surfaced[1]. First, while convergence in terms of per capita GDP has been achieved among a restricted set of industrialized countries, i.e. the so-called *convergence club* (Baumol, 1986), divergence has been the rule for the GDP distribution taken as a whole (see e.g. Pritchett, 1997). Second, the density function of the cross-country GDPs distribution has moved from a unimodal shape in the 1960s to a "twin-peaks" shape in the 1990s (see e.g. Quah, 1993; 1996).

In this paper we aim to add a new perspective to this literature, by discussing a third stylized fact regarding the world GDPs distribution which to our knowledge has been largely neglected so far[2]. We show that the GDP per capita of countries comprised between the 30th and the 85th percentiles of the distribution follows a power law, and that this result is extremely robust as we move from 1960 to 1997. Furthermore, over the same period the exponent of such a power law distribution displays a downward trend.

Our findings have interesting implications for theories of growth and the business cycle. The emergence of scaling in the steady state distribution of GDPs per capita can be easily explained by means of country-specific random growth processes obeying a modified version of the well-known Gibrat's law of proportional growth[3]. This suggests the existence of a significant range of the world GDP distribution where countries share a common, size-independent average growth rate. Size is likely to matter for the volatility of growth rates, however, as can be inferred by noting that our estimates return an exponent of the power law distribution always different from one. In particular, the estimated variance of growth rates scales down at a rate lower than that predicted by the Law of Large Numbers, suggesting that microeconomic direct interactions are likely to be important for explaining business cycle fluctuations.

The remainder of this paper is organized as follows. In section 2 we present some evidence on the emergence of scaling behavior for a significant range of the world GDPs distribution. In section 3 we briefly discuss how this evidence relates to models of economic growth and business cycle fluctuations. Finally, section 4 concludes.

2. Empirical Evidence

We study the world distribution of per capita GDPs as taken from the Penn World Table (PWT) Mark 6.1 (Summers, Heston and Ater, 2002), from 1960 to 1997. For the sake of brevity, in what follows we will refer to this object as the world income distribution. Though the PWT dataset contains estimates for some countries extending from 1950 to 2000, a restriction of the time horizon has been imposed in order to minimize the trade-off between the cross-section dimension and the time dimension of the panel.

The empirical methodology we employ is extremely simple. Let the distribution of GDP per capita of M countries at year t be $\underline{\boldsymbol{x}}_t = (x_{1t}, \ldots, x_{Mt})$. Suppose each observation x_{it} is a particular realization of a random variable x with cumulative distribution function $F_t(x)$. Furthermore, let the observations in $\underline{\boldsymbol{x}}_t$ to be ordered from the largest to the smaller, so that the index i corresponds to the rank of x_{it}. It follows that the ranking returns the sample countercumulative distribution of x, which in log reads:

[1] Interesting reviews are e.g. Parente and Prescott (1993) and Jones (1997).
[2] For an example of work very close in spirit to ours, see Sinclair (2001).
[3] For a comprehensive discussion on the Gibrat's law, see Sutton (1997).

$$\ln i = \ln M + \ln (1 - \hat{F}_t(x)). \quad (1)$$

When we make use of (1) to graphically represent the income distribution, which operationally corresponds to a scatter plot of the log of rank against the log of GDP per capita, we obtain a so-called Zipf plot (Stanley *et al.*, 1995). As a matter of example, in figure 1 we show the Zipf plot of the world income distribution for t = 1980. Qualitatively similar findings hold for all the other years in our sample[4].

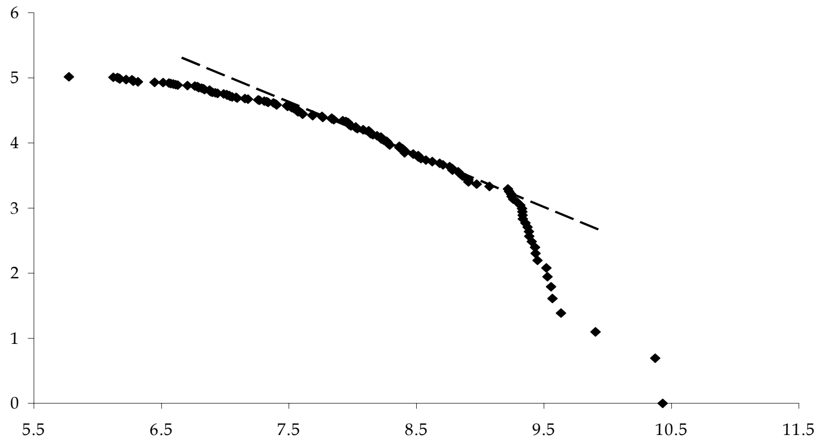

Figure 1. Zipf's plot of the world income distribution (GDP per capita) in 1980.

In figure 1 we superpose a dashed line, which helps us in visually isolating four different regions of the distribution: *i*) starting from the early 1970s, in several years there is a small group of richest countries - typically, scarcely populated oil-producing ones - behaving as outliers; *ii*) the remainder of the left tail, which is typically composed of high income OECD countries, plus other more thickly populated oil-producing nations; *iii*) the central part of the distribution, containing roughly 55% of the countries, where the log of per capita income is arranged along a line; *iv*) the left tail, which can be identified, for any practical purpose, with Africa.

The most intriguing feature emerging from this analysis is undoubtedly the regularity characterizing region *iii*), that is the fact that the data on GDP per capita for middle income economies fit a downward sloping straight line remarkably well. This fact holds invariably for the range comprised between the 30^{th} and the 85^{th} percentiles[5] of the world income distribution in each single year comprised in our time horizon, though the slope of the fitting line tends to sensibly change over time, as one can easily recognize from figure 2.

[4] All Zipf plots and estimation results are available upon request from the authors.
[5] This range has been obtained on a pure data-dependent basis.

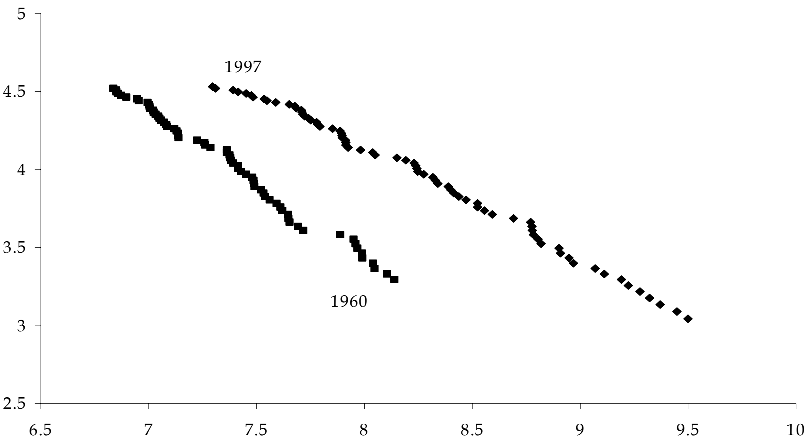

Figure 2. Zipf's plot of the 30^{th}-85^{th} percentiles of the world income distribution (GDP per capita) in 1960 and 1997.

In terms of the associated distribution, the levels of GDP per capita of middle income economies can be seen as Pareto distributed random variables, that is $F(x) = 1 - \left(\frac{x_l}{x}\right)^{\gamma}$, with x_l>0 being the minimum size, and γ>0. As we insert this CDF into (1), we obtain the linear relationship:

$$\ln i = \alpha - \gamma \ln x_i \qquad (2)$$

with $\alpha = \ln M + \gamma \ln x_l$. We used specification (2) to run an OLS regression for each year of the time span 1960-1997, for the data comprised between the 30^{th} and the 85^{th} percentiles of the world income distribution. Results are summarized in figure 3, where we plot the estimated value of the scaling exponent γ (continuous line)[6], and a measure of the goodness of fit expressed in terms of R^2 (dashed line).

The hypothesis that the central part of the world income distribution follows a power law seems to be corroborated by the extremely good fit of linear regressions, as one can appreciate by noting that the value of the OLS R^2 is never below 0.978. Furthermore, note that γ shows a clear tendency to decrease over time. Both features have interesting implications for theory, as briefly discussed in the following section.

[6] The coefficient γ was always statistically significant at the 1% level.

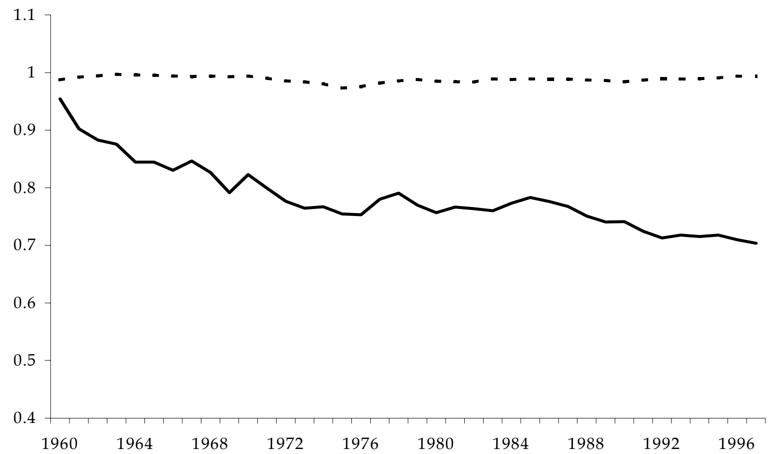

Figure 3. Temporal path of the power exponent γ (continuous line), and goodness of fit of OLS estimates in terms of R^2 (dashed line).

3. Discussion

Power law distributions are well known objects in economics. As a matter of example, both the distributions of city sizes (Krugman, 1996) and firm sizes (Axtell, 2001) follow a power law, with $\gamma \approx 1$[7]. Among the many explanations proposed so far in urban economics and industrial organization, the most convincing ones are probabilistic models based on size-independent growth processes[8]. Our suggestion is that these models could provide interesting insights also if applied to macroeconomics.

The argument can be made in its most general form by following Cordoba (2001). Let us assume continuity both of GDP per capita levels and of time. Let $p(x, t; x_0)$ be the probability density function for x_t, where x_0 represents the initial condition. The law of motion of $p(x, t; x_0)$ is given by the following Fokker-Plank diffusion equation:

$$\frac{\partial p(x,t;x_0)}{\partial t} = -\frac{\partial[x\mu(x)p(x,t;x_0)]}{\partial x} + \frac{1}{2}\frac{\partial[x^2\sigma^2(x)p(x,t;x_0)]}{\partial x^2}. \qquad (3)$$

where $\mu(x)$ and $\sigma(x)$ are the drift and the diffusion coefficients, respectively. Cordoba demonstrates (Theorem 2, p.14) that for $F(x)$ to be Pareto with exponent γ, necessary conditions are that: *i*) the conditional mean, or drift, is constant, $\mu(x) = \phi$; *ii*) the diffusion coefficient takes the form $\sigma(x) = Ax^{\gamma-1}$, where A is a positive constant.

What these two conditions say is that countries belonging to the range of the world income distribution which scales down as a Pareto distribution are characterized by a common average growth rate ϕ, and that the variance of growth decreases with size as soon as $|\gamma| < 1$. The first condition, in particular, states a precise relationship between scale and growth, in that growth rates have to be scale-invariant. This result is in line with the prediction of a recent stream of R&D endogenous growth, according to which scale effects show up on GDP per capita levels, but not on growth rates[9]. Furthermore, the conjecture of a common average

[7] Although in both cases the result of a scaling exponent equal to 1 has been largely disputed. See e.g. Rosen and Resnik (1980) for cities, and e.g. Gaffeo *et al.* (2003) for firms.
[8] See e.g. Ijiri and Simon (1977), Gabaix (1999) and Blank and Solomon (2000).
[9] See e.g. Dinopoulos and Thompson (1998) and Segerstrom (1998). Jones (1999) surveys the topic.

growth rate is consistent with panel data estimations provided by Evans (1998), who shows that the null hypothesis of different trend growth rates among a sample of countries with well-educated populations is rejected at standard statistical levels. While steady state growth without scale effects seems to characterize countries with GDP per capita in the middle of the distribution, however, from our analysis it turns out that the mechanics of growth is likely to differ widely for very rich and very poor countries. In particular, the finding that growth processes for countries comprised in the first 15% of the world income distribution seem to differ from those of the other high and middle income countries is somehow puzzling, and it deserves further research.

If the assumptions at the core of model (3) hold true, our estimates of γ imply that the variance of growth rates scales down on average as $\sigma^2(x) \sim x^{-0.22}$, meaning that the standard deviation follows a Pareto distribution with exponent $\beta = -0.11$. This guess is strikingly close to direct estimates of $\sigma(x)$ reported in Canning *et al.* (1998) and Lee *et al.* (1998), where $\beta = -0.15 \pm 0.03$. Notice that if an economy J is composed of $j > 1$ identical and independently distributed units of size x_0 [10], $x_j = jx_0$, the volatility of its growth tends to decrease with the square root of its size, so that for the whole vector $\underline{\boldsymbol{x}}$ fluctuations as a function of size should scale down with an exponent $\beta = -0.5$ (Buldyrev *et al.*, 1997). Therefore, an average β smaller (in absolute value) than 0.5 can be read as suggesting the existence of long-range correlation between an economy's components, like in models of the business cycle based on direct interactions[11].

Furthermore, the negatively sloped trend of the estimated parameter γ signals that the volatility of fluctuations in countries in the lowest part of the 30^{th}-85^{th} range of the distribution has been increasing in relative terms all over the span 1960-1997, so that β has actually increased over the same period. Of course, our analysis is unsuited to ascertain whether this fact is due to an increase in the amplitude of output fluctuations in low-income countries or to a decrease of volatility in countries with higher incomes. Independent evidence (Agenor *et al.*, 2000; IMF, 2001), however, seems to suggest that the first conjecture is likely to be the right one, probably reflecting a strengthening of the inverse relationship between income levels and vulnerability to financial and debt crisis.

4. Conclusions

In this paper, we have presented some evidence on a particular feature of the world income distribution measured in terms of GDP per capita, that is that the range comprised between the 30^{th} and the 85^{th} percentiles of the distribution scales as a power law for each year from 1960 up to 1997. This regularity has interesting implications for theory. In particular, our results are consistent with the following traits: *i*) all countries with GDP per capita comprised in 30^{th} - 85^{th} percentiles range share a common average growth rate; *ii*) richer countries are characterized by fluctuations of smaller amplitude if compared to poorer countries; *iii*) such a difference in terms of the relative amplitude of fluctuations between richer and poorer countries is increasing over time; *iv*) the volatility of fluctuations decreases with size at a rate suggesting the presence of direct interactions among economic agents. Successful models of growth and the business cycle should return predictions in line with these facts.

[10] Think, e.g., to the multi-sector RBC model of Long and Plosser (1983).
[11] As a matter of example, the models by Durlauf (1996) and Aoki (1998).

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