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Monopoly Profit in a Cournot oligopoly

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Abstract

A Cournot oligopoly with at least three firms is considered, where one of the firms has a cost−reducing innovation. A general version of royalty contract is proposed, and it is shown that this contract enables the innovator firm to earn the monopoly profit with the reduced cost.

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1. Introduction

Consider an oligopolistic industry where each firm produces with an identical marginal cost. Now suppose that a cost-reducing innovation has taken place and the owner of the innovation wants sell it to the firms. How much are the firms willing to pay to the owner for the innovation? Is it possible for the owner to come up with a selling mechanism such that he reaps all the benefits of the innovation? This note shows that if the owner of the innovation is one of the firms in the industry, then there is a simple contract that enables him to earn the monopoly profit with the reduced cost.

The relation between market structure and incentives for innovation was first explored by Arrow (1962). There is a significant number of papers considering various aspects of licensing of innovation. We refer to Kamien (1992) for an excellent survey of these issues. The interaction between the owner of the innovation (the innovator) and the firms has been modelled as a three-stage game where the innovator acts as a Stackelberg leader and announces a selling mechanism in the first stage. In the next stage, the firms simultaneously decide whether or not to buy the innovation, and the set of buyers become commonly known at the end of the second stage. In the final stage, the firms compete, where the buyers of the innovation have the reduced cost, and all other firms operate with the old cost. In the literature, three standard licensing policies, viz., fixed upfront fee, per-unit uniform royalty, and auction, have been considered. It has been shown by Wang (1998) in the context of a duopoly, and by Kamien and Tauman (2002) under a oligopolistic set-up that the nature of the optimal licensing policies could be significantly different if the innovator is one of the firms instead of being an outsider to the industry.

Kamien, Oren and Tauman (1992) have proposed a general licensing policy, where an outsider innovator can earn a payoff that is sufficiently close to the maximum industry profit that can be made with the innovation. This policy, referred by Kamien (1992) as the "chutzpah" mechanism, is complicated and proceeds in a certain number of stages. In this note, we show that when the innovator is one of the firms, there is a simple policy based on royalty by which the innovator can earn the monopoly profit with the reduced cost. The underlying reason is that the very fact that the innovator is an incumbent firm acts as an important instrument in the strategic interaction between the innovator and other firms. The rest of this note is organized as follows. We present the model and our main proposition in Section 2. We conclude in Section 3.

2. The Model

Consider a Cournot oligopoly with $n+1$ firms producing the same product. The firms are denoted by $0, 1, \ldots, n$. The firms compete in quantities. For $i = 0, 1, \ldots, n$, let q_i be the quantity produced by firm i and let $Q = \sum_{i=0}^{n} q_i$. The inverse demand function of the industry is linear and is given by $Q = a-p$, for $p \leq a$ and $Q = 0$, otherwise. Initially, all $n+1$ firms produce at the identical marginal cost c, where $0 < c < a$. Firm 0 has a cost-reducing innovation, which reduces the marginal cost from c to $c - \varepsilon$, where $0 < \varepsilon < c$. Firm 0, the innovator, considers licensing the innovation to some other firms in the industry. We then have the following three-stage licensing game.

In stage 1, the innovator announces a licensing contract. In stage 2, n firms in $\{1, \ldots, n\}$ simultaneously decide whether to accept the contract or not, and the set of licensees become commonly known at the end of the second stage. In stage 3, all $n+1$ firms compete in quantities, where the marginal cost of the innovator and each licensee firm is $c - \varepsilon$ and that of each non-licensee firm is c. The licensees pay to the innovator according to the licensing contract.

Before proceeding further, let us mention the notion of drastic innovation, due to Arrow (1962). An innovation is said to be *drastic* if the monopoly price under the reduced cost does not exceed the competitive price under the old cost; otherwise, it is *non-drastic*. In the model under consideration, an innovation is drastic if $a - c \leq \varepsilon$, and it is non-drastic if $a - c > \varepsilon$. It is immediately seen that when the innovation is drastic and the innovator is one of the firms, all other firms drop out, and, the innovator earns the monopoly profit with the reduced cost. So, the issue under question is interesting only when the innovation is non-drastic. Throughout this note, we assume that the innovation is non-drastic, that is, $a - c > \varepsilon$. Our main proposition is based on a general version of the royalty contract, which can be described as follows.

General Royalty Contract. For a *general royalty contract*, the rate of royalty depends on the number of licensees. Specifically, a typical contract C_r is given by an n-tuple $\langle r(1), \ldots, r(n) \rangle$, where for $m = 1, \ldots, n$, $r(m)$ denotes the per-unit linear royalty that each licensee has to pay when the number of licensees is m. When $r(i) = r$ for all i, we have the usual royalty contract.

Proposition 1. Let $n \geq 2$ and $(a-c) > \varepsilon$. There exists a general royalty contract $C_r = \langle r(1), \ldots, r(n) \rangle$ such that the following holds for the three-stage licensing game.

[1] If the innovator offers the contract C_r, it is weakly dominant for every firm to accept the contract. Consequently, in the unique subgame-perfect equilibrium, every firm $i \in \{1, \ldots, n\}$ becomes a licensee.

[2] When there are n licensees, then, every firm $i \in \{1, \ldots, n\}$ finds it optimal not to produce. So, in the unique subgame-perfect equilibrium, the innovator is the only producer, and he earns the monopoly profit $(a-c+\varepsilon)^2/4$.

Proof. There is more than one contract that satisfies the proposition. We shall prove the proposition by providing a specific contract. Consider the contract $C_r = \langle r(1), \ldots, r(n) \rangle$, where

$$r(m) = \frac{c-a+(m+1)\varepsilon}{m}, \text{ for } m \in \{1, \ldots, n-1\}, \text{ and } r(n) = \frac{a-c+\varepsilon}{2}. \quad (1)$$

Suppose there are m licensees, for $m \in \{0, 1, \ldots, n\}$. From symmetry it follows that any two licensees will produce the same quantity and earn the same payoff. So will any two non-licensee firms. Let $\Pi_0(m)$, $\Pi_L(m)$ and $\Pi_N(m)$ denote the payoff of firm 0, a licensee and a non-licensee respectively under the contract C_r when there are m licensees. To prove [1], then, we need to show the following.

$$\Pi_L(m) \geq \Pi_N(m-1) \text{ for all } m \in \{1, \ldots, n\}, \text{ and,}$$

$$\Pi_L(m) > \Pi_N(m-1) \text{ for at least one } m \in \{1, \ldots, n\}. \quad (2)$$

In what follows, we show that $\Pi_L(m) > \Pi_N(m-1)$ for all $m \in \{1, \ldots, n-1\}$ and $\Pi_L(n) = \Pi_N(n-1)$. First consider the case when there is no licensee. For $i \in \{0, 1, \ldots, n\}$, let Π_i denote the payoff of firm i. Then, in this case,

$$\Pi_0 = (a - \sum_{j=0}^{n} q_j)q_0 - (c-\varepsilon)q_0; \; \Pi_i = (a - \sum_{j=0}^{n} q_j)q_i - cq_i \text{ for } i \in \{1, \ldots, n\}. \quad (3)$$

Solving for optimal quantities, from (3) we get the following.

$$q_0 = \frac{a-c+(n+1)\varepsilon}{n+2}, q_i = \frac{a-c-\varepsilon}{n+2} \text{ for } i \in \{1, \ldots, n\}. \quad (4)$$

Further, for $i \in \{0, 1, \ldots, n\}$, $\Pi_i = q_i^2$. Hence,

$$\Pi_N(0) = \left[\frac{a-c-\varepsilon}{n+2}\right]^2. \quad (5)$$

Next, consider the case when there are m licensees, for $m \in \{1, \ldots, n-1\}$. Let $\{1, \ldots, m\}$ be the set of licensees. Then,

$$\Pi_0 = (a - \sum_{j=0}^{n} q_j)q_0 - (c - \varepsilon)q_0 + r(m)\sum_{j=1}^{m} q_j,$$

$$\Pi_i = (a - \sum_{j=0}^{n} q_j)q_i - (c - \varepsilon + r(m))q_i \text{ for } i \in \{1, \ldots, m\}, \text{ and,}$$

$$\Pi_i = (a - \sum_{j=0}^{n} q_j)q_i - cq_i \text{ for } i \in \{m+1, \ldots, n\}. \tag{6}$$

Solving for optimal quantities, from (6) we get the following.

$$q_0 = \frac{a - c + (n - m + 1)\varepsilon + mr(m)}{n+2},$$

$$q_i = \frac{a - c + (n - m + 1)\varepsilon - (n - m + 2)r(m)}{n+2} \text{ for } i \in \{1, \ldots, m\}, \text{ and,}$$

$$q_i = \frac{a - c - (m + 1)\varepsilon + mr(m)}{n+2} \text{ for } i \in \{m+1, \ldots, n\}. \tag{7}$$

Replacing the value of $r(m)$ from (1) in (7) we conclude that

$$q_0 = \varepsilon, q_i = \frac{a - c - \varepsilon}{m} \text{ for } i \in \{1, \ldots, m\}, q_i = 0 \text{ for } i \in \{m+1, \ldots, n\}. \tag{8}$$

Further, for $i \in \{1, \ldots, n\}$, $\Pi_i = q_i^2$, so that from (8) we conclude the following.

$$\Pi_L(m) = \left[\frac{a - c - \varepsilon}{m}\right]^2, \Pi_N(m) = 0 \text{ for } m \in \{1, \ldots, n-1\}. \tag{9}$$

Since $a - c > \varepsilon$, we conclude from (9) that $\Pi_L(m) > \Pi_N(m-1)$ for all $m \in \{2, \ldots, n-1\}$. Further, from (5) and (9) it follows that

$$\Pi_L(1) - \Pi_N(0) = \frac{(n+1)(n+3)(a - c - \varepsilon)^2}{(n+2)^2} > 0. \tag{10}$$

Thus, we have shown that $\Pi_L(m) > \Pi_N(m-1)$ for all $m \in \{1, \ldots, n-1\}$. Now we show that $\Pi_L(n) = \Pi_N(n-1)$. Let us consider the case when every firm $i \in \{1, \ldots, n\}$ is a licensee. In that case,

$$\Pi_0 = (a - \sum_{j=0}^{n} q_j)q_0 - (c - \varepsilon)q_0 + r(n)\sum_{j=1}^{n} q_j,$$

$$\Pi_i = (a - \sum_{j=0}^{n} q_j)q_i - (c - \varepsilon + r(n))q_i \text{ for } i \in \{1, \ldots, n\}. \tag{11}$$

Solving for optimal quantities, from (11) we get the following.

$$q_0 = \frac{a - c + \varepsilon + nr(n)}{n + 2}, q_i = \frac{a - c + \varepsilon - 2r(n)}{n + 2} \text{ for } i \in \{1, \ldots, n\}. \tag{12}$$

Recall from (1) that $r(n) = (a - c + \varepsilon)/2$. Replacing the value of $r(n)$ in (12) we conclude that

$$q_0 = \frac{a - c + \varepsilon}{2}, q_i = 0 \text{ for } i \in \{1, \ldots, n\}. \tag{13}$$

From (9) and (12) we conclude that $\Pi_L(n) = \Pi_N(n - 1) = 0$. This proves [1]. Also, from (13) it follows that under the contract C_r, when there are n licensees, only the innovator produces, and he earns the monopoly profit $(a - c + \varepsilon)^2/4$. This proves [2]. ■

Remark. When $n = 1$, the innovator cannot earn the monopoly profit. With $n \geq 2$, the innovator generates competition among other firms, and, in the process extracts all the surplus. The element of competition is absent with $n = 1$. Specifically, firm 1 can always ensure the payoff $(a-c-\varepsilon)^2/9$. Thus, the maximum payoff that the innovator can get is $[(a - c + \varepsilon)^2/4 - (a - c - \varepsilon)^2/9]$. This can be achieved with a royalty plus fixed fee contract $\langle r, f \rangle$, where r is sufficiently large (which ensures that the licensee does not produce), and $f = -(a-c-\varepsilon)^2/9$ (firm 1 is paid $-f$ so that it is indifferent between accepting and rejecting the contract).

3. Conclusion

In this note, we have shown that in a Cournot oligopoly with at least three firms, if the owner of cost-reducing innovation is an incumbent firm, there is a general version of royalty contract that enables the innovator to earn the monopoly profit with the reduced cost. Our result depends on two factors: first, the innovator is one of the firms, which implies that irrespective of the other firms' decisions, there always exists an efficient firm in form of the innovator. Second, there must be at least two firms other than the innovator. With just one more firm, the innovator cannot generate the kind of competition that drives our result.

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