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Unusual behaviour of Dickey−Fuller tests in the presence of trend misspecification: comment

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Abstract

In this paper we present explanation on the phenomenon pointed out in Cook and Manning (2002) on the unusual behaviour of the Dickey−Fuller test in the presence of trend misspecification. It appears that the rejection frequency of the unit root tests in the presence of trend misspecification is very sensitive to the number of the initial observations that need to be discarded. Based on the evidence from the Monte Carlo simulations, we show that for the DGP in Cook and Manning (2002), the unusual behaviour of the Dickey−Fuller test disappears as the number of the discarded initial observations becomes sufficiently large.

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1 Introduction.

The literature on unit root testing in economic time series is already extensive and well established. Nevertheless, it is too early to say whether it is entirely complete. Although the properties of the most popular unit root tests have been well described and understood, there are still nuances that remain to be explained and possibly discovered by future research.

One example is the recent paper by Cook and Manning (2002), where the behaviour of the two unit root tests – the $\tau_\tau-$ and $K-$tests suggested in Fuller (1976) – is compared under trend misspecification. The authors point out that the former test exhibits unusual behaviour whereas the latter test performs as expected.

In this paper, we address the issue and provide an explanation for this seemingly puzzling finding. In particular, we extend the Monte Carlo results of Cook and Manning (2002) by examining to what extent the rejection frequency of the examined unit root tests depends on the number of observations that are discarded to lessen the influence of the initial values.

We find that the unusual behaviour of the τ_τ-test eventually disappears as one discards an increasing number of initial observations. Moreover, for a rather large number of the discarded observations, the rejection frequency of the τ_τ- and K-tests is quite similar. This is rather remarkable since when rather small number of initial observations is discarded, their behaviour varies considerably, as reported in Cook and Manning (2002).

The rest of the paper is organized as follows. Section 2 discusses the motivation behind Cook and Manning (2002) and explains their Monte Carlo setup. Section 3 presents the Monte Carlo setup used in the simulations and presents the obtained results. The final section concludes.

All graphics as well as the computational results were obtained using the programs Ox 3.30 Professional and Givewin 2.10, see Doornik (2001) and Hendry and Doornik (2001), respectively.

2 Results of Cook and Manning (2002).

Consider the unit root process with drift

$$y_t = \mu_1 + y_{t-1} + e_t = y_0 + \mu_1 t + \sum_{i=1}^{t} e_i, e_i \sim i.i.d.(0, \sigma^2) \tag{1}$$

As seen from the corresponding MA-representation, this process exhibits trending behaviour. Hence it has been suggested in the literature (e.g. see Banerjee et al., 1994; Harris, 1995) that the following auxiliary Dickey-Fuller (DF) regression be used for unit root testing

$$y_t = a + bt + \phi y_{t-1} + e_t \tag{2}$$

in order to gain power against the trend-stationarity of a process. Observe that by using equation (2) we ensure similar behaviour of the time series both under the null and alternative hypotheses.

Fuller (1976) suggests the following ways of testing for unit root hypothesis ($\phi = 1$). The first is $\tau_\tau-$test in the form of $t-$ratio statistic

$$\tau_\tau = \frac{\widehat{\phi} - 1}{se(\widehat{\phi})} \tag{3}$$

and the $K-$test in the coefficient form

$$K = T(\widehat{\phi} - 1). \tag{4}$$

Under the null hypothesis of unit root, both tests have nonstandard Dickey-Fuller distributions, tabulated in Fuller (1976). As usual, the rejection/acceptance decision of the null hypothesis of unit root is based on comparing the obtained values of the respective test statistics with the tabulated critical values for a chosen significance level.

However, since in reality the true DGP remains unknown, there always exists a possibility for running the DF-regression (2) under a false assumption that the data have been generated in accordance with equation (1). Cook and Manning (2002) investigate the power properties of the unit root test based on equation (2) when the data have been generated by the following DGP

$$y_t = \alpha + \beta t + \rho y_{t-1} + \xi_t, t = 1, ..., T. \tag{5}$$

It is easy to verify that under the null hypothesis of unit root there is a quadratic deterministic trend in the MA-form of equation (5). Hence, an appropriate set of deterministic terms in the DF-regression should include a quadratic trend as well. In the opposite case, the misspecification of the test regression is apparent.

Cook and Manning (2002) study the consequences of such trend misspecification. They report the empirical rejection frequency of the $\tau_\tau-$ and $K-$forms of the Dickey-Fuller tests using the relevant critical values that correspond to the nominal 5% significance level. Although the authors indicate that in their Monte Carlo experiments they have tried different parameter combinations, they specifically report the results of only one particular parameter set $\{\alpha, \beta, T\} = \{0, 0.08, 250\}$ and $\rho = \{0.9600, 0.9605, .., 0.9995, 1\}$, which illustrate their main finding. In particular they consider the initial value $y_0 = 0$ and discard first $N = 100$ observations to lessen its influence on the obtained results. Observe that the error term is $\xi_t \sim i.i.d.N(0, 1)$. Figure 1 replicates their results. Notice that in the presence of the trend misspecification it is expected that the DF tests will be biased towards nonrejection of the hypothesis of the unit root especially for values of ρ close to one. The behaviour of the $K-$test conforms with the expectations, whereas the DF τ_τ-test displays rather

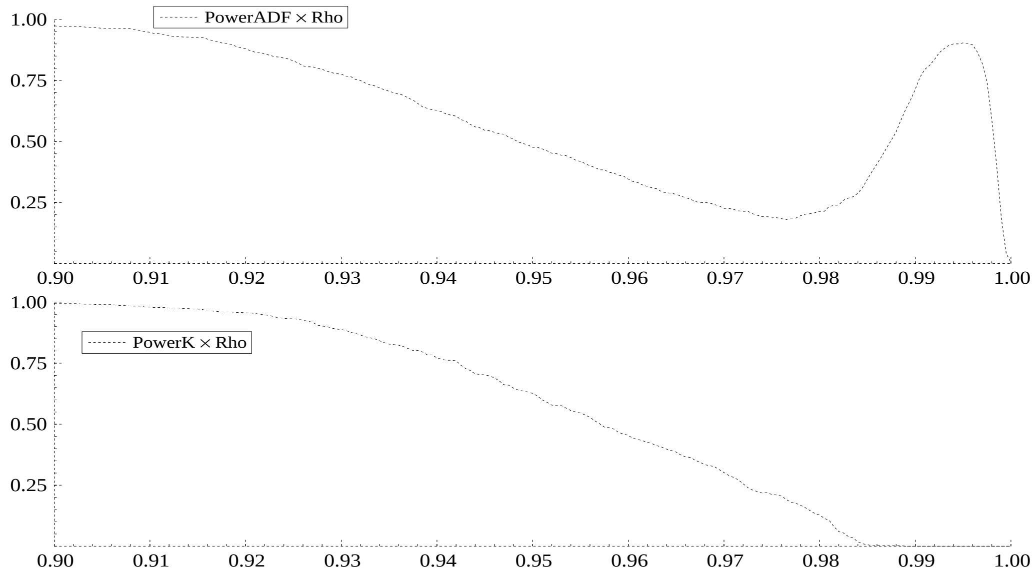

Figure 1: Upper panel: rejection frequencies of $\tau_\tau-$ test; Lower panel: rejection frequencies of $K-$test. Replicated results of Cook and Manning (2002): $\{\alpha, \beta, T\} = \{0, 0.08, 250\}$ and $\rho = \{0.9000, 0.9005, .., 0.9995, 1\}$, first $N = 100$ observations discarded. Each curve has been obtained using 5000 replications.

unusual behaviour. As seen, the rejection frequency after the initial decline rises sharply and then falls rapidly towards zero at $\rho = 1$.

In the subsequent section we provide an explanation of this apparently aberrant behaviour. The main argument is that this puzzling finding is caused by the fact that the removal of the first $N = 100$ observations as done by Cook and Manning (2002) is not enough to remove the influence of the initial observation $y_0 = 0$ as claimed by the authors. We show that once one removes enough initial observations in the starting period, then this puzzling result disappears. In the limit for the large number of discarded initial observations, the rejection frequencies of both τ_τ- and K-tests are very similar.

3 Our results.

In this section, we extend the Monte Carlo results of Cook and Manning (2002) by considering the rejection frequencies of the τ_τ- and K-tests for different number of startup observations. Note that otherwise the DGP is the same as in Cook and Manning (2002), see above. To this end we consider the following numbers of initial N observations removed $N = \{50,100,200,10000\}$. The results of the Monte Carlo simulations are presented in Figure 2. As seen in Figure 1, the unusual behaviour

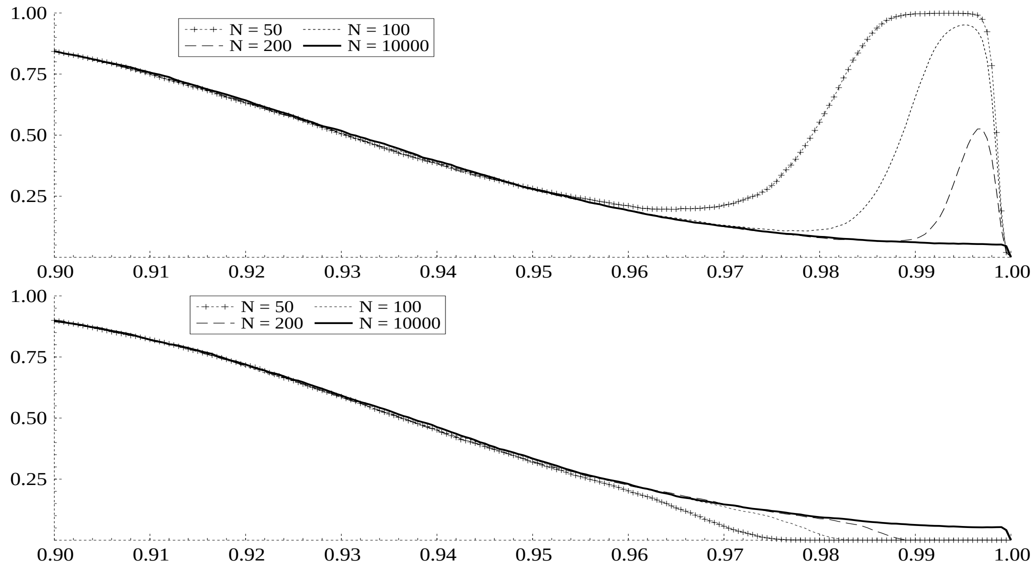

Figure 2: Upper panel: rejection frequencies of $\tau_\tau-$ test; Lower panel: rejection frequencies of $K-$test. DGP of Cook and Manning (2002): $\{\alpha, \beta, T\} = \{0, 0.08, 250\}$ and $\rho = \{0.9000, 0.9005, .., 0.9995, 1\}$, first $N = \{50,100,200,10000\}$ observations discarded. Each curve has been obtained using 5000 replications.

of the Dickey-Fuller τ_τ-test fades away as one eliminates more and more initial observations. Also notice that the rejection frequencies of the τ_τ- and K-tests, so different for rather small values of N, become very similar for rather large values of N, e.g. $N = 10000$.

Observe that in the example of Cook and Manning (2002), the trend misspecification is introduced by omitting the quadratic deterministic trend from the test regression. It also possible to construct a case where the trend misspecification occurs due to the absence of a linear deterministic trend in the DF regression. That is, the testing regression is given by

$$y_t = a + \phi y_{t-1} + e_t, \tag{6}$$

but the data are generated as follows

$$y_t = \alpha + \rho y_{t-1} + \xi_t. \tag{7}$$

Observe that similarly to the case considered in Cook and Manning (2002), equation (6) is misspecified as it omits a linear deterministic trend that is present in the MA-form of equation (7) under the null hypothesis of unit root $\rho = 1$.

In order to investigate the properties of these two unit root tests under this misspecification type, we chose the following parameter values $\{\alpha, \beta, T\} = \{2.0, 0, 250\}$

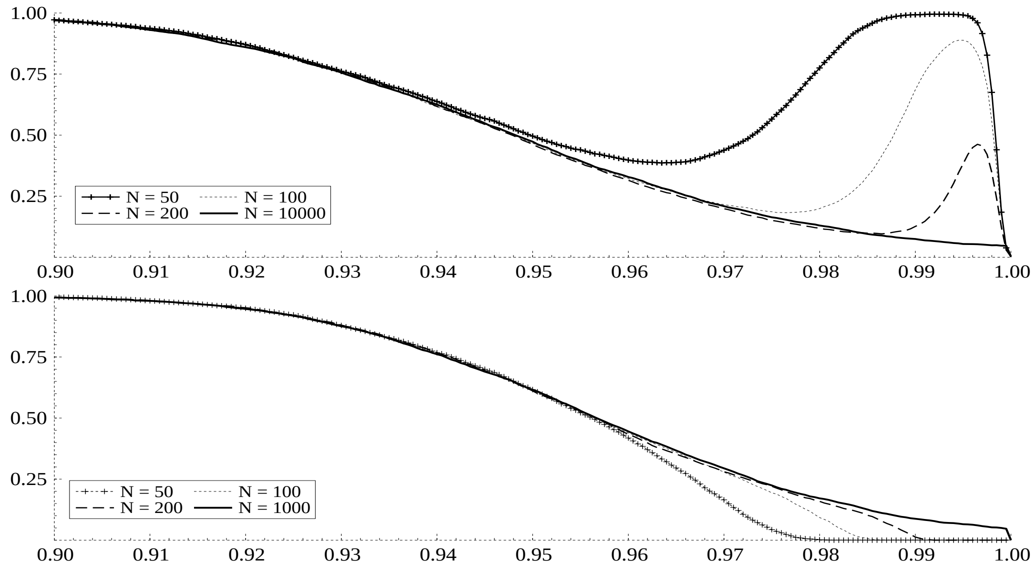

Figure 3: Upper panel: rejection frequencies of $\tau_\tau-$ test; Lower panel: rejection frequencies of $K-$test. DGP: $\{\alpha, \beta, T\} = \{2.0, 0, 250\}$ and $\rho = \{0.9000, 0.9005, .., 0.9995, 1\}$, first $N = \{50,100,200,10000\}$ observations discarded. Each curve has been obtained using 5000 replications.

and $\rho = \{0.9000, 0.9005, .., 0.9995, 1\}$ with the following values of the discarded startup period $N = \{50,100,200,10000\}$. Figure 3 displays the rejection frequencies obtained for both τ_τ- and $K-$tests.

As seen in Figure 3, the qualitatively similar picture to that displayed in Figure 2 emerges here as well. That is, for rather small values of N, the behaviour of the $\tau_\tau-$test is counterintuitive and differs markedly from that of the $K-$tests. However, this difference is decreasing as the value of N grows, and eventually for rather large values of N, the behaviour of both tests is very similar.

What can we learn from this Monte Carlo evidence? We observe the unusual behaviour of the τ_τ-test for unit roots only when a relatively small number of initial observations is discarded. In this case, when a situation with trend misspecification as described in Cook and Manning (2002) – see equations (2) and (5) as well as equations (6) and (7) – arises, the two unit root tests suggested in Fuller (1976) display remarkably different behaviour. However, when there is a sufficient number of initial observations being discarded, the behaviour of these two unit root tests is rather similar.

4 Conclusions.

In this paper, we analyze the results reported in Cook and Manning (2002) on the unusual behaviour of the unit root test in the presence of trend misspecification. We are able to replicate the results of their study when the quadratic deterministic trend has been omitted from the testing regression for unit roots. Furthermore, we provide evidence of the impact of trend misspecification when the linear deterministic trend has been omitted from the testing regression for unit roots. The results of the latter case fully mirror the results obtained for the former case.

Thus, when the number of the initial observations that are discarded is rather small, the τ_τ-test exhibits very unusual behaviour. For the range of values of the autoregressive parameter that are considerably smaller than one, the rejection frequency of the null hypothesis of unit root declines as this parameter approaches unity. Then for the range of values that are quite close to unity, the rejection frequency of the τ_τ-test surprisingly increases, such that it exhibits an unusually high rejection rate of the null hypothesis. On the other hand, the rejection frequency of the $K-$test declines monotonically as the value of the autoregressive parameter approaches unity.

As the main contribution of our paper, we show that the reason for these two unit root tests exhibiting such remarkably different behaviour in the Monte Carlo setup used by Cook and Manning (2002) is that an insufficient number of initial observations has been discarded. We show that when a sufficiently large number of initial observations have been discarded, there is no evidence for the abnormal behaviuor of the τ_τ-test recorded in Cook and Manning (2002). In fact, under these conditions, the rejection frequency of both the τ_τ- and $K-$tests steadily declines as the autoregressive parameter value approaches unity.

References

Banerjee, A., J. Dolado, D. Hendry, and J. W. Galbraith (1994). *Co-Integration, Error Correction, and the Econometric Analysis of Non-Stationary Data.* Advanced Texts in Econometrics. Oxford University Press.

Cook, S. and N. Manning (2002). Unusual behaviour of Dickey-Fuller tests in the presence of trend misspecification. *Economics Bulletin 3*(8), 1–7.

Doornik, J. A. (2001). *Ox: An Object-Oriented Matrix Language* (4th ed.). London: Timberlake Consultants Press.

Fuller, W. A. (1976). *Introduction to Statistical Time Series.* New York: John Wiley.

Harris, R. I. D. (1995). *Using Cointegration Analysis in Econometric Modelling.* Prentice Hall PTR.

Hendry, D. F. and J. A. Doornik (2001). *GiveWin: An Interface to Empirical Modelling* (3rd ed.). London: Timberlake Consultants Press.