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# On price uncertainty, nominal assets and uninsurable idiosyncratic risks

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## *Abstract*

The paper discusses a way in which price uncertainty may affect the extent of idiosyncratic, uninsurable risks in an incomplete markets economy with nominal assets and thereby affect output and welfare. Although the returns on these assets are constant and riskfree in nominal terms, price uncertainty causes their real returns to be stochastic. This affects the ability of households to diversify their idiosyncratic risks using these assets and consequently the extent of uninsurable risks in the economy. The paper establishes a relationship between the volume of trade in nominal assets, the stochastic characteristics of the price shocks and the covariance between the price and idiosyncratic shocks.

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# 1 Introduction

The paper points to a new channel through which price uncertainty affects output. Economists have long been interested in price and inflation uncertainty and their effect on output. The last two decade in particular has seen a substantial growth in studies related to the measurement and costs of price instability and monetary policies targeting price stability (see Andres and Hernando (1999) and Woodford (2005) for surveys). The interest on the issue notwithstanding, there has not been many theoretical attempts to explain the effect of price uncertainty within a fully general equilibrium set up. The present work attempts to partially fill this gap.

Two features in the present set up creates this channel of influence - the presence of uninsurable idiosyncratic risks and nominal assets. Although the returns on these assets are constant in nominal terms, price shocks cause their real returns to be stochastic. The covariance between the stochastic price level and the idiosyncratic productivity and endowment shocks determine the extent of uninsurable risks in the economy and the volume of trade in these nominal assets. Thus output is affected.

We use an existing dynamic general equilibrium model with incomplete markets, CARA preferences and normal shocks, developed in a series of recent papers ( see Angeletos and Calvet (2001,2003), Calvet (2003) and Athanasoulis (2005)) to prove our point. Although this specification has certain known limitations, it is mathematically tractible (equilibrium has closed form) and hence useful in this preliminary attempt to understand an effect of price uncertainty. Analyzing this in the more traditional framework of CRRA preferences is left for future.

Dotsey and Sarte (2000) shows that shocks to real balances generate precautionary effects if agents face cash in advance constraints and these can positively affect growth. The present work is close in spirit to their work though not in content. The only role money plays here is as a unit of account (not medium of exchange) as some assets are denominated in nominal units. Further unlike Dotsey and Sarte, we are looking at an economy with incomplete markets. Price uncertainty will have no real effects within the same set up if markets are complete.

## 2 The Model

The economy consists of a continuum of households $h \in [0,1]$, living for $T$ periods where $T$ may be finite or infinite, and each having a stochastic endowment at date $t$, denoted by $e_t^h$ (labor income). Further each household has access to a specific and risky production technology which uses capital as an only input. The same good is used for both consumption and investment. Capital is not subject to depreciation. The $h$th household's production function is given by, $y_t^h = \eta_t^h f(k_t^h)$, where $\eta_t^h$ is a household specific productivity shock and $k_t^h$ is physical capital. The production function satisfies the usual neoclassical assumptions of concavity and Inada conditions.

Households are allowed to trade in financial assets and without loss of generality we assume that there are two short term bonds, a real and a nominal one, indexed $j = 0,1$ respectively. The amount purchased of the $j$th asset by the $h$th household at time $t$ is denoted by $\theta_{j,t}^h$. Asset payoff for the $j$th asset, at date $t$, measured in units of the consumption good is denoted by $d_{j,t}$. Since the 0th bond is a real, riskfree bond, we have $d_{0,t} = 1$. Since the bond indexed 1 is a nominal bond which pays one unit of money every period, its real returns are $d_{1,t} = \frac{1}{p_t}$, where $p_t$ is the exogenously given stochastic price level.

**Assumption 1** *All households have identical CARA preferences and maximize* $E_0 \sum_{t=0}^{T} \beta^t (-\frac{1}{A} \exp(-Ac_t^h))$.

where $A$ is the degree of absolute risk aversion, $\beta$ the discount factor and $c_t^h$ the level of consumption of the $h$th household.

Denoting nominal asset prices by $\pi_{j,t}$, the date $t$ budget constraint of the $h$th household in real terms is given by,

$$e_t^h + \eta_t^h f(k_t^h) + \sum_{j=0}^{1} d_{j,t}\theta_{j,t-1}^h = c_t^h + (k_{t+1}^h - k_t^h) + \sum_{j=0}^{1} \frac{pi_{j,t}}{p_t}\theta_{j.t}^h \tag{1}$$

The model has closed form solution under normality assumptions for all shocks and the assumption of no aggregative risks. Hence,

**Assumption 2** *(i)* $(e_t^h, \eta_t^h)$ *are jointly normal, identically and independently distributed over time.* $(e_t^h, \eta_t^h)$ *have the same mean and variance across agents and* $Cov(e_t^h, \eta_t^h) = 0$.

*(ii)* $d_{1,t} = \frac{1}{p_t}$ *is normal with mean* $\mu$ *and variance* $\sigma^2$.[1]

We can perform the following OLS decompositions of the idiosyncratic real shocks on the returns stream of the risky asset (the nominal bond),

$$\eta_t^h = \eta + \kappa^h d_{1,t} + \tilde{\eta}_t^h \tag{2}$$

$$e_t^h = e + \xi^h d_{1,t} + \tilde{e}_t^h \tag{3}$$

where $\eta = E(\eta_t^h)$, $e = E(e_t^h)$, $\kappa^h = \mathrm{Cov}(\eta_t^h, d_{1,t})/\mathrm{Var}(d_{1,t})$ and $\xi^h = \mathrm{Cov}(e_t^h, d_{1,t})/\mathrm{Var}(d_{1,t})$. Since

[1] The assumption of normal shocks may seem empirically unrealistic but note that normality may be used as an approximation for many other distributions.

idiosyncratic shocks are identically distributed across agents and by the properties of the OLS decomposition, $E(\tilde{\eta}_t^h) = 0$, $E(\tilde{e}_t^h) = 0$, $\mathrm{Var}(\tilde{\eta}_t^h) = \sigma_p^2$ and $\mathrm{Var}(\tilde{e}_t^h) = \sigma_e^2$.

The residuals $\tilde{\eta}_t^h$ and $\tilde{e}_t^h$ represent the non-diversifiable component of the idiosyncratic risks, in the economy. The variances, $\mathrm{Var}(\tilde{\eta}_t^h)$ and $\mathrm{Var}(\tilde{e}_t^h)$ measure the extent of uninsurable risks. Finally we assume that idiosyncratic shocks cancel each other out. Hence,

**Assumption 3** $\int_H \tilde{\eta}_t^h = \int_H \tilde{e}_t^h = 0.$

which ensures that there is no aggregate risks and in fact that in equilibrium aggregate output is deterministic. We end this section by defining a dynamic competitive equilibrium for this economy.

**Definition 1** *A competitive equilibrium for the economy is a sequence of individual allocations,* $(\{c_t^h, k_{t+1}^h, \{\theta_{j,t}^h\}_{j=0,1})$ *and (relative) asset prices,* $(\frac{\pi_{1,t}}{\pi_{0,t}}, \frac{\pi_{0,t}}{p_t})$*, such that*

*(i) A household maximixes its intertemporal utility subject to its budget constraint at each date.*

*(ii) the goods and asset markets clear at each date, that is*

$$\int_H (c_t^h + k_{t+1}^h) = \int_H (e_t^h + \eta_t^h f(k_t^h) + k_t^h) \tag{4}$$

$$\int_H (\theta_{j,t}^h) = 0, \forall j \tag{5}$$

A closed form solution of the dynamic equilibrium exists for this set up. That and the method of finding it, is briefly discussed in the Appendix. The interested reader is also referred to the papers mentioned in the introduction.

## 3 Price uncertainty, nominal assets and uninsurable risks

In this section we show how price shocks influence the extent of uninsurable risks in the economy, hence the volume of trade in the nominal bond in equilibrium and hence ouput. The first task is to characterize the equilibrium demand for the nominal asset.

**Proposition 1** *The household's demand for the nominal bond is given by,*

$$\theta^h_{1,t} = -\xi^h - \kappa^h f(k^h_{t+1}) \tag{6}$$

**Proof**: See proof of Proposition 4 in Appendix.

The household's demand for the nominal asset depends only on the covariance of the return of the asset with the idiosyncratic income shocks. In particular they hold long positions in this asset if the covariances are negative and short positions if the covariances are positive.

We now try to establish a connection between the stochastic characteristics of the price shocks and the extent of uninsurable risks. We start with the following relationships between the variances of the non-diversifiable risks and the stochastic characteristics of the nominal asset, which follow from definitions and OLS decompositions ( 2) and ( 3).

$$\sigma_p^2 = \mathrm{Var}(\eta_t^h) - (\kappa^h)^2 \mathrm{Var}(\frac{1}{p_t}) \tag{7}$$

$$\sigma_e^2 = \mathrm{Var}(e_t^h) - (\xi^h)^2 \mathrm{Var}(\frac{1}{p_t}) \tag{8}$$

The following theorem connects $\sigma_p^2, \sigma_e^2$ to the covariance between price shocks and productivity and endowment shocks and the variance and expectations of the former.

**Proposition 2**

$$\sigma_p^2 = Var(\eta_t^h) - \frac{(Cov(\eta_t^h, p_t))^2}{Var(\frac{1}{p_t})(E(p_t)^2)^2} \tag{9}$$

$$\sigma_e^2 = Var(e_t^h) - \frac{(Cov(e_t^h, p_t))^2}{Var(\frac{1}{p_t})(E(p_t)^2)^2} \tag{10}$$

**Proof**: $\mathrm{Cov}(\eta_t^h, p_t) = \mathrm{Cov}(\eta_t^h, \frac{1}{\frac{1}{p_t}})$

Using Stein's Lemma, the right hand side of the above equation reduces to,

$$\begin{aligned} & -E((p_t)^2)\mathrm{Cov}(\eta_t^h, \frac{1}{p_t}) \\ = & -E((p_t)^2)\kappa^h \mathrm{Var}(\frac{1}{p_t}) \end{aligned}$$

Transposing,

$$\kappa^h = -\frac{\mathrm{Cov}(\eta_t^h, p_t)}{\mathrm{Var}(\frac{1}{p_t})E((p_t)^2)}$$

Substituting the above expression for $\kappa^h$ into the right hand side of equation ( 7) and simplifying gives us equation ( 9). Equation ( 10) is derived using similar steps. $\Delta$

Proposition 2 reveals that the values of $\sigma_p^2, \sigma_e^2$ depend (i) on the absolute size of the covariance between the price shocks and the endowment and productivity shocks and (ii) on the product

$\mathrm{Var}(\frac{1}{p_t})(E(p_t)^2)^2$. The variance of non-insurable risks increases if $\mathrm{Cov}(e_t^h, p_t)$ and $\mathrm{Cov}(\eta_t^h, p_t)$, diminish in absolute terms. We discuss the implications below.

Note that the demand for the risky nominal asset is given by,

$$\theta_{1,t}^h = -\xi^h - \kappa^h f(k_{t+1}^h)$$

Substituting the values of $\xi^h$ and $\kappa^h$ from Theorem 1, we get,

$$\theta_{1,t}^h = \frac{\mathrm{Cov}(e_t^h, p_t)}{\mathrm{Var}(\frac{1}{p_t})E((p_t)^2)} + \frac{\mathrm{Cov}(\eta_t^h, p_t)}{\mathrm{Var}(\frac{1}{p_t})E((p_t)^2)} f(k_{t+1}^h)$$

Households borrow or lend using the nominal bond depending upon whether their endowment and productivity shocks are negatively or positively correlated with the price shocks. For households with both covariance terms positive, $\theta_{1,t}^h$ is positive. Such households lend by buying the nominal bond from other households because when their realizations of $\eta_t^h$ and $e_t^h$ are low, their return $\frac{1}{p_t}$ on the nominal asset is better since $p_t$ is low also. Similarly, for households with both covariance terms negative, $\theta_{1,t}^h$ is negative. Such households borrow by selling the nominal bond to other households because when their realizations of $\eta_t^h$ and $e_t^h$ are low, the real interest they pay $\frac{1}{p_t}$ on the nominal asset to other households is low since $p_t$ is high. Note that when these covariances diminish in absolute terms, agents use the nominal bond less. The extent of uninsurable risks increase as a result.

# 4 Appendix: Characterizing the dynamic equilibrium

The date $t$ Euler equations are,

$$\begin{aligned}
\frac{\pi_{0,t}}{p_t}u_c^h(c_t^h) &= \beta E_t(u_c^h(c_{t+1}^h) && (11)\\
\frac{\pi_{1,t}}{p_t}u_c^h(c_t^h) &= \beta E_t(u_c^h(c_{t+1}^h)\frac{1}{p_{t+1}}) && (12)\\
u_c^h(c_t^h) &= \beta E_t(u_c^h(c_{t+1}^h)\eta_{t+1}^h f'(k_{t+1}^h) && (13)
\end{aligned}$$

**Proposition 3** *The equilibrium prices of the nominal (risky) and real (riskfree) bonds are given by*

$$\begin{aligned}
\frac{\pi_{1,t}}{\pi_{0,t}} &= E(\frac{1}{p_{t+1}}) && (14)\\
\log(\frac{\pi_{0,t}}{p_t}) &= A((Y_t - Y_{t+1}) + (K_t - 2K_{t+1} + K_{t+2})) \\
&\quad + \frac{A^2}{2}\int_H Var(c_{t+1}^h) + \log\beta && (15)
\end{aligned}$$

*where $Y_t$ denotes aggregate output and $K_t$, aggregate capital stock, at date $t$.*

**Proof**: The Euler equation ( 12) for risky assets can be written as,

$$\frac{\pi_{1,t}}{p_t}u_c^h(c_t^h) = E(u_c^h(c_{t+1}^h))E(d_{1,t+1}) + \mathrm{Cov}(u_c^h(c_{t+1}^h), d_{1,t+1})$$

Assuming normality of $c_t^h$ for all $t$ (we prove this below) and using Stein's lemma, the above expression can be expanded as

$$\frac{\pi_{1,t}}{p_t}u_c^h(c_t^h) = E(u_c^h(c_{t+1}^h))E(d_{1,t+1}) + E(u_{cc}^h(c_{t+1}^h)\mathrm{Cov}(c_{t+1}^h), d_{1,t+1})$$

where $u^h_{cc}(.)$ represents the derivative of $u^h_c(.)$. Dividing the above expression by the Euler equation 1 for the riskfree asset, and noting that $\frac{E(u^h_{cc}(c^h_{t+1})}{E(u^h_c(c^h_{t+1}))} = -A$ we get,

$$\frac{\pi_{1,t}}{\pi_{0,t}} = E(d_{1,t+1}) - A\text{Cov}(c^h_{t+1}), d_{1,t+1})$$

Aggregating over households, and noting that aggregate output is deterministic, the covariance term becomes zero and we get the required expression.

To find the equilibrium price of the riskfree asset, we first evaluate the definite integral $E(u^h_c(c^h_{t+1}))$ in Euler equation ( 11). Assuming that $c^h_t$ is $N \sim (\bar{c}, \sigma^2_c)$, it can be shown that,

$$\begin{aligned} E_t(u^h_c(c^h_t)) &= \int_{\infty}^{\infty} \text{Exp}(-Ac^h_t)\frac{1}{\sigma_c\sqrt{(2\pi)}}\text{Exp}(-\frac{(c^h_t - \bar{c})^2}{2\sigma^2_c})d(c^h_t) \\ &= \text{Exp}(-AE_t(c^h_t) + \frac{A^2}{2}\text{Var}_t(c^h_t)) \end{aligned}$$

Substituting for $u^h_c(c^h_t) = e^{-Ac^h_t}$ on the left hand side and for $E_t(u^h_c(c^h_t))$ on the right hand side and simplifying we have,

$$\log(\frac{\pi_{0,t}}{p_t}) = Ac^h_t - AE(c^h_{t+1}) + A^2\text{Var}(c^h_{t+1}) + \log\beta$$

We aggregate over households and note that aggregate consumption equals aggregate ouput minus investment, both of which are deterministic in equilibrium. Substituting and simplifying gives us the required expression.

**Proposition 4** *The optimal consumption $c^h_t$ is normally distributed at each t and is of the form,*

$$c^h_t = a_t W^h_t - b^h_t \tag{16}$$

*where the marginal propensity to consume is given by*

$$a_t = \frac{1}{1 + \frac{1}{a_{t+1}} \frac{\pi_{0,t}}{p_t}}$$

*and is deterministic, $W_t^h$ is the wealth or permanent income of the household at date $t$ and the term $b_t^h$ depends on a complex of household specific factors but is deterministic, in particular.*

**Proof**: To derive the optimal consumption rule we define $y_t^h$ as the household income at date $t$ from all sources and $\tilde{y}_t^h$ as the stochastic component of income at date $t$. We define $W_t^h$ or wealth as the current income and present value of future expected income. Thus,

$$\begin{aligned}
y_t^h &= e_t^h + \eta_t^h f(k_t^h) + \sum_{j=0}^{1} d_{j,t} \theta_{j,t-1}^h \\
\tilde{y}_t^h &= e_t^h + \eta_t^h f(k_t^h) + d_{1,t} \theta_{1,t-1}^h \\
W_t^h &= y_t^h + \Pi_t^{t+1} E(\tilde{y}_{t+1}^h) + \Pi_t^{t+2} E(\tilde{y}_{t+2}^h) + \ldots + \Pi_t^T E(\tilde{y}_T^h) \\
\Pi_t^{t+n} &= \frac{\pi_{0,t}}{p_t} . \frac{\pi_{0,t+1}}{p_{t+1}} \ldots \frac{\pi_{0,t+n}}{p_{t+n}}
\end{aligned}$$

Note that $\Pi_t^{t+n}$ is equal to the price at date $t$ of a $n$ period real bond (although not explicitly included in the model) and is therefore the inverse of the gross real rate on such a bond.

We first assume a finte $T$. At date $T$,

$$c_T^h = e_T^h + \eta_T^h f(k_T^h) + k_T^h + \sum_{j=0}^{1} d_{j,T} \theta_{j,T-1}^h$$

Since $e_T^h$, $\eta_T^h$ and $d_{1,T}$ are normal, $c_T^h$ is normal. Hence equation ( 16) is true for date $T-1$ with $a_T = 1$, $b_T^h = 0$, and $W_T^h$ given by the right hand side. At date $T-1$, for the risky asset

$$
\begin{aligned}
\frac{\pi_{1,T-1}}{\pi_{0,T-1}} &= E(d_{1,T}) - A\mathrm{Cov}(c_T^h, d_{1,T})) \\
&= E(d_{1,T}) - A\mathrm{Cov}((e_T^h + \eta_T^h f(k_T^h) + k_T^h + \sum_{j=0}^{1} d_{j,T}\theta_{j,T-1}^h), d_{1,T})
\end{aligned}
$$

On simplifying, the demand for the risky asset at date $T-1$ is given by,

$$\theta_{1,T-1}^h = -\xi^h - \kappa^h f(k_T^h)$$

We shall see below that along the dynamic path $k_t^h$ is deterministic for all $t$. Hence $\theta_{1,T-1}^h$ is non stochastic.

To derive the demand for the risk free asset, we note that for date $T-1$,

$$\log(\frac{\pi_{0,T-1}}{p_T-1}) = Ac_{T-1}^h - AE(c_T^h) + A^2\mathrm{Var}(c_T^h) + \log\beta$$

Substituting from the household's budget constraint for $c_T^h$ and $c_{T-1}^h$ into the above expression and simplifying, the demand for the risk free asset at date $T-1$ is given by,

$$\theta_{0,T-1}^h = \frac{1}{1+\frac{\pi_{0,T-1}}{p_{T-1}}}[y_{T-1}^h - k_T^h - \sum_{j=1}^{1}\frac{\pi_{j,T-1}}{p_{T-1}}\theta_{j,T-1}^h - E(\tilde{y}_T^h) + \frac{A}{2}\mathrm{Var}(c_T^h) + \frac{\log\beta}{A} - \frac{1}{A}\log(\frac{\pi_{0,T-1}}{p_{T-1}})$$

where $\tilde{y}_T^h$ is as defined in the text. The demand for the risk free asset at date $T-1$ does depend on $y_{T-1}^h$ which is stochastic. Thus the demand for the risk free asset is stochastic.

Consumption at date $T-1$ is found by eliminating $\theta_{0,T-1}^h$ from the following definition and simplifying. Thus,

$$
\begin{aligned}
c^h_{T-1} &= y^h_{T-1} - (k^h_T - k^h_{T-1}) - \sum_{j=0}^{1} \frac{\pi_{j,T-1}}{p_{T-1}} \theta^h_{j,T-1} \\
&= y^h_{T-1} - (k^h_T - k^h_{T-1}) - \frac{\pi_{1,T-1}}{p_{T-1}} \theta^h_{1,T-1} - \frac{\pi_{0,T-1}}{p_{T-1}} \theta^h_{0,T-1} \\
&= \frac{1}{1+\frac{\pi_{0,T-1}}{p_{T-1}}} [y^h_{T-1} + \frac{\pi_{0,T-1}}{p_{T-1}} . p_T E(\tilde{y}^h_T)] \\
&\quad - \frac{\frac{\pi_{0,T-1}}{p_{T-1}}}{1+\frac{\pi_{0,T-1}}{p_{T-1}}} [\frac{\pi_{1,T-1}}{\pi_{0,T-1}} \theta^h_{1,T-1} + \frac{A^2}{2} \mathrm{Var}(c^h_T) + \frac{\log\beta}{A} \\
&\quad - \frac{1}{A} \log(\frac{\pi_{0,T-1}}{p_{T-1}}) + \frac{p_{T-1}}{\pi_{0,T-1}} k^h_T] \qquad (17)
\end{aligned}
$$

Note that $y^h_{T-1} + \frac{\pi_{0,T-1}}{p_{T-1}} . p_T E(\tilde{y}^h_T) = W^h_{T-1}$, by our previous definition. $W^h_{T-1}$ is normal since $y^h_{T-1}$ is normal.

We let

$$
\begin{aligned}
a_{T-1} &= \frac{1}{1+\frac{\pi_{0,T-1}}{p_{T-1}}} \\
b^h_{T-1} &= \frac{\frac{\pi_{0,T-1}}{p_{T-1}}}{1+\frac{\pi_{0,T-1}}{p_{T-1}}} [\frac{\pi_{1,T-1}}{\pi_{0,T-1}} \theta^h_{1,T-1} + \frac{A^2}{2} \mathrm{Var}(c^h_T) + \frac{\log\beta}{A} \\
&\quad - \frac{1}{A} \log(\frac{\pi_{0,T-1}}{p_{T-1}}) + \frac{p_{T-1}}{\pi_{0,T-1}} k^h_T]
\end{aligned}
$$

Then, consumption at date $T-1$ can then be written as,

$$c^h_{T-1} = a_{T-1} W^h_{T-1} - b^h_{T-1}$$

where $a_{T-1}$ and $b^h_{T-1}$ are non stochastic. Date $T-1$ consumption is thus affine in $W^h_{T-1}$ and hence normal.

At date $T-2$, the price of the risky asset is given by

$$\begin{aligned}\frac{\pi_{1,T-2}}{\pi_{0,T-2}} &= E(d_{1,T-1}) - A\mathrm{Cov}(c^h_{T-1}, d_{1,T-1})) \\ &= E(d_{1,T-1}) - A\mathrm{Cov}(a_{T-1}W^h_{T-1} - b^h_{T-1}, d_{1,T})\end{aligned}$$

Substituting for $W^h_{T-1}$, and simplifying and rearranging terms we can express the demand for the risky asset as,

$$\theta^h_{1,T-2} == -\xi^h - \kappa^h f(k^h_{T-1})$$

Once again since $k^h_{T-1}$ is non-stochastic, the demand for the risky asset is non-stochastic.

The demand for the riskfree bond can be found to be,

$$\begin{aligned}\theta^h_{0,T-2} &= \frac{1}{a_{T-1} + \frac{\pi_{0,T-2}}{p_{T-2}}}[y^h_{T-2} - (k^h_{T-1} - k^h_{T-2}) - \frac{\pi_{1,T-2}}{p_{T-2}}\theta^h_{1,T-2} - a_{T-1}(E(\tilde{y}^h_{T-1}) + \frac{\pi_{0,T-1}}{p_{T-1}}E(\tilde{y}^h_T) \\ &\quad + b^h_{T-1} + \frac{A}{2}\mathrm{Var}(c^h_{T-1}) + \frac{\log\beta}{A} - \frac{1}{A}\log(\frac{\pi_{0,T-2}}{p_{T-2}})]\end{aligned}$$

To characterize $c^h_{T-2}$, we define,

$$
\begin{aligned}
a_{T-2} &= \frac{1}{1+\frac{1}{a_{T-1}}\frac{p_{T-1}}{p_{T-2}}} \\
b^h_{T-2} &= \frac{\frac{\pi_{0,T-2}}{p_{T-2}}}{a_{T-1}+\frac{\pi_{0,T-2}}{p_{T-2}}}[a_{T-1}\frac{\pi_{j,T-2}}{\pi_{0,T-2}}\theta^h_{j,T-2}+b^h_{T-1}+\frac{A}{2}\mathrm{Var}(c^h_{T-1})+\frac{\log\beta}{A} \\
&\quad -\frac{1}{A}\log(\frac{\pi_{0,T-2}}{p_{T-2}})+a_{T-1}\frac{p_{T-2}}{\pi_{0,T-2}} \\
W^h_{T-2} &= y^h_{T-2}+\frac{\pi_{0,T-2}}{p_{T-2}}(E(\tilde{y}^h_{T-1})+\frac{\pi_{0,T-1}}{p_{T-1}}p_T E(\tilde{y}^h_T)
\end{aligned}
$$

Substituting for $\theta^h_{0,T-2}$ into the definition of $c^h_{T-2}$, and simplifying and using the above definitions, equilibrium consumption at date $T-2$ can be expressed as,

$$c^h_{T-2} = a_{T-2}W^h_{T-2} - b^h_{T-2}$$

where $a_{T-2}$ and $b^h_{T-2}$ are deterministic and $W^h_{T-2}$ is normal. Thus $c^h_{T-2}$ is affine in $W^h_{T-2}$ and normal.

Generalizing, we get proposition 4. The form of the consumption function for a finite $T$ generalizes to $T=\infty$ under the assumption of bounded asset prices.

To derive the household's demand for physiacl capital, expand the Euler equation ( 13) in the same way as we did in Proposition above, use Stein's lemma, Euler equation ( 11) and the affine form of the optimal consumption rule and simplify. The optimal choice of capital stock ia given by,

$$\frac{p_t}{\pi_{0,t}} = f'(k^h_{t+1})(\eta - Aa_{t+1}f(k^h_{t+1})\sigma^2_p) \tag{18}$$

Finally, we show that,

**Proposition 5** *Aggregate ouput is deterministic along the equilibrium path.*

**Proof:** From equation ( 18) it is clear that in equilibrium $k_t^h$ is uniform across all households. This together with the assumption that idiosyncratic productivity and endowment shocks cancel each other gives us the result.

We further note that along the equilibrium path the variance of consumption, $\mathrm{Var}(c_t^h) = (a_t)^2(\sigma_e^2 + (f(k_t^h))^2\sigma_p^2)$ is uniform across households also.

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