E C O N O M I C S   B U L L E T I N

``The choice between emission taxes and output taxes under imperfect monitoring'': a comment

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Abstract

We consider a special case of Schmutzler's and Goulder's (1997) analysis of output taxes vs emission taxes as environmental policy instruments. We identify new necessary conditions for the existence of an optimum. We also show that, in this case, it is always optimal to have a mixed tax with positive enforcement effort.

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1 Introduction

In an interesting and stimulating paper, Schmutzler and Goulder (1997) have compared output taxes with emission taxes as environmental policy instruments.

In the most important section of the paper, they assume that monitoring a polluting firm's output is free. However, for the regulator, the cost of monitoring polluting emissions is assumed to depend on monitoring intensity.

Our comment is motivated by the following observation. For illustrative purposes, Schmutzler and Goulder start their analysis with a specific profit function for the polluting firm. However, they quickly move on to a more abstract setting. Their analysis therefore gains in generality but also loses somehow in intuition.

In this model, we stick to (a slightly modified version of) the illustrative profit function Schmutzler and Goulder introduced in the beginning of their paper.

Our results have intuitively appealing properties. Moreover, we obtain several surprising new results. First, we identify new *necessary* conditions for the optimality of a mixed tax. Second, we show that it is never optimal to have a *pure* (output or emission) tax.

We retain all the notational assumptions used by Schmutzler and Goulder. x is the output level of the polluting firm, p is the output price, t_0 is the unit output tax, $C(e,x)$ is the cost function, e are real emissions, e_d are declared emissions and t_e is the unit tax on declared emissions.

We use the following specific fine function for false self-reports: $f(e, e_d, m)$ (discussed by the authors in footnote 13 of the original article). Thus, the expected fine depends in a non-specified, but exogenous way on real emissions, undeclared emissions and monitoring effort m. [1] Schmutzler and Goulder assume that "without any monitoring, the firm will behave as if there was no emission tax at all". We interpret this assumption as: if $m = 0$, then $t_e = 0$ and $f(e, e_d, m) = 0$ for all e and e_d.

The firm's expected profits are then:

$$\Pi = (p - t_0)x - C(e, x) - t_e e_d - f(e, e_d, m)$$

To simplify notation, let:

- $\frac{\partial C(e,x)}{\partial x} = C_x$ and $\frac{\partial C(e,x)}{\partial e} = C_e$
- $\frac{\partial f(e,e_d,m)}{\partial e} = f_e$, $\frac{\partial f(e,e_d,m)}{\partial e_d} = f_d$ and $\frac{\partial f(e,e_d,m)}{\partial m} = f_m$
- $\frac{\partial^2 C(e,x)}{\partial x^2} = C_{xx}$, $\frac{\partial^2 C(e,x)}{\partial x \partial e} = C_{xe}$ and $\frac{\partial^2 C(e,x)}{\partial e^2} = C_{ee}$
- $\frac{\partial^2 f(e,e_d,m)}{\partial e^2} = f_{ee}$, $\frac{\partial^2 f(e,e_d,m)}{\partial (e_d)^2} = f_{dd}$, $\frac{\partial^2 f(e,e_d,m)}{\partial e_d \partial e} = f_{ed}$, $\frac{\partial^2 f(e,e_d,m)}{\partial e_d \partial m} = f_{dm}$ and $\frac{\partial^2 f(e,e_d,m)}{\partial m \partial e} = f_{me}$.

[1] For instance, in Schmutzler and Goulder, "monitoring effort" corresponds to the number of firms that are monitored. The regulator is then assumed to observe perfectly the amount of actual emissions by the firms that have indeed been monitored.

2 The firm's decision problem

It seems reasonable to limit the attention to interior solutions for x and e, but to consider explicitly the possibility of a corner solution for e_d.

The firm's FOC are then:

$$\frac{\partial \Pi}{\partial x} = p - t_0 - C_x \quad = \quad 0 \tag{1}$$

$$\frac{\partial \Pi}{\partial e} = -C_e - f_e \quad = \quad 0 \tag{2}$$

and:

$$\frac{\partial \Pi}{\partial e_d} = -t_e - f_d \leq 0 \quad e_d \geq 0 \quad (t_e + f_d)e_d = 0 \tag{3}$$

Now consider the Hessian corresponding to the second-order conditions:

$$\begin{vmatrix} -C_{xx} & -C_{xe} & 0 \\ -C_{xe} & -C_{ee} - f_{ee} & -f_{ed} \\ 0 & -f_{ed} & -f_{dd} \end{vmatrix}$$

Let $|D_{ij,kl}|$ be the minor with upper-left element ij and lower-right element kl.

The SOC are satisfied if the three principal minors alternate in sign:

- $|D_{11,11}| = -C_{xx} < 0$,
- $|D_{11,22}| = \begin{vmatrix} -C_{xx} & -C_{xe} \\ -C_{xe} & -C_{ee} - f_{ee} \end{vmatrix} > 0$
- $|D_{11,33}| < 0$, or $(f_{ed})^2 C_{xx} - f_{dd}|D_{11,22}| < 0$.

We shall from now on assume that these conditions are fulfilled. Note that the second SOC can only be satisfied if $C_{ee} + f_{ee} > 0$. The third SOC requires $f_{dd} > 0$. This implies that the firm's SOC not only impose restrictions on the firm's cost function, but also on the fine for undeclared emissions!

3 The regulator's problem

Following Schmutzler and Goulder, suppose that the regulator's objective function takes the following form:

$$B(x, e) - C(x, e) - C^{\mu}(m) \tag{4}$$

where $B(x, e)$ are the social benefits of the produced good and environmental quality, and $C^{\mu}(m)$ is the cost of monitoring with intensity m.

From Conditions 1, 2 and 3, we see that t_0, t_e and m determine x, e and e_d.

We will now have to distinguish between interior and corner solutions for declared emissions.

4 Interior solution for declared emissions

Remember that if $m = 0$, then $t_e = 0$ and $f(e, e_d, m) = 0$ for all e and e_d.

Therefore, $e_d > 0$ requires both $m > 0$ and $t_e > 0$.

If there is an interior solution for e_d, then Condition 3 reduces to $-f_d = t_e$.

This condition can only be fulfilled if $f_d < 0$. This condition requires that, in the optimum, and for given real emissions and monitoring effort, an increase in declared emissions leads to a decrease in the expected fine. It has not been identified in the original paper, but it is clearly a *necessary* condition for the existence of an interior solution. As the functional relation between the fine and undeclared emissions is exogenous in the model, there is no compelling reason why this condition *should* be fulfilled.

We can therefore reformulate the problem as follows: the decision problem for the regulator is to choose the values of x, e, e_d and m in order to maximize Expression 4 subject to Equation 2. Solving Equations 1 and 3 then gives the desired values of t_0 and t_e.[2]

The Lagrangian is therefore (where μ is the Lagrange multiplier):

$$\mathcal{L} = B(x,e) - C(x,e) - C^{\mu}(m) + \mu\left(-f_e - C_e\right) \tag{5}$$

This implies immediately that the FOC with respect to x, e and e_d are given by:

$$\frac{\partial \mathcal{L}}{\partial x} = B_x - C_x - \mu C_{xe} = 0 \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial e} = B_e - C_e - \mu\left(f_{ee} + C_{ee}\right) = 0 \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial e_d} = -\mu f_{ed} = 0 \tag{8}$$

This implies immediately that an interior solution for e_d requires that there exist an e and an e_d such that $f_{ed} = 0$. Again, we obtain here a *necessary* condition for the existence of an interior solution that has not been identified in the original paper either.

Now remember that the third SOC for the firm, $|D_{11,33}| < 0$, can only be fulfilled if $f_{dd} > 0$. Thus, if the expected fine would take the specific form proposed in Equation 3 in the Schmutzler and Goulder analysis, then $f_{dd} = -f_{ed}$. Thus, in that particular formulation of the problem, the SOC are not satisfied.

[2] Alternatively, we could also add t_0 and t_e as choice variable. However, it is straightforward to verify that, in this alternative formulation of the problem, Equations 1 and 3 do not bind as constraints. The reason for this is that the regulator does not care about the tax rates themselves: they are merely instruments that induce the firm to choose the optimal output and emission levels.

However, suppose the FOC are fulfilled. Then $f_{ed} = 0$ implies that the firm should declare emissions up to the point where they minimize the expected marginal (relative to real emissions) fine.

From Equations 6 and 7, we see that $\frac{B_x - C_x}{C_{xe}} = \frac{B_e - C_e}{f_{ee} + C_{ee}}$. Schmutzler and Goulder assume that $C_{xe} < 0$. We have seen above that the firm's SOC can only be satisfied if $f_{ee} + C_{ee} > 0$. Thus, we obtain that, in the equilibrium, $B_x - C_x$ and $B_e - C_e$ have opposite signs.

The regulator's FOC with respect to m are given by (remember that we require $m > 0$):

$$\frac{\partial \mathcal{L}}{\partial m} = -\frac{\partial C^{\mu}}{\partial m} - \mu f_{em} \quad = \quad 0 \tag{9}$$

Schmutzler and Goulder assume that $\frac{\partial C^{\mu}}{\partial m} > 0$. Therefore, an interior solution for m is only possible if $\mu f_{em} < 0$.

Suppose first that, in the optimum, $f_{em} < 0$. This condition requires that, in equilibrium, the marginal impact of an increase in *real* emissions on the expected fine decreases when monitoring effort increases. A possible interpretation of this condition is that an increase in monitoring effort leads to a decrease in the probability that the regulator overestimates the firm's emissions.

In this case, an interior solution for m is thus only possible if $\mu > 0$. This implies:

Proposition 4.1 *An equilibrium with a strictly positive enforcement effort where $f_{em} < 0$ is only possible if, in that equilibrium, $C_x > B_x$ and that $B_e > C_e$.*

Suppose next that $f_{em} > 0$. This condition requires that, in equilibrium, the marginal impact of an increase in *real* emissions on the expected fine increases when monitoring effort increases. In this case, a possible interpretation is that an increase in monitoring effort leads to a decrease in the probability that the regulator underestimates the firm's emissions. An interior solution for m is then only possible if $\mu < 0$. Following the same argument as above, $\mu < 0$ implies:

Proposition 4.2 *An equilibrium with a strictly positive enforcement effort where $f_{em} > 0$ is only possible if, in that equilibrium, $B_x > C_x$ and that $C_e > B_e$.*

Let us address the problem whether it could ever be optimal to have a pure emission tax.

If this were the case, then $t_0 = 0$ and Equation 1 requires that, in the optimum, $p = C_x$. If, as Schmutzler and Goulder assume, the "price always adjusts in such a way that it equals the marginal benefits of output" (formally, $C_x = B_x$), then Propositions 4.1 and 4.2 imply:

Proposition 4.3 *In an equilibrium with strictly positive enforcement effort, it is always optimal to have a mixed tax.*

Compare this with Proposition 5 in the Schmutzler and Goulder paper. In that proposition, the authors had identified a sufficient condition for this result. Our analysis shows that the result always hold with the specific profit function that has been used here if the necessary conditions we have identified are fulfilled.

Essentially, the point is that in order to induce the firm to declare any emissions at all, the regulator must always exert a positive monitoring effort. This always creates distortions compared to the first-best solution where $B_x = C_x$ and that $C_e = B_e$. Introducing an output tax then leads to a marginal decrease in the total distortion.

5 Corner solution for declared emissions

Consider now the possibility that it might be optimal to induce a corner solution for declared emissions: $e_d = 0$.

We see again that, for any desired value of x and e, solving Equations 1 and 3 gives the value of t_0 and the minimal required value of t_e.

We can therefore reformulate the problem as follows: the decision problem for the regulator is to choose the values of x, e and m in order to maximize Expression 4 subject to $e_d = 0$.

The Lagrangian is therefore (where μ is the Lagrange multiplier):

$$\mathcal{L} = B(x,e) - C(x,e) - C^{\mu}(m) - \mu e_d \tag{10}$$

Therefore, the FOC with respect to x and e are given by:

$$\frac{\partial \mathcal{L}}{\partial x} = B_x - C_x = 0 \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial e} = B_e - C_e = 0 \tag{12}$$

The regulator's FOC with respect to m are given by:

$$\frac{\partial \mathcal{L}}{\partial m} = -\frac{\partial C^{\mu}}{\partial m} \leq 0 \quad m \geq 0 \quad \frac{\partial \mathcal{L}}{\partial m} m = 0 \tag{13}$$

We see immediately that $\frac{\partial C^{\mu}}{\partial m} > 0$ implies that there is no interior solution for m.

Now remember that if $m = 0$, then $t_e = 0$ and $f(e, e_d, m) = 0$ for all e and e_d. There is then no t_0 such that the regulator's FOC with respect to e are compatible with the firm's FOC. We thus obtain:

Proposition 5.1 *It is impossible to have an optimum with zero declared emissions.*

6 Conclusion

From Proposition 4.3 and 5.1, we see that there is only one possible equilibrium: a mixed tax with positive enforcement effort. A pure tax is thus never optimal.

In order for such a mixed tax to be optimal, the following conditions must be satisfied in the optimum: $f_d < 0$, $f_{dd} > 0$ and $f_{ed} = 0$. These conditions have not been identified in the original paper, but have appealing intuitive properties. However, as these conditions are related to the (exogenous) fine function, there is no guarantee that they are fulfilled.

Moreover, with a mixed tax, there exists a one-to-one relationship between the sign of f_{em} and the sign of $B_x - C_x$ and $B_e - C_e$.

References

Schmutzler, A. and L.H. Goulder (1997), The Choice between Emission Taxes and Output Taxes under Imperfect Monitoring, *Journal of Environmental Economics and Management* **32**, 51-64