E C O N O M I C S   B U L L E T I N

A note on commitment when there are errors in communication

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Abstract

In this note we analyze the viability of a commitment strategy when there are errors in communication. We consider an entry deterrence game where with a certain probability the Incumbent's decision is either perfectly observed by the Potential Intruder or, with complementary probability, nothing is observed. We find that in equilibrium the Incumbent benefits as much from a decision to accommodate entry as a commitment to fight entry being observed with sufficiently high probability by the potential intruder. Indeed, there is an equilibrium where the Incumbent commits to fight entry with probability one even when this action is observed with zero probability.

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1 Introduction

The game-theoretic literature on commitment under imperfect observability has formalised imperfect observability in two different ways. The first approach, initiated by Bagwell (1995), assumes that the follower observes a noisy signal where there is some strictly positive probability of receiving a false signal. This approach is sometimes called 'errors in perception' (van Damme and Hurkens, 1987). Bagwell shows that with errors in perception the Stackelberg outcome ceases to be a pure Nash equilibrium. Imperfect observability therefore destroys the advantage from commitment. See also van Damme and Hurkens (1997), Güth, Kirchsteiger, and Ritzberger (1998), Oechssler and Schlag (2000), and Güth, Müller, and Spiegel (2006).

The second approach, 'errors in communication', assumes that the follower either perfectly observes the leader's output decision or does not observe anything. See Güth, Müller, and Spiegel (2006) for a discussion of the two technologies. Bonanno (1992) analyzes an entry deterrence game with errors in communication (see also Bonanno, 1988 and Chakravorti and Spiegel, 1993) and he shows that if the probability with which the Potential Intruder observes a decision of the Incumbent to fight entry is sufficiently high then there is a unique and mixed Nash equilibrium where the probability with which the incumbent fights entry is increasing in the probability that the decision will be observed by the potential intruder.[1]

While Bonanno's result is intuitive, there is, however, a certain asymmetry in his errors in communication information technology: While a decision by the incumbent to commit to fight entry is with some probability observed by the potential intruder, a decision by the incumbent to accommodate entry is never observed by the potential intruder. This can be contrasted with a more general information technology, where both a decision by the incumbent to fight entry as well as a decision to accommodate are observed by the potential intruder with a certain probability and with complementary decision nothing is observed. The purpose of this note is to analyze this alternative and more general information technology and to draw out the implications for the viability of a commitment strategy.

In order to motivate the difference between these two information technologies let us as an example, as do Güth, Müller, and Spiegel (2006), consider the Brander and Spencer (1985) model of strategic trade policy. A home firm and a foreign firm competes in the same market in a third country. The home government can by giving the home firm a subsidy shift the home firm's best reply function outwards which, if observed by the foreign firm, has a strategic impact on the latter and will benefit the domestic government. Bonanno's information technology captures a situation where, if the subsidy is granted to the home firm the foreign firm perfectly observes the size of the subsidy with some probability, and with complementary probability the foreign firm observes nothing, while if no subsidy is granted then the foreign firm learns nothing. The information technology we study in this note describes the same situation as Bonanno's when the government grants the home firm a subsidy. But if no subsidy is granted the foreign firm learn this with some probability and otherwise

[1]Bonanno (1992) also studies the case where the Potential Intruder is initially unaware that the Incumbent can commit to fight and only becomes aware of it if the commitment to fight is observed. In what follows we assume the availability of the commitment option is common knowledge.

learns nothing.[2] In other words, both a decision to commit to a subsidy *and* a decision to not grant a subsidy will with some probability be perfectly observed by the foreign firm. It is of course in general not a priori obvious which information technology (or which combination of the two set-ups) is more plausible; this depends on the specific situation that is studied (the same observation of course applies to the more fundamental choice between an errors in perception or an errors in communication information technology).

Our game is an extension and generalization of the entry deterrence game studied in Bonanno (1992). Our results are the following. The outcome where the incumbent commits to fight entry and the intruder stays out is the unique equilibrium when both the decision to accommodate entry and the decision to fight entry are observed with sufficiently high probability. Moreover, the outcome where the incumbent commits to fight entry and the intruder stays out is an equilibrium even when a decision to fight entry will never be observed by the intruder, as long as a decision to accommodate entry would be observed with sufficiently high probability. These outcomes are qualitatively different from those found in the existing literature.

One way to make sense of these results is to hypothetically assume that the incumbent could control the probabilities with which a decision to fight entry or to accommodate entry would be observed by the intruder. Consider the following recommendation: The incumbent should ensure that a commitment to fight would be observed with high probability, but any decision to accommodate entry should be kept secret since it would be exploited by the intruder. In the situation studied by Brander and Spencer (1985) the domestic government should widely advertize the introduction of an export subsidy, but a decision not to introduce any subsidy should be kept secret. Our analysis shows that this recommendation is wrong: The domestic government should make sure that either decision is highly likely to be observed by the foreign firm. Intuitively, the cost of accommodating the intruder is raised by making it more likely that such accommodation is observed, and this in turn makes it more likely that in equilibrium the incumbent commits to fight entry and the intruder stays out.

The rest of the note is structured as follows. Section 2 describes the entry deterrence game. Section 3 describes the set of Nash equilibria. Finally, Section 4 summarizes the analysis.

2 The Entry Deterrence Game

The game is shown in Figure 1 below. There are two players, an Incumbent and a Potential Intruder. The Incumbent moves first and decides whether to commit to fight entry (F) or to accommodate any entry (A). Nature reveals the Incumbent's decision to the Potential Intruder as follows: If the Incumbent chose to commit to fight entry, the Potential Intruder learns this with an exogenous probability, α, where $\alpha \in [0, 1]$. With complementary probability, $1 - \alpha$, the Potential Intruder does not learn the Incumbent's decision. These probabilities are shown in the game tree. If the

[2] The case of errors in perception captures the situation where the foreign firm observes that a subsidy has been chosen, but the observed level of the subsidy may not be the actually chosen level.

Incumbent decides to accommodate entry the Potential Intruder learns this with some fixed probability β, where $\beta \in [0, 1]$, and with probability $1 - \beta$ he does not learn the Incumbent's decision. The Potential Intruder then decides whether to enter (E) or to stay out (SO).

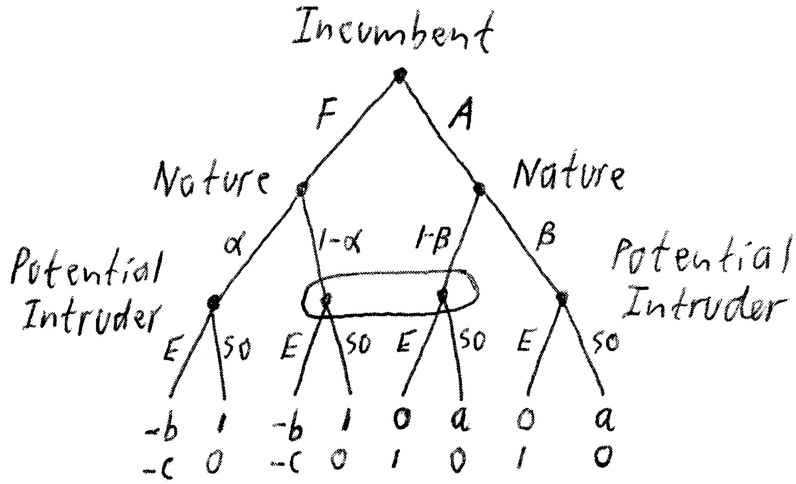

Figure 1: The game tree.

The payoff numbers (the top ones being those of the Incumbent) satisfy the following inequalities: $a > 1$, $b > 0$ and $c > 0$. The first inequality means that achieving commitment to fight entry is costly for the Incumbent: If the Potential Intruder stays out, the Incumbent prefers to not commit. The second inequality expresses that if the Potential Intruder enters, the Incumbent prefers to accommodate entry. The third inequality says that if the Incumbent commits to fight entry, the Potential Intruder prefers to stay out. Our game can be interpreted as a reduced–form version of a more complicated game where the Incumbent has some specific commitment technology available, such as those described in, for example, Dixit (1982).

Our payoffs are the same as in the game studied in Bonanno (1992), where a decision by the Incumbent to accommodate entry is never revealed to the Potential Intruder: $\beta \equiv 0$. In Bonanno's game the Potential Intruder can receive two signals: S1: 'The Incumbent committed to fight entry' and S2: 'There is no information about the Incumbent's decision'. In our model there are three signals, S1, S2, and now S3: 'The Incumbent decided to accommodate entry'.

We can interpret this game as a simple duopoly game. The Incumbent is the leader and the Potential Intruder is the follower. The F action is the Stackelberg output and A is the Cournot output. For the follower E is the Cournot output and SO is the Stackelberg follower output. We can also interpret the game as a very simple version of the Spencer and Brander (1985) model discussed in the Introduction. The Incumbent is the combined home government and home firm. The Potential Intruder is the foreign firm. The F action means choosing the subsidy and output level that is optimal under perfect observability, while A means not introducing any subsidy and choosing a correspondingly lower output level. The SO action for the foreign firm refers to choosing the output level that would be optimal if the subsidy and output chosen by the home government–firm are perfectly observed by the foreign firm, and E is the 'Cournot' output level.

3 Equilibrium

Subgame-perfection implies that the Potential Intruder stays out if he observes that the Incumbent committed himself to fight entry, and enters if he observes that the Incumbent accommodates entry. We can therefore restrict attention to just two strategies for the Potential Intruder: (enter when observing A, stay out when observing F, and enter when not observing anything) and (enter when observing A, stay out when observing F, and stay out when not observing anything). Denoting these strategies by E and SO, we obtain the following strategic form representation of our game:

	E	SO
F	$\alpha-(1-\alpha)b, -(1-\alpha)c$	$1, 0$
A	$0, 1$	$(1-\beta)a, \beta$

Table 1: The strategic form.

The Nash equilibria of this game are described in the following result. We exclude the case of $(\alpha, \beta) = (1, 1)$ from consideration; in this case the Incumbent optimally chooses to commit to fight entry and the Potential Intruder reacts by staying out, as outlined above.

Proposition 1 *Fix a pair* $(\alpha, \beta) \in [0,1]^2$*. Set* $\alpha^* \equiv b/(1+b)$ *and* $\beta^* \equiv (a-1)/a$*.*

(i). If $\alpha \leq \alpha^*$ *and* $\beta \leq \beta^*$*, (A,E) is the unique Nash equilibrium.*

(ii). If $\alpha > \alpha^*$ *and* $\beta > \beta^*$*, but* $(\alpha, \beta) \neq (1,1)$*, the unique Nash equilibrium is (F,SO).*

(iii). Suppose $\alpha \leq \alpha^*$ *and* $\beta^* < \beta < 1$*. Then there are two Nash equilibria, (A,E) and (F,SO). There is also a mixed Nash equilibrium, where the Incumbent fights entry with probability* p^* *and the Potential Intruder when not observing the Incumbent's decision enters with probability* r^**. If* $\beta = 1$*, only (A,E) and (F,SO) are equilibria.*

(iv). Finally, suppose $\alpha^* < \alpha < 1$ *and* $\beta \leq \beta^*$*. Then there is a unique and mixed Nash equilibrium,* (p^*, r^*)*.*

$$p^* = \frac{1-\beta}{1+c(1-\alpha)-\beta} \tag{1}$$

$$r^* = \frac{(1-\beta)a-1}{a-1-b+\alpha(1+b)-a\beta}. \tag{2}$$

The result is illustrated in Figure 2, where region I corresponds to part (i) of Proposition 1, region II to part (ii), and so on.

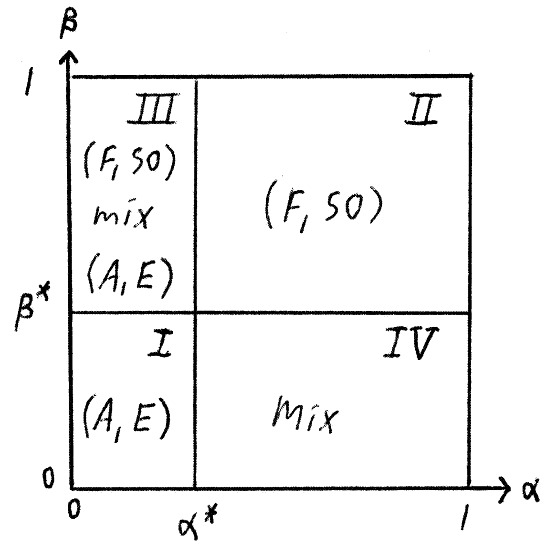

Figure 2: The set of Nash equilibria.

Proposition 1 gives our two results mentioned in the Introduction: If both the probability α that a decision to fight entry is observed, and the probability β that accommodation is observed are sufficiently large ($\alpha > \alpha^*$ and $\beta > \beta^*$), then there is a unique equilibrium where entry will be fought and the Potential Intruder stays out (region II). Moreover, when, for a given probability α, the probability β is sufficiently large ($\beta > \beta^*$), there is a Nash equilibrium where the Incumbent commits to fight entry and the Potential Intruder stays out (regions II and III). This is the case even if α is very small, or zero (region III). The novel finding is that a commitment to fight entry can be part of an equilibrium even when the Intruder stays out, and this remains the case even when the commitment decision will never be observed. Commitment is optimal for the Incumbent since the alternative, accommodation, is sufficiently likely to be observed and exploited by the potential Intruder.

For any given probability α the Incumbent's equilibrium welfare is increasing in β. This result is evident from Figure 2: If $\alpha \leq \alpha^*$, increasing β improves the situation for the Incumbent since both the (F,SO) equilibrium and the mixed equilibrium are better for the Incumbent than the (A,E) equilibrium. Moreover, if $\alpha > \alpha^*$ the outcome is either the mixed equilibrium [Region IV] or the (F,SO) equilibrium [Region II]. The Incumbent is strictly better off in the latter equilibrium than when $\beta = 0$. Finally, in the mixed equilibrium in Region IV the Incumbent commits to fight entry less and the Potential Intruder enters less when $\beta > 0$ than when $\beta = 0$. Thus *both* the Incumbent and the Potential Intruder are better off in Region IV when $\beta > 0$ than when $\beta = 0$.

4 Conclusion

In this note we analyze the viability of commitment in an entry deterrence game played by an incumbent and a potential intruder under imperfect observability, using an 'errors in communication' information technology. One possibility for such a scenario, studied in Bonanno (1992), is that if the incumbent commits to fight entry then the potential intruder observes this with a certain probability, but the potential intruder never observes a decision of the Incumbent to accommodate entry. We extend this analysis by using a more general information technology where the Potential Intruder can in addition with some probability learn if the Incumbent decided to accommodate entry. We obtain two results: Whenever both the probability that a decision to fight entry and to accommodate entry are high enough, there is a unique equilibruim where any entry is fought and the intruder stays out. Moreover, as long as accommodation is sufficiently likely to be observed this outcome remains an equilibrium, even if a decision to fight entry will never be observed. The logic underlying these observations is simple: The more likely it is that any decision to accommodate entry will be observed and exploited by the potential intruder the higher is the cost of accommodation, and this then makes the commitment to fight entry optimal.

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