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Bargaining with partially revocable commitments: a simple model

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Abstract

Fershtman and Seidmann (1993) showed that the presence of an irrevocable endogenous commitment with a fixed deadline results in the so called deadline effect. We examine the robustness of this result to the presence of a much more general class of commitments: partially revocable commitments.

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1 Introduction.

Delays are obtained in Fershtman and Seidmann (1993) (henceforth FS) in a complete information framework because of the conjunction of irrevocable endogenous commitment and a deadline. The aim of this note is to study the robustness of FS's result to the presence of a more general class of commitment which encompasses as a particular case the irrevocable commitment considered by FS. In particular, we assume that commitments are partially revocable, that is, they can be revoked up to a certain extent with finite costs but beyond a given point they become irrevocable.

In this sense, FS do not specify the way by which the completely irrevocable commitment is achieved and we do not discuss the usefulness of working with a reduced-form model. However, we do believe that many interesting cases are omitted by the complete irrevocability assumption. In real life, as it was pointed out by Muthoo (1999), commitments are always partial and revocable, although at a finite cost. Therefore, the reduced-form analysis is appropriate only if we work with a wide class of commitment mechanisms, which are captured by different induced costs of revoking such partial commitments.

For simplicity, we restrict our analysis to a two-period negotiation. The main insights of this note arise even in this simple framework. Any bargaining model with a larger number of periods yields the same results at the expense of complicating the analysis.

We obtain that the degree of irrevocability crucially determines the outcome of the negotiation. Delays only arise if at least one of the players possesses a partially revocable commitment with a minimal amount of irrevocability. However, when only one player has such a commitment a negative externality arises. Whereas the player possessing a commitment below the minimal degree of irrevocability obtains the same payoff, the other one gets a lower payoff than in the case without commitment. In contrast, when neither player reaches such degree of irrevocability there is no delay in equilibrium and the game is equivalent to that without commitments.

2 The model.

Consider a two-period negotiation between a seller (S) and a buyer (B) who bargain over the partition of a cake of unit size. At the beginning of each period, a fair lottery determines the identity of the proposer.

We make the following crucial assumption: players cannot accept an offer less generous than those rejected in the past without incurring a cost. Fershtman and Seidmann (1993) assumed that such costs were infinite, leading to completely irrevocable commitments.

We depart from these authors by considering partially revocable commitments. In our model if a player accepts an offer worse than any previously rejected offer, he will incur an infinite cost if this offer is lower than a specific critical value. If, however, this offer is higher than this critical value, he faces an infinite cost. Namely, commitments are revocable up to a certain extent and with a finite cost. Beyond a given point, they become irrevocable.

Formally, denote by $x_i^t \in [0,1]$, $i \in \{B,S\}$ and $t \in \{0,1\}$, as the share offered by player j to player i, $j \in \{B,S\}$ and $j \neq i$, in period t.

We define z_i^t as player i's commitment in period $t \in \{0,1\}$. We assume that players do not carry any commitment into the game, that is, $z_i^0 \equiv 0$. Moreover, player i's commitment in the second period is $z_i^1 \equiv z_i \equiv x_i^0$, whenever x_i^0 has been rejected by player i. If player i accepts any offer x_i^1, he will face a cost for revoking such a commitment which can be defined by the following function,

Definition 1 *Let the cost of revoking function,* $C_i\left(x_i^t, z_i, \varepsilon_i, \lambda_i\right)$*, be defined as:*

$$C_i\left(x_i^t, z_i, \varepsilon_i, \lambda_i\right) = \begin{cases} 0 & \text{if } x_i^t \geq z_i \\ \lambda_i\left(z_i - x_i^t\right) & \text{if } (1-\varepsilon_i)\, z_i \leq x_i^t < z_i \\ \infty & \text{if } x_i^t < (1-\varepsilon_i)\, z_i \end{cases}$$

where $1 > \varepsilon_i > 0$ *and* $\lambda_i > 0$.

We assume that players share the common discount factor $0 < \delta < 1$. Strategies and subgame perfect equilibria (SPE) for this game are defined in the usual way.

3 The results.

The parameter ε_i measures the share of the cake that player i is ready to give up in relation to the commitment at a finite cost, that is, the degree of irrevocability of the commitment. When ε_i goes to zero, the commitment becomes completely irrevocable and we return to FS's case. In contrast, when ε_i goes to one, the commitment becomes completely revocable although at a unitary cost of λ_i.

Our main interest is the case in which $\varepsilon_i \in (0,1)$, that is, partially revocable commitments. We will show that the degree of irrevocability determines crucially the outcome of the negotiation. For these purposes, we distinguish three different cases.

3.1 A high degree of irrevocability in both players.

We show that when both players possess a partially revocable commitment with a minimal amount of irrevocability, $\varepsilon_i < \frac{1}{1+\lambda_i}$ and $\varepsilon_j < \frac{1}{1+\lambda_j}$, there is a unique subgame perfect equilibrium with delay identical to that obtained by FS.

Let us solve the game by backward induction. We obtain that a player's payoff in the second period is increasing (decreasing) in his (the other player) commitment.

Lemma 1 *If $\varepsilon_i < \frac{1}{1+\lambda_i}$ and $\varepsilon_j < \frac{1}{1+\lambda_j}$, the players' equilibrium utility for any z_i and z_j in the second period are:*

$$U_i(z_i, z_j) = \frac{1}{2} + \frac{1}{2}\left(z_i A_i - z_j(1-\varepsilon_j)\right), \quad (1)$$

$$U_j(z_i, z_j) = \frac{1}{2} + \frac{1}{2}\left(z_j A_j - z_i(1-\varepsilon_i)\right). \quad (2)$$

where $A_i = 1 - \varepsilon_i(1+\lambda_i)$ and $A_j = 1 - \varepsilon_j(1+\lambda_j)$.

Proof. In equilibrium, if player i becomes the proposer, $x_j^1 \geq \max\left\{z_j(1-\varepsilon_j), \frac{\lambda_j}{1+\lambda_j} z_j\right\}$. When $\varepsilon_j < \frac{1}{1+\lambda_j}$, $x_j^1 = z_j(1-\varepsilon_j)$, which is accepted by player j (the same when player j is the proposer). A fair lottery determines the identity of the proposer, then, player i obtains a utility of $1 - z_j(1-\varepsilon_j)$ when proposer, and $z_i A_i$ when responder. ■

Let us now analyse the equilibrium behavior in the first period. If player i gets to be the proposer, on the one hand, any positive offer increases player j's payoff of the next period since $z_j = x_j^0$ and, on the other hand, since he does not establish any commitment, $z_i = 0$. Therefore, player j will accept x_j^0 only if:

$$x_j^0 \geq \frac{\delta}{2} + \frac{\delta}{2}\left(x_j^0 A_j\right). \tag{3}$$

That is,

$$x_j^0 \geq \frac{\delta}{2 - \delta A_j}. \tag{4}$$

Then, player i obtains by making an offer that would be accepted at most,

$$1 - \frac{\delta}{2 - \delta A_j}. \tag{5}$$

On the other hand, by offering $x_i^0 = 0$, player i obtains $\frac{\delta}{2}$.

Define $\widehat{\delta}_i(\varepsilon_i)$ as the solution to $1 - \frac{\delta}{2-\delta A_j} = \frac{\delta}{2}$. Therefore, for sufficiently high discount factors $\left(\delta > \widehat{\delta}_i(\varepsilon_i)\right)$, $1 - \frac{\delta}{2-\delta A_j} < \frac{\delta}{2}$ and, hence, there will be no agreement in the first period. The following Lemma summarises this result.

Lemma 2 *When $\varepsilon_i < \frac{1}{1+\lambda_i}$ and $\varepsilon_j < \frac{1}{1+\lambda_j}$, if $\delta > \max\left\{\widehat{\delta}_i(\varepsilon_i), \widehat{\delta}_j(\varepsilon_j)\right\}$, there is no agreement in equilibrium in the first period of the negotiation and the payoff is $\frac{\delta}{2}$ for each player.*

The inefficient delay obtained by FS holds for partially revocable commitments with a minimal amount of irrevocability. It can be checked that $\delta_{FS} = 3 - \sqrt{5}$ is the critical discount factor necessary for the existence of delays in FS's model for two periods. It is easy to check that $1 > \widehat{\delta}_i(\varepsilon_i) > \delta_{FS}$. Moreover,

Corollary 1 *When ε_i tends to zero, $\widehat{\delta}_i(\varepsilon_i) \rightarrow \delta_{FS} = 3 - \sqrt{5}$.*

Therefore, when commitments have a revocable component the appearance of delays requires a higher discount factor. Intuitively, the responder demands a lower payoff (compared with the completely irrevocable case) in order to reach an agreement. Consequently, the proposer's payoff from making an offer that would be accepted by the responder increases and it will be required a higher discount factor in order to reach an agreement in the second period of the negotiation.

3.2 A low degree of irrevocability in both players.

We show that when commitment is highly revocable for both players, $\varepsilon_i \geq \frac{1}{1+\lambda_i}$ and $\varepsilon_j \geq \frac{1}{1+\lambda_j}$, there is a unique equilibrium where delays disappear and the players obtain the same equilibrium payoffs as in the game with no commitments.

First, we show that when the irrevocable part of the commitment is not large enough, a player's payoff in the second period does not depend on his commitment.

Lemma 3 *If $\varepsilon_i \geq \frac{1}{1+\lambda_i}$ and $\varepsilon_j \geq \frac{1}{1+\lambda_j}$, the players' equilibrium utility for any z_i and z_j in the second period are:*

$$U_i(z_j) = \frac{1}{2}\left(1 - z_j \frac{\lambda_j}{1+\lambda_j}\right), \tag{6}$$

$$U_j(z_i) = \frac{1}{2}\left(1 - z_i \frac{\lambda_i}{1+\lambda_i}\right). \tag{7}$$

Proof. When player i gets to be the proposer, $x_j^1 \geq \max\left\{z_j(1-\varepsilon_j), \frac{\lambda_j}{1+\lambda_j} z_j\right\}$. When $\varepsilon_j \geq \frac{1}{1+\lambda_j}$, $x_j^1 = z_j \frac{\lambda_j}{1+\lambda_j}$, where $U_j\left(z_j \frac{\lambda_j}{1+\lambda_j}\right) = 0$. A fair lottery determines the identity of the proposer, then, player i obtains a utility of $1 - z_j \frac{\lambda_j}{1+\lambda_j}$ when proposer, and 0 when responder. ■

In the first period, if player i gets to be the proposer, he does not establish any commitment, $z_i = 0$. Therefore, by (7) player j will accept any:

$$x_j^0 \geq \frac{\delta}{2}. \tag{8}$$

Consequently, player i will offer $\left(1 - \frac{\delta}{2}, \frac{\delta}{2}\right)$ and it is accepted by player j.

Lemma 4 *If $\varepsilon_i \geq \frac{1}{1+\lambda_i}$ and $\varepsilon_j \geq \frac{1}{1+\lambda_j}$, the equilibrium of the negotiation coincides with that of the game with no commitments, that is, the game ends with an immediate agreement and the expected payoff is $\frac{1}{2}$ for each player.*

Summarising, when commitments possess a sufficiently large revocable part, they do not affect the bargaining outcome at all. Notice that when ε tends to one, the commitment is completely revocable and the same result holds.

3.3 A high degree of irrevocability only in one player.

We show that if only one of the players (player i) possesses a sufficiently large irrevocable commitment, the delay will arise only if the other player gets to be the proposer in the first period.

By Lemma 1 and 3, only player i's payoff depends on his commitment in the second period.

Lemma 5 *If $\varepsilon_i < \frac{1}{1+\lambda_i}$ and $\varepsilon_j \geq \frac{1}{1+\lambda_j}$, the players' equilibrium utility for any z_i and z_j in the second period are:*

$$U_i(z_i, z_j) = \frac{1}{2} + \frac{1}{2}\left(z_i A_i - \frac{\lambda_j}{1+\lambda_j} z_j\right), \tag{9}$$

$$U_j(z_i) = \frac{1}{2}(1 - z_i(1-\varepsilon_i)). \tag{10}$$

When player i becomes the proposer in the first period, player j accepts any:

$$x_j^0 \geq \frac{\delta}{2} \tag{11}$$

Player i will offer the partition $\left(1 - \frac{\delta}{2}, \frac{\delta}{2}\right)$, which player j accepts.

By contrast, when player j gets to be the proposer,

$$x_i^0 \geq \frac{\delta}{2} + \frac{\delta}{2} x_i^0 A_i. \tag{12}$$

that is,

$$x_i^0 \geq \frac{\delta}{2 - \delta A_i}. \tag{13}$$

When $\delta > \widehat{\delta}_i(\varepsilon_i)$, player j prefers to offer $x_i^0 = 0$, and there is no agreement in the first period.

Summarising,

Lemma 6 *If* $\varepsilon_i < \frac{1}{1+\lambda_i}$ *and* $\varepsilon_j \geq \frac{1}{1+\lambda_j}$ *and* $\delta > \widehat{\delta}_i(\varepsilon_i)$*, there is an immediate agreement (delay) in equilibrium in the first period when player* i *(player* j*) gets to be the proposer. Player* i *and player* j *obtain an expected payoff of* $\frac{1}{2}$ *and* $\frac{\delta}{2}$*, respectively.*

In this case, the presence of a sufficiently irrevocable commitment in one player gives rise to a negative externality. Notice that the player with a low degree of irrevocability in his commitment gets a lower payoff than in the game without commitments. This does not imply that the player with a high degree of irrevocability improves his payoff. In fact, his commitment has no value.

References

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