Asset Prices and Monetary Policy: A New View of the Cost Channel

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Abstract

Should the central bank act to prevent "excessive" asset price dynamics or should it wait until the boom spontaneously turns into a crash and intervene afterwards to attenuate the the fallout on the real economy? The standard "three equation" New Keynesian framework is inadequate to analyse this issue for the very simple reason that asset prices are not explicitly included in the model. There are two straightforward ways to take into account asset price dynamics in this framework. First of all, the objective function of the central bank – usually defined in terms of inflation and the output gap – could be "augmented" to take into account asset price inflation. Second, expected asset price inflation can affect the IS curve through a wealth effect. In this paper we follow a different route. In our model in fact, the expected asset price dynamics will be eventually incorporated into

the NK Phillips curve. This is due to the assumption of a cost channel for monetary policy which is activated whenever monetary policy affects asset prices and dividends. In fact they determine the cost of external finance in the simple "equity only" financing model we consider, abstracting for simplicity from internal funds and the credit market. We analyze the design and the transmission mechanism of monetary policy in this simplified setting, both in the case of an instrument rule (with or without a feedback from asset prices) and in the case of optimal monetary policy.

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1 Introduction

Should the central bank prevent "excessive" asset price dynamics raising interest rates to halt a Stock market boom or a bubble in house prices or should it wait until the boom spontaneously turns into a crash and intervene afterwards to attenuate the pains of the market adjustment and the fallout on the real economy?

The debate over this crucial issue is at least a decade old – if we date it from the Bernanke-Gertler (1999) vs Cecchetti et al. (2000) exchange – but it has not settled yet, expecially in the light of the 2007-08 financial crisis.

The standard framework to analyse the transmission mechanism and the optimal design of monetary policy, i.e. the "three equation" New Keynesian (NK) DSGE framework is of course too simple and therefore inadequate to analyse this issue for the very good reason that asset prices do not show up anywhere in the model.

In order to make the model operational from this viewpoint, asset prices should be explicitly included in the macroeconomic model of interest. There have been many insightful attempts in this direction (Bernanke, Gertler and Gilchrist, 1999; Bean, 2004; Carlstrom and Fuerst 2007; Gilchrist and Saito, 2006; Iacoviello, 2005; Monacelli, 2006; Airaudo, Nisticò and Zanna, 2007; Santoro and Pfajfar, 2007; De Grauwe, 2009 to name just a few) but there is much room for improvement in our opinion.

So far, two approaches have been adopted in the literature to take into account asset price dynamics. First of all, the objective function of the central bank – usually defined in terms of inflation and the output gap – could be "augmented" to take into account asset price inflation. This is for instance the route followed by Cecchetti (2000).

Second, asset price inflation can show up as a factor "augmenting" the IS curve. In fact an asset price shock can impact on the macroeconomy basically through two channels: (i) a Tobin *q effect* on investment expenditure; (ii) a *wealth effect* on consumption and/or on investment. The wealth effect on investment takes the form of a *net worth or balance sheet effect.* In both cases, the asset price shock affects aggregate demand and leads to an *"Augmented" (optimizing) IS curve.* This is essentially the route followed by Bernanke-Gertler-Gilchrist – who emphasize the net worth effect – and

Airaudo-Nisticò-Zanna, who stress the role of the wealth effect on consumption. The impact of asset price changes on inflation is only *indirect* through changes in demand driven output gap changes.

In this paper we follow a different route. In our model in fact, the expected asset price dynamics will be eventually incorporated into the NK Phillips curve. This is due to the assumption of a *cost channel* for monetary policy (Walsh and Ravenna, 2006) which is activated whenever monetary policy affects asset prices and dividends. The latter in fact are the cost of external finance in our model.

In simplified economy we consider, in fact, firms have to anticipate wages to workers before they can cash in sales proceeds. Therefore they need funds at the moment wages have to be paid. For simplicity, we assume that firms do not accumulate internal funds and have to issue new equities to raise external finance ("equity only" financing). The novelty of the analysis consists in a peculiar treatment of financing decisions, which aims at bringing to the fore the relationship between pricing of goods and pricing of assets.

In the end we obtain an *"Augmented" NK Phillips curve.* The impact of asset price changes on inflation is in this case *direct* through changes in the cost structure of the corporate sector. In a sense this is a variant of the *cost channel* NK-DSGE model. While in Ravenna-Walsh monetary policy impacts on inflation *directly* because the interest rate is a determinant of the firm's cost, in our setting the cost channel is activated *indirectly* whenever monetary policy affects – through changes in the interest rate – asset price inflation.

In this context, optimal monetary policy should take into account asset price dynamics, essentially because it signals future changes in inflation. In a sense, we are exactly in the conditions emphasized by Bernanke and Gertler: "... policy should not respond to changes in asset prices, *except insofar as they signal changes in expected inflation...*" (emphasis added).

The toy economy we consider is of course a far cry from reality. For reasons of tractability and as a very preliminary step towards a more satisfactory – and necessarily more complicated – setting, in fact, we abstract from a wide range of crucial imperfections of financial markets. The implications of the model, however, are surprisingly far reaching. We analyse the design and the transmission mechanism of monetary policy in three regimes:(a) an instrument rule with no-reaction to asset prices (IR-NAP), (b) an instrument rule with reaction to asset prices (IR-RAP) and (c) an optimal monetary policy rule (OR). In cases (a) and (c), by construction, monetary policy does not respond to asset prices. This is essentially due to the fact that the model has a built in tendency to dichotomize into 2 independent subsystems (one for output, inflation and the interest rate and the other for asset prices). In case (b) this tendency is overcome by the explicit consideration of asset prices as an argument of the "Augmented" Taylor rule.

In the case of a supply shock, the policy prescription and the transmission mechanism are qualitatively the same both with an instrument rule and with in an optimal monetary policy setting. The central bank is "leaning against the wind": the interest rate goes up, asset prices fall, the output gap turns negative, the return on shares increases. The magnitude of the effect, however, is indeed different. When the central bank takes into account asset prices – i.e. in the IR-RAP case – the impact of the shock on both inflation and the output gap is milder than in the IR-NAP. In the OR case, if the central banker is sufficiently "hard nosed", a supply shock can even turn into a deflationary shock.

The results are even more intriguing in the case of a demand shock. The same (demand) shock has opposite effects on the output gap. In the IR regime, it has a positive effect – as we are led to think in a standard short run macro setting – while in the OR regime it has a negative effect. When the central bank takes into account also asset prices, i.e.in the IR-RAP case, output grows more than in the IR-NAP case but inflation will be milder.

Our simplified model, therefore, can account for a wide range of possible real world outcomes. We consider these results as an encouragement to enrich the model to explore more realistic environments.

The paper is organized as follows. Sections 2 and 3 describe households' and firms' decision rules. Section 4 is devoted to the determination of the flex-price equilibrium. The log-linearization around the steady state is carried out in section 5. In section 6 we derive the Augmented NK Phillips curve. In section 7 we evaluate the impact of a Taylor type instrument rule for monetary policy,with and without asset prices. We design optimal monetary policy in section 8. In section 9 we derive the optimal inflation targeting rule. We compare the results in the IR vs. OR regime in section 10. Finally, in section 11 we study the properties of the model (under different rules for dividends) in terms of stability and learnability in an adaptive learning environment à la Evans and Honkapojia. Section 12 concludes.

2 Households

The economy is populated by households and firms. The former decide on consumption, asset holdings (money, bonds, shares) and labour supply. The latter produce differentiated goods in a monopolistic setting à la Dixit-Stiglitz, using only labour as an input. Pricing decisions are characterized by Calvo type nominal rigidity. Therefore there are five markets: labor, goods, money, bonds, shares.

There is a continuum of unit mass of infinitely lived identical households which discount the future at the factor β. Period by period utility is represented by a standard CRRA function:

$$U(C_t, m_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-\zeta} m_t^{1-\zeta} - \chi \frac{N_t^{1+\eta}}{1+\eta}$$

where σ, γ, ζ, χ, η are positive parameters with the usual interpretation, C_t is a CES aggregator of consumption goods,[1] $m_t := M_t/P_t$ are real money balances [2] and N_t represents hours worked. Real money balances show up in the utility function because they provide liquidity services.

The households' portfolio consists of money, bonds and shares. The nominal value in t of money balances (resp. Government bonds) carried

[1] C_t consists of differentiated consumption goods produced by monopolistically competitive firms and is defined as follows:

$$C_t = \left[\int_0^1 c_{jt}^{\frac{e-1}{e}} dj\right]^{\frac{e}{e-1}}$$

with $e > 1$ governs the price elasticity of demand of each good.

[2] The price level is defined as a CES aggregator of the individual prices: $P_t = \left[\int_0^1 p_{jt}^{1-e} dj\right]^{\frac{1}{e-1}}$

over from the past is denoted by M_{t-1} (B_{t-1}). Moreover the household owns A_{t-1} shares, whose price is Q_t. In period t the household receives a flow of interest payments on Government bonds $i_{t-1}B_{t-1}$ where i_{t-1} is the nominal interest rate decided upon in t-1. Moreover we assume that firms pay in t (nominal) dividends equal to D_t per share (more on this in a while) held in t-1.

The household employs "resources" consisting of wage income, interest payments, and dividends to consume and increase money, bond and shareholdings according to the following budget constraint in real terms:

$$C_t + m_t + b_t + q_t A_t = w_t N_t + \frac{1}{1+\pi_t}\left[m_{t-1} + (1+i_{t-1})\, b_{t-1}\right] + (q_t + d_t)\, A_{t-1} \tag{1}$$

where $b_t := B_t/P_t$ are real bond holdings; $q_t := Q_t/P_t$ is the real price of each share (asset price or Stock price for short in the following); $w_t := W_t/P_t$ is the real wage; $\pi_t := \dfrac{P_t}{P_{t-1}} - 1$ is the inflation rate and d_t are dividends per share.

Liquidity injections (withdrawals) are implemented (by the central bank) by means of open market purchases (sales) of bonds: $M_t - M_{t-1} = -\left[B_t - (1+i_{t-1})\, B_{t-1}\right]$.Taking into account this procedure, the budget constraint of the representative household boils down to: $P_t C_t + Q_t\,(A_t - A_{t-1}) = W_t N_t + D_t A_{t-1}$ Recalling that $Q_t A_t = W_t N_t$ it turns out that

$$P_t C_t = (Q_t + D_t)\, A_{t-1} \tag{2}$$

In period t, the representative household maximizes:

$$E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-\zeta} \left(m_{t+s}\right)^{1-\zeta} - \chi \frac{N_{t+s}^{1+\eta}}{1+\eta} \right] \tag{3}$$

subject to a sequence of budget constraints of the form (1). From the first order conditions (see the appendix for details) one can derive the usual optimal relations, i.e. the consumption Euler equations for consumption, money and labour supply:

$$\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = \beta \left(1+i_t\right) E_t \left(\frac{P_t}{P_{t+1}}\right) \tag{4}$$

$$\frac{i_t}{1+i_t} = \gamma \frac{C_t^{\sigma}}{m_t^{\zeta}} \tag{5}$$

$$\chi C_t^{\sigma} N_t^{\eta} = w_t \tag{6}$$

Moreover we get one additional optimal relation that we interpret as a *No-Arbitrage Condition*

$$\frac{1+i_t}{1+E_t \pi_{t+1}} = \frac{E_t \left(q_{t+1} + d_{t+1}\right)}{q_t} \tag{7}$$

Equation (7) establishes the equality between the return on bonds, i.e. the real interest rate, and the return on equities, i.e. the sum of the dividend yield and the capital gain (in real terms). By simple algenra, this condition can be turned into an *asset price equation*:

$$q_t = \frac{E_t\left(q_{t+1} + d_{t+1}\right)}{1 + i_t}\left(1 + E_t\pi_{t+1}\right) \tag{8}$$

Consolidating the No-arbitrage condition and the Consumption Euler equation we get:

$$C_t^{-\sigma} q_t = \beta E_t C_{t+1}^{-\sigma}\left(q_{t+1} + d_{t+1}\right) \tag{9}$$

This optimality condition states the equality between the marginal utility the agent gives up by saving in order to purchase one share and the present value of the marginal utility the agent will gain one period ahead by transforming into consumption the dividend and the capital gain the share yields.

3 Firms

As in the standard New Keynesian model the corporate sector consists of J firms, indexed by j, which produce differentiated goods in a monopolistically competitive setting à la Dixit and Stiglitz (1977) using only labour.Therefore firms incur only the production cost represented by the wage bill.

We depart from the standard setting in assuming the following

1. *Production takes time.* Technology is represented by the CRS production function $Y_{jt+1} = Z_t N_{jt}$ where Z_t is a technological shock (uniform across firms). Since firms hire workers in period t and sell output in t+1, they cannot pay wages out of sales proceeds: at the beginning of each period they have to anticipate the wage bill to employees.

2. *No internal funds*: firms do not accumulate internal finance so that the financing gap coincides with the wage bill. They have to raise external finance to fill the financing gap.

In order to concentrate on the role of asset prices in macroeconomic performance, we adopt the following simplifying shortcut:

3. *"Equity only" financing:* there is only one source of external funds, the Stock market.

Assumptions 2. and 3. allow us to get rid, in the following, of the complications due to the accumulation of net worth and to ignore the credit market. This is patently unrealistic. We consider the present framework as only a first step towards a more satisfactory and realistic model.

From the *"equity only" financing* assumption, follows that the j-th firm raises funds issuing new shares and the amount of shares sold is equal to the wage bill: [3]

$$w_t N_{jt} = q_t A_{jt} \tag{10}$$

4. *Dividend and buy-back policy:* Shareholders are remunerated by means of dividends (distributed in t+1 on shares held in t), which represent the cost of external funds for the firms. Furthermore firms buy back all the shares outstanding in t+1.

[3] In principle, each firm issues its own shares so that there should be an entire range of heterogeneous asset prices, one for each firm. In order to simplify the argument, we will impose from the start the symmetry among firms which is built-in the model and assume that *the asset price is uniform across equity-issuing firms:* Alternatively, one can think of q as the average Stock market index and assume that each individual share prices q_j is not too far from the average. In the end, however, firms will behave uniformly, so that the individual share price will coincide with the average.

The time schedule can be summarized as follows. At the beginning of period t, the firm issues equities and uses the proceeds to hire workers and start production. Since production takes an entire period, output will be available for sale in t+1. Sale proceeds are used in t+1 to pay dividends and buy back shares issued in t. In fact, as shown above – see (2) – $P_{t+1}C_{t+1} = (Q_{t+1} + D_{t+1}) A_t$.At the beginning of period t+1, the cycle starts again.

In the end, therefore, we are assuming that in the same period (t+1) the firm is paying dividends and reimbursing shareholders for the shares they bought in t and it is issuing new equities to finance production in t+1. This is clearly unrealistic but simplifies the analysis to a great extent.

The firm's total disbursement occur in t+1 but are related to operating costs incurred in t. The firm's total cost in real terms, therefore, is $TC_{jt} = E_t (q_{t+1} + d_{t+1}) A_{jt}$. [4]Substituting (10) into this expression we obtain: $TC_j = \frac{E_t (q_{t+1} + d_{t+1})}{q_t} w_t N_{jt}$.Hence the real marginal cost is:

$$\phi_t = \frac{E_t (q_{t+1} + d_{t+1})}{q_t} \frac{w_t}{Z_t} \tag{11}$$

The expression

$$\frac{E_t (q_{t+1} + d_{t+1})}{q_t} = ROS$$

is the novelty of this approach. With respect to the standard setting, whereby $\phi_t = \frac{w_t}{Z_t}$, the marginal cost must be augmented by a term which represents the *cost of external finance* for the firm. This, in turn, coincides

[4]Since disbursement will occur one period ahead, in t the firm has to form expectations on the total gross return in t+1 of each share issued in t. This gross return in real terms if the sum of the asset price and dividends in t+1.

with the *return on stock* for the shareholder, i.e. the sum of the dividend yield $\frac{E_t d_{t+1}}{q_t}$ and the capital gain $\frac{E_t q_{t+1}}{q_t}$.

4 Flexprice equilibrium

As in the original New Keynesian framework, in a *symmetric flexprice equilibrium* all the firms charge the same price P_t equal to a markup $\mu > 1$ over nominal marginal cost $P_t \phi_t$.[5]. Therefore $\phi_t = \frac{1}{\mu}$. Recalling (11) we get:

$$w_t = \frac{Z_t}{\mu} \frac{q_t}{E_t (q_{t+1} + d_{t+1})} \tag{12}$$

Log-linearizing around the steady state (s.s.) and denoting s.s. values with the suscript s and percent deviations from the s.s. with a hat, from the equation above we derive:

$$\hat{w}_t = \hat{Z}_t - \left[\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1} - \hat{q}_t\right] \tag{13}$$

where

$$\theta := \frac{q_s}{q_s + d_s} = \beta$$

is the inverse of the s.s. ROS, which is equal to the discount factor (see below).[6] Equation (13) is the *price rule* in the present context. It is repre-

[5] The mark-up, in turn, depends on price elasticity: $\mu = \frac{e}{e-1}$.

In the optimum, in period the firm charges a price P_t which is a multiple of the contemporaneous marginal cost evaluated at prices of period t: $P_t \phi_t$. The real marginal cost in t, in turn, reflects expected real disbursements which will occur in t+1: $\phi_t = \frac{E_t(q_{t+1}+d_{t+1})}{q_t} \frac{w_t}{Z_t}$.Sales proceeds in t will then be used to validate commitments towards shareholders originated in t-1 (see (2)).

[6] The expression $\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1}$ – i.e. the weighted sum of the percent deviations of dividends and the future asset price. is equal to the percent deviation of the sum

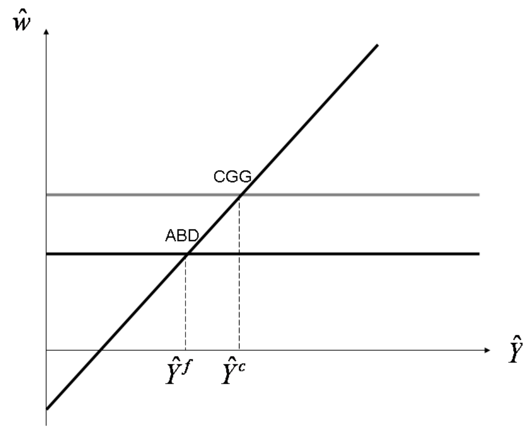


Figure 1: Price rule and wage rule

sented by the black horizontal line in figure 1. The grey line is the price rule in the canonical CGG model, whose equation is $\hat{w}_t = \hat{Z}_t$.

The expression

$$\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1} - \hat{q}_t = \widehat{ROS} \tag{14}$$

is the deviation of the ROS from the s.s.

In order to derive the wage rule, we start from the optimality condition (6), which states that the real wage should be equal to the marginal rate of substitution between labour and leisure. Plugging the goods market equilibrium condition $C_t = Y_t$ and the labour requirement function $N_t = Y_t/Z_t$ into

of dividends and future asset price, i.e. $E_t \left(\widehat{q_{t+1} + d_{t+1}} \right)$.

(6) and rearranging we get:

$$w_t = \chi \frac{Y_t^{\eta+\sigma}}{Z_t^{\eta}} \tag{15}$$

Log-linearizing around the steady state we get:

$$\hat{w}_t = (\eta + \sigma) \hat{Y}_t - \eta \hat{Z}_t \tag{16}$$

This is the *wage rule*, represented by the upward sloping black line in figure 1. The wage rule is the same as in the canonical model.

Equating (13) and (16) we obtain the flexprice equilibrium deviation of output from the s.s.

$$\hat{Y}_t^f = \hat{Y}_t^c - \frac{1}{\eta + \sigma} \left[\theta E_t \hat{q}_{t+1} + (1 - \theta) E_t \hat{d}_{t+1} - \hat{q}_t \right] \tag{17}$$

where $\hat{Y}_t^c = \frac{1+\eta}{\eta+\sigma} \hat{Z}_t$ is the flexprice equilibrium in the standard (canonical) NK model.From figure 1 it is clear that when $\widehat{ROS}$ is positive: (i) $\hat{Y}_t^f < \hat{Y}_t^c$ i.e. the flexprice equilibrium is smaller than in the canonical case, and (ii) $\hat{w}_t < \hat{w}_t^c = \hat{Z}_t$ – see (13) – i.e. the real wage is smaller than in the standard case. The reason why both the real wage and output (in log deviations) are smaller in the present context is simple: The ROS represents the cost of external finance for the firm. In the presence of this additional cost, the firm is producing less at a higher price (a lower real wage).

5 Steady states and log-linearization

The economy consists of five markets: labor, goods, money, bonds, shares. The equilibrium condition on the goods market is $C_t = Y_t$.Moreover, $Y_t = Z_t N_t$.Imposing the s.s. condition in (4), it turns out that

$$\frac{1+i_t}{1+E_t\pi_{t+1}} = \beta^{-1} = 1+r \tag{18}$$

i.e. in the steady state the real interest rate is anchored to the rate of time preference r.

Using (18) and imposing the s.s. condition in the asset price equation (8) we get

$$\frac{d_s}{q_s} = \beta^{-1} - 1 = r \tag{19}$$

i.e. in the s.s. the dividend yield is constant and equal to the rate of time preference. From the equation above follows $q_s = d_s/r$ i.e. a pure *dividend discount* model of asset price determination: in the steady state, the asset price is the discounted sum of an infinite stream of dividends.

Therefore the s.s. ROS is:

$$ROS_s = \frac{1}{\theta} = \frac{q_s + d_s}{q_s} = 1 + r$$

This is obvious: Because of the no-arbitrage condition, the real interest rate should be equal to the ROS.

Equating (15) and (12) we obtain the *level* of the flexprice equilibrium:

$$Y_t^f = \left(\frac{Z_t^{1+\eta}}{\chi\mu} \frac{q_t}{E_t\left(q_{t+1} + d_{t+1}\right)} \right)^{\frac{1}{\eta+\sigma}}$$

which in the steady state is equal to

$$Y_s^f = \left(\beta \frac{Z_s^{1+\eta}}{\chi\mu} \right)^{\frac{1}{\eta+\sigma}} \tag{20}$$

Notice that in the standard case we have $Y_s^c = \left(\frac{Z_s^{1+\eta}}{\chi\mu} \right)^{\frac{1}{\eta+\sigma}}$. In the present setting, therefore, the s.s. flexprice equilibrium output is a fraction $\beta^{\frac{1}{\eta+\sigma}}$ of the standard one.

From the consumption Euler equation (4) through linearization aroud the s.s. and taking into account the equilibrium condition $C_t = Y_t$ we get

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}\right)$$

For the sake of comparison with the standard NK-DSGE model, we rewrite the equation above as

$$x_t = E_t x_{t+1} - \Gamma\left(i_t - E_t \pi_{t+1}\right) + g_t \tag{21}$$

where x_t denotes the output gap, defined as the difference between current output and flexprice equilibrium output *in the canonical model,* i.e. $x_t := \hat{Y}_t - \hat{Y}_t^c$. Equation (21) represents the optimizing IS curve. We have appended a demand shock to the IS curve . As usual g_t follows an AR(1) process:$g_t = \psi g_{t-1} + \widetilde{g}_t$ with $\widetilde{g}_t$ ~iid$(0, \sigma_g^2)$.

From the asset price equation (8) through linearization we get the Asset Price (AP) schedule:

$$\hat{q}_t = -(i_t - E_t\pi_{t+1}) + \left[\theta E_t\hat{q}_{t+1} + (1-\theta)\, E_t\hat{d}_{t+1}\right] \tag{22}$$

6 The "augmented" NK Phillips curve

From the linearization of (11) around the s.s. we get

$$\hat{\phi}_t = \hat{w}_t - \hat{Z}_t + \left[\theta E_t\hat{q}_{t+1} + (1-\theta)\, E_t\hat{d}_{t+1} - \hat{q}_t\right]$$

Plugging (16) into the expression above and rearranging we get:

$$\hat{\phi}_t = (\eta + \sigma)\left\{x_t + \frac{1}{\eta+\sigma}\left[\theta E_t\hat{q}_{t+1} + (1-\theta)\, E_t\hat{d}_{t+1} - \hat{q}_t\right]\right\} \tag{23}$$

where x_t is the output gap as defined in the canonical New Keynesian framework.

In each period a fraction ω of firms is unable to adjust its price. As usual in a Calvo pricing context, therefore, ω is a measure of the degree of *nominal rigidity*. The j-th firm's pricing decision problem therefore is

$$\max_{p_{jt}} E_t \sum_{s=0}^{\infty} \omega^s \Delta_{s,t+s} \left[\left(\frac{p_{jt}}{P_{t+s}}\right)^{1-e} - \phi_{t+s}\left(\frac{p_{jt}}{P_{t+s}}\right)^{-e}\right] C_{t+s}$$

where $\Delta_{s,t+s} = \beta^s \left(\frac{C_{t+s}}{C_t}\right)^{-\sigma}$ is the consumption based discount factor, $\left(\frac{p_{jt}}{P_{t+s}}\right)^{-e} C_{t+s} = c_{jt}$ is demand for the j-th firm's product and ϕ_t is the marginal (and average) cost.

The optimal relative price of the good produced by the adjusting firm in period t, therefore, takes into account the stream of future marginal costs, which, in our framework, depends on *current and future asset prices and dividends* (see (11)).

From the standard microfoundations of the NK Phillips curve, after linearization we get $\pi_t = k\hat{\phi}_t + \beta E_t \pi_{t+1}$.Substituting (23) and rearranging we get

$$\pi_t = \lambda x_t + k\left[\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1} - \hat{q}_t\right] + \beta E_t \pi_{t+1} + u_t \qquad (24)$$

with $\lambda := k(\eta + \sigma)$.Equation (24) is the NK Phillips curve in the new setting. We have appended a supply shock u_t to the NK Phillips curve in order to avoid the "divine coincidence". As usual u_t follows an AR(1) process:$u_t = \rho u_{t-1} + \widetilde{u}_t$ with $\widetilde{u}_t \sim$iid$(0, \sigma_u^2)$.

The difference w.r.t. the canonical NK-PC is the term in brackets, i.e. $\widehat{ROS}$ (see equation (14). In fact, the cost channel and the equity-only financing assumptions imply that the cost of external finance, which coincides with the ROS, is affecting the firms' pricing decisions and therefore inflation. This is the reason why we will define the equation above the *Augmented* New Keynesian-Phillips Curve (A-NKPC).

7 An instrument rule for monetary policy

We will first explore the design and the transmission mechanism of monetary policy in the case in which the central bank adopts a simple Taylor-type instrument rule. For the sake of simplicity and without loss of generality, let's assume that this rule is activated exclusively by the feedback from inflation (in other words, the central bank does not take into account the output gap in devising its policy). Hence, the rule is $i_t = r + \gamma_\pi \pi_t$ where r is the real interest rate (equal to the rate of time preference in the steady state). In the following, in order to get rid of unnecessary complications, we will ignore the real interest rate so that the instrument rule becomes

$$i_t = \gamma_\pi \pi_t \tag{25}$$

This is the simplest rule one can imagine. In subsection 7.2 we will consider an instrument rule augmented by the asset price.

7.1 Model I-1

The macroeconomic model in structural form consists of the No-Arbitrage Condition (22), Augmented NK Phillips curve (24), IS curve (21) and Taylor rule (25) which we reproduce here for the reader's convenience.

$$\begin{gathered}
\hat{q}_t = -(i_t - E_t\pi_{t+1}) + \left[\theta E_t \hat{q}_{t+1} + (1-\theta)\, E_t \hat{d}_{t+1}\right] \\
\pi_t = \lambda x_t + k\left[\theta E_t \hat{q}_{t+1} + (1-\theta)\, E_t \hat{d}_{t+1} - \hat{q}_t\right] + \beta E_t \pi_{t+1} + u_t \\
x_t = E_t x_{t+1} - \Gamma\left(i_t - E_t \pi_{t+1}\right) + g_t \\
i_t = \gamma_\pi \pi_t
\end{gathered} \tag{M I-1}$$

This is "model I-1", a system of four linear difference equations in five state variables, $x_t, \hat{q}_t, \pi_t, i_t, \hat{d}_t$.

Model I-1 therefore is incomplete: There is one degree of freedom in modelling the macroeconomy in the present setting, which we can exploit to specify the dynamic pattern of dividends. [7] We will specify the dividend policy of firms in sections ....

The system above is recursive. Using the no-arbitrage condition, in fact, we obtain:

$$
\begin{aligned}
\pi_t &= \lambda x_t + k\left(i_t - E_t\pi_{t+1}\right) + \beta E_t\pi_{t+1} + u_t && \text{(M I-0)} \\
x_t &= E_t x_{t+1} - \Gamma\left(i_t - E_t\pi_{t+1}\right) + g_t \\
i_t &= \gamma_\pi \pi_t
\end{aligned}
$$

These equations form "model I-0", a system of three equations in x_t, π_t, i_t.

Notice that we can solve for these variables without any reference to $\widehat{ROS}$ and therefore to asset prices and dividends. In fact, we have replaced $\widehat{ROS}$ with the real interest rate $i_t - E_t\pi_{t+1}$, exploiting the no-arbitrage condition. In other words

Remark 1 *If the economy is described by model I-1 the determination of the asset price and dividends can be separated from the determination of all the other state variables. The equilibrium values of x_t, π_t, i_t. can be logically determined by solving model I-0 before determining asset prices and*

[7] We will not get entangled at this stage of the analysis in the debate on the *dividend puzzle* and simply borrow from the real world the stylized fact that firms do pay dividends even if the reasons for this behavior are not exactly clear.

dividends.

The Rational Expectations Equilibrium (REE) of model I-0 is computed in appendix B. In the following we illustrate the transmission of shocks within model I-0 by means of simple diagrams.

In order to do so, notice first that from (21), recalling that $\Gamma = 1/\sigma$ follows that $i_t - E_t\pi_{t+1} = \sigma\left(E_t x_{t+1} - x_t + g_t\right)$.Second, notice that in order to solve the system by the method of undetermined coefficient, we guess $s_1 = s_1 u_t + s_2 g_t$ for each state variable $s = \pi, x, i$. Therefore $E_t s_{t+1} = s_1 \rho u_t + s_2 \psi g_t$.

Assumption 1. Let's assume, for the sake of discussion, that $\rho = \psi$. This assumption is of course restrictive and may entail a modest loss of generality. It greatly simplifies the calculations, however, and yields very neat results since $E_t s_{t+1} = \rho\left(s_0 u_t + s_1 g_t\right) = \rho s_t$ for each and every state variable.

Because of assumption 1, the RE of a state variable taken in t for $t+1$ is a fraction of the current value of the variable. The expected rate of change therefore is decreasing with the current value: $E_t s_{t+1} - s_t = -\left(1-\rho\right)s_t$. This implicitly determines a *mean reverting* behaviour of that variable. If a shock hits a variable, causing a departure from the s.s., a negative (stabilizing) feedback is activated.

From assumption 1 follows that $E_t x_{t+1} = \rho x_t, E_t\pi_{t+1} = \rho\pi_t$. Hence the real interest rate is

$$i_t - E_t\pi_{t+1} = \left(\gamma_\pi - \rho\right)\pi_t \tag{26}$$

Using (M I-0), the system above boils down to:

$$x_t = -\Gamma \frac{\gamma_\pi - \rho}{1-\rho} \pi_t + \frac{1}{1-\rho} g_t \quad (27)$$

$$\pi_t = \frac{\lambda}{1-\beta\rho - k(\gamma_\pi - \rho)} x_t + \frac{1}{1-\beta\rho - k(\gamma_\pi - \rho)} u_t \quad (28)$$

Equation (27) represents the AD schedule in the present setting. Equation (28) represents the AS schedule.

Assumption 2. We assume

$$\rho < \gamma_\pi < \rho + \frac{1-\beta\rho}{k} \quad (29)$$

The inequality on the LHS of (29) is the equivalent, in the present setting, of the *Taylor principle*: in fact it assures that the real interest rate is positive when there is a burst of inflation (and viceversa). Thanks to the Taylor principle, the AD schedule is downward sloping on the (x_t, π_t) plane. The inequality on the RHS of (29) assures, on the other hand, that the AS schedule is upward sloping.When the AD and the AS curves are well behaved (i.e. they have the "appropriate slopes"), the solutions of M I-0 make sense (the system is "viable"). This means that

Remark 2 *The reaction of the central bank to current inflation must be neither too weak (Taylor principle: $\rho < \gamma_\pi$)* ***nor too strong*** *($\gamma_\pi < \rho + \frac{1-\beta\rho}{k}$) to assure the viability of the model solution.*

The RHS of (29) is the truly novel feature of this setting. In the absence of the cost channel, in fact, model M I-0 would boild down to the canonical model which we will label M I-0(c):

$$\begin{aligned} \pi_t &= \lambda x_t + \beta E_t \pi_{t+1} + u_t \\ x_t &= E_t x_{t+1} - \Gamma\left(i_t - E_t \pi_{t+1}\right) + g_t \\ i_t &= \gamma_\pi \pi_t \end{aligned} \tag{M I-0(c)}$$

which, after incorporating the model-consistent expectations, becomes:

$$x_t = -\Gamma \frac{\gamma_\pi - \rho}{1-\rho} \pi_t + \frac{1}{1-\rho} g_t \tag{30}$$

$$\pi_t = \frac{\lambda}{1-\beta\rho} x_t + \frac{1}{1-\beta\rho} u_t \tag{31}$$

Hence only $\rho < \gamma_\pi$ must be assumed to assure that the AD curve is downward sloping; the AS curve in model M I-0(c) is upward sloping **for any** value of γ_π.

Notice moreover that, as a consequence of the Taylor principle, the AS curve in model M I-0 is steeper that in model M I-0(c):

$$\left.\frac{\partial \pi_t}{\partial x_t}\right|_{AS} = \frac{\lambda}{1-\beta\rho - k\left(\gamma_\pi - \rho\right)} > \left.\frac{\partial \pi_t}{\partial x_t}\right|_{AS(c)} = \frac{\lambda}{1-\beta\rho}$$

In a sense this is obvious. In fact, an increase in output (with respect to the flexprice equilibrium) brings about an increase of inflation equal to $\left.\frac{\partial \pi_t}{\partial x_t}\right|_{AS(c)}$ in the canonical model. In the presence of the cost channel, the reaction of the central bank, i.e. the increase of the interest rate due to inflation, will make the increase of inflation even bigger, as shown by $\left.\frac{\partial \pi_t}{\partial x_t}\right|_{AS}$.

Solving (27) (28) gives x_t and π_t as linear functions of the shocks. Substituting the solution for π_t into (25) one gets the fundamentals based in-

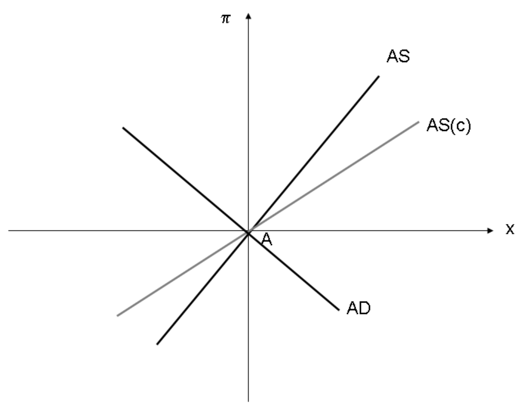


Figure 2: AS and AD schedules

strument rule (see again appendix B for details).

We are now ready to examine the transmission and the effects of shocks.

Suppose initially there are no shocks: $g_t = u_t = 0$. In figure 2 we represent the AD and the AS schedules in the present setting (black) and in the canonical one (grey). In the absence of shocks in both settings the two lines intersect in the origin, point A.

Suppose a (temporary) *supply shock* hits the economy. In a canonical setting, inflation goes up by $\frac{1}{1-\beta\rho}u_t$ on impact (see point B in figure 3).

In the presence of the cost channel, the reaction of the central bank to the increase in inflation – i.e. the increase of the interest rate – adds to inflation on impact. This is the reason why inflation goes up by $\frac{1}{1-\beta\rho-k\left(\gamma_\pi-\rho\right)}u_t$on impact in the presence of the cost channnel (see point B' in figure 3). In other words, the AS curve augmented with the cost channel shifts up more

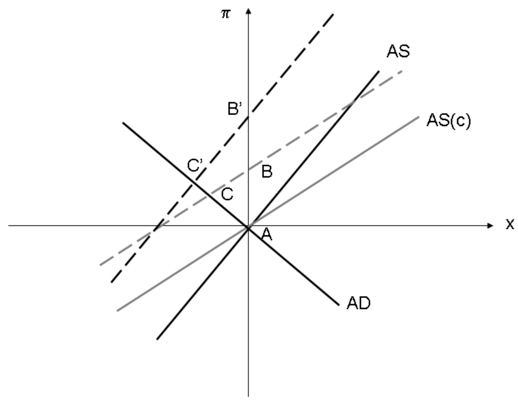


Figure 3: Effects of a supply shock

than in the canonical case.[8]

The central bank reaction steers the economy to C'. In the end, therefore, there will be more inflation an a more acute recession than in the canonical case (compare with C). In the case of a supply shock, therefore, the cost channel works as an *amplification mechanism* of the shock.[9] Of course, since the shock is temporary, with the passing of time the economy will move back to point A.

In the case of a *demand shock*, the new (short run) equilibrium will be at the intersection B' as shown in figure 4. The output gap turns positive but, in the presence of the cost channel, the expansion is weaker and inflation is

[8]It is easy to see, however, that the intercepts on the x-axis of the AS and AS(c) schedules after the shock coincide.

[9]In fact, in the RE solution – the coefficients of inflation and the output gap w.r.t. the supply shock are greater in absolute value in the presence of the cost channel. In symbols: $b_1 > b_1^c$, $|a_1| > |a_1^c|$ as shown in appendix B.

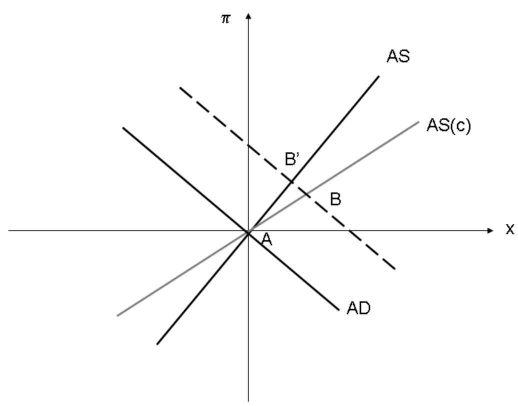


Figure 4: Effects of a demand shock

higher than in the canonical case (compare with B).

What happens to the stock price? As we said, since the system is recursive we can solve for the asset price after having solved for the ouput gap, inflation and the interest rate. Suppose, as a very convenient special case, that firms do not distribute dividends. In this case $\widehat{ROS} = E_t\hat{q}_{t+1} - \hat{q}_t$ i.e. the deviation of ROS from the s.s. is equal to the deviation of the capital gain from the s.s. [10] Notice that, because of assumption 1, $E_t\hat{q}_{t+1} - \hat{q}_t = -(1-\rho)\hat{q}_t$.Using this definition in (22) we get:

$$\hat{q}_t = -\frac{\gamma_\pi - \rho}{1-\rho}\pi_t \tag{32}$$

[10] It is easy to see that this difference is the *expected (real) asset price inflation,* i.e. the difference between (nominal) expected asset price inflation $E_t\varphi_t := \frac{Q_t}{Q_{t-1}} - 1$ and expected inflation $E_t\pi_t := \frac{P_t}{P_{t-1}} - 1$. Hence: $\widehat{ROS} = E_t\hat{q}_{t+1} - \hat{q}_t = E_t\varphi_t - E_t\pi_t$

Hence, thanks to the Taylor principle, a burst of inflation has a negative impact on the asset price. This is, once again, in a sense obvious. When the economy is hit by an inflationary shock, the central bank raises the interest rate prompting a flight from equities. Asset prices fall bringing about an increase of the return on shares such as to match the increase of the interest rate. This is how the no-arbitrage condition is re-established. Hence $\hat{q}$ is a linear decreasing function of g and u because they both bring about an increase of inflation. Both types of shocks therefore, are detrimental for the Stock market.

7.2 Model I-2

Let's consider now an *augmented* interest rate rule for monetary policy which takes into account not only inflation but also the asset price deviation from the s.s.

$$i_t = \gamma_\pi \pi_t + \gamma_q \hat{q}_t \tag{33}$$

In this case, the macroeconomic model in structural form consists of equations (22), (24), (21) and (33). For the sake of discussion, let's assume away the problem of providing at least a behavioral assumption for dividends confining ourselves to the very convenient scenario in which firms do not pay out dividends. In this special case the model becomes:

$$\begin{aligned}
\hat{q}_t &= -(i_t - E_t\pi_{t+1}) + E_t\hat{q}_{t+1} \\
\pi_t &= \lambda x_t + k\left(E_t\hat{q}_{t+1} - \hat{q}_t\right) + \beta E_t\pi_{t+1} + u_t \\
x_t &= E_t x_{t+1} - \Gamma\left(i_t - E_t\pi_{t+1}\right) + g_t \\
i_t &= \gamma_\pi \pi_t + \gamma_q \hat{q}_t
\end{aligned} \tag{M I-2}$$

This is "model I-2", i.e. a system of four linear difference equations in four state variables, $x_t, \hat{q}_t, \pi_t, i_t$.

This system is **not recursive.** In other words, when the central bank reacts to the asset price, the system does not dichotomize into 2 independent subsystems (one for x_t, π_t, i_t and the other for $\hat{q}_t$) as in model I-1. [11]

The REE of model I-2 is computed in appendix ... In this new setting the real interest rate is

$$i_t - E_t\pi_{t+1} = \left(\gamma_\pi - \rho\right)\pi_t + \gamma_q \hat{q}_t \tag{34}$$

In order to solve this model, it is convenient to plug (34) into (22). Using assumption 1 (so that $Ex^{'} = \rho x$, $E\pi^{'} = \rho\pi$, $E\hat{q}^{'} = \rho\hat{q}$) we get

$$\hat{q}_t = -\frac{\gamma_\pi}{1+\gamma_q}\pi_t + \frac{1}{1+\gamma_q}E_t\pi_{t+1} + \frac{1}{1+\gamma_q}E_t\hat{q}_{t+1} \tag{35}$$

Hence, substituting (35) into M I-2 and using assumption 1, the system

[11] A similar dichotomy occurs also in Carlstrom and Fuerst (2007) albeit in a different context.

becomes:

$$\begin{gathered}
\pi_t = \frac{\lambda}{1-\beta\rho-k\left(\gamma_\pi-\rho\right)}x_t + \frac{k\gamma_q}{1-\beta\rho-k\left(\gamma_\pi-\rho\right)}\hat{q}_t + \frac{1}{1-\beta\rho-k\left(\gamma_\pi-\rho\right)}u_t \\
x_t = -\frac{\Gamma\left(\gamma_\pi-\rho\right)}{1-\rho}\pi_t - \frac{\Gamma\gamma_q}{1-\rho}\hat{q}_t + \frac{g_t}{1-\rho} \\
\hat{q}_t = -\frac{\gamma_\pi-\rho}{1+\gamma_q-\rho}\pi_t
\end{gathered} \tag{M I-2bis}$$

Notice that asset prices impact directly on inflation (see the first equation). These equations form a system in $x_t, \pi_t, \hat{q}_t$.Substituting the asset price equation into the other equations we get:

$$x_t = -\frac{\Gamma\left(\gamma_\pi-\rho\right)}{1+\gamma_q-\rho}\pi_t + \frac{g_t}{1-\rho} \tag{36}$$

$$\begin{aligned}
\pi_t \quad = \quad & \frac{\lambda}{1-\beta\rho-k\left(\gamma_\pi-\rho\right)\left(1+\frac{\gamma_q}{1-\rho}\right)^{-1}}x_t + \\
& + \frac{1}{1-\beta\rho-k\left(\gamma_\pi-\rho\right)\left(1+\frac{\gamma_q}{1-\rho}\right)^{-1}}u_t
\end{aligned} \tag{37}$$

Equation (36) represents the AD schedule in model I-2. Equation (37) represents the AS schedule.

Assumption 3. We assume

$$\rho < \gamma_\pi < \rho + \frac{1-\beta\rho}{k} + \frac{1-\beta\rho}{k\left(1-\rho\right)}\gamma_q \tag{38}$$

The inequality on the LHS of (38) is, of course, the *Taylor principle.* The inequality on the RHS of (38) assures that the AS schedule is upward slop-

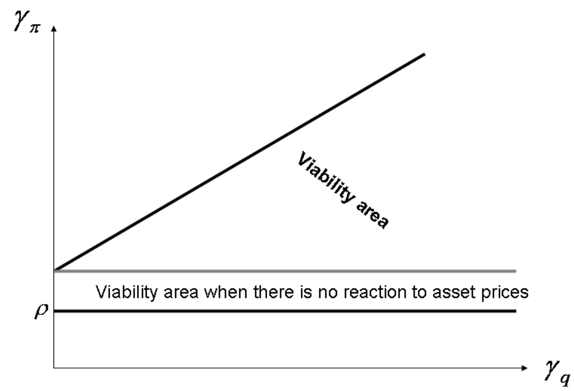


Figure 5: Viability area

ing. As already noted in Remark 2 above, this means that the reaction of the central bank to inflation must not be too strong. In the present case, moreover, it should fulfill an additional requirement concerning the sensitivity of monetary policy to asset prices. In figure 5 we represent the viability area in the presence of the cost channel. When the central bank does not react to asset prices, viability is confined to the area between the grey and the black horizontal lines (i.e. the area that fulfills (29)). When the central bank reacts to asset prices the viability area (i.e. the area that fulfills (38)) expands to the area between the upward sloping and the horizontal black lines.

Notice that the AS schedule in the presence of reaction to asset prices is flatter than in the case of no asset price (AP) reaction. In fact

$$\left.\frac{\partial \pi_t}{\partial x_t}\right|_{AS} = \frac{\lambda}{1-\beta\rho-k\left(\gamma_\pi-\rho\right)} > \left.\frac{\partial \pi_t}{\partial x_t}\right|_{AS(q)} = \frac{\lambda}{1-\beta\rho-k\left(\gamma_\pi-\rho\right)\left(1+\frac{\gamma_q}{1-\rho}\right)^{-1}}$$

In order to understand why, recall that, absent the cost channel, a positive output gap brings about an increase of inflation equal to $\left.\frac{\partial \pi_t}{\partial x_t}\right|_{AS(c)}$. In the presence of the cost channel, the reaction of the central bank, i.e. the **increase of the interest rate** due to inflation, will make the increase of inflation bigger, as shown by $\left.\frac{\partial \pi_t}{\partial x_t}\right|_{AS}$. According to the third equation in M I-2, asset prices go down as a consequence of inflation. If the central bank targets also asset prices, the contraction of the asset price will induce a monetary easing, i.e. a **reduction of the interest rate**, which, in the presence of the asset price cost channel, translates into a reduction of inflation. Overall, there will be an increase in the interest rate also in the case of reaction to asset prices, but this increase will be smaller than in the case of no reaction. Notice that the higher the reaction to asset prices, the flatter the AS curve becomes. As γ_q increases, the slope tends asymptotically to that of the AS curve in the canonical (i.e. no cost channel) case. in fact

$$\lim_{\gamma_q\to\infty}\left.\frac{\partial \pi_t}{\partial x_t}\right|_{AS(q)} = \frac{\lambda}{1-\beta\rho} = \left.\frac{\partial \pi_t}{\partial x_t}\right|_{AS(c)}$$

Notice moreover that when the central bank reacts to $\hat{q}_t$ also the slope of the AD curve changes w.r.t. the case of no reaction. In absolute value:

$$\left.\frac{\partial \pi_t}{\partial x_t}\right|_{AD} = \frac{1-\rho}{\Gamma\left(\gamma_\pi-\rho\right)} < \left.\frac{\partial \pi_t}{\partial x_t}\right|_{AD(q)} = \frac{1+\gamma_q-\rho}{\Gamma\left(\gamma_\pi-\rho\right)}$$

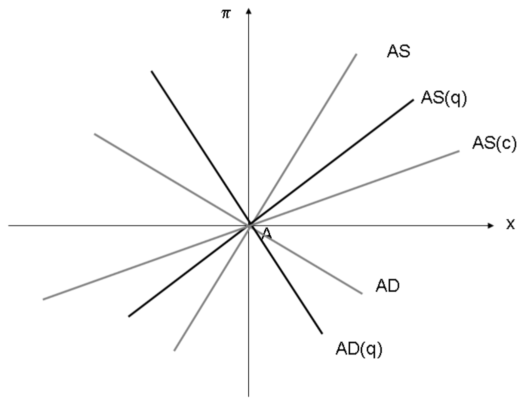
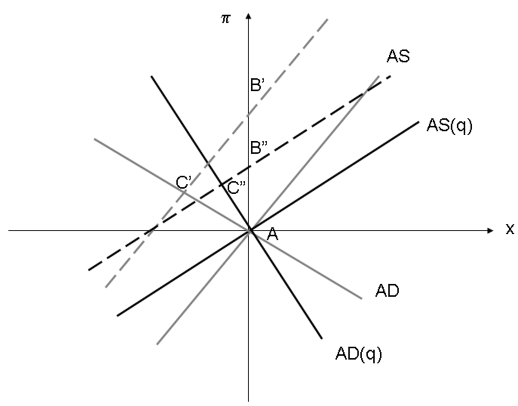


Figure 6: Slopes of the AD and AS curves

In words, the AD curve when $\gamma_q > 0$ is steeper – on the (x_t, π_t) plane – than in the case $\gamma_q = 0$. In order to understand why, recall that, in the case $\gamma_q = 0$, an increase of inflation brings about a contraction of output whose magnitude is $\left.\frac{\partial x_t}{\partial \pi_t}\right|_{AD} = \dfrac{\Gamma(\gamma_\pi - \rho)}{1-\rho}$. This is due to the reaction of the central bank to inflation, i.e. to the **increase of the interest rate**. Inflation leads to a fall of asset prices (due to arbitrage). In the case $\gamma_q > 0$, the central bank contrasts this tendency by "easing" a bit, i.e. reducing the interest rate marginally w.r.t. the previous interest rate hike. This will make the contractionary impact of the increase of the interest rate smaller, as shown by $\left.\frac{\partial x_t}{\partial \pi_t}\right|_{AD(q)} = \dfrac{\Gamma(\gamma_\pi - \rho)}{1+\gamma_q-\rho}$.

In figure 6 we report the AD and AS curves in the different cases.

Solving (36) (37) gives x_t and π_t as linear functions of the shocks. Substituting the solution for π_t into (33) one gets the fundamentals based in-



strument rule (see again appendix ... for details)).

We are now ready to examine the transmission and the effects of shocks.

Suppose initially there are no shocks: $g_t = u_t = 0$. In figure 6 we represent the AD and the AS schedules in the case in which $\gamma_q > 0$ (AS(q) in black) and in the case in which there is no reaction to the asset price (AS in grey). In the absence of shocks in both settings the two lines intersect in the origin, point A.

Suppose a *supply shock* hits the economy. In the no asset price reaction case, inflation goes up by $\dfrac{1}{1-\beta\rho-k\left(\gamma_\pi-\rho\right)}u_t$ (see point B' in figure 7.2, which corresponds to B' in figure 3). This burst of inflation incorporates the fact that the central bank reacts to the shock *raising* the interest rate, which adds to inflation on impact. The increase in inflation makes asset prices go down. When $\gamma_q > 0$, the central bank reacts to the fall of asset prices easing a bit so that the increase of the interest rate – and the additional inflation due to the cost channel – will be smaller than in the no reaction case. In other words, targeting asset prices will reduce the impact on inflation of a contractionary monetary policy in the presence of the cost channel.

The central bank then steers the economy to C". Notice that the AD curve is now steeper than in the no AP reaction. In the end, therefore, there will be less inflation and a milder recession than in the case in which the central bank does not react to asset prices (compare with C'). When a supply shock hits the economy, therefore, the reaction of the central bank to asset prices has a *mitigating effect* on both the change in output and inflation, curbing the *amplification mechanism* activated by the cost channel in the no reaction case.

In the case of a *demand shock*, the new short run equilibrium will be at the intersection B" as shown in figure 4. The output gap turns positive. But with the cost channel and the reaction to asset prices the expansion is stronger and inflation is higher than in the previous case. [12]When a demand shock hits the economy, therefore, the reaction of the central bank to asset prices has a *mitigating effect* on inflation, but an *amplification mechanism* on output with respect to the no reaction case.

What happens to the stock price? Recall that in the previous case: $\hat{q}_t = -\dfrac{\gamma_\pi - \rho}{1-\rho}\pi_t$ while now $\hat{q}_t = -\dfrac{\gamma_\pi - \rho}{1+\gamma_q-\rho}\pi_t$. Hence, a burst of inflation has a negative impact on the asset price but smaller than in the previous case.In fact, when the economy is hit by an inflationary shock, asset prices fall.

[12] Points B' and B" lie on an upward sloping straight line (not shown in the figure), whose equation –i.e. (44) – is obtained by consolidating the Augmented NK-PC as defined in equation (41) and of the IS curve (42). We will refer to this curve as the Augmented NK-PC in section 8).

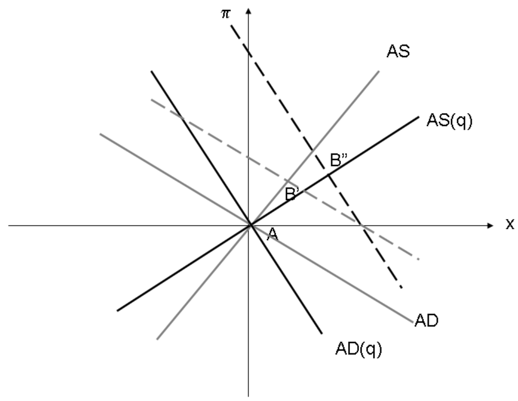


Figure 7: Effect of a demand shock

When $\gamma_q > 0$, the central bank eases a bit mitigating the fall of asset prices.

8 Optimal Monetary Policy

In this section we turn our attention to the case in which monetary policy is determined optimally. In order to do so we need to specify the central bank's preferences. We represent them by means of a quadratic loss function whose arguments are the deviations of inflation and the output gap from the target values, which we can set to zero for simplicity.

The output gap which should show up in the loss function is the difference between current output and the flexprice equilibrium output as defined *in the present context, i.e.*$\bar{x}_t := \hat{Y}_t - \hat{Y}_t^f$. Unless the central bank is myopic, in fact, it is straightforward to assume that it wants to minimize the difference between current output and the relevant notion of flexprice equilibrium with reference to the economy under scrutiny.

There is an obvious relationship between the canonical notion of output gap $x_t := \hat{Y}_t - \hat{Y}_t^c$ and the relevant notion for the policy maker $\bar{x}_t$:

$$\bar{x}_t = x_t + \left(\hat{Y}_t^c - \hat{Y}_t^f\right) = x_t + \frac{\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1} - \hat{q}_t}{\eta + \sigma}$$

so that the loss fuction can be written as follows:

$$L = E_t \sum_{s=0}^{\infty} \beta^s \left[\pi_{t+s}^2 + \alpha \left(x_{t+s} + \frac{\theta E_t \hat{q}_{t+s+1} + (1-\theta) E_t \hat{d}_{t+s+1} - \hat{q}_t}{\eta + \sigma}\right)^2\right]$$

Hence $\widehat{ROS} = \theta E_t \hat{q}_{t+s+1} + (1-\theta) E_t \hat{d}_{t+s+1} - \hat{q}_t$ shows up in the loss function in a straightforward (non ad-hoc) way. In the canonical model, under discretion, the loss is minimized subject only to the New Keynesian Phillips curve assuming that agents' expectations are given. In the present setting the optimization problem is more complicated. Not only the (Augmented) New Keynesian Phillips Curve but also the optimizing IS curve should play the role of constraints in the optimization problem.

Therefore the intertemporal optimization problem boils down to a sequence of period by period minimization problems of the type:

$$\begin{aligned}
\min_{\pi_t, x_t, \hat{q}_t} L &= \pi_t^2 + \alpha \left(x_t + \frac{\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1} - \hat{q}_t}{\eta + \sigma}\right)^2 + C_0 \\
\text{s.t.} \quad \pi_t &= \lambda x_t - k \hat{q}_t + C_1 \\
x_t &= \Gamma \hat{q}_t + C_2
\end{aligned}$$

where we treat expectations and shocks as given as it is customary in the discretionary regime. Hence:

$$
\begin{aligned}
C_0 &: = E_t \sum_{s=1}^{\infty} \beta^s \left[\pi_{t+s}^2 + \alpha \left(x_{t+s} + \frac{\theta E_t \hat{q}_{t+s+1} + (1-\theta) E_t \hat{d}_{t+s+1} - \hat{q}_t}{\eta + \sigma} \right)^2 \right] \\
C_1 &: = \beta E_t \pi_{t+1} + k \left[\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1} \right] + u_t \\
C_2 &: = E_t x_{t+1} - \Gamma \left[\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1} \right] + g_t
\end{aligned}
$$

are treated as constants in the minimization problem. From the FOCs of the problem above one gets the Social Expansion Path (SEP):

$$
x_t = -\frac{\lambda}{\alpha} \pi_t - \frac{\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1} - \hat{q}_t}{\alpha (\eta + \sigma)} \tag{39}
$$

The first component of the RHS of (39) is exactly the same as in the canonical Clarida-Galì-Gertler (CGG) model. In our setting, the SEP is "augmented" by a factor proportional to $\widehat{ROS}$.[13]

8.1 Model O-1

The macroeconomic model in structural form consists of the Social Expansion Path (39), Augmented NK Phillips curve (24), No-Arbitrage Condition (22) and IS curve (21) which we reproduce here for the reader's convenience.

[13] Notice, however, that $\widehat{ROS}$ is equal to the real interest rate due to the No-arbitrage condition and that the real interest rate is equal to $\frac{E_t x_{t+1} + g_t - x_t}{\Gamma}$. Taking these considerations into account would yield a SEP whose slope in the end is different with respect to the standard one. We will make use of this considerations below (see subsection **??**).

$$
\begin{aligned}
x_t &= -\frac{\lambda}{\alpha}\pi_t - \frac{\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1} - \hat{q}_t}{\alpha(\eta+\sigma)} \\
\hat{q}_t &= -(i_t - E_t \pi_{t+1}) + \left[\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1}\right] \\
\pi_t &= \lambda x_t + k\left[\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1} - \hat{q}_t\right] + \beta E_t \pi_{t+1} + u_t \\
x_t &= E_t x_{t+1} - \Gamma(i_t - E_t \pi_{t+1}) + g_t
\end{aligned}
\tag{M 0-1}
$$

This is "model O-1", a system of four linear difference equations in five state variables, $x_t, \hat{q}_t, \pi_t, i_t, \hat{d}_t$.

For the moment we do not need to specify dividend policy for the very good reason that the system is recursive. In fact, plugging (22) into (39), and (24), in fact, we obtain:[14]

$$
x_t = -\frac{\lambda}{\alpha}\pi_t - \frac{i_t - E_t \pi_{t+1}}{\alpha(\eta+\sigma)} \tag{40}
$$

$$
\pi_t = \lambda x_t + k(i_t - E_t \pi_{t+1}) + \beta E_t \pi_{t+1} + u_t \tag{41}
$$

These equations, together with (21) form "model O-0", a system of three equations in x_t, π_t, i_t.

Notice that we can solve for these variables without any reference to $\widehat{ROS}$ and therefore to asset prices and dividends.In other words we have the

[14] Plugging (22) into (21) we get:

$$
x_t = E_t x_{t+1} - \Gamma\left[\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1} - \hat{q}_t\right] + g_t
$$

Since there is a negative relationship between $\widehat{ROS}$ and the output gap, in the end the current output gap is increasing with the current asset price. This positive relationship is not new in the literature. The optimizing IS curve in fact may incorporate this positive relationship due a wealth effect. However we obtain this result for entirely different reasons: The higher is $\hat{q}_t$, the smaller will be $\widehat{ROS}$ and correspondingly smaller, in equilibrium, will be the real interest rate; the associated increase in consumption will boost output.

same dichotomy as in model I-0 (see remark 1 above).

We assume, as in the previous section, that $\rho = \psi$ (assumption 1) so that $E_t x_{t+1} = \rho x_t, E_t \pi_{t+1} = \rho \pi_t$. Hence, from the IS curve follows:

$$i_t - E_t \pi_{t+1} = -\sigma (1-\rho) x_t + g_t \tag{42}$$

Plugging (42)into the system above and rearranging we get:

$$\begin{aligned} x_t &= -\frac{\lambda(\eta+\sigma)}{\alpha(\eta+\sigma) - \sigma(1-\rho)} \pi_t - \frac{\sigma}{\alpha(\eta+\sigma) - \sigma(1-\rho)} g_t & (43) \\ \pi_t &= k \frac{\eta + \sigma\rho}{1-\beta\rho} x_t + \frac{k\sigma}{1-\beta\rho} g_t + \frac{1}{1-\beta\rho} u_t & (44) \end{aligned}$$

Solving (43) (44) gives x_t and π_t as linear functions of the shocks. Substituting the solutions for x_t and π_t into (21) one gets the fundamentals based optimal interest rate rule (see again appendix D for details).

For the sake of comparison, we recall that the standard CGG model – incorporating the model-consistent expectations into the system – boils down to:

$$\begin{aligned} x_t &= -\frac{\lambda}{\alpha} \pi_t & (45) \\ \pi_t &= k \frac{\eta+\sigma}{1-\beta\rho} x_t + \frac{1}{1-\beta\rho} u_t & (46) \end{aligned}$$

Assumption 4. Let $\alpha(\eta+\sigma) - \sigma(1-\rho) > 0$ i.e. $\alpha > \hat{\alpha}_1 := \frac{\sigma}{\eta+\sigma}(1-\rho)$ [15].

[15] Notice that $\hat{\alpha}_1 < 1$.

In this case the SEP of equation (43) – which we will label SEP-ABD – is downward sloping and flatter – on the (x_t, π_t) plane – than the SEP in CGG represented by (45):

$$\left.\frac{\partial \pi_t}{\partial x_t}\right|_{SEP-ABD} = \frac{\alpha - \frac{\sigma(1-\rho)}{\eta+\sigma}}{\lambda} < \left.\frac{\partial \pi_t}{\partial x_t}\right|_{SEP-CGG} = \frac{\alpha}{\lambda}$$

This means that for any given inflation shock, the policy-induced recession necessary to steer the macroeconomy on the optimal inflation-output gap locus is bigger in the present setting than in CGG. Moreover the SEP is affected by demand shocks, which was not the case in CGG.

As to the Phillips curve, the Augmented NK-PC of equation (44) is flatter than the NK-PC represented by (46). Moreover the the Augmented NK-PC is affected by demand shocks, which was not the case in CGG: in the present setting there is an *indirect supply shock* induced by the increase in demand through the cost channel.

The fact that the Augmented NK-PC is flatter than the canonical NK-PC puzzling. After all, one would expect inflation to be higher – for a given increase in output (with respect to the flexprice equilibrium) – in the presence of the cost channel. In order to explain the puzzle, notice that equation (44) is the consolidation of the Augmented NK-PC as defined in equation (41) and of the IS curve (42). In the absence of shocks, a positive output gap brings about inflation equal to $\left.\frac{\partial \pi_t}{\partial x_t}\right|_{NKPC} = \frac{k\left(\eta+\sigma\right)}{1-\beta\rho}$ in the CGG model. Notice that the output gap turns positive, according to (42) only if the central bank has engineered a **reduction** of the real interest rate. In the presence of the cost channel, this reduction of the real interest rate

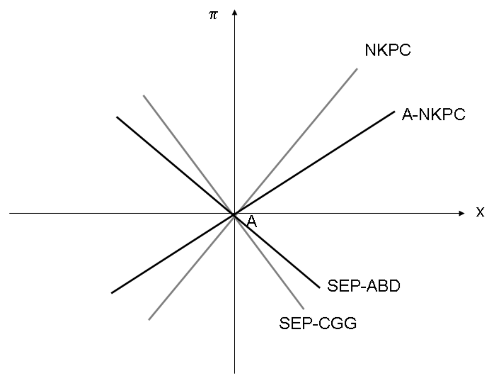


Figure 8: NK-PC and SEP in different cases.

will make the increase of inflation associated to a given increase of output smaller: $\frac{\partial \pi_t}{\partial x_t}\Big|_{A-NKPC} = \dfrac{k(\eta + \sigma\rho)}{1 - \beta\rho}$.

We are now ready to examine the transmission and the effects of shocks. Suppose initially there are no shocks: $g_t = u_t = 0$. In figure 8 we represent the SEP and the Phillips curve in CGG and in the present (ABD) setting. In the absence of shocks in both settings the two lines intersect in the origin, point A, which is also the bliss point.

Suppose a *supply shock* hits the economy. In a CGG setting, the NK-PC shifts up by $\dfrac{1}{1 - \beta\rho} u_t$ on impact (see point B in figure 9). The central bank reacts *raising* the interest rate to steer the macroeconomy on the SEP and the new short run equilibrium will be in C. The output gap turns negative. Also in our setting the Augmented NK-PC shifts up by $\dfrac{1}{1 - \beta\rho} u_t$on impact (once again see point B). The central bank reacts *raising* the interest rate

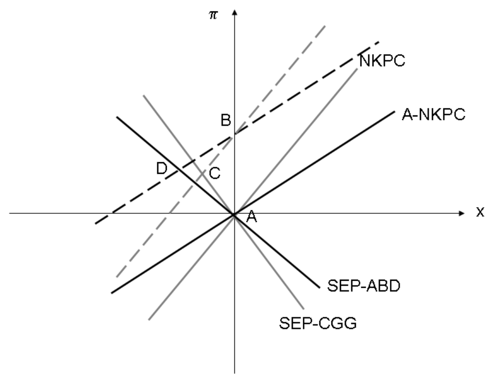


Figure 9: Effect of a supply shock

and the new short run equilibrium will be at the intersection D. Qualitatively we have the same prescription in favour of a *leaning against the wind* policy as in the standard setting.

Notice however that in the present setting the central bank is implicitly targeting the ROS. In fact $\widehat{ROS} = i_t - E_t\pi_{t+1}$ in equilibrium (no-arbitrage). In other words, by changing the policy rate the central bank steers the $\widehat{ROS}$ in such a way as to obtain a target level of $\widehat{ROS}$ consistent with the SEP.

The quantitative impact moreover is different. Due to the smaller slopes of the schedules involved, in our setting the contraction induced by the leaning against the wind policy is bigger while the effect on inflation may be smaller.

Things are more complicated and more interesting in case a *demand shock* occurs. In a CGG economy, the demand shock does not affect either inflation or the output gap because it is completely offset by the central bank.In the present model, the Augmented NK-PC shifts up by $\frac{k\sigma}{1-\beta\rho}g_t$ on impact (see point B in figure 10). This is the *indirect supply shock* induced by the increase in demand through the cost channel.[16] The central bank reacts *raising* the interest rate to steer the macroeconomy on the SEP. The output gap turns negative. In our setting the SEP shifts down due to the demand shock, making the recession more acute. This is actually lowering inflation. The short run equilibrium will be in C. In the figure inflation is still positive in C.

If the SEP shifts down "enough", however, one can well have a deflation, i.e. a negative rate of growth of the price level. In appendix ...we show that this is the case if the central banker is (relatively) conservative, i.e $\hat{\alpha}_1 < \alpha < 1$.

What happens to the stock price? As we said, since the system is recursive we can solve for the asset price after having solved for the ouput gap, inflation and the interest rate. Suppose that firms do not distribute dividends. In this case

$$\widehat{ROS} = E_t\hat{q}_{t+1} - \hat{q}_t = -(1-\rho)\,\hat{q}_t \tag{47}$$

[16] Consolidating the IS and A-NKPC curves through the cost channel we have:

$$\pi_t = k\eta x_t + k\left[\sigma\left(E_t x_{t+1} + g_t\right)\right] + \beta E_t\pi_{t+1} + u_t$$

so that the effect of a demand shock on inflation given expected inflation is $\left.\frac{\partial \pi_t}{\partial g_t}\right|_G = k\sigma$. If we take rational expectations into account, i.e. $E_t x_{t+1} = \rho x_t, E_t\pi_{t+1} = \rho\pi_t$, we end up with (44) so that effect of a demand shock on inflation with rational expectations is bigger: $\left.\frac{\partial \pi_t}{\partial g_t}\right|_{RE} = \frac{k\sigma}{1-\beta\rho}$.

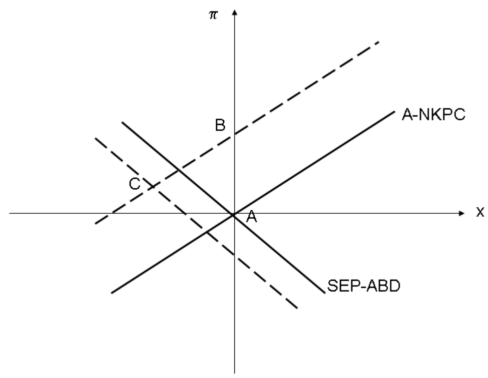


Figure 10: Effect of a demand shock

From the no-arbitrage condition (22) and the IS (21), moreover one gets

$$\widehat{ROS} = i_t - E_t \pi_{t+1} = -\sigma (1-\rho) x_t + g_t \tag{48}$$

Therefore:

$$\hat{q}_t = \sigma x_t - \frac{g_t}{1-\rho} \tag{49}$$

In the end, therefore, we have a new schedule on the $(x_t, \hat{q}_t)$ plane which is upward sloping and subject to a shock. We can think of this schedule as an *Asset Price Phillips curve (AP-PC)*: when the output gap is positive there will be a burst of asset price inflation and viceversa. A sudden increase of demand translates into a negative shock for the Stock market.

In order to understand why an *AP-PC* is implicit in our setup, let's represent equations (47) and (48) on the $\left(\hat{q}_t, \widehat{ROS}\right)$ plane as in figure 11.

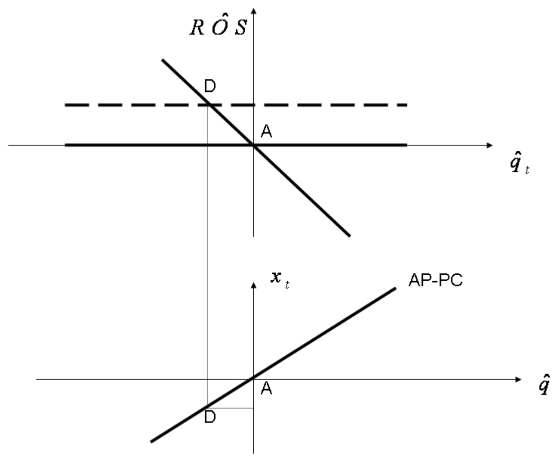


Figure 11: Effect of a supply shock on $\hat{q}$

(47) is represented by a downward sloping straight line passing through the origin, which represents the equilibrium (point A). If the economy is in the flexprice equilibrium ($x_t = 0$) and there are no shocks, (48) coincides with the x-axis ($\widehat{ROS} = 0$ for any $\hat{q}_t$).

Suppose a supply shock occurs so that the central bank steers the economy on the SEP by raising the interest rate. Hence the output gap becomes negative. The horizontal line representing (48) shifts up.The new equilibrium is D: the asset price has gone down. Figure 11 should be thought of as a complement to figure 9. Points A and D in the former corresponds to points A and D on the latter.

Suppose now a demand shock occurs so that the central bank raises the interest rate. The output gap becomes negative. The horizontal line of equation (48) shifts up twice – as shown in figure 12 – because of the shock

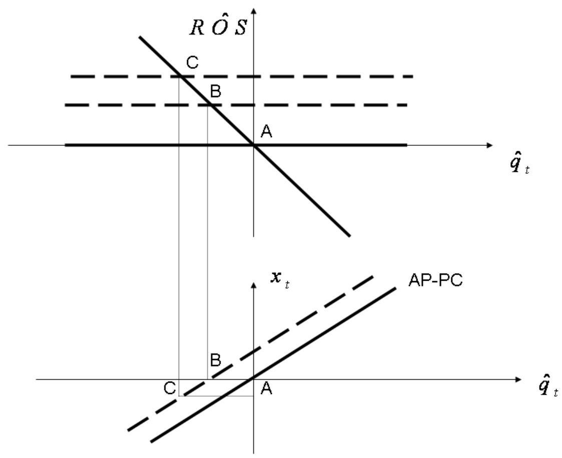


Figure 12: Effects of a demand shock on $\hat{q}$

(from A to B) and because of the recession (from B to C). The asset price has gone down.Figure 12 is a complement to figure 10. Points A and C in the former correspond to points A and C on the latter.

Substituting the solutions for x_t in (49) we obtain $\hat{q}_t$ as a linear function of the shocks (see again appendix D for details). It turns out that $\hat{q}$ is a linear decreasing function of g and u. Both types of shocks therefore, are detrimental for the Stock market.

8.2 Model O-1.1: dividends and profits

So far we have not specified how firms set dividends. The specification of dividend policy allows to complete model O-1 – which is, as we said above, dichotomous – in a satisfactory way. In section 8.1 in fact, for the sake of discussion, we have closed the model (and derived the solution for the asset price) by assuming that firms do not pay dividends so that the return on shares coincides with the capital gain.

In the present section, we will derive an explicit solution for dividends and the asset price assuming that firms' real profits are paid out to households in the form of dividends: $d_t = Y_t - \frac{w_t}{Z_t} Y_t$.Substituting the real wage as defined in (15) into the expression above we get

$$d_t = Y_t - \chi \frac{Y_t^{1+\eta+\sigma}}{Z_t^{1+\eta}} \tag{50}$$

Log-linearizing (50) around the steady state and rearranging we get:

$$\hat{d}_t = \left[1 + \frac{\beta}{\mu - \beta} (\eta + \sigma) \right] x_t + \frac{1+\eta}{\eta+\sigma} \hat{z}_t \tag{51}$$

where $\frac{1+\eta}{\eta+\sigma}\hat{z}_t = \hat{Y}_t^c$ and $\frac{\beta}{\mu-\beta} = \frac{Y_s - d_s}{d_s}$. [17]We assume that the technology shock $\hat{z}_t$ follows a AR(1) process $\hat{z}_t = \rho_z \hat{z}_{t-1} + \tilde{z}_t$, with $0 < \rho_z < 1$ and $\widetilde{z}_t$ ∼iid$(0, \sigma_z^2)$. Hence dividends are an increasing linear function of the output gap subject to a technology shock. Our complete system therefore consists of the equations of model O-1 supplemented by (51), i.e.

$$\begin{aligned}
x_t &= -\frac{\lambda}{\alpha}\pi_t - \frac{\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1} - \hat{q}_t}{\alpha(\eta+\sigma)} \\
\hat{q}_t &= -(i_t - E_t \pi_{t+1}) + \left[\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1} \right] \\
\pi_t &= \lambda x_t + k \left[\theta E_t \hat{q}_{t+1} + (1-\theta) E_t \hat{d}_{t+1} - \hat{q}_t \right] + \beta E_t \pi_{t+1} + u_t \\
x_t &= E_t x_{t+1} - \Gamma (i_t - E_t \pi_{t+1}) + g_t \\
\hat{d}_t &= \left[1 + \frac{\beta}{\mu-\beta} (\eta+\sigma) \right] x_t + \frac{1+\eta}{\eta+\sigma} \hat{z}_t
\end{aligned} \tag{M O-1.1}$$

[17]Notice that $d_s/Y_s = \frac{\mu-\beta}{\mu}$is the s.s. share of profits in total income.

This is "model O-1.1". If we iterate (51) one period ahead and take the expected value we get

$$E_t\hat{d}_{t+1} = \left[1 + \frac{\beta}{\mu - \beta}(\eta + \sigma)\right] E_t x_{t+1} + \frac{1+\eta}{\eta + \sigma}\rho_z \hat{z}_t. \tag{52}$$

Substituting out i_t and $E_t\hat{d}_{t+1}$ from (21) and (52) into (22), we get

$$\begin{aligned} \hat{q}_t &= \theta E_t\hat{q}_{t+1} + \left\{(1-\theta)\left[1 + \frac{\beta}{\mu - \beta}(\eta + \sigma)\right] - \frac{1}{\Gamma}\right\} E_t x_{t+1} \\ & \quad + \frac{1}{\Gamma}x_t - \frac{1}{\Gamma}g_t + (1-\theta)\frac{1+\eta}{\eta+\sigma}\rho_z\hat{z}_t \end{aligned} \tag{53}$$

In the present setting, the $\widehat{ROS}$ becomes

$$\widehat{ROS} = \theta E_t\hat{q}_{t+1} + (1-\theta)\left[1 + \frac{\beta}{\mu - \beta}(\eta + \sigma)\right] E_t x_{t+1} + (1-\theta)\frac{1+\eta}{\eta+\sigma}E_t\hat{z}_{t+1} - \hat{q}_t$$

The Rational Expectation of $\hat{q}_{t+1}$ taken in t is $E_t\hat{q}_{t+1} = \rho\hat{q}_t$ due to assumption 1. This implicitly determines a *mean reverting* behaviour of the asset price too. Taking model-consistent expectations into account the expression above boils down to:

$$\widehat{ROS} = (1-\theta)\left[1 + \frac{\beta}{\mu - \beta}(\eta + \sigma)\right]\rho x_t + (1-\theta)\frac{1+\eta}{\eta+\sigma}\rho_z\hat{z}_t - (1-\theta\rho)\hat{q}_t \tag{54}$$

As shown in section 8.1, when firms do not pay dividends $\widehat{ROS}$ is represented by equation to (47). Equation (54) shows that, when firms pay dividends out of profits, $\widehat{ROS}$ is not only decreasing with $\hat{q}_t$ (because of the capital gain) but also increasing with x_t (because of the distribution of dividends).

From equations (21) and (22), moreover we obtain (48), i.e. $\widehat{ROS} = -\sigma(1-\rho)x_t + \sigma g_t$.Equating (54) and (48) one gets:

$$\hat{q}_t = \frac{(1-\theta)\left[1+\frac{\beta}{\mu-\beta}(\eta+\sigma)\right]\rho+\sigma(1-\rho)}{1-\theta\rho}x_t + \\ +\frac{(1-\theta)\frac{1+\eta}{\eta+\sigma}\rho_z}{1-\theta\rho}\hat{z}_t - \frac{\sigma}{1-\theta\rho}g_t \tag{55}$$

In the end, therefore, we have a new equation for the *Asset Price Phillips curve* which take into accounts also dividends and the productivity shock (compare with (49)).[18] While the demand shock will have a negative impact on the Stock market, a productivity shock will boost asset prices, ceteris paribus.

In the case of a supply shock, the central bank steers the economy on the SEP by raising the interest rate. The output gap becomes negative. The horizontal line representing (48) shifts up.Profits and dividends decrease because of the recession. Hence (**??**) shifts down. The new equilibrium is D: the asset price has gone down. Figure 13 should be thought of as a complement to figure 9. Points A and D in the former corresponds to points A and D on the latter.

Suppose now a demand shock occurs so that the central bank raises the interest rate. The output gap becomes negative. The horizontal line of equation (**??**) shifts up twice – as shown in figure 14 – because of the shock

[18] Notice that (55) can be derived from (53) by incorporating model consistent expectations.

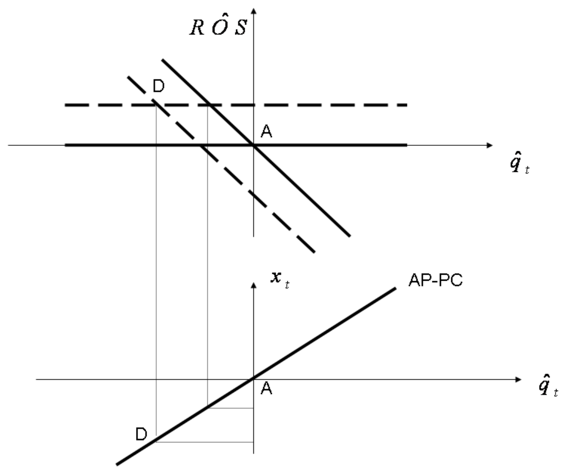


Figure 13: Effect of a supply shock on the asset price.

and because of the recession.

Profits and dividends decrease. (**??**) shifts down. Equilibrium is D: the asset price has gone down.Figure 14 should be thought of as a complement to figure 10. Points A and D in the former corresponds to points A and D on the latter.

Finally consider a productivity shock. The central bank does not intervene because the output gap is zero (but the flexprice equilibrium output has gone up). (**??**) shifts up as shown in figure. Equilibrium is B. The asset price has gone up.

Another way of visualizing the impact of different shocks consists in simulating the impulse response function. For the sake of comparison we use the CGG parameterization, i.e. $\Gamma = 4$ so that $\sigma = 0.25$. This is admittedly a controversial assumption but it considered somehow acceptable in the

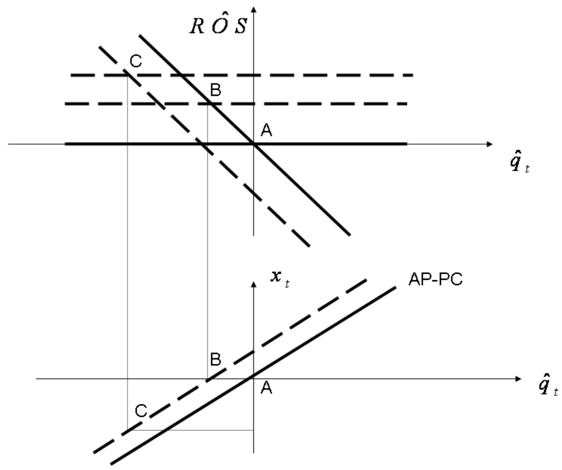


Figure 14: Effect of a demand shock on the asset price

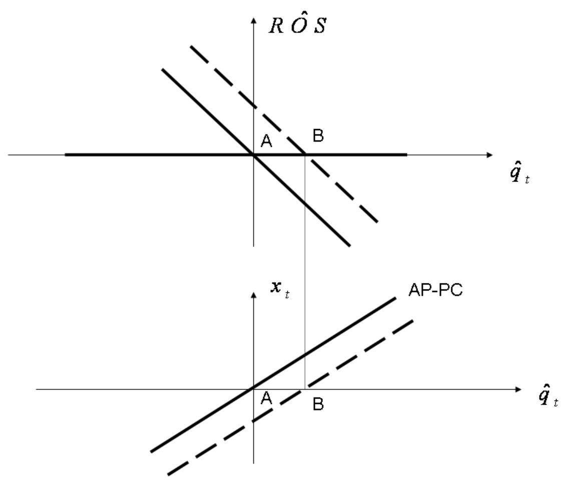


Figure 15: Effect of a productivity shock on the asset price

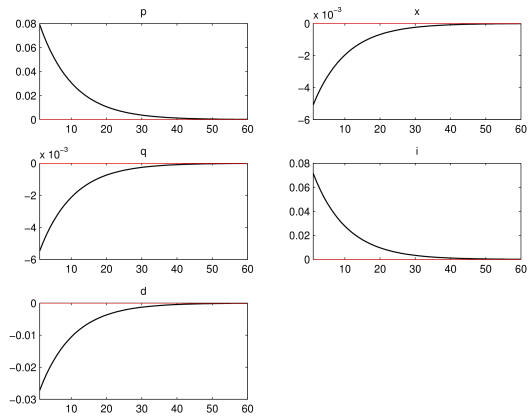


Figure 16: Effects of a supply shock.

literature. As to the other parameters, we assume

$$\begin{aligned} k &= 0.03; \eta = 2; \sigma = \Gamma^{-1}; \lambda = k(\eta + \sigma) = 0.075 \\ \alpha &= 1.2; \beta = 0.99; \psi = \rho = 0.9; \rho_z = 0.95 \end{aligned}$$

The effect of a temporary supply shock is shown in figure 16. The reaction of the central bank is contractionary as expected. The asset price and dividends follow the dynamic pattern of the output gap.

The impulse response function for a temporary demand shock is shown in figure 17. The dynamic patterns of all the variables of interest are similar to the ones recorded in the case a supply shock. This is not surprising because – as explained above – in the present context the demand shock plays the role of an indirect supply shock.

As a consequence of recursive structure of the system, a temporary tech-

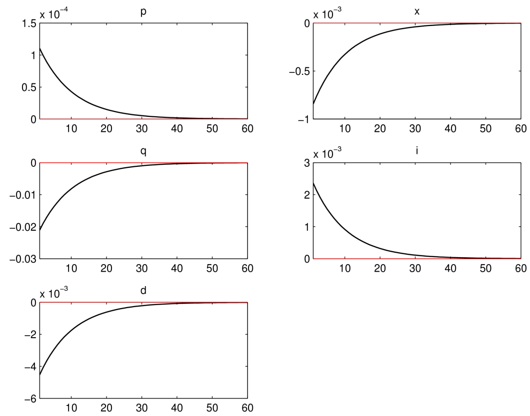


Figure 17: Effects of a demand shock

nology shock affects only the asset price and dividends as shown in figure 18.

8.3 Model O-1.2: dividends and asset prices

In this section we will explore an alternative approach to dividends, i.e. we assume that firms pay dividends on the basis of the following behavioral rule:[19]

$$d_t = q_t^{\delta} \tag{56}$$

where δ is the elasticity of dividends to asset prices.

Log-linearizing around the steady state we get: $\hat{d}_t = \delta \hat{q}_t$. In this case

[19] Of course, the amount of dividends paid out following (56) should be no greater than realized profits.

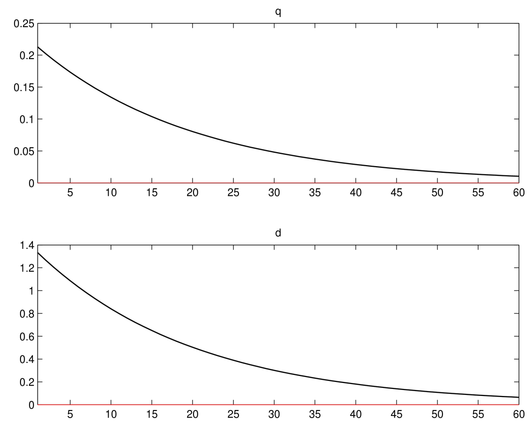


Figure 18: Effects of a technology shock

$\widehat{ROS}$ becomes:

$$\widehat{ROS} = \delta' E_t \hat{q}_{t+1} - \hat{q}_t \tag{57}$$

where $\delta' := \theta + (1 - \theta)\,\delta$.

In figure 19 we report the scatter diagram of dividends paid in the US by non-farm and non-financial enterprises and the Dow Jones from 1970 to 2008 on a log-log scale. In the data, the elasticity δ is smaller than (but close to) one.

Using this fact, we can assume that $\delta' := \theta + (1 - \theta)\,\delta$ is positive but smaller than one.[20] Equating (57) and (48) we get:

$$\hat{q}_t = \delta' E_t \hat{q}_{t+1} + \sigma(x_t - E_t x_{t+1} - g_t) \tag{58}$$

Incorporating rational expectations into the definition (57) above, we

[20] In fact δ' is a weighted average of 1 and δ.

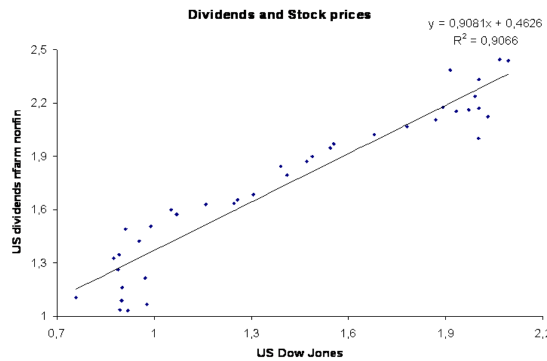


Figure 19: Dividends and Stock prices

have

$$\widehat{ROS} = -\left(1 - \rho\delta'\right)\hat{q}_t \tag{59}$$

Hence $\widehat{ROS}$ is decreasing with the current asset price. Equating (59) and (48) we get

$$\hat{q}_t = \frac{1-\rho}{1-\rho\delta'}\sigma x_t - \frac{\sigma}{1-\rho\delta'}g_t \tag{60}$$

which is the equation of the AP-PC with this particular definition of dividends. Qualitatively, the same discussion we have proposed at the end of section 8.1 applies also here.

We simulate the model using the following parameters:

$$\lambda = 0.075; \alpha = 1.2; \beta = 0.99; \Gamma = 4; \eta = 2; k = 0.003; \rho = \psi = 0.9$$

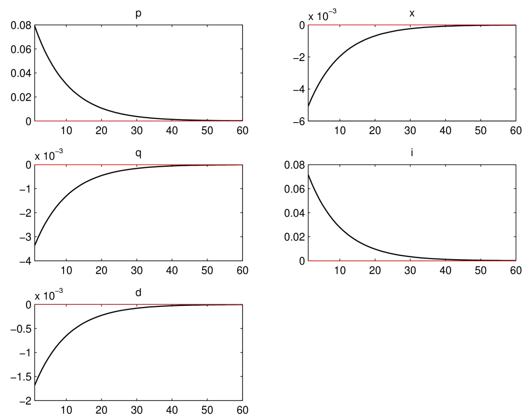


Figure 20: Supply shock. $\delta = 0.5$

The impulse response function for a temporary supply shock is shown in figure 20. The central bank reacts to an inflationary shock by raising the interest rate. Both the output gap and the asset price go down (and therefore also dividends decrease). Over time, all the variables converge, albeit with a certain persistence, to the steady state.

The impulse response function for a temporary demand shock is shown in figure 21. The central bank reacts to the shock by raising the interest rate. Contrary to the standard case, in this scenario the central bank is unable to offset completely the shock and to anchor output at the flexprice equilbrium. Both the output gap and the asset price go down (and therefore also dividends go down). Over time, all the variables converge, albeit with a certain persistence, to the steady state.

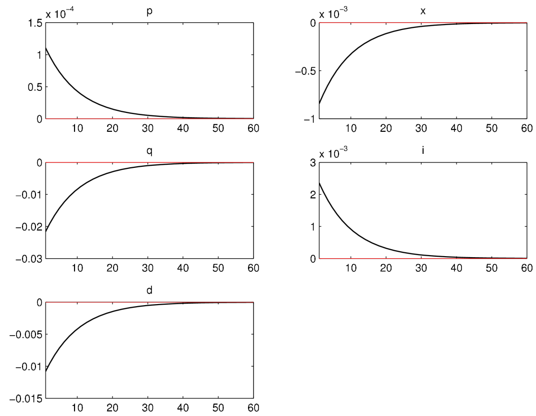


Figure 21: Demand shock. $\delta = 0.5$

9 The augmented inflation targeting rule

From model O-0, i.e. (40)(41)(21),after some algebra (see appendix E for details), we get the optimal *expectations based monetary policy rule:*

$$i_t = \gamma_\pi E_t \pi_{t+1} + \gamma_x E_t x_{t+1} + \gamma_u u_t + \gamma_g g_t \tag{61}$$

where

$$\begin{aligned}
\gamma_\pi &= 1 + \frac{\sigma}{1 - \sigma A_0} \times \frac{\lambda \beta}{\alpha + \lambda^2} \\
\gamma_x &= \frac{\sigma}{1 - \sigma A_0} \\
\gamma_u &= \frac{\sigma A_2}{1 - \sigma A_0} \\
\gamma_g &= \frac{\sigma}{1 - \sigma A_0}
\end{aligned}$$

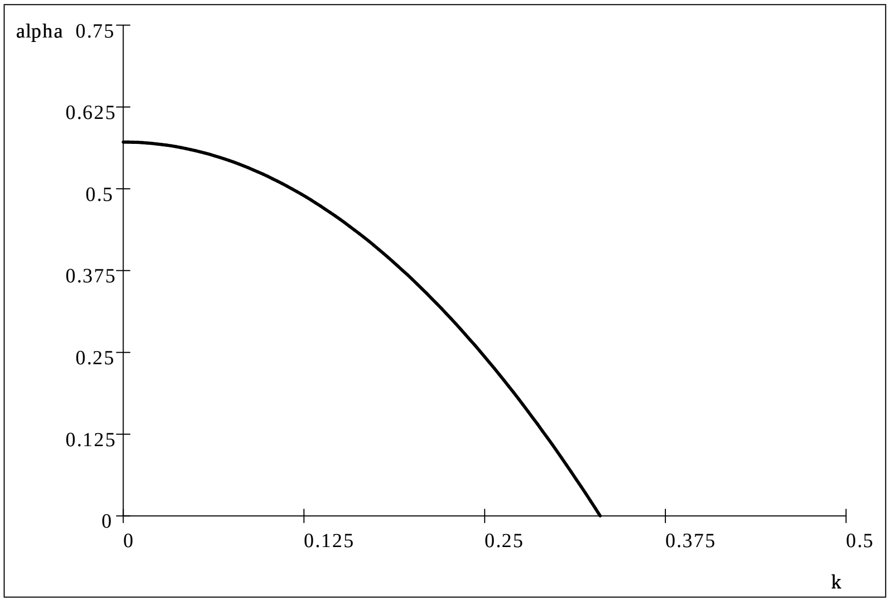
and $A_0 = \dfrac{\lambda k + (\eta + \sigma)^{-1}}{\alpha + \lambda^2}$.Notice that $sign\,(\gamma_x) = sign\,(\gamma_u) = sign\,(\gamma_g) = sign\,(1 - \sigma A_0)$.

Assumption 5: We assume that

$$\alpha > \hat{\alpha}_0 := \sigma\,(\eta + \sigma)^{-1} - \eta\,(\eta + \sigma)\,k^2 \tag{62}$$

Recalling that $\Gamma = 1/\sigma$ and $\lambda = k\,(\eta + \sigma)$ it is easy to verify that (62) implies $1 > \sigma A_0$.Hence assumption 5 assures that the response of the interest rate to a policy shock goes in the familiar direction $(\gamma_x > 0, \gamma_u > 0, \gamma_g > 0)$ and $\gamma_\pi > 0$.

It is interesting to note that 62 is satisfied if the central banker is not "too conservative", i.e. if the aversion to output dispersion is high enough, greater than a threshold $\hat{\alpha}_0$ which is in turn a function of k, the sensitivity of inflation to the cost channel. In figure 9, the condition is fulfilled for all the points of the (k, α) plane above the curve. Notice that for relatively high values of k this condition is always satisfied.



$\hat{\alpha}_0$ as a function of k

If we "translate" $E_t x_{t+1}$ into a function of expectations of inflation (see again appendix E for details) we obtain

$$i_t = \gamma_\pi E_t \pi_{t+1} + \gamma'_\pi E_t \pi_{t+2} + \gamma_u u_t + \gamma_g g_t \tag{63}$$

where

$$\begin{aligned}
\gamma_\pi &= 1 + \left(\frac{\lambda\beta}{\alpha + \lambda^2} - \frac{\lambda k + (\eta + \sigma)^{-1}}{k(\alpha - 1)} \right) \gamma_g \\
\gamma'_\pi &= \frac{\beta (\eta + \sigma)^{-1}}{(\alpha - 1) k} \gamma_g \\
\gamma_u &= \left[\frac{\rho (\eta + \sigma)^{-1}}{(\alpha - 1) k} + \frac{\lambda}{\alpha + \lambda^2} \right] \gamma_g \\
\gamma_g &= (\Gamma - A_0)^{-1}
\end{aligned}$$

Equation (63) is the optimal **augmented** *inflation targeting rule* according to which the central bank should respond to changes in the inflation expectations (formed in t) not only for t+1 but also for t+2.

In order to discuss the response of the central bank to expectations and shocks we focus first on the expression $(\Gamma - A_0)^{-1}$ which is the response γ_g of the policy rate to a demand shock and shows up in all the other coefficients of the inflation targeting rule. Notice that $\gamma_g > 0$ if $\Gamma > A_0$ which is satisfied if we adopt assumption 5, i.e. if the central banker is sufficiently "accommodating".

This is puzzling: the central banker should be "wet" (enough) for the policy rate to *increase* in response to a demand shock – i.e. for γ_g to be positive. Even when $\gamma_g > 0$, moreover, in this model the central bank responds *less aggressively* to a demand shock than in the standard New Keynesian setting – such as in Clarida-Galì-Gertler – where $\gamma_g^{CGG} = \Gamma^{-1}$. However, the rationale for this is clear. When a demand shock hits the economy, in fact, the attempt of the central bank to stabilize output by increasing the interest rate translates into an *(indirect) supply shock* – through the cost channel – that boosts inflation. A conservative central banker would "fight" against this inflation shock by raising the policy rate *less* than an accommodating central banker exactly because the former is more concerned with inflation than the latter.

The response of the central banker to expectations of inflation *one period ahead* γ_π can well be smaller than 1. As to the response of the central bank to expectations of inflation two periods ahead, it is worth noting that γ_π' is positive, i.e. the policy rate *increases* in response to $E_t\pi_{t+2}$ if $\alpha > 1$.

10 Instrument rule vs. optimal monetary policy

We are now in a position to sum up the discussion of monetary policy so far. We have basically three regimes:

- an instrument rule with no-reaction to asset prices (IR-NAP),
- an instrument rule with reaction to asset prices (IR-RAP)
- an optimal monetary policy rule (OR) which, by construction, does not respond to asset prices.

In the case of a supply shock, the policy prescription and the transmission mechanism is the same both in the IR and OR regimes. The central bank reacts by raising the interest rate, asset prices fall, the output gap turns negative, the return on shares increases (even if dividends fall both in case dividends are linked to output through profits and in case they are linked to the asset price through the asset price elasticity). The magnitude of the effect, however, is indeed different. When it takes into account asset price changes – i.e. in the IR-RAP case – the central bank usually mitigates the impact on price and quantity of its contractionary policy in a instrument rule setting. The IR-RAP regime, therefore, is characterized by milder variations in inflation and output. In the asymptotic case of an infinite reaction to asset prices, the policy prescription and the transmission mechanism are the same as in a CGG economy without cost channel.

Things are more complicated and more interesting in the case of a demand shock. To compare the effects of a demand shock in the IR and OR cases, notice first that the Augmented NK-PC of equation (44), being the consolidation of the Augmented NK-PC as defined in equation (41) and of the IS curve (42), can be employed not only in the OR case (as we have done in section 8) but also in the IR case. In the IR case, the intersection between the AD and AS schedules should lie on the Augmented NK-PC.

In order to avoid messy diagrams, suppose for the sake of the argument that the SEP of equation (43) and the AD curve (in the IR-NAP case) of equation (27) are graphically coincident. In other words, by a fluke the slopes of the downward sloping loci on the (x, π) plane in the OR setting (i.e. the SEP) and in the IR context (i.e. the AD curve) are the same. A demand shock shifts the AD curve up (see the dashed downward sloping black line) but the SEP down (see the dotted downward sloping line), as shown in figure 22. Moreover, the Augmented NK-PC of equation (44) wil shift up (see the dotted upward sloping line). The new short run equilibrium will be B in the IR case and C in the OR case.

Hence the same (demand) shock has opposite effects on the output gap. In the IR regime, it has a positive effect – as we are accustomed to think in a standard short run macro setting – while in the OR regime it has a negative effect. In the latter case, in fact, the downward sloping locus – i.e. the SEP – incorporates the attempt of the central bank to to stabilize output and inflation in a setting characterized by the presence of a cost channel. The contractionary reaction of the central bank to the shock translates into a downward shift of the locus.

When the central bank takes into account also asset prices, i.e. is in the IR-RAP case characterized by (q) in the figure, the new short run equilibrium will be in B(q). Of course both B and B(q) lie on the new Augmented

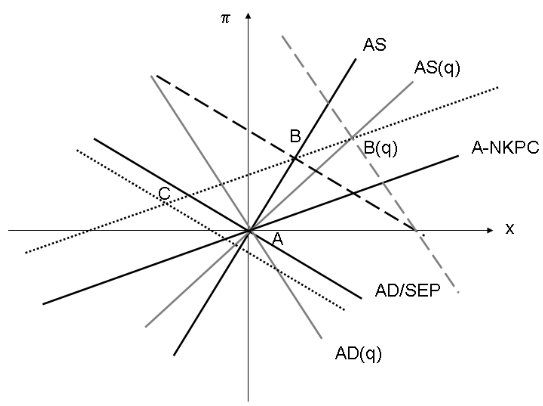


Figure 22: Effects of a demand shock in the IR and OR cases

NK-PC. Output grows more than in the IR-NAP case but inflation will be milder.

11 Learning

An interesting research question we want to answer is whether the properties of the basic NK model change – in terms of determinacy and learnability of the RE equilibrium – once we introduce asset prices through a cost channel. Carlstrom and Fuerst (2007) introduce asset prices in an augmented Taylor rule (but not in the structural equations for supply and demand in the economy). They show that in this case indeterminacy is more likely. Airaudo et al. (2007), instead, introduce asset prices in the demand side of the economy through a wealth effect and find that a central bank that responds to expected stock prices can induce multiple sunspot-driven equilibria. Moreover E-instability of the fundamental equilibrium is more likely.

In order to explore this issue we start from the system

$$y_t = AE_t y_{t+1} + Bw_t \tag{64}$$

where matrices A and B will depend on the specific policy rule adopted, y_t is the vector of state variables and w_t is the vector of exogenous shocks. We write the agents' Perceived Law of Motion (PLM) in matrix form as follows

$$y_t = Hw_t$$

Hence the Actual Law of Motion (ALM) is

$$y_t = (AHF + B)\, w_t$$

where F is the (diagonal) matrix of autoregressive parameters for the shocks.

This set-up implies the following map from PLM to ALM

$$\dot{H} = AHF + B - H \tag{65}$$

whose fix point ($\bar{H}$) represents the Rational Expectations Equilibrium (REE). REE is E-stable if the matrix differential equation (65) is locally stable at $\bar{H}$.

To evaluate stability, we have to vectorize the matrix differental equation and derive its Jacobian

$$J = \left(F' \otimes A - I\right). \tag{66}$$

The REE is E-stable iff the eigenvalues of J have all negative real parts.

We analyze determinacy and E-stability under two alternative policy rules, the Fundamentals based and the Expectations based policy rule.

11.1 Model O-1.1

In the REE solution of model O-0 x_t, π_t, i_t are linear functions of the shocks as shown in appendix D.1. The solution for i_t takes the form:

$$i_t = \gamma_u u_t + \gamma_g g_t + \gamma_z \hat{z}_t \tag{67}$$

where $\gamma_i, i = u, g$ are functions of the structural parameters and $\gamma_z = 0$.This is the *fundamentals based* optimal interest rate rule. The same rule applies to both model O-1.1 and O-1.2.

In the case of model O-1.1 we get the solution for $\hat{q}_t$ as a linear function of the shocks (demand, supply, productivity) from the Asset Price Phillips schedule (55) (see appendix D.3).

In order to evaluate determinacy and E-stability, we use the fundamentals based policy rule (67) to substitute out i_t from the system consisting of the IS curve (21), Augmented New Keynesian Phillips curve (24) and Asset Price Phillips curve (53) and obtain a 3 dimensional dynamic system of the form (64)with

$$
\begin{aligned}
y_t &= \begin{bmatrix} x_t \\ \pi_t \\ q_t \end{bmatrix}; w_t = \begin{bmatrix} u_t \\ g_t \\ \hat{z}_t \end{bmatrix} \\
A &= \begin{bmatrix} 1 & \Gamma & 0 \\ \lambda & \lambda\Gamma + \beta - k & 0 \\ (1-\theta)\left[1 + \dfrac{\beta}{\mu - \beta}(\eta + \sigma)\right] & 1 & \theta \end{bmatrix} \\
B &= \begin{bmatrix} -\Gamma\gamma_u & -\Gamma\gamma_g + 1 & 0 \\ (k - \lambda\Gamma)\gamma_u + 1 & (k - \lambda\Gamma)\gamma_g + \lambda & 0 \\ -\gamma_u & -\gamma_g & \dfrac{1+\eta}{\eta+\sigma}\rho_z \end{bmatrix}
\end{aligned}
$$

Since none of the variables are predetermined, determinacy requires all the eigenvalues of A to be within the unit circle. Finding the eigenvalues requires to find the roots of the characteristic polynomial, which is of 3rd degree. Not much can be said analytically, so we calibrate the model and solve numerically. For the parameters in the standard NK model, we use the parameterization suggested by Clarida, Gali and Gertler (2000)[21].E-stability requires the negative part of all eigenvalues of matrix J as defined in (66)to be negative.

Numerical results with the parameterization chosen show that the MSV REE is *indeterminate* and *not learnable* (E-unstable). This result is consistent with what found previously in the literature (see Evans and Honkapo-

[21] We also check the robustness of our results by considering the alternative calibrations suggested by McCallum and Nelson (1999) and Woodford (1999).

hja, 2003). In this sense, asset prices do not help solve the problem of indeterminacy of equilibria in the model when an optimal fundamental based policy rule is implemented by the central bank.

We move therefore to the analysis with the Expectations based rule. In order to find it, we use (40)(41)(21) to express i_t in terms of expectations only. We obtain:

$$i_t = \gamma_x E_t x_{t+1} + \gamma_\pi E_t \pi_{t+1} + \gamma_g g_t + \gamma_u u_t \tag{68}$$

with

$$\begin{aligned}
\gamma_x &= \frac{1}{\Delta\Gamma} \\
\gamma_\pi &= \frac{s}{\Delta} \\
\gamma_g &= \frac{1}{\Delta\Gamma} \\
\gamma_u &= -\frac{\lambda}{\Delta\Gamma\left(\alpha+\lambda^2\right)}
\end{aligned}$$

and

$$\begin{aligned}
\Delta &: = 1 - \frac{1}{\Gamma}\left[\frac{\lambda k}{\alpha+\lambda^2} + \frac{1}{\left(\alpha+\lambda^2\right)(\eta+\sigma)}\right] \\
s &: = 1 - \frac{1}{\Gamma}\left[\frac{\lambda k}{\alpha+\lambda^2} + \frac{1}{\left(\alpha+\lambda^2\right)(\eta+\sigma)} - \frac{\beta\lambda}{\alpha+\lambda^2}\right]
\end{aligned}$$

as shown in appendix E. Note that the Expectations based rule does not respond to expected future asset price, but only to expected output and inflation. This is due to the dichotomy inherent in the system.

Using policy rule (68) to solve out i_t we obtain a 3-dimensional system of the form (64) with matrices

$$A = \begin{bmatrix} 1-\Gamma\gamma_x & \Gamma(1-\gamma_\pi) & 0 \\ \lambda\left(1-\Gamma\gamma_x\right)+k\gamma_x & \beta+\left(\lambda\Gamma-k\right)\left(1-\gamma_\pi\right) & 0 \\ -\gamma_x+\left(1-\theta\right)\left[1-\xi\left(\eta+\sigma\right)\right] & 1-\gamma_\pi & \theta \end{bmatrix}$$

$$B = \begin{bmatrix} -\Gamma\gamma_u & -\Gamma\gamma_g+1 & 0 \\ \left(k-\lambda\Gamma\right)\gamma_u+1 & \left(k-\lambda\Gamma\right)\gamma_g+\lambda & 0 \\ -\gamma_u & -\gamma_g & \dfrac{1+\eta}{\eta+\sigma}\rho_z \end{bmatrix}$$

With the same parameterization chosen above, numerical results show that when an expectations based policy is employed the MSV REE is *determinate* and *learnable* (E-stable). This outcome is consistent with previous findings that an expectations based rule can help to solve the problem of indeterminacy.

11.2 Model O-1.2

The REE for x_t, π_t, i_t are the same as in the case of model O-1.1 because they are derived as solutions of model O-0 as shown in appendix D.1. Therefore also the fundamentals based rule is the same.

In the case of model O-1.2 we get the solution for $\hat{q}_t$ as a linear function of the shocks (demand, supply) from the Asset Price Phillips schedule (60) (see appendix D.4).

Using the fundamentals based policy rule:$i_t = \gamma_u u_t + \gamma_g g_t$ to substitute out i_t from the system consisting of the IS curve (21), Augmented New Keynesian Phillips curve (24) and Asset Price Phillips curve (58) we obtain a 3 dimensional dynamic system of the form (64) with matrices

$$A = \begin{bmatrix} 1 & \Gamma & 0 \\ \lambda & \lambda\Gamma + \beta - k & 0 \\ 0 & 1 & \delta' \end{bmatrix}$$

$$B = \begin{bmatrix} -\Gamma\gamma_u & -\Gamma\gamma_g + 1 \\ (k - \lambda\Gamma)\,\gamma_u + 1 & (k - \lambda\Gamma)\,\gamma_g + \lambda \\ -\gamma_u & -\gamma_g \end{bmatrix}$$

Numerical results with the parameterization chosen show that the MSV REE is *indeterminate* and *not learnable* (E-unstable).

Following the same procedure as before, we find the Expectations based rule, which is the same as (68). We obtain therefore the 3-dimensional system of the form (64) to be analyzed for determinacy and E-stability with matrices

$$A = \begin{bmatrix} 1 - \Gamma\gamma_x & \Gamma(1 - \gamma_\pi) & 0 \\ \lambda\,(1 - \Gamma\gamma_x) + k\gamma_x & \beta + (\lambda\Gamma - k)\,(1 - \gamma_\pi) & 0 \\ -\gamma_x & 1 - \gamma_\pi & [\theta + (1 - \theta)\,\delta] \end{bmatrix}$$

$$B = \begin{bmatrix} -\Gamma\gamma_u & -\Gamma\gamma_g + 1 \\ (k - \lambda\Gamma)\,\gamma_u + 1 & (k - \lambda\Gamma)\,\gamma_g + \lambda \\ -\gamma_u & -\gamma_g \end{bmatrix}$$

The numerical analysis shows that the MSV REE is determinate and learnable (E-stable) when an expectations based policy is employed.

Finally, in the particular case in which the firm does not pay dividends,

$\widehat{ROS} = E_t\hat{q}_{t+1} - \hat{q}_t$, from the numerical results we infer that the MSV REE is indeterminate and non-learnable (E-unstable) when using a fundamentals based policy rule,while if an expectations based policy rule is employed we find that the MSV REE is still indeterminate but it becomes learnable (E-stable).

In the present setting therefore we find that calibrating the model à la Clarida, Gali and Gertler, the REE solution is *indeterminate and E-unstable* when a fundamentals based policy rule is implemented – whatever approach we use to modelling dividends – while it generally becomes both determinate and stable if we use an expectations based rule. There is in fact one notable exception: In the no-dividend case, the problem of indeterminacy cannot be overcome by resorting to the expectations based rule (see table 1).

Table 1: Properties of the RE solution

	Fundamentals B.	Expectations B.
Dividends=Profits	indeterminate E-unstable	determinate E-stable
Dividends elastic to asset prices	indeterminate E-unstable	determinate E-stable
No-dividends	indeterminate E-unstable	indeterminate E-stable

12 Conclusion

In this paper we have presented a NK-DSGE model in which asset prices will be eventually incorporated into the NK Phillips curve. This is due to the assumption of a cost channel for monetary policy which is activated whenever monetary policy affects asset prices and therefore the return on shares. The latter in fact is the cost of external finance in our model. The novelty of the analysis consists in this peculiar treatment of financing decisions, which brings to the fore the relationship between pricing of goods and pricing of assets.

We analyse three monetary policy regimes:(a) an instrument rule with no-reaction to asset prices (IR-NAP), (b) an instrument rule with reaction to asset prices (IR-RAP) and (c) an optimal monetary policy rule (OR).

In the case of a supply shock, the policy prescription and the transmission mechanism are qualitatively the same both with an instrument rule and with in an optimal monetary policy setting. The results are more complicated but also more interesting in the case of a demand shock, which has opposite effects on the output gap. In the IR regime, it has a positive effect while in the OR regime it has a negative effect. In the IR-RAP case, output grows more than in the IR-NAP case but inflation will be lower.

We consider these results encouraging even if this is a very preliminary exploration of the properties of the model. We want to pursue an appropriate generalization because the model has to be enriched to explore more realistic environments. The most straightforward extension will consist in incorporating credit markets and credit market imperfections because they have a major role to play in our "story". The list of possible extensions that one can imagine, however, is quite long and will figure on top of our research agenda in the near future.

A The household's maximization problem

The representative household's problem consists in :

$$\max_{C_t,m_t,N_t,A_t,b_t} E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-\zeta} \left(m_{t+s}\right)^{1-\zeta} - \chi \frac{N_{t+s}^{1+\eta}}{1+\eta} \right]$$

subject to a sequence of budget constraints defined as it follows:

$$C_{t+s} + m_{t+s} + b_{t+s} + A_{t+s} q_{t+s} = w_{t+s} N_{t+s} + q_{t+s} A_{t-1+s} +$$

$$+ m_{t-1+s} \frac{1}{1+\pi_{t+s}} + \frac{1+i_{t-1+s}}{1+\pi_{t+s}} b_{t-1+s} + d_{t+s} A_{t-1+s}$$

The Lagrangian therefore is:

$$\begin{aligned} L \;=\; & E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-\zeta} \left(m_{t+s}\right)^{1-\zeta} - \chi \frac{N_{t+s}^{1+\eta}}{1+\eta} \right] + \\ & - E_t \sum_{s=0}^{\infty} \beta^s \lambda_{t+s} \left[C_{t+s} + m_{t+s} + b_{t+s} + A_{t+s} q_{t+s} + \right. \\ & - w_{t+s} N_{t+s} - m_{t-1+s} \frac{1}{1+\pi_{t+s}} - \frac{1+i_{t-1+s}}{1+\pi_{t+s}} b_{t-1+s} + \\ & \left. - q_{t+s} A_{t-1+s} - d_{t+s} A_{t-1+s} \right] \end{aligned}$$

Solving the above problem we get the following $FOCs$ that hold $\forall t$:

$$\frac{\partial L}{\partial C_t} = 0 \Longrightarrow \quad C_t^{-\sigma} - \lambda_t = 0$$

$$\frac{\partial L}{\partial m_t} = 0 \Longrightarrow \gamma \left(m_t\right)^{-\zeta} - \lambda_t + \beta \lambda_{t+1} \frac{1}{1+\pi_{t+1}} = 0$$

$$\frac{\partial L}{\partial N_t} = 0 \Longrightarrow \quad -\chi N_t^{\eta} + \lambda_t w_t = 0$$

$$\frac{\partial L}{\partial A_t} = 0 \Longrightarrow \quad -\lambda_t q_t + \beta \lambda_{t+1} \left(E_t q_{t+1} + E_t d_{t+1}\right) = 0$$

$$\frac{\partial L}{\partial b_t} = 0 \Longrightarrow \quad -\lambda_t + \beta \lambda_{t+1} \frac{1+i_t}{1+E_t \pi_{t+1}} = 0$$

From the above conditions we get the the Euler equations (4)(5)(6) and the asset price equation (7) as defined in section 2.

B Model I-1

We proceed to the solution of model I-0, which boils down to equations (27) and (28) by the method of undetermined coefficients. We "guess" the following:

$$\begin{aligned} x_t &= a_1 u_t + a_2 g_t \\ \pi_t &= b_1 u_t + b_2 g_t \end{aligned}$$

So that, under assumption 1,

$$\begin{aligned} E_t x_{t+1} &= \rho\left(a_1 u_t + a_2 g_t\right) \\ E_t \pi_{t+1} &= \rho\left(b_1 u_t + b_2 g_t\right) \end{aligned}$$

After some algebra we verify that the conjecture is indeed correct and we get the following solutions:

$$\begin{aligned} a_1 &= -\frac{\Gamma\left(\gamma_\pi - \rho\right)}{K_0} \\ a_2 &= \frac{1 - \beta\rho - k\left(\gamma_\pi - \rho\right)}{K_0} \\ b_1 &= \frac{1-\rho}{K_0} \\ b_2 &= \frac{\lambda}{K_0} \end{aligned}$$

where

$$K_0 := (1 - \beta\rho)(1 - \rho) + \frac{k}{\sigma}(\gamma_\pi - \rho)(\eta + \sigma\rho)$$

Under assumption 2 it turns out that $K_0 > 0$ and $a_1 < 0, a_2 > 0, b_1 > 0, b_2 > 0$.

The coefficients for the fundamentals based interest rate rule:

$$i_t = \gamma_u u_t + \gamma_u g_t$$

can be computed as follows: $\gamma_u = \gamma_\pi b_1 > 0; \gamma_g = \gamma_\pi b_2 > 0$.

Finally, from (32) follows that, under the special case of no-dividends, the solution for $\hat{q}_t$ is $\hat{q}_t = c_1 u_t + c_2 g_t$ where

$$\begin{aligned} c_1 &= -\frac{\gamma_\pi - \rho}{1 - \rho} b_1 = -\frac{\gamma_\pi - \rho}{K_0} \\ c_2 &= -\frac{\gamma_\pi - \rho}{1 - \rho} b_2 = -\frac{\lambda(\gamma_\pi - \rho)}{(1 - \rho) K_0} = \frac{\lambda}{1 - \rho} c_1 \end{aligned}$$

Therefore $c_1 < 0, c_2 < 0$.This completes the solution of model I-1.

The canonical model (without the cost channel) M I-0(c) consists of equations (30) and (31). The RE solution is:

$$\begin{aligned} a_1^c &= -\frac{\Gamma(\gamma_\pi - \rho)}{K_1} \\ a_2^c &= \frac{1 - \beta\rho}{K_1} \\ b_1^c &= \frac{1 - \rho}{K_1} \\ b_2^c &= \frac{\lambda}{K_1} \end{aligned}$$

where

$$K_1 := (1 - \beta\rho)(1 - \rho) + \frac{k}{\sigma}(\gamma_\pi - \rho)(\eta + \sigma)$$

The coefficients have the same sign as the corresponding coefficients of the model with the cost channel. Moreover $K_1 > K_0$. Therefore $|a_1^c| < |a_1|, a_2^c > a_2, b_1^c < b_1, b_2^c < b_2$.

The coefficients for the fundamentals based interest rate rule are: $\gamma_u = \gamma_\pi b_1^c > 0; \gamma_g = \gamma_\pi b_2^c > 0$.Finally, from (32) follows that, under the special case of no-dividends, $\hat{q}_t = -\frac{\gamma_\pi - \rho}{1 - \rho}(b_1^c u_t + b_2^c g_t)$. This completes the solution of model I-1(c).

C Model I-2

In order to find the RE solution of model I-2bis we "guess" the following:

$$\begin{aligned}
x_t &= a_1 u_t + a_2 g_t \\
\pi_t &= b_1 u_t + b_2 g_t \\
\hat{q}_t &= c_1 u_t + c_2 g_t
\end{aligned}$$

So that, under assumption 1,

$$\begin{aligned}
E_t x_{t+1} &= \rho(a_1 u_t + a_2 g_t) \\
E_t \pi_{t+1} &= \rho(b_1 u_t + b_2 g_t) \\
E_t \hat{q}_{t+1} &= \rho(c_1 u_t + c_2 g_t)
\end{aligned}$$

After some algebra we verify that the conjecture is indeed correct and we get the following solutions:

$$
\begin{aligned}
a_1 &= -\frac{\Gamma\left(\gamma_\pi-\rho\right)}{K_2} \\
a_2 &= \frac{(1-\rho)\left[1-\beta\rho-k\left(\gamma_\pi-\rho\right)\right]+\gamma_q(1-\beta\rho)}{(1-\rho)K_2} \\
b_1 &= \frac{1+\gamma_q-\rho}{K_2} \\
b_2 &= \frac{\lambda\left(1+\gamma_q-\rho\right)}{(1-\rho)K_2}=\frac{\lambda}{1-\rho}b_1 \\
c_1 &= -\frac{\gamma_\pi-\rho}{K_2}=\sigma a_1 \\
c_2 &= -\frac{\lambda\left(\gamma_\pi-\rho\right)}{(1-\rho)K_2}=\frac{\lambda}{1-\rho}c_1=\frac{\lambda}{1-\rho}\sigma a_1
\end{aligned}
$$

where

$$K_2 := (1-\beta\rho)\left(1+\gamma_q-\rho\right)+\frac{k}{\sigma}\left(\gamma_\pi-\rho\right)(\eta+\sigma\rho)$$

Under assumption 3 it turns out that $K_2 > 0$ and $a_1 < 0, a_2 > 0, b_1 > 0, b_2 > 0, c_1 < 0, c_2 < 0$.

We can determine the coefficients for the interest rate in the fundamentals based rule:

$$i_t = \gamma_u u_t + \gamma_u g_t$$

as follows: $\gamma_u = \gamma_\pi b_1 + \gamma_q c_1; \gamma_g = \gamma_\pi b_2 + \gamma_q c_2 \gamma_g$. Using the expressions above for b_i and c_i $i = 1, 2$ we get:

$$\begin{aligned}
\gamma_u &= \frac{\gamma_q \rho + \gamma_\pi (1-\rho)}{K_2} \\
\gamma_g &= \frac{\lambda \left[\gamma_q \rho + \gamma_\pi (1-\rho)\right]}{(1-\rho) K_2} = \frac{\lambda}{1-\rho} \gamma_u
\end{aligned}$$

This completes the solution of model I-2.

D Model O-1

D.1 Model O-0

We proceed to the solution of model O-0, which boils down to equations (43) and (44) by the method of undetermined coefficients. We "guess" the following:

$$\begin{aligned}
x_t &= a_1 u_t + a_2 g_t \\
\pi_t &= b_1 u_t + b_2 g_t
\end{aligned}$$

so that, under assumption 1,

$$\begin{aligned}
E_t x_{t+1} &= \rho (a_1 u_t + a_2 g_t) \\
E_t \pi_{t+1} &= \rho (b_1 u_t + b_2 g_t)
\end{aligned}$$

After some algebra we verify that the conjecture is indeed correct and we get the following solutions:

$$
\begin{aligned}
a_1 &= -\frac{\lambda(\eta+\sigma)}{(1-\beta\rho)K_3+\lambda^2(\eta+\sigma\rho)} \\
a_2 &= -\frac{\sigma}{K_3}\left\{\frac{\lambda^2(\eta+\sigma)(\alpha-1)}{(1-\beta\rho)K_3+\lambda^2(\eta+\sigma\rho)}+1\right\} \\
b_1 &= \frac{K_3}{(1-\beta\rho)K_3+\lambda^2(\eta+\sigma\rho)} \\
b_2 &= \frac{k\sigma(\eta+\sigma)(\alpha-1)}{(1-\beta\rho)K_3+\lambda^2(\eta+\sigma\rho)}
\end{aligned}
$$

where

$$K_3 := \alpha(\eta+\sigma)-\sigma(1-\rho)$$

Assuming $K_3 := \alpha(\eta+\sigma)-\sigma(1-\rho)>0$; i.e. $\alpha<\hat{\alpha}_1 := \frac{\sigma}{\eta+\sigma}(1-\rho)$, $a_1<0, a_2<0, b_1>0, b_2>0$ if $\alpha>1$ and viceversa.

Recall that

$$i_t = \rho\pi_t - \sigma(1-\rho)x_t + \sigma g$$

Therefore, the coefficients for the *fundamentals based optimal interest rate rule:*

$$i_t = \gamma_u u_t + \gamma_u g_t$$

can be computed as follows:

$$
\begin{aligned}
\gamma_u &= \rho b_1 - \sigma(1-\rho)a_1 \\
\gamma_g &= \rho b_2 - \sigma(1-\rho)a_2 + \sigma
\end{aligned}
$$

It turns out that $\gamma_u>0; \gamma_g>0$.

D.2 No dividends

From (49) follows that, *under the special case of no-dividends*, the solution for $\hat{q}_t$ is $\hat{q}_t = c_1 u_t + c_2 g_t$ where

$$
\begin{aligned}
c_1 &= \sigma a_1 \\
c_2 &= \sigma a_2 - \frac{1}{1-\rho}
\end{aligned}
$$

Therefore $c_1 < 0, c_2 < 0$.This completes the solution of model O-1 in the no dividends case.

D.3 Model O-1.1

In order to find the solution of model O-1.1 we guess

$$
\begin{aligned}
x_t &= a_1 u_t + a_2 g_t + a_3 \hat{z}_t \\
\pi_t &= b_1 u_t + b_2 g_t + b_3 \hat{z}_t \\
\hat{q}_t &= c_1 u_t + c_2 g_t + c_3 \hat{z}_t
\end{aligned}
$$

Since the system is recursive, $a_i, b_i$with $i = \{1, 2\}$ are the same as in model O-0 (see above) and $a_3 = b_3 = 0$.

From (55) follows that the solution for $\hat{q}_t$ is $\hat{q}_t = c_1 u_t + c_2 g_t$ where

$$
\begin{aligned}
c_1 &= \frac{\Omega\rho + \sigma(1-\rho)}{1-\theta\rho} a_1 \\
c_2 &= \frac{\Omega\rho + \sigma(1-\rho)}{1-\theta\rho} a_2 - \frac{\sigma}{1-\theta\rho} \\
c_3 &= \frac{\Omega\rho_z + \sigma(1-\rho_z)}{1-\theta\rho_z} a_3 + \frac{(1-\theta)(1+\eta)}{(\eta+\sigma)(1-\theta\rho_z)}\rho_z
\end{aligned}
$$

where $\Omega = (1-\theta)\left[1 + \frac{\beta}{\mu - \beta}(\eta + \sigma)\right]$ (recall that $\beta = \theta$ and $\Gamma = \frac{1}{\sigma}$).

D.4 Model O-1.2

In order to find the solution of model O-1.2 we guess

$$\begin{aligned} x_t &= a_1 u_t + a_2 g_t \\ \pi_t &= b_1 u_t + b_2 g_t \\ \hat{q}_t &= c_1 u_t + c_2 g_t \end{aligned}$$

Since the system is recursive, $a_i, b_i$with $i = \{1, 2\}$ are the same as in model O-0 (see above).

$$\begin{aligned} c_1 &= \frac{1-\rho}{1-\rho\delta'}\sigma a_1 \\ c_2 &= \frac{\sigma}{1-\rho\delta'}\left[(1-\rho)\, a_2 - 1\right] \end{aligned}$$

E The expectations based rule

The system consists of equations (40)(41)(21), which we reproduce here for the reader's convenience

$$\begin{aligned} x_t &= -\frac{\lambda}{\alpha}\pi_t - \frac{i_t - E_t\pi_{t+1}}{\alpha(\eta+\sigma)} \\ \pi_t &= \lambda x_t + k\,(i_t - E_t\pi_{t+1}) + \beta E_t\pi_{t+1} + u_t \\ x_t &= E_t x_{t+1} - \Gamma\,(i_t - E_t\pi_{t+1}) + g_t \end{aligned}$$

Solving the first 2 equations we get:

$$x_t = -A_0 i_t + A_1 E_t \pi_{t+1} - A_2 u_t \tag{69}$$

and

$$\pi_t = A_3 i_t + A_4 E_t \pi_{t+1} + A_5 u_t \tag{70}$$

where

$$\begin{aligned}
A_0 &= \frac{\lambda k + (\eta + \sigma)^{-1}}{\alpha + \lambda^2} \\
A_1 &= \frac{-\lambda (\beta - k) + (\eta + \sigma)^{-1}}{\alpha + \lambda^2} = \frac{-\lambda \beta}{\alpha + \lambda^2} + A_0 \\
A_2 &= \frac{\lambda}{\alpha + \lambda^2} \\
A_3 &= \frac{(\alpha - 1) k}{\alpha + \lambda^2} \\
A_4 &= \frac{\alpha \beta + (\alpha - 1) k}{\alpha + \lambda^2} \\
A_5 &= \frac{\alpha}{\alpha + \lambda^2}
\end{aligned}$$

In order to derive the *expectations based rule* we plug (69) into the IS curve (21). After rearranging we obtain:

$$i_t = \gamma_\pi E_t \pi_{t+1} + \gamma_x E_t x_{t+1} + \gamma_u u_t + \gamma_g g_t$$

where

$$\begin{aligned}
\gamma_\pi &= 1+\frac{\sigma}{1-\sigma A_0}\times\frac{\lambda\beta}{\alpha+\lambda^2} \\
\gamma_x &= \frac{\sigma}{1-\sigma A_0} \\
\gamma_u &= \frac{\sigma A_2}{1-\sigma A_0} \\
\gamma_u &= \frac{\sigma}{1-\sigma A_0}
\end{aligned}$$

and $1-\sigma A_0 > 0$ provided $\Gamma > \frac{\lambda k+(\eta+\sigma)^{-1}}{\alpha+\lambda^2}$.

In order to obtain the optimal inflation targeting rule, we update (69), take the expectation and using the law of iterated projections and recalling that $E_t u_{t+1} = \rho u_t$ we get:

$$E_t x_{t+1} = -A_0 E_t i_{t+1} + A_1 E_t \pi_{t+2} - A_2 \rho u_t$$

We can retrieve $E_t i_{t+1}$ from (70), taking the expectation and using the law of iterated projections:

$$E_t i_{t+1} = \frac{E_t \pi_{t+1}}{A_3} - \frac{A_4}{A_3} E_t \pi_{t+2} - \frac{A_5}{A_3}\rho u_t$$

Substituting the second expression into the first one and rearranging:

$$E_t x_{t+1} = -\frac{A_0}{A_3} E_t \pi_{t+1} + \left(A_1 + A_0\frac{A_4}{A_3}\right) E_t \pi_{t+2} + \left(A_0\frac{A_5}{A_3} - A_2\right)\rho u_t \quad (71)$$

Substituting (69) and (71) into (**??**) and solving for it yields:

$$i_t = \gamma_\pi E_t \pi_{t+1} + \gamma'_\pi E_t \pi_{t+2} + \gamma_u u_t + \gamma_g g_t$$

where

$$\begin{aligned}
\gamma_{\pi} &= 1+\left(\frac{\lambda\beta}{\alpha+\lambda^{2}}-\frac{\lambda k+(\eta+\sigma)^{-1}}{k(\alpha-1)}\right)\left(\Gamma-\frac{\lambda k+(\eta+\sigma)^{-1}}{\alpha+\lambda^{2}}\right)^{-1} \\
\gamma_{\pi}' &= \frac{\beta(\eta+\sigma)^{-1}}{(\alpha-1)k}\left(\Gamma-\frac{\lambda k+(\eta+\sigma)^{-1}}{\alpha+\lambda^{2}}\right)^{-1} \\
\gamma_{u} &= \left[\frac{\rho(\eta+\sigma)^{-1}}{(\alpha-1)k}+\frac{\lambda}{\alpha+\lambda^{2}}\right]\left(\Gamma-\frac{\lambda k+(\eta+\sigma)^{-1}}{\alpha+\lambda^{2}}\right)^{-1} \\
\gamma_{g} &= \left(\Gamma-\frac{\lambda k+(\eta+\sigma)^{-1}}{\alpha+\lambda^{2}}\right)^{-1}
\end{aligned}$$

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