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Advertising and price signaling of quality in a duopoly with endogenous locations*

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Abstract

We analyze a two-sender quality-signaling game in a duopoly model where goods are horizontally and vertically differentiated. While locations are chosen under quality uncertainty, firms choose prices and advertising expenditures being privately informed about their types. We show that pure price separation is impossible, and that dissipative advertising is necessary to ensure existence of separating equilibria. Equilibrium refinements discard all pooling equilibria and select a unique separating equilibrium. When vertical differentiation is not too high, horizontal differentiation is maximum, the high-quality firm advertises, and both firms adopt prices that are distorted upwards (compared to the symmetric-information benchmark). When vertical differentiation is high, firms choose identical locations and ex post, only the high-quality firm obtains positive profits. Incomplete information and the subsequent signaling activity are shown to increase the set of parameters values for which maximum horizontal differentiation occurs.

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Key-words: advertising, location choice; quality; incomplete information; multi-sender signaling game.

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1 Introduction

Markets for experience goods are characterized by the uncertainty consumers face regarding the utility they will get from purchasing and consuming these goods. In many situations, consumers do not observe the objective quality of the commodity they intend to buy. In order to avoid the standard "lemons" problem, emphasized by Akerlof (1970), producers may find incentives to transmit some information to consumers. Since the pioneering work of Spence (1973), an extensive literature has looked at the various strategies firms can employ to reveal information about their products' quality. In particular, in their pathbreaking paper, Milgrom and Roberts (1986) show how a monopolist can signal the high quality of a new experience good by distorting his strategy (price, advertising intensity) compared to the benchmark situation of complete information.[1] Hence, signaling high quality is costly for the monopolist because distorting strategies implies a reduction of profits.

However, the literature we refer to in analyzing a price signaling game has almost exclusively focused on the monopoly case, where a producer with private information about his high quality only "compete" with his "ghost" of low quality. To our knowledge, very few papers are studying a similar price signaling game in a competitive environment. Among these few, we can cite Kihlstrom and Riordan (1984), Bagwell (1990), Bagwell and Ramey (1991), Fluet and Garella (2002), Hertzendorf and Overgaard (1998, 2001a, 2001b, 2002). As Hertzendorf and Overgaard (1998) mention, this emphasis on the monopoly case does not rely on the belief that monopolized markets are more realistic, or common, neither does it stem from the intuition that a monopoly is more likely to signal its type. The lack of work on oligopolistic markets reflects the absence of a model, or a game, as tractable and coherent as the monopoly game. Each of the papers treating the oligopoly case studies a particular setting, in order to keep a balance between introducing competition and ensure tractability.

In this paper, we contribute to this emerging literature by considering the issue of signaling quality in a duopolistic setting. More precisely, we study the impact of (horizontal) product differentiation on the existence and features of separating and pooling equilibria. We analyze a model where both firms choose their locations (variety) before competing in price and purely dissipative advertising. One of the duopolists will have a quality advantage, which is

[1] See also Bagwell and Riordan (1991).

exogenously selected by Nature, but the identity of the better quality product will remain unknown to consumers before actual consumption.[2] We will assume that qualities are perfectly negatively correlated, to illustrate the fact that consumers, knowing goods will be vertically differentiated, face some uncertainty concerning which firm produces the better quality, and to figure out whether the high-quality firm will be able to provide them with enough information to be unambiguously identified. An important feature of our model is that the duopolists will have to choose their locations under uncertainty, that is before knowing which quality they will be able to produce. Hence, only the probability distribution of quality is known at the location-choice stage.

This assumption about the timing can be justified as follows.[3] Consider a market on which two firms enter into a Research and Development race. One of the duopolists will win this race and be able to benefit from a new technology which will ensure the production of a good of quality higher than the competitor's. Nevertheless, firms have to make a long-term decision as to the variety of their goods (for example, one can think of a choice of ingredients, components, or marketing or distribution channels) before knowing the result of the R&D race and thus their exact quality. After location decisions are made, which firm wins the R&D race and thus produces which quality is exogenously determined. Finally, both firms simultaneously compete in price and dissipative advertising. Consumers perfectly observe locations, prices and advertising expenditures, but not qualities. Note that our assumption that quality is revealed to firms only after locations are chosen rules out the possibility that duopolists may use location as a signal of their types.[4]

Another motivation for this timing may be derived from some stylized facts observed in several sectors. For instance, producers of agricultural goods or food commodities will have to make long-term decision on the variety of their produce, before knowing the actual quality of the final good. Indeed, quality is often variable for many food commodities because it is linked to the quality of raw inputs which is largely influenced by exogenous factors such as

[2] It is worth noting that Gabszewicz and Grilo (1992) studied price competition in a duopoly when consumers are uncertain about which firm sells which quality. A significant difference with our paper and H&O's is that Gabszewicz and Grilo assume that consumers' beliefs are *exogenous*, such that no inference from prices can be considered.

[3] We thank Esther Gal-or for suggesting this motivation of the timing of the game.

[4] This alternative timing where duopolists may signal their type through two instruments, location and price, is studied by Vettas (1999). Vettas shows that a high-quality firm will signal its type by choosing to locate closer to its rival (relative to the complete information benchmark).

climatic ones (seasonal quality of fresh inputs such as fruits). Nevertheless, food processors have to take observable long term decisions such as packaging, content per unit, the type of retailing channel (supermarkets versus specialized retailers), before processing the product.

To further motivate our timing, consider also the often invoked example of restaurants with no established reputation in a tourist resort. Restaurant owners might have to choose the style of cuisine (Italian versus French cuisine) before hiring the chef whose ability will determine the quality of food. We argue that this commitment to a specific variety, i.e. a choice of location, is a chance for a firm to benefit from a specific market (a niche), on which it might benefit from enough market power to be able to signal through prices the true quality of the product to consumers. Nonetheless, at the time the firm chooses a variety or location, it does not know what the actual quality of his commodity will be at the final stage.

Our main results are as follows. We first prove that any separating equilibrium involves strictly positive advertising expenditures by the high-quality firm, whatever the quality differential. This first result is interesting in itself. Contrary to Fluet and Garella (2002) or Hertzendorf and Overgaard (2001a), dissipative advertising is necessary to signal high quality, even when the quality differential is high, and despite the fact that marginal costs of production increase with quality. This latter characteristic of the model would ensure existence of pure price separation in a monopoly game, but does not in the present context. Second, our results show that both low- and high-quality prices will be distorted upwards, compared to the symmetric information benchmark. This finding contrasts with Hertzendorf and Overgaard (2001a) where it is shown that prices may be distorted downwards (compared to the relevant symmetric information case) when the quality differential is relatively small and the separating equilibrium involves positive advertising expenditures to reveal high quality.

Were advertising prohibited, or just impossible, the high-quality firm would have to distort its price, downward or upward, as the only way to reveal its actual type. An upward distortion is excluded because it would result in such a high price that all consumers would give up purchasing the high-quality good and switch to the low-quality one. Signaling high quality with a high price would drive all costumers away and thus be too costly for the producer. A downward price distortion would have the opposite effect: all consumers would purchase the high-quality good, and the low-quality firm would have no demand to serve and hence would make zero profit. As a consequence, this latter firm has no opportunity cost of cheating and mimicking its rival's choice. The high-quality firm cannot prevent its low-quality competitor from mimicking its choice of price, and thus a low price cannot constitute a separating strategy to reveal high quality.

As is usually the case in signaling games, a multitude of separating and pooling equilibria of the price-advertising subgame are characterized. In order to solve the first stage of the game, it is crucial to be able to select the ones that are the most plausible, on which firms will most likely focus. We introduce two refinements criteria, which require consumers out-of-equilibrium beliefs to be first resistant to a deviation to another equilibrium strategy, then intuitive. The first requirement places restrictions on beliefs after a deviation off the equilibrium path that is still consistent with an alternative separating equilibrium. Consumers beliefs should reflect the opportunity of receiving two messages, such that if they observe a deviation by one firm, they can still rely on the information transmitted by the non-deviating one, particularly when the deviation is to an action that could belong to another separating profile. This refinement yields a unique separating equilibrium profile where the high-quality firm reveals its type with the least costly pair of price and advertising expenditures. This strategy profile is also characterized by an upward distortion in both low- and high-quality prices, compared to the symmetric information benchmark. The second requirement on beliefs is reminiscent from the Intuitive Criterion (Cho and Kreps 1987), and discards all pooling equilibria as implausible.

Equilibrium selection enables us to characterize equilibrium location choices. Maximal or minimal horizontal differentiation occurs, depending on parameters. In the present model, a crucial parameter is the relative degree of vertical differentiation, measured by the ratio between quality discrepancy and transportation cost. When this parameter's value is lower than a certain threshold, horizontal differentiation is maximum. Above this threshold, quality differential is so large that firms choose identical locations: The perspective of obtaining the entire market outweighs the incentive to horizontally differentiate.

Our results also shed some light on the impact of asymmetric information on horizontal differentiation. On this topic, Boyer *et al.* (1994, 1995), Bester (1998) and Vettas (1999) all conclude that incomplete information about the type of a duopolist yields less horizontal differentiation. On the contrary, in our setting, the likelihood of observing maximum differentiation increases under incomplete information. It then appears that asymmetric information allows the low-quality firm to better resist an increasingly efficient competitor. Note however that in the papers cited above, location is distorted in order to signal a firm's type, which is not the case in our framework.

The remainder of the paper is organized as follows. Section 2 presents the model and Section 3 is devoted to the benchmark situation of symmetric information. Section 4 introduces asymmetric information and examines the interactions between location choices and signaling through price and advertising. Section 5 concludes.

2 The Model

Consider a continuum of consumers whose locations are uniformly distributed over the unit interval $[0, 1]$. Two firms, labeled by $i = 1, 2$, choose a location $y_i \in [0, 1]$. Without loss of generality, we assume that $y_1 \leq y_2$.

A consumer located in $x \in [0, 1]$ gets a utility from purchasing a unit of good i that is specified as

$$U(p_i, q_i) = R + q_i - p_i - t(x - y_i)^2, \text{ for } i = 1, 2, \tag{1}$$

where $R > 0$ is the basic utility obtained by a consumer purchasing any of the two goods, and q_i and p_i represent respectively the quality and the price of the good. A consumer's transportation cost of visiting a firm located in y_i is quadratic, and the parameter t reflects the degree of horizontal product differentiation.[5] We assume that R is sufficiently large that consumers always choose to purchase either from firm 1 or firm 2. Notice that we also assume perfect homogeneity of consumers with respect to their valuation of quality.

We assume that $q_1 \neq q_2$: the goods are exogenously vertically differentiated. For the sake of simplicity, we will suppose that quality can only take one of two values, L or H. Therefore, we will have $q_i = L$ and $q_j = H$, where L and H stand for low and high quality respectively. We will denote $\Delta \equiv H - L$ the quality discrepancy. The assumption that

[5] This specification of the utility function is widely retained in the literature. See for instance Bester (1998), Vettas (1999), Christou and Vettas (2005).

qualities are perfectly negatively correlated illustrates the interesting case where consumers are faced with two goods of distinct qualities, one for which they are all willing to pay more, everything else being equal. The issue at stake is whether or not they will be provided with enough information to avoid being wronged by the firm producing low quality.[6]

Firms have constant marginal cost of production $c(q_i)$, and we assume that this cost is increasing with quality. Indeed, producing high quality requires more inputs, or inputs of better quality that are more costly than the ones necessary to produce low quality. We normalize the unit cost of low quality $c(L)$ to 0 and we denote the unit cost of high quality by $c(H) = c > 0$. We assume that $\Delta > c$, which basically means that the production of high quality is socially valuable.[7] Adverting expenditures, a_i, enter profit functions as a sunk cost and do not influence demand functions.

For further reference, we introduce the following crucial parameter, $\rho \equiv \frac{\Delta - c}{t}$. This ratio captures the importance of vertical differentiation relative to horizontal differentiation, and will be referred to as the *relative quality differential.*

We analyze the following game:

- In a first stage, firms simultaneously choose their locations, while they are still uncertain of their respective type.
- Then, in a second stage, they learn the quality of the good they will produce.
- Finally, they simultaneously compete in price and dissipative advertising, and consumers decide which good to purchase.

The choice of location is made under uncertainty as to the quality of the good. In stage 2, firms privately learn their own and rival's type. Consumers do not directly observe qualities, but

[6]Recall that the production of high-quality results from a R&D race that only one firm can win. The winning firm will offer quality H greater than its rival's quality, L. Another way to justify the negative correlation between the two qualities is the following. Suppose that firms qualities q_i, $i = 1, 2$, were two independent random variables, each following a normal cdf $N(0, \sigma_i)$. Then, $(q_i - q_j)$ would also follow a normal cdf, with the probability that $q_i = q_j$ equal to zero. Furthermore, Prob $(q_i - q_j > 0) = 0.5$. To represent this situation in a simple and tractable way, we consider only two cases, $q_i = L$ and $q_j = H$, $i \neq j$, with Prob $(q_i = H) = \frac{1}{2}$, such that Prob $(q_i - q_j > 0) = 0.5$, and will discuss how our results depend on $\Delta = H - L$.

[7]In the inverse case where $c > \Delta$, it can be shown that the problem rapidly degenerates to one in which only the low-quality firm is profitable. In particular, it can be shown that the set of parameter values for which a separating equilibrium could exist requires that $t > \Delta$. This latter inequality implies that horizontal differentiation effects dominate vertical differentiation ones. In such a case, the high-quality firm is never going to be willing to use costly signals to reveal a quality that consumers will not find interesting to purchase anyway.

they know that products are vertically differentiated. In other words, consumers perfectly observe locations but remain uninformed about which firm offers the highest quality. The question is whether firms will manage to signal their types at the last stage.

There is no reason to believe a priori that one firm is more likely to produce high quality, hence we will assume that Nature draws firms' types from a prior, commonly known, probability distribution Prob $(q_i = H) = 1/2$. Since firms are *ex ante* symmetric, and consumers' preferences follow a symmetric density, it is quite natural to think that incentives to horizontally differentiate will be symmetric, and that we could focus on symmetric locations, $y_1 = 1 - y_2 \equiv y$. As a matter of fact, symmetric equilibria will encompass all the qualitatively interesting situations. Christou and Vettas (2005) analyze how locations are chosen by duopolists under quality uncertainty, in a model very similar to ours.[8] Their results exhibit equilibria in which horizontal differentiation is either maximum or minimum. In the latter case, firms can choose identical locations which can be (almost) anywhere on the linear city. They do not find any equilibria in which firms would locate asymmetrically by choosing distinct strategies.[9] By restricting ourselves to symmetric equilibria, we restrict ourselves to minimum differentiation equilibria where firms are necessarily located at the center. We exclude equilibria where firms are located at some other point on the line. Nevertheless, it will be easy to check that those latter equilibria would be qualitatively identical to the symmetric one we derive. When differentiation is minimum, whether firms are located at the center or at some other point on the line does not influence equilibrium prices and profits. Therefore, assuming symmetric locations from the start simplifies the exposition of the model and of its resolution, while still capturing all qualitatively interesting outcomes.

Considering only symmetric locations also means that the identity of the firm (that is, whether we are considering firm 1 or firm 2) is irrelevant when we look for firms' optimal strategies. What matters is the quality each firm produces. Therefore, we redefine location as $y \equiv y_1 = 1 - y_2$, and denote p_L and p_H prices of the low- and high-quality firm, respectively.

[8] They assume that the random variable $q_i - q_j$ is uniformly distributed on some interval whereas we assume that it follows a Bernoulli distribution.

[9] We find no reasons to believe that our setting would induce such asymmetric equilibria.

3 Benchmark case: symmetric information

Building on Vettas (1999) and Christou and Vettas (2005), this section proposes a solution to the game under symmetric information. It has two purposes. One is to introduce some notations and to derive some results that will turn out to be useful later in the paper, when solving the game under incomplete information. The other one is to show what are the forces driving firms to choose their locations and prices.

We solve the game by backward induction, starting with the last stage of the game, where firms simultaneously compete in price and advertising, for given locations y and $1-y$. Here, firms as well as consumers observe the actual quality of the two goods. Note that since advertising is purely dissipative, it has no effect on demand, such that firms will optimally choose not to advertise. Then in a second step we will solve the location stage, where both firms rationally anticipate equilibrium prices and choose locations, not knowing their types. We analyze the case where Firm 1 produces low quality and Firm 2 high quality. As the game is symmetric, we can denote p_L and p_H the respective prices of the goods of low and high quality.

3.1 Price equilibria

Demand functions depend on the location of the marginal consumer who is indifferent between the two products. We assume that parameter values are such that, in equilibrium, the market is entirely covered. We denote $z^B(p_L, p_H) \in [0,1]$ the location of the marginal consumer.[10] Its identity is given by

$$L - p_L - t(z^B - y)^2 = H - p_H - t(z^B - 1 + y)^2.$$

Solving this equation in z^B gives

$$z^B(p_L, p_H) = \begin{cases} 0 & \text{if } p_L \geq p_H - \Delta + td \\ 1 & \text{if } p_L \leq p_H - \Delta - td \\ \frac{1}{2} + \frac{p_H - p_L - \Delta}{2td} & \text{otherwise.} \end{cases} \quad (2)$$

[10] In what follows, the superscript B denotes benchmark values of all variables.

where $d \equiv 1 - 2y$ denotes the distance between the two firms.

We denote profit functions $\pi_L^B = p_L z^B$ and $\pi_H^B = (p_H - c)(1 - z^B)$. Looking for a Nash equilibrium in price, we first obtain each firm's best response to its competitor's price:

$$\begin{aligned} p_L^B(p_H) &= \frac{1}{2}\left[p_H + td - \Delta\right], \\ p_H^B(p_L) &= \frac{1}{2}\left[p_L + td + \Delta + c\right]. \end{aligned}$$

We find that equilibrium prices are

$$p_L^B(d) = \begin{cases} 0 & \text{if} \quad d \leq \frac{\rho}{3}, \\ \frac{1}{3}\left[3td - \Delta + c\right] & \text{otherwise,} \end{cases} \tag{3}$$

and

$$p_H^B(d) = \begin{cases} \Delta - td & \text{if} \quad d \leq \frac{\rho}{3}, \\ \frac{1}{3}\left[3td + \Delta + 2c\right] & \text{otherwise.} \end{cases}$$

The condition on d means that if firms are too close to each other (the distance between them is too small) the low-quality firm will not have a positive market share. In this case, the high-quality firm charges a limit price and gets the entire demand. Note that there is a market for the low quality good if $d > \frac{\rho}{3}$, which can happen only if $\rho < 3$.

Note also that if qualities were identical (i.e., $\Delta = 0$ and $c = 0$), then price competition would correspond to a standard Hotelling setting with quadratic transportation costs and homogeneous products (see D'Aspremont *et al.* (1979)). When qualities differ (i.e. $\Delta > 0$, $c > 0$), each firm obtains a strictly positive demand only when firms are located sufficiently far apart. If firms are too close to each other, such that $d < \frac{\rho}{3}$, the high-quality firm gets the whole demand. In particular if $d = 0$, then $p_L^B = 0$ and $p_H^B = \Delta$, and the low-quality firm is excluded. This is the standard outcome in a duopoly where differentiation is only vertical and consumers have homogenous tastes for quality.

Equilibrium profits, for given locations, are:

$$\pi_L^B(d) = \begin{cases} \frac{t}{18d}\left[3d - \rho\right]^2 & \text{if} \quad d > \frac{\rho}{3}, \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

and

$$\pi_H^B(d) = \begin{cases} \frac{t}{18d}\left[3d+\rho\right]^2 & \text{if} \qquad d > \frac{\rho}{3}, \\ t(\rho - d) & \text{otherwise.} \end{cases} \tag{5}$$

3.2 Location equilibria

We now turn to the first stage of the game, where firms choose their locations while still uncertain about the quality of the good they will produce later. A firm's expected profit, evaluated at the beginning of the game, is

$$E\Pi(d) = \frac{1}{2}\pi_L^B(d) + \frac{1}{2}\pi_H^B(d)$$

An equilibrium where both firms are active is $d > \rho/3$ maximizing $E\Pi(d)$. Using (4) and (5), we have

$$E\Pi(d) = \frac{1}{2}\left[\frac{t}{18d}(3d-\rho)^2 + \frac{t}{18d}(3d+\rho)^2\right] = \frac{t}{18d}(9d^2+\rho^2), \tag{6}$$

Differentiating (6) with respect to d yields

$$\frac{dE\Pi(d)}{dd} = \frac{1}{2}\left(1 - \frac{\rho^2}{9d^2}\right).$$

Note that for $d > \rho/3$, $\frac{dE\Pi(d)}{dd} > 0$: firms have incentives to maximize distance between them. Conversely, for $d \leq \rho/3$, only the high-quality firm is active and $E\Pi(d) = \frac{1}{2}\pi_H^B(d) = \frac{1}{2}t(\rho - d)$ decreases in d. We thus have to compare $E\Pi(1)$ with $E\Pi(0)$:

$$E\Pi(1) = \frac{t}{18}(9+\rho^2) \geq E\Pi\,(0) = \frac{t}{2}\rho \Leftrightarrow \rho \leq \frac{9-3\sqrt{5}}{2} \approx 1.146$$

Therefore, we can state the following result:

Proposition 1 *The outcome of the game under symmetric information involves either maximal differentiation or minimal differentiation depending on the value of ρ. More precisely,*

(i) *for $\rho \in [0, 1.146]$, there is maximum horizontal differentiation ($d = 1$) and both firms are active and obtain strictly positive profits,*

(ii) *for $\rho > 1.146$, there is minimum horizontal differentiation ($d = 0$) and only the high-quality firm obtains positive profits.*

The values of equilibrium prices and profits are summarized in Table 1. Ex post, firms are asymmetric: one of them will have the advantage of producing higher quality. Nevertheless, the uncertainty concerning the quality of the good they will produce implies that firms are *ex ante* symmetric, such that they have identical incentives to differentiate. The parameter ρ indicates a relative quality advantage: the higher it is, the more likely firms will want to take the chance of locating in the middle of the market, with the hope of monopolizing the market at the final stage.

	prices		**expected profits**	**ex-post profits**	
Values of ρ	p_L^B	p_H^B	$E\Pi$	π_L^B	π_H^B
$\rho \in [0, 1.146]$	$t(1-\rho/3)$	$c + t(1+\rho/3)$	$\frac{t}{18}(9+\rho^2)$	$\frac{t}{18}(3-\rho)^2$	$\frac{t}{18}(3+\rho)^2$
$\rho > 1.146$	0	Δ	$\frac{t}{2}\rho$	0	$t\rho$

Table 1: Equilibrium values for prices and profits

4 Signaling quality

We now turn to the incomplete information framework in which consumers do not ascertain goods qualities before purchase. At the last stage of the game, consumers have to choose between commodities offered by firm 1 and firm 2. They observe locations and are aware these were chosen before firms learned their types, and they know that goods are of different qualities. But consumers do not know which firm produces which quality. Their prior belief about firm 1 producing high quality is $\mu_0 = 1/2$. The question is whether firms can signal their types, and if so, how the possibility of signaling at the last stage of the game influences their previous choice of locations.

4.1 Strategies, beliefs, and equilibrium definition

Firms first choose locations, before they learn their types. Each firm knows it will produce high quality with probability 1/2. Then firms privately learn their types. They each offer a price for their good and simultaneously determine their expenditures in dissipative advertising. Finally, consumers, observing these variables, try to infer some information about which firm produces which quality and make their choice. Strategies for firms are:

Location Firm 1 and firm 2 simultaneously choose their location y and $1-y$, which determines the degree of horizontal differentiation d.

Price and advertising expenditures Each firm i, $i = 1, 2$, can have one of two types L or H. Firm i's strategy profile is a vector of price-advertising actions $((p_{iL}, a_{iL}), (p_{iH}, a_{iH}))$. Since the game is symmetric, $((p_{iL}, a_{iL}), (p_{iH}, a_{iH})) = ((p_{jL}, a_{jL}), (p_{jH}, a_{jH}))$. We will hence be interested in strategy profiles of the form $((p_L, a_L), (p_H, a_H))$, identical for both firms.

We assume that advertising expenditures a_i enter firm i's profit function as a fixed cost. Given locations, consumers observe two pairs of price and advertising expenditures $((p_Q, a_Q), (p_K, a_K))$, $Q, K \in \{L, H\}$, and update their belief about which firm produces which quality. Denote $\mu((p_Q, a_Q), (p_K, a_K)) \in [0, 1]$ the probability that the firm choosing the pair (p_Q, a_Q) produces high quality, conditional on $((p_Q, a_Q), (p_K, a_K))$ being observed.

This two-stage game will be solved by backward induction. At the final stage of the game, for given locations, each firm chooses price and advertising expenditures maximizing its profit function, that we denote $\pi_Q(p_Q, a_Q, p_K, a_K, \mu)$ when it produces quality Q, and is *perceived* by consumers as producing high quality with probability $\mu = \mu((p_Q, a_Q), (p_K, a_K))$.

At the first stage, firms compute their expected profits as $E\Pi(d) \equiv \frac{1}{2}\Pi_L(d) + \frac{1}{2}\Pi_H(d)$, where $\Pi_Q(d)$ is the subgame payoff of the firm producing quality Q at the final stage.

The game under study is a game with imperfect information and observed actions, for which we are looking for perfect Bayesian equilibria (PBE). We limit our analysis to pure strategy equilibria. These can be of two types: separating or pooling. In a separating equilibrium, firms choose different pairs of price and advertising expenditure that truly reveal their type. Strategies are such that consumers correctly infer the true quality of each good. Conversely, in a pooling equilibrium, both firms charge the same price and choose the same level of advertising, which does not allow consumers to infer any more information than the one they have *a priori* .

Definition 1 *A vector* $\{d, p_L, a_L, p_H, a_H, \mu(.,.)\}$ *characterizes a pure strategy perfect Bayesian equilibrium if:*

(a) *For any given* d, $(p_L, a_L) = \arg\max_{p,a} \pi_L\left(p, a, p_H, a_H, \mu((p,a),(p_H, a_H))\right)$.

(b) *For any given d,* $(p_H, a_H) = \arg\max_{p,a} \pi_H\left(p, a, p_L, a_L, \mu((p,a),(p_L,a_L))\right).$

(c) *If* $(p_H, a_H) \neq (p_L, a_L)$ *then* $\mu((p_H, a_H),(p_L, a_L)) = 1 = 1 - \mu((p_L, a_L),(p_H, a_H)).$

(d) *If* $(p_H, a_H) = (p_L, a_L)$ *then* $\mu((p_H, a_H),(p_L, a_L)) = \mu((p_L, a_L),(p_H, a_H)) = \frac{1}{2}.$

(e) $d \in \arg\max E\Pi(d) = \frac{1}{2}\Pi_L(d) + \frac{1}{2}\Pi_H(d).$

Conditions (a) and (b) require that each firm maximizes its profit, given its rival's strategy and consumers posterior beliefs. Requirements (c) and (d) state that beliefs must be consistent with the structure of the game and firms' strategies. Namely, when firms choose different actions, consumers will correctly infer which firm produces which quality. Conversely, if actions are identical, consumers cannot update their beliefs and must revert to their prior ones. Note that those requirements on beliefs only refer to observations of equilibrium strategies. Finally, Condition (e) requires each firm to choose its location optimally, anticipating equilibrium prices and advertising strategies, and consumers' beliefs.

In order to write conditions for the existence of separating equilibria, we need to complement Definition 1 by restricting beliefs a little further. As we will see in the next subsection, a firm always has the possibility of mimicking its rival's strategy. But contrary to a monopoly game, this deviation does not necessarily constitute a deviation on the equilibrium path, such that Definition 1 does not specify beliefs in such a situation. We replace **(d)** in Definition 1 by:

(d') If $(p'_H, a'_H) = (p'_L, a'_L)$ then $\mu((p'_H, a'_H),(p'_L, a'_L)) = \mu((p'_L, a'_L),(p'_H, a'_H)) = \frac{1}{2},$
where $((p'_H, a'_H),(p'_L, a'_L))$ refers to any arbitrary pairs of identical actions. Since the firms are ex ante symmetric, observing two identical pairs of actions does not enable consumers to infer any new information or to believe that one firm was more likely to deviate than the other.

4.2 Separating equilibria

When consumers observe a profile $((p_Q, a_Q),(p_K, a_K))$, they update their beliefs such that $\mu \equiv \mu((p_Q, a_Q),(p_K, a_K)) = \text{Prob}\ (Q = H \,|\, (p_Q, a_Q),(p_K, a_K))$. The indifferent consumer located in z^* is such that

$$R + \mu H + (1-\mu)L - p_Q - t(z^* - y)^2 = R + (1-\mu)H + \mu L - p_K - t(z^* - 1 + y)^2$$

which implies that

$$z^*(p_Q, p_K, \mu) = \begin{cases} 0 & \text{if} \quad p_Q \geq p_K + td - (1-2\mu)\Delta \\ 1 & \text{if} \quad p_Q \leq p_K - td - (1-2\mu)\Delta \\ \frac{1}{2} - \frac{p_Q - p_K + (1-2\mu)\Delta}{2td} & \text{otherwise.} \end{cases} \tag{7}$$

Profit functions are denoted

$$\begin{aligned} \pi_Q(p_Q, a_Q, p_K, \mu) &= (p_Q - c(Q))z^*(p_Q, p_K, \mu) - a_Q \\ \pi_K(p_K, a_K, p_Q, \mu) &= (p_K - c(K))(1 - z^*(p_Q, p_K, \mu)) - a_K \end{aligned}$$

where $\mu = \mu((p_Q, a_Q), (p_K, a_K))$.[11]

The necessary conditions for the strategy profile $((p_L, a_L), (p_H, a_H))$ to be a separating profile are:

$$\text{IC}_L: \ \pi_L(p_L, a_L, p_H, 0) \geq \pi_L\left(p_H, a_H, p_H, \frac{1}{2}\right) \tag{8}$$

$$\text{IC}_H: \ \pi_H(p_H, a_H, p_L, 1) \geq \pi_H\left(p_L, a_L, p_L, \frac{1}{2}\right) \tag{9}$$

$$(p_L, a_L) \in \arg\max_{p,a} \pi_L(p, a, p_H, 0). \tag{10}$$

Conditions (IC_L) and (IC_H) are implied by (a), (b) and (d') of Definition 1. Namely, by construction of the belief system, each type of firm has the possibility of mimicking the strategy of its rival of the other type, thereby confusing consumers. Condition (10) stipulates that in a separating equilibrium the low-quality firm, facing the worst possible beliefs and being perfectly identified by consumers, should optimize accordingly.

[11] Note that we have suppressed the variable a_i in π_j since π_j only depends on a_i through beliefs μ.

The no-mimicking conditions (8) and (9) can be rewritten as:

$$\text{IC}_L: \quad a_H \geq \max\left\{\underline{a}_H\left(p_H, p_L, a_L\right), 0\right\} \tag{11}$$

$$\text{IC}_H: \quad 0 \leq a_H \leq \overline{a}_H\left(p_H, p_L, a_L\right) \tag{12}$$

where

$$\underline{a}_H\left(p_H, p_L, a_L\right) = \frac{1}{2}p_H + a_L - p_L z^*(p_L, p_H, 0)$$

and

$$\overline{a}_H\left(p_H, p_L, a_L\right) = \left(p_H - c\right)\left(1 - z^*(p_L, p_H, 0)\right) - \frac{1}{2}(p_L - c) + a_L.$$

The expenditure $\underline{a}_H$ represents the minimum level of dissipative advertising that the high-quality firm must incur to deter a low-quality firm from mimicking its strategy. Conversely, $\overline{a}_H$ is the highest level of advertising sustainable for a high-quality firm in a separating equilibrium, above which the high-quality firm would rather be mistaken for a low-quality firm. Hence, on the one hand, advertising expenditures must be sufficiently high to prevent the low-quality firm from mimicking, and on the other hand, they must not be so high that separation is no longer profitable for the high-quality firm.

Recall from Condition (10) that in a separating equilibrium, the low-quality firm faces the worst possible beliefs and is perfectly identified by consumers. Therefore, this firm will maximize its profit accordingly and consequently, its best response to its competitor is identical to the one that would prevail under symmetric information. This yields to the following result.

Lemma 1 *In any separating equilibrium profile, the low-quality firm chooses* $p_L = p_L^B(p_H) = \frac{1}{2}[p_H + td - \Delta]$ *and* $a_L = 0$.

Intuitively, the low-quality firm has no better choice than playing its symmetric-information best price response and hence is not willing to spend money in wasteful advertising. Using Lemma 1, we are now able to characterize the set of potential separating equilibrium profiles $((p_H, a_H), (p_L, a_L)) = ((p_H, a_H), (p_L^B(p_H), 0))$, using the following necessary condition on p_H and a_H,

$$\max\left\{\underline{a}_H\left(p_H, p_L^B(p_H), 0\right), 0\right\} \leq a_H \leq \overline{a}_H\left(p_H, p_L^B(p_H), 0\right) \tag{13}$$

together with the non-negativity condition for firms' demands,

$$0 \leq z^*(p_L^B(p_H), p_H, 0) \leq 1, \tag{14}$$

and the condition $p_H \geq c$ for positive margin.

Lemma 2 *The set Ω of admissible separating prices p_H and advertising expenditures a_H, such that conditions (13) and (14) are satisfied, is non empty and defined as follows:*

$$\underline{a}_H\left(p_H, p_L^B(p_H), 0\right) \leq a_H \leq \overline{a}_H\left(p_H, p_L^B(p_H), 0\right)$$
$$\text{and} \begin{cases} p_H \in [\Delta - td, \Delta + td] & \text{if} \quad d \leq \rho \\ p_H \in [2c + td - \Delta, \Delta + td] & \text{if} \quad d > \rho \end{cases}$$

Proof: See Appendix A.■

Given this characterization, we are now able to state the main result on the existence of separating equilibria.

Proposition 2 *Any strategy profile $((p_H, a_H), (p_L, a_L))$ such that $(p_H, a_H) \in \Omega$ and $(p_L, a_L) = \left(\frac{1}{2}[p_H + td - \Delta], 0\right)$ can be paired with a system of beliefs to form a separating equilibrium.*

Proof: See Appendix B.■

The proof of Proposition 2 shows that considering beliefs that satisfy Definition 1 and the following requirements:

$$\begin{aligned} \mu((p,a),(p_H,a_H)) &= 0 \text{ for any } (p,a) \notin \{((p_H,a_H),(p_L,a_L)\} \\ \mu((p_L,a_L),(p,a)) &= 1 \text{ for any } (p,a) \notin \{(p_H,a_H),(p_L,a_L)\} \end{aligned}$$

is sufficient to support any profile $((p_H, a_H), (p_L, a_L))$ such that $(p_H, a_H) \in \Omega$ and $(p_L, a_L) = \left(\frac{1}{2}[p_H + td - \Delta], 0\right)$ as part of a separating equilibrium. With such beliefs, consumers infer from any deviation that the deviating firm offers low quality and that the non deviating firm offers high quality.

Figures 1, 2 and 3 illustrate the set of separating equilibria in the space (p_H, a_H), depending on the different values that the parameter ρ can take. An important consequence of our result is that purely dissipative advertising is absolutely necessary for the high-quality firm to reveal its type in a separating equilibrium. This holds whatever the gap between qualities or the distance between firms. This contrasts with the results obtained by Hertzendorf and Overgaard (2001a) and Fluet and Garella (2002), who suggest that when the quality differential is sufficiently high, price signaling alone is sufficient to get separation.

An intuition for our result unfolds as follows. It can be shown that if advertising were not possible the high-quality firm would have to distort its price *downward* in order to prevent imitation by the low-quality firm (one can observe this by extrapolating the curves $\underline{a}_H$ and $\overline{a}_H$ in Figures 1, 2 and 3 until they cross the horizontal axis). But such a low price would then allow the high-quality firm to appropriate the entire demand, leaving its competitor with zero profit. As the low-quality firm gets nothing, it has nothing to lose from choosing any other price: Its opportunity cost of deviating is null. Therefore, its incentive compatible constraint not to deviate cannot be satisfied, since it is always worthwhile imitating any price the high-quality firm could choose and have a chance to get a positive share of the demand.

4.3 Pooling equilibria

We now turn to the study of pooling equilibria. In a pooling equilibrium, both firms set the same price p and the same advertising expenditure a. Consumers are therefore unable to infer any information about quality, so that their posterior beliefs are identical to their prior ones: $\mu((p,a),(p,a)) = \frac{1}{2}$. Firms will therefore split the market equally. Let us denote Γ the set of pooling equilibrium strategies (p,a). We define out-of-equilibrium beliefs as being pessimistic, such that given a putative pooling equilibrium strategy profile $((p,a),(p,a))$, consumers infer from any unilateral deviation $(p',a') \neq (p,a)$ that the deviating firm produces low quality: $\mu((p',a'),(p,a)) = 1 - \mu((p,a),(p',a')) = 0$. Hence, the set Γ is characterized by the following necessary and sufficient conditions:

$$\pi_L\left(p,a,p,\frac{1}{2}\right) \geq \max_{p',a'} \pi_L\left(p',a',p,0\right) \tag{15}$$

$$\pi_H\left(p,a,p,\frac{1}{2}\right) \geq \max_{p',a'} \pi_H\left(p',a',p,0\right) \tag{16}$$

Studying conditions (15) and (16) gives rise to the following Proposition.

Proposition 3 *For $d \geq c^2/16\Delta t$, the set Γ is non empty and defined by*

$$
\begin{aligned}
0 &\leq a \leq \min\left\{\underline{a}_H(p,p,0), \underline{a}_H(p-c,p-c,0)\right\} \\
td + \Delta + c - 2\sqrt{\Delta td} &\leq p \leq td + \Delta + 2\sqrt{\Delta td}.
\end{aligned}
$$

Any $(p,a) \in \Gamma$ can be supported as a pooling equilibrium with the following system of beliefs: $\mu((p,a),(p,a)) = \frac{1}{2}$ and $\mu((p',a'),(p,a)) = 1 - \mu((p,a),(p',a')) = 0$ for any $(p',a') \neq (p,a)$.

Proof: See Appendix C.■

When the distance between firms is sufficiently high, there is an infinity of pooling equilibria, some of which are characterized by positive advertising expenditures. Note however that when $c^2/16\Delta t > 1$, there does not exist any pooling equilibrium.

4.4 Selecting an equilibrium

As we ultimately want to solve the first stage of the game, i.e. the location stage, we need to select a plausible equilibrium for the price-advertising competition subgame. Given the results contained in the previous two subsections, we are left with two different regimes. First, when the gap between qualities is sufficiently small ($\Delta < c^2/16t$) only separating equilibria exist and second, when there is sufficient distance between firms ($1 \geq d \geq c^2/16\Delta t$), pooling and separating equilibria co-exist.

The multiplicity of equilibria is a common feature in signaling games, and is due to the lack of restrictions on beliefs after a deviation off the equilibrium path. In monopoly signaling games, well-known refinements of the equilibrium concept are applied to prune the set of equilibria, such as the Intuitive Criterion (see Cho and Kreps (1987)). The idea of the Intuitive Criterion is to eliminate equilibria that rely on implausible beliefs, where beliefs are considered implausible if they put a positive probability on one type of the sender deviating to a strategy that is equilibrium dominated for him, while it is not for his alter ego of the other type.

But in our context, there are two senders, and consumers receive two messages. This implies that we cannot apply the Intuitive Criterion without some modifications, but also that we might not have to apply it so straightforwardly. For instance, consider a separating equilibrium from which one of the two senders deviates. The new feature with respect to a monopoly signaling game is that consumers can still infer some information from the non deviating sender, and can (and should) use this to construct beliefs off the equilibrium path. Therefore, a deviation from an informative equilibrium might not have any consequences for consumers, because their beliefs can remain unchanged as long as the other sender's strategy remains consistent with a separating equilibrium. If we require consumers to follow such a logic, separating equilibria can be tested without systematically testing for equilibrium domination (as the Intuitive Criterion requires). Such a test, requiring beliefs to be *unprejudiced*, can be found in Bagwell and Ramey (1991) and Schultz (1999). As Hertzendorf and Overgaard (2001a), we will apply a restricted version of Bagwell and Ramey's unprejudiced beliefs, requiring an equilibrium profile to be Resistant to Equilibrium Defections (REDE). This requirement will discipline beliefs following not any deviation, but those ones that *belong to the set of potential separating profiles.*

The definition of an equilibrium profile that is REDE happens to have a flavor close to the definition of undefeated equilibrium by Mailath *et al.* (1993). The idea common to these two definitions is that, when observing a firm deviating from a proposed equilibrium, consumers should consider whether this deviation and its subsequent outcome is consistent with another equilibrium. If so, their beliefs should reflect this consistency. In the present framework, if consumers observe a deviation from a separating profile to a price-advertising pair that could be chosen by the high-quality firm in an alternative separating equilibrium, while the other price-advertising pair is consistent with a low-quality firm's choice, then consumers' out-of-equilibrium beliefs should put a positive probability on the high-quality firm deviating. As we show below, imposing this requirement on beliefs reduces the set of separating equilibrium profiles to a unique one, in which the high-quality firm signals its type with the least costly pair of price and advertising expenditures.

The requirement on beliefs to be REDE has no grip on pooling equilibria, since in that case a deviation from the equilibrium leaves consumers with an equilibrium message from which they cannot infer any information. In that case, the logic of the Intuitive Criterion will be relevant, and consumers will be required not to put a positive belief on deviations (or messages) that are equilibrium dominated for one type of the deviating sender, but not for the other type.

4.4.1 Refining separating equilibria

In a separating equilibrium as defined in Proposition 2, consumers expect to observe $((p_L, a_L), (p_H, a_H))$ where $(p_H, a_H) \in \Omega$ and $(p_L, a_L) = \left(p_L^B(p_H), 0\right)$. Equilibrium beliefs are thus $\mu\left((p_L, a_L), (p_H, a_H)\right) = 0$. This profile is supported as a separating equilibrium by the following belief system:

$$\begin{aligned} \mu((p,a),(p_H,a_H)) &= 0 \text{ for any } (p,a) \notin \{(p_H,a_H),(p_L,a_L)\} \\ \mu((p_L,a_L),(p,a)) &= 1 \text{ for any } (p,a) \notin \{(p_H,a_H),(p_L,a_L)\}. \end{aligned}$$

The second part of this system requires consumers to believe that the firm that chose (p_L, a_L) is of the high quality type, whereas the one that chose the strategy (p, a) is of the low quality type, even if $(p, a) \in \Omega$, that is even if the high-quality firm deviates to a strategy consistent with an alternative separating equilibrium. Such a belief off the equilibrium path induces consumers to believe they have observed *two* deviations from the tested separating equilibrium. We claim that this should not occur, and that consumers should hold beliefs consistent with the information contained in the strategy of the non deviating firm as well as the information of the deviation itself, whenever possible.

Following Hertzendorf and Overgaard (2001a), we require candidate separating equilibria to be Resistant to Equilibrium Defections.

Definition 2 *Consider a profile* $((p_L, a_L), (p_H, a_H))$ *such that* $(p_H, a_H) \in \Omega$ *and* $(p_L, a_L) = \left(p_L^B(p_H), 0\right)$ *as defined in Proposition 2. Consider also an alternative profile* $((p'_L, a'_L), (p'_H, a'_H)) \neq ((p_L, a_L), (p_H, a_H))$ *where* $(p'_L, a'_L) \neq (p'_H, a'_H)$*. An equilibrium profile* $((p_L, a_L), (p_H, a_H))$ *is resistant to equilibrium defections (REDE) if beliefs satisfy* $\mu((p'_L, a'_L), (p'_H, a'_H)) = 0$ *whenever*

(1) $(p'_L, a'_L) = \left(p_L^B(\tilde{p}_H), 0\right)$ *for some* $(\tilde{p}_H, \tilde{a}_H) \in \Omega$ *and,*

(2) $(p'_H, a'_H) \in \Omega$.

The idea behind this refinement is as follows. Suppose that consumers expect to see (p_H, a_H) from the high-quality producer and $\left(p_L^B(p_H), 0\right)$ from the low-quality producer. Suppose instead that they observe $((p'_L, a'_L), (p'_H, a'_H))$. If (p'_L, a'_L) is consistent with some alternative separating equilibrium play of a low-quality firm, and if (p'_H, a'_H) is consistent with some (possibly different) separating equilibrium play of a high-quality firm, then consumers have enough information to infer that (p'_H, a'_H) is played by the high-quality firm with probability one. When the refinement REDE is silent, the beliefs are still specified as being pessimistic as in Proposition 2.

Consider the low-quality firm first. The imposition of the REDE criterion on out-of-equilibrium beliefs has no consequences for that producer. Indeed, any unilateral and non-mimicking deviation from a putative equilibrium profile $((p_L, a_L), (p_H, a_H))$ leads consumers to infer that the deviating firm produces low quality. But the equilibrium has been constructed such that those deviations are suboptimal. From the low-quality firm perspective, this inference remains unchanged under REDE. The same is true for any mimicking deviation.

Now, consider the high-quality firm. A deviation from $(p_H, a_H) \in \Omega$ to $(p'_H, a'_H) \notin \Omega$ falls under the belief system defined for Proposition 2. Such a deviation is, by construction, not profitable. But the REDE criterion has a large impact on the high-quality firm when considering deviations to $(p'_H, a'_H) \in \Omega$, since such deviations are no longer believed to come from a low-quality firm. Therefore, there could exist deviations to strategies in Ω that are profitable to the high-quality firm, thus eliminating the separating equilibrium under scrutiny. To survive the REDE criterion, a separating equilibrium profile $((p_L, a_L), (p_H, a_H))$ must be such that the high-quality firm does not want to deviate to some other strategy $(p'_H, a'_H) \in \Omega$, given $(p_L, a_L) = (p_L^B(p_H), 0)$. In other words, (p_H, a_H) must be, in the set Ω, a best response to $(p_L, a_L) = (p_L^B(p_H), 0)$.

To establish the next Proposition, let us define $D_1 = \{d \mid 1 \geq d > \rho\}$ (note that if $\rho > 1$, then $D_1 = \varnothing$). Similarly, let us denote $D_2 = \{d \mid \rho \geq d > \rho/3\}$ and $D_3 = \{d \mid \rho/3 \geq d \geq 0\}$. Also define $D = D_1 \cup D_2 \cup D_3$.

Proposition 4 *There is a unique separating equilibrium profile $((p_L^*, a_L^*), (p_H^*, a_H^*))$ that satisfies the REDE refinement criterion, where $(p_L^*, a_L^*) = \left(p_L^B(p_H^*), 0\right)$, $a_H^* = \underline{a}_H(p_H^*, p_L^*, 0)$ and*

(i) *when $d \in D_1 \neq \varnothing$, then $p_H^* = \Delta + td$,*

(ii) *when $d \in D_2 \neq \varnothing$, then $p_H^* = c + 2td$,*

(iii) *when $d \in D_3 \neq \varnothing$, then $p_H^* = \Delta - td$.*

Proof: See Appendix D.■

4.4.2 Refining pooling equilibria

As we have mentioned earlier, requiring Resistance to Equilibrium Defection has no effect on the set of pooling equilibrium profiles. We thus apply the logic of the Intuitive Criterion as follows. If $(p', a') \neq (p, a)$ is a deviation which is equilibrium dominated for the low-quality firm but not for the high-quality firm, then consumers should not put a positive weight on the possibility that such a deviation comes from a low-quality firm. Formally, if $(p', a') \neq (p, a)$ is such that

$$\pi_L\left(p', a', p, 1\right) \leq \pi_L\left(p, a, p, \frac{1}{2}\right) \tag{17}$$

$$\pi_H\left(p', a', p, 1\right) > \pi_H\left(p, a, p, \frac{1}{2}\right) \tag{18}$$

then we must have $\mu((p', a'), (p, a)) = 1$.

It means that if there exists a deviation to (p', a') that would, with the best possible beliefs (or the most optimistic beliefs), make the high quality firm better off but not the low-quality firm, then consumers should understand that this deviation can only come from the high-quality firm, and thus should infer $\mu((p', a'), (p, a)) = 1$. This contradicts the system of beliefs that supports pooling equilibria in Proposition 3, where we assumed that consumers infer that any deviating firm is of low-quality.

Let us show that such a deviation from a pooling equilibrium profile always exists. We can restrict our search to a deviation (p', a), which is profitable for the high-quality firm but not for the low-quality one, with respect to any $(p, a) \in \Gamma$. Conditions (17) and (18) require p' to be such that:

$$p'\left(\frac{1}{2} + \frac{p - p' + \Delta}{2td}\right) - a \leq \frac{1}{2}p - a \tag{19}$$

$$(p' - c)\left(\frac{1}{2} + \frac{p - p' + \Delta}{2td}\right) - a > \frac{1}{2}(p - c) - a. \tag{20}$$

Take a closer look at inequality (19), and let us define $\mathcal{D}$ as the set of prices p' satisfying it. Denoting $\underline{p}'$ and $\overline{p}'$ the lower and upper roots, solutions (in p') of $p'\left(\frac{1}{2} + \frac{p-p'+\Delta}{2td}\right) = \frac{1}{2}p$, we can write $\mathcal{D} = [0, \underline{p}'] \bigcup [\overline{p}', \infty)$. Using the fact that $\overline{p}'\left(\frac{1}{2} + \frac{p-\overline{p}'+\Delta}{2td}\right) = \frac{1}{2}p$, we can write inequality (20) for $p' = \overline{p}'$ as follows:

$$
\begin{aligned}
& (\overline{p}' - c)\left(\frac{1}{2} + \frac{p - \overline{p}' + \Delta}{2td}\right) - \frac{1}{2}(p-c) > 0 \\
\Leftrightarrow \quad & -c\frac{p - \overline{p}' + \Delta}{2td} > 0.
\end{aligned}
$$

Straightforward algebra shows that $\overline{p}'$ is greater that $p+\Delta$, implying that the latter inequality is satisfied.[12]

In conclusion, a deviation from $(p, a) \in \Gamma$ to $(\overline{p}', a)$ would be, with the most optimistic beliefs, strictly profitable for the high-quality firm, while it would leave the profit of the low-quality firm unchanged. To avoid any ambiguity, we can consider a deviation to $(\overline{p}' + \epsilon, a)$, ϵ positive but small, such that this deviation strictly benefits the high-quality firm while the low-quality firm can only be strictly worse off. Therefore, any pooling equilibrium $(p, a) \in \Gamma$ can be shown to be non intuitive, because there always exists a deviation to a different price that could benefit the high-quality firm but would be detrimental to the low-quality firm.

This result relies on the single-crossing property of profit curves that also holds in the standard signaling literature. This property reflects that a high-quality firm is more willing to signal than the low-quality one. In other words, it is less costly for the high-quality firm to distort its actions (notably its price) than for the low-quality firm. Formally, a firm's marginal profit (for given locations and price-advertising pair $(\bar{p}, \bar{a})$ chosen by its competitor) of distorting its price is $\frac{\partial \pi_L(p,a,\bar{p},\mu)}{\partial p} = \frac{1}{2} + \frac{\bar{p} - 2p + (1-2\mu)\Delta}{2td}$ if it is of the low-quality type, and $\frac{\partial \pi_H(p,a,\bar{p},\mu)}{\partial p} = \frac{1}{2} + \frac{\bar{p} - 2p + (1-2\mu)\Delta + c}{2td}$ if it is of the high-quality type, implying the single-crossing property

$$
\frac{\partial \pi_H(p, a, \bar{p}, \mu)}{\partial p} \geq \frac{\partial \pi_L(p, a, \bar{p}, \mu)}{\partial p}.
$$

Observe that this property is satisfied as long as marginal costs are type-dependent. A marginal upward price distortion, translating into a marginal reduction in sales, involves a loss that amounts to p for the low-quality firm, but only to $p-c$ for the high-quality firm. Were marginal costs not type-dependent, intuitive beliefs would have no impact on the multiplicity of pooling equilibria, and some other refinements would have to be applied to address this

[12] Indeed $\overline{p}' = \frac{1}{2}\left(p + \Delta + td + \sqrt{(p + \Delta + td)^2 - 4tdp}\right)$.

issue.[13]

4.5 Location equilibria

Making use of the results from the preceding subsection, we first note that the type of separating equilibrium depends in particular on which set D_i ($i = 1, 2$ or 3) the parameter ρ belongs to. Denoting $\Pi_L(d)$ and $\Pi_H(d)$ firms profits evaluated at the separating equilibrium described in Proposition 4, we can compute a firm's expected profit $E\Pi(d) = \frac{1}{2}\Pi_H(d) + \frac{1}{2}\Pi_L(d)$ and look for its maximum with respect to d. We get the following Proposition.

Proposition 5 *Whenever $\Delta \geq c$, the equilibrium is separating and involves either maximal or minimal horizontal differentiation. More precisely,*

(i) *when $0 \leq \rho \leq 1$, horizontal differentiation is maximal, $d^* = 1$, with $p_H^* = \Delta + t$,*

(ii) *when $1 < \rho \leq 3 - \sqrt{2}$, horizontal differentiation is also maximal, $d^* = 1$, but with $p_H^* = c + 2t$,*

(iii) *and when $3 - \sqrt{2} < \rho$, horizontal differentiation is minimal, $d^* = 0$, with $p_H^* = \Delta$ and only the high-quality firm is active.*

Proof: See Appendix E.■

Tables 2 and 3 summarize equilibrium prices, advertising expenditures, locations and ex-post profits. As in the symmetric information situation, consumers face a duopoly or a monopoly, depending on the relative quality differential ρ. Nevertheless, the critical threshold above which a high-quality monopoly will arise is different, as emphasized in the following Corollary.

Corollary 1 *The equilibrium of the signalling game exhibits maximum horizontal differentiation for a larger interval of ρ compared to the symmetric information benchmark.*

Recall from Section 3 that both firms are active for $\rho \in [0, 1.146]$. As Proposition 5 shows, the separating equilibrium involves a duopoly for $\rho < 3 - \sqrt{2} \approx 1.586$. Hence, for $\rho \in [1.146, 1.586]$, asymmetric information modifies the structure of the market.

[13] In Hertzendorf and Overgaard (2001a), marginal costs are type-independent, and the authors define *impartial* out-of-equilibrium beliefs that restrict consumers inferences after a deviation from a pooling equilibrium.

As summarized in Table 2, signaling high quality involves positive advertising, whatever the relative quality differential ρ. In addition, as long as both firms are active in the market, the high-quality firm must distort its price upward to truthfully reveal its type. Consequently, the low-quality firm adopts a higher price in the separating equilibrium than under symmetric information, since prices are strategic complements.

Overall, the low-quality duopolist benefits from asymmetric information. In regions Σ_1 and Σ_2, in which its market share is positive, its profits are higher than under symmetric information. Not only the price of the low quality good is higher in the separating equilibrium, but also its market share is larger. Under symmetric information, $z^B(p_L^B, p_H^B) = \frac{1}{2} - \frac{\rho}{6}$, whereas $z^*(p_L^*, p_H^*, 0) = \frac{1}{2}$ when $\rho \in [0, 1]$ and $z^*(.) = \frac{3}{4} - \frac{\rho}{4} > \frac{1}{2} - \frac{\rho}{6}$ when $\rho \in \Sigma_2$. Higher prices drive some consumers to purchase the low-quality good, whereas they would have bought the high-quality one, had information been symmetric.

Conversely, the high-quality firm loses from asymmetric information, since it bears the signaling activity and its subsequent cost. The upward distortion in price has two opposite effects on variable profits. On the one hand, the price charged to customers is higher. But on the other hand, the market share of the high-quality firm is reduced at the benefit of its low-quality competitor. Furthermore, the high-quality firm has to burn money on dissipative advertising. Overall, it is easily checked that the last two effects outweigh the first one, and that the high-quality firm's profits are lower under asymmetric information.

It is interesting to note that, under asymmetric information, the high-quality firm will not always earn less than the low-quality one, as it would be the case in a monopoly signaling game, in which by construction the high-quality monopolist would realize a profit lower than its low-quality alter ego. In the present context, the high-quality duopolist will obtain a profit higher than its low-quality competitor as long as the relative quality differential ρ is greater than $1 + \frac{c}{t}$. Let us stress that this finding follows from the definition of consumers utility function in which it is assumed that R, the basic surplus obtained by purchasing any of the two commodities, is high enough that all consumers always buy a unit of good and the entire market is always covered.

From the consumers' perspective, asymmetric information has a negative impact when $\rho < 1.146$. Both firms charge higher prices than under symmetric information, thus reducing consumers surplus unambiguously. When $\rho > 1.586$, the high-quality firm monopolizes the market and charges the same price whatever the informational structure, and consumers surplus is not affected. For $\rho \in [1.146, 1.586]$, asymmetric information enables the low-quality firm to remain active, which could have a positive effect on consumers surplus. Nevertheless, straightforward calculations show that consumers surplus is lower under asymmetric information and a duopoly regime than under symmetric information with a high-quality monopoly.

In region Σ_3, Δ is so large that producing low-quality is not profitable at the final stage of the game. Ex ante, both firms choose identical locations, each of them expecting to monopolize the market with a high quality good. In this region, prices are not distorted, compared to the symmetric information benchmark (in both cases, $p_H = \Delta$), and dissipative advertising only signals high quality. As a result, consumers do not suffer from the signaling activity. However, the high-quality firm earns less because of the necessity to signal its type through costly advertising. Finally, the cost of advertising is increasing in the quality differential in region Σ_3.

The evolution of advertising a_H depends on the region we consider. While it is clearly decreasing in Δ in region Σ_2, it is increasing in Δ in regions Σ_1 and Σ_3. Hence the cost of advertising is non monotonic in the quality gap Δ, which is reminiscent of results obtained by Hertzendorf and Overgaard (2001a) in a different setting.

Parameters	**Locations**	**Prices**		**Advertising**	
region	d^*	p_L^*	p_H^*	a_H^*	a_L^*
$\Sigma_1 : 0 \leq \rho \leq 1$	1	t	$\Delta + t$	$\frac{\Delta}{2}$	0
$\Sigma_2 : 1 < \rho \leq 3 - \sqrt{2}$	1	$\frac{t}{2}(3-\rho)$	$c + 2t$	$\frac{c+2t}{2} - \frac{t}{8}(3-\rho)^2$	0
$\Sigma_3 : 3 - \sqrt{2} < \rho$	0	0	Δ	$\frac{\Delta}{2}$	0

Table 2: Equilibrium values for locations, prices, and advertising expenditures

Parameters	**Ex-post profits**	
region	Π_L^*	Π_H^*
$\Sigma_1 : 0 \leq \rho \leq 1$	$\frac{t}{2}$	$\frac{t-c}{2}$
$\Sigma_2 : 1 < \rho \leq 3 - \sqrt{2}$	$\frac{t}{8}(3-\rho)^2$	$\frac{1}{2}(\Delta - 2c - t) + \frac{t}{8}(3-\rho)^2$
$\Sigma_3 : 3 - \sqrt{2} < \rho$	0	$t\rho - \frac{\Delta}{2}$

Table 3: Equilibrium values for ex-post profits

5 Conclusion

We have constructed a duopoly model in which goods are horizontally and vertically differentiated. Vertical differentiation is exogenous, and which firm produces which quality is exogenously and randomly determined. Firms must choose their location (which can be interpreted geographically or in terms of product design) before quality uncertainty is resolved. But at the final stage, they are privately informed of their types, and want to reveal information to their customers.

This paper contributes to the emerging literature on quality signaling in an imperfectly competitive environment. We show that pure price signaling is impossible, whatever the degree of vertical differentiation. In the present context, pure price signaling could be achieved thanks to a *downward* price distortion, resulting in the high-quality firm monopolizing the market. Consequently, the low-quality firm would make zero profit, and thus would have strong incentives to deviate and mimic any strategy its high-quality rival would choose.

Dissipative advertising is crucial to the existence of separating equilibria. The possibility to combine price and advertising expenditures to signal high quality results in an upward distortion of both prices, except when vertical differentiation is so large that only producing high quality is profitable. In this latter case, the high-quality price is not distorted, and dissipative advertising alone is used to deter the low-quality firm from mimicking its high-quality rival. In the former case where vertical differentiation is not too large, an upward distortion in price endows the high-quality firm with a profit sufficiently high that it can afford to make dissipative expenditures on advertising. Since prices are strategic complements, the increase of the price of the low-quality good follows from the signaling activity of the high-quality firm.

Asymmetric information also has some consequences on firms location choices. Indeed, compared to the symmetric information benchmark, maximum horizontal differentiation occurs for a larger set of parameter values. Therefore, in our setting, incomplete information yields more horizontal differentiation.

Some possible extensions of the present paper come to mind that seem rather natural. One of them would be to study an alternative timing, where location choice can act as a signal of quality, as in Vettas (1999). Consumers would receive two sequential messages, first firms location choices, then their decisions on the combination of price and advertising expenditures. This timing might give more opportunities for a high-quality firm to reveal its type, but would also involve complex inferences on the consumer side.

Another possible extension would generalize the present setting to include the possibility that firms can make (potentially observable) decisions like investment early in the game, that could affect the realization of quality which would remain private information to the firms. Such a setting would involve both moral hazard and adverse selection issues, and studying their interactions would undoubtedly yield interesting results.

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Appendix

A Proof of Lemma 2

Let us start with condition (14). From (7) and Lemma 1, we have

$$z^*(p_L^B(p_H), p_H, 0) = \frac{1}{2} - \frac{\frac{1}{2}(p_H + td - \Delta) - p_H + \Delta}{2td} = \frac{1}{4td}(p_H + td - \Delta)$$

and $0 \leq z^*(p_L^B(p_H), p_H, 0) \leq 1$ is then equivalent to the following condition

$$\max\{c, \Delta - td\} \leq p_H \leq \Delta + 3td.$$

This gives us a lower and upper bound for p_H, such that demands are positive for both firms. In addition, given the definition of $\underline{a}_H$ and Lemma 1, we can write

$$\underline{a}_H\left(p_H, p_L^B(p_H), 0\right) = \frac{1}{2}p_H - \frac{1}{8td}(p_H + td - \Delta)^2.$$

This expression shows that $\underline{a}_H$ is an inverted parabola function of p_H, which is symmetric on the interval $I = [\Delta - td, \Delta + 3td]$, and reaches its maximum in $p_H = \Delta + td$.
Finally, given the definition of $\overline{a}_H$ and Lemma 1, we obtain

$$\overline{a}_H\left(p_H, p_L^B(p_H), 0\right) = \frac{1}{4td}(p_H - c)(3td - p_H + \Delta) - \frac{1}{4}(p_H + td - \Delta - 2c)$$

which is also an inverted parabola. Its maximum is reached in $p_H = \frac{\Delta + c}{2} + td$, which is lower than $\Delta + td$, since by assumption $c < \Delta$.

Let us compute the difference between $\overline{a}_H$ and $\underline{a}_H$:

$$\begin{aligned}\overline{a}_H - \underline{a}_H &= \frac{1}{4td}(p_H - c)(3td - p_H + \Delta) - \frac{1}{4}(p_H + td - \Delta - 2c) - \frac{1}{2}p_H + \frac{1}{8td}(p_H + td - \Delta)^2 \\ &= \frac{1}{8td}(p_H - td - \Delta)(2c - p_H + td - \Delta).\end{aligned}$$

This is again an inverted parabola which is positive only between the roots $\Delta + td$ and $2c + td - \Delta$. Given that $c < \Delta$, the upper root is $\Delta + td > 0$.

Hence the set of *admissible* separating prices p_H, i. e. high-quality prices satisfying necessary conditions (11) and (12), is non empty and such that

$$\max\{c,\ \Delta - td,\ 2c + td - \Delta\} \leq p_H \leq \Delta + td.$$

More precisely, as we have the following inequalities:

$$\begin{aligned} 2c + td - \Delta &> c \Leftrightarrow d > \rho = \frac{\Delta - c}{t} \\ \Delta - td &> c \Leftrightarrow d < \rho \\ 2c + td - \Delta &> \Delta - td \Leftrightarrow d > \rho, \end{aligned}$$

it follows that when $\rho \geq d$, the set of admissible p_H is the interval $[\Delta - td, \Delta + td]$, while when $\rho < d$, the set of admissible p_H is $[2c + td - \Delta, \Delta + td]$. This completes the proof.

B Proof of Proposition 2

Consider a putative equilibrium $((\tilde{p}_H, \tilde{a}_H), (\tilde{p}_L, \tilde{a}_L))$. We choose the following out-of-equilibrium beliefs:

$$\begin{aligned} \mu(p, a, \tilde{p}_H, \tilde{a}_H) &= 0 \text{ for any } (p, a) \notin \{(\tilde{p}_H, \tilde{a}_H), (\tilde{p}_L, \tilde{a}_L)\} \\ \mu(\tilde{p}_L, \tilde{a}_L, p, a) &= 1 \text{ for any } (p, a) \notin \{(\tilde{p}_H, \tilde{a}_H), (\tilde{p}_L, \tilde{a}_L)\}. \end{aligned}$$

These beliefs are such that, if one firm is playing according to a separating equilibrium strategy (whether the high- or low-quality firm), and the other one is deviating to a strategy off the equilibrium path, then the latter is immediately perceived as a low-quality firm by consumers. These beliefs are the most pessimistic for a deviating firm, and support the largest possible set of separating profiles.

We first check that the low-quality firm has no incentives to deviate. Indeed, it is clear from the above definition of beliefs that any deviation to $(p, a) \neq (\tilde{p}_H, \tilde{a}_H)$ by this firm does not affect consumers perception of its type. From the necessary conditions for the existence of separating equilibria, the best response to these beliefs and the rival's action is $(\tilde{p}_L, \tilde{a}_L) \in \arg\max_{p,a} \pi_L(p, a, \tilde{p}_H, 0)$, implying that (p, a) is suboptimal. In addition, deviating by mimicking the high-quality behavior $(\tilde{p}_H, \tilde{a}_H)$ leads to $\mu((\tilde{p}_H, \tilde{a}_H), (\tilde{p}_H, \tilde{a}_H)) = \frac{1}{2}$. However, by virtue of necessary condition (IC_L), this deviation is suboptimal.

We now show that the high-quality firm has no incentives to deviate. First, deviating by mimicking the low-quality firm's equilibrium strategy leads to $\mu((\tilde{p}_L, \tilde{a}_L), (\tilde{p}_L, \tilde{a}_L)) = \frac{1}{2}$ and from necessary condition (IC_H), this deviation is suboptimal.

Second, if the high-quality firm deviates to $p \neq \tilde{p}_L$, then by the definition of out-of-equilibrium beliefs, consumers will believe that it is selling low quality (and that the rival firm is offering the high-quality product), as $\mu((\tilde{p}_L, \tilde{a}_L), (p, a)) = 1$. It then follows that the high-quality firm has no incentives to spend money in useless advertising, so that it will choose $a = 0$. In such a deviation, the demand $D_{L/H}$ addressed to the high-quality firm when perceived by consumers as offering a low-quality product is given by:

$$\begin{aligned} D_{L/H} &= 1 - z^*(\tilde{p}_L, p, 1) \\ &= 1 - \left(\frac{1}{2} - \frac{\tilde{p}_L - p - \Delta}{2td}\right) \end{aligned}$$

and the corresponding (concave) payoff is:

$$\pi^d_{L/H}(p) = (p - c) D_{L/H} = (p - c)\left(\frac{1}{2} + \frac{\tilde{p}_L - p - \Delta}{2td}\right). \tag{21}$$

The best deviation $p^d (\neq \tilde{p}_L)$ from $\tilde{p}_H$ is such that $p^d \in \arg\max_p \pi^d_{L/H}(p)$ which leads to:

$$p^d = \frac{td}{2} + \frac{\tilde{p}_L + c - \Delta}{2}. \tag{22}$$

Recall from Lemma 1 that $\tilde{p}_L = \frac{1}{2}[\tilde{p}_H + td - \Delta]$. Substituting this expression into (22), we can derive $p^d(\tilde{p}_H)$ and, after substitution in (21), we can finally write the payoff from the best (non-mimicking) deviation:

$$\pi^d_{L/H}(p^d) = \frac{1}{32td}\left(\tilde{p}_H + 3(td - \Delta) - 2c\right)^2$$

which is defined only for $\tilde{p}_H \geq p^1_H \equiv 3(\Delta - td) + 2c$ to ensure positive demand and markup.

We now prove that playing the best deviation $(p^d, 0)$ is dominated by playing the putative equilibrium strategy $(\tilde{p}_H, \tilde{a}_H)$. For this, it suffices to prove that $(p^d, 0)$ is dominated by the mimicking strategy $(\tilde{p}_L, \tilde{a}_L) = (\frac{1}{2}[\tilde{p}_H + td - \Delta], 0)$. Indeed, by Condition ($\text{IC}_H$), mimicking a low quality producer is already suboptimal.

The profit obtained when a high-quality producer mimics the low-quality producer is

$$\pi_H\left(\tilde{p}_L, \tilde{a}_L, \tilde{p}_L, \frac{1}{2}\right) = \left(\frac{1}{2}[\tilde{p}_H + td - \Delta] - c\right)\frac{1}{2}$$

which is only defined for $\tilde{p}_H \geq p_H^2 \equiv 2c + \Delta - td$. Now, computing the difference between $\pi^d_{L/H}(p^d)$ and $\pi_H\left(\tilde{p}_L, \tilde{a}_L, \tilde{p}_L, \frac{1}{2}\right)$, we obtain a convex quadratic form in $\tilde{p}_H$ which roots are given by $2c + td + 3\Delta \pm 4\sqrt{td\Delta}$. This difference is thus non positive between the roots.

There are two cases depending on whether d is lower or greater than Δ/t. First, when $d \leq \Delta/t$, then $\Delta - td \geq 0$. Hence, $p_H^2 \leq p_H^1$, which means that the profit from mimicking is higher than the profit from deviating, $\pi^d_{L/H}$, whenever $\tilde{p}_H$ is lower than the upper root $(2c + td + 3\Delta + 4\sqrt{td\Delta})$. But recall from Lemma 2 that the maximal admissible $\tilde{p}_H$ for a separating equilibrium to exist is $\Delta + td$, which is clearly lower than this upper root. It follows that for the admissible values of $\tilde{p}_H$, the mimicking strategy is always preferred to the best (non mimicking) deviation strategy.

Second, when $d \geq \Delta/t$, then $\Delta - td \leq 0$. In that case, recall that from Lemma 2 that the set of admissible $\tilde{p}_H$ for separation is $[2c + td - \Delta, \Delta + td]$. We just have shown that the maximal value $\Delta + td$ is lower than the upper root. It remains to show that the lowest admissible value $2c + td - \Delta$ is higher than the lower root $2c + td + 3\Delta - \sqrt{td\Delta}$:

$$2c + td - \Delta - \left(2c + td + 3\Delta - 4\sqrt{td\Delta}\right) = 4\sqrt{\Delta}(\sqrt{td} - \sqrt{\Delta}) \geq 0$$

as $d \geq \Delta/t$. Hence, once again, for the admissible values of $\tilde{p}_H$, the mimicking deviation is always preferred to the best (non mimicking) deviation. This concludes the proof.

C Proof of Proposition 3

Let us start with condition (15). The deviating profit is

$$\max_{p',a'} \pi_L\left(p', a', p, 0\right) = \max_{p',a'}\left(p' z^*(p', p, 0) - a'\right)$$

with $z^*(p', p, 0) = \left(\frac{1}{2} - \frac{p'-p+\Delta}{2td}\right)$ from (7). Solving this maximization problem, we find that the optimal deviation is $(p', a') = \left(\frac{1}{2}(p + td - \Delta), 0\right)$. Consequently, after substituting, the optimal profit from deviating for a low-quality firm is

$$\max_{p',a'} \pi_L \left(p', a', p, 0\right) = \frac{1}{8td} \left(p + td - \Delta\right)^2$$

which is only defined for $p \in [\max(c, \Delta - td), \Delta + 3td]$.[14] The pooling equilibrium profit for a low-quality firm is

$$\pi_L \left(p, a, p, \frac{1}{2}\right) = \frac{1}{2}p - a.$$

Hence, condition (15) reduces to:

$$0 \leq a \leq \underline{a}_H(p, p, 0) = \frac{1}{2}p - \frac{1}{8td} \left(p + td - \Delta\right)^2. \tag{23}$$

The function $\underline{a}_H(p, p, 0)$ is an inverted parabola function of p which is positive on the interval between the roots, $[\alpha, \beta] \equiv [td + \Delta - 2\sqrt{\Delta td}\,,\ td + \Delta + 2\sqrt{\Delta td}]$.

Looking at condition (16), it is easy to see that it yields a similar region in space (p, a) but horizontally translated to the right by an amount c. Indeed, we have:

$$\begin{aligned} \pi_H \left(p, a, p, \frac{1}{2}\right) &= \frac{1}{2} (p - c) - a \\ \max_{p',a'} \pi_H \left(p', a', p, 0\right) &= \max_{p',a'} \left[(p' - c) \left(1 - z^*(p, p', 1)\right) - a'\right] \end{aligned}$$

with $z^*(p, p', 1) = \left(\frac{1}{2} - \frac{p-p'-\Delta}{2td}\right)$. Solving this maximization problem, we obtain

$$\left(\frac{1}{2}(p + td + c - \Delta), 0\right) \in \arg\max_{p',a'} \pi_H \left(p', a', p, 0\right)$$

and consequently, $\max_{p',a'} \pi_H \left(p', a', p, 0\right) = \frac{1}{8td} \left(p + td - c - \Delta\right)^2$ which is only defined for $p \in [\max(c, \Delta - td + c), \Delta + 3td + c]$. Finally, condition (16) reduces to:

$$0 \leq a \leq \underline{a}_H(p - c, p - c, 0) = \frac{1}{2} (p - c) - \frac{1}{8td} \left(p - c + td - \Delta\right)^2. \tag{24}$$

Similarly, the function $\underline{a}_H(p - c, p - c, 0)$ defines an inverted parabola function of p, positive

[14] This definition set ensures that $p' \geq 0$ and that $z^*(p', p, 0) \in [0, 1]$.

on the interval between the roots $[\gamma, \delta] = [td + \Delta + c - 2\sqrt{\Delta td}\,, \, td + \Delta + c + 2\sqrt{\Delta td}]$. We are done if we can prove that the two regions defined by (23) and (24) in the space (p, a) intersect. Because $\alpha < \gamma$ and $\beta < \delta$, $[\alpha, \beta] \bigcap [\gamma, \delta] \neq 0$ if and only if $\gamma \leq \beta$:

$$td + \Delta + c - 2\sqrt{\Delta td} \leq td + \Delta + 2\sqrt{\Delta td}$$

which reduces to $d \geq c^2/16\Delta t$. Finally, note that $\gamma = td + \Delta + c - 2\sqrt{\Delta td}$ is greater than c, since

$$td + \Delta + c - 2\sqrt{\Delta td} - c = (\sqrt{\Delta} - \sqrt{td})^2 > 0.$$

This ensures that the high-quality firm gets a positive markup at any pooling equilibrium. This concludes the proof.

D Proof of Proposition 4

We have already explained in the text that the imposition of REDE on the low-quality firm has no impact on its incentive to deviate from the equilibrium profile. Therefore, the refined equilibrium profile will involve $(p_L, a_L) = \left(p_L^B(p_H), 0\right)$.

On the contrary, for the high-quality firm, the imposition of REDE makes some deviations profitable. Quite obviously, any separating equilibrium profile $((p_H, a_H), (p_L, a_L))$ with $a_H > \underline{a}_H(p_H, p_L^B(p_H), 0)$ does not resist to a deviation to $((p_H, a_H'), (p_L, 0))$ with $a_H' = \underline{a}_H(p_H, p_L^B(p_H), 0)$. Indeed, beliefs are required to be $\mu\left((p_L, 0), (p_H, a_H)\right) = \mu\left((p_L, 0), (p_H, a_H')\right) = 0$ (since $(p_H, a_H') \in \Omega$), and thus implies that the high-quality firm obtains a strictly higher profit by reducing its advertising expenditures. Clearly, for a given price p_H, there is no incentives for the high-quality producer to spend more than the minimum level $\underline{a}_H(p_H, p_L^B(p_H), 0)$ of dissipative advertising required for separation. Given this remark, we study the payoff of the high-quality firm:

$$\begin{aligned}
\pi_H^s(p_H) &= (p_H - c)\left(1 - z^*(p_L^B(p_H), p_H, 0)\right) - \underline{a}_H(p_H, p_L^B(p_H), 0) \\
&= (p_H - c)\left(1 - z^*(p_L^B(p_H), p_H, 0)\right) - \frac{1}{2}p_H + p_L^B(p_H) z^*(p_L^B(p_H), p_H, 0) \\
&= \frac{1}{4td}(p_H - c)\,(3td - p_H + \Delta) - \frac{1}{2}p_H + \frac{1}{8td}\,(p_H + td - \Delta)^2 .
\end{aligned}$$

It can easily be checked that $\pi_H^s(p_H)$ is an inverted parabola with its maximum at $p_H = c+2td$. We then have to rank $c+2td$ with the lower and upper bounds of admissible prices p_H. Given that we have the following inequalities:

$$\begin{aligned}
c+2td &> \Delta - td \Leftrightarrow d > \rho/3 \\
c+2td &> 2c + td - \Delta \Leftrightarrow d + \rho > 0 \text{ which is always true} \\
c+2td &> \Delta + td \Leftrightarrow d > \rho,
\end{aligned}$$

we are left with three possible regimes.

- for $d > \rho$, the set of admissible prices p_H is $[2c + td - \Delta, \Delta + td]$ and as $c+2td > \Delta+td$, clearly $\pi_H^s(p_H)$ is increasing on this interval. The highest payoff for the high-quality firm is then obtained when $p_H = \Delta + td$, i.e.

$$\pi_H^s(\Delta + td) = \frac{td - c}{2}.$$

- for $\rho \geq d > \rho/3$, the set of admissible prices p_H is $[\Delta - td, \Delta + td]$ and $c + 2td$ belongs to this interval. Consequently, the highest payoff for the high-quality firm is obtained for $p_H = c + 2td$, i.e.

$$\pi_H^s(c + 2td) = \frac{1}{2}(\Delta - 2c - td) + \frac{1}{8td}(c + 3td - \Delta)^2.$$

- for $\rho/3 \geq d \geq 0$, the set of admissible prices p_H is still $[\Delta - td, \Delta + td]$, but $\pi_H^s(p_H)$ is now decreasing on this interval because $c + 2td < \Delta - td$. Consequently, the highest payoff for the high-quality firm is obtained when $p_H = \Delta - td$, i.e.

$$\pi_H^s(\Delta - td) = \frac{1}{2}(\Delta - td) - c.$$

This concludes the proof.

E Proof of Proposition 5

From Proposition 4, we have for:

- $d \in D_3$, $p_H^* = \Delta - td$ so that $\Pi_H = \frac{1}{2}(\Delta - td) - c$. In addition, $p_L^* = p_L^B(\Delta - td) = \frac{1}{2}[\Delta - td + td - \Delta] = 0$ and hence $\Pi_L = 0$. We thus have

$$E\Pi(d) = \frac{1}{4}(\Delta - td) - \frac{c}{2}$$

which is decreasing in d over the set D_3.

- $d \in D_2$, $p_H^* = c + 2td$ and $\Pi_H = \frac{1}{2}(\Delta - 2c - td) + \frac{1}{8td}(c + 3td - \Delta)^2$. Moreover, $p_L^* = p_L^B(c + 2td) = \frac{1}{2}[c + 3td - \Delta]$ and we have $\Pi_L = \frac{1}{8td}(c + 3td - \Delta)^2$. Hence,

$$\begin{aligned} E\Pi(d) &= \frac{1}{4}(\Delta - 2c - td) + \frac{1}{8td}(c + 3td - \Delta)^2 \\ &= \frac{1}{8td}\left(7t^2d^2 + 2t(c - 2\Delta)d + (\Delta - c)^2\right). \end{aligned}$$

Studying this function for positive values of d reveals that it is convex, first decreasing, reaching a unique minimum at $d = \rho/\sqrt{7}$, and then increasing.

- $d \in D_1$, $p_H^* = \Delta + td$ and $\Pi_H = \frac{td-c}{2}$. Moreover, $p_L^* = p_L^B(\Delta + td) = td$ and $\Pi_L = \frac{td}{2}$. Hence, we have for $d \in D_1$,

$$E\Pi(d) = \frac{td - c}{4} + \frac{td}{4} = \frac{td}{2} - \frac{c}{4}$$

which is clearly increasing in d.

Given that the function $E\Pi(d)$ is continuous over $[0, 1] \cap D$, it is clear from those results that $E\Pi(d)$ is first decreasing then increasing. Hence, the maximum of $E\Pi(d)$ is obtained either for $d = 0$ or for $d = 1$. Computing these values, we have

$$\begin{aligned} E\Pi(0) &= \frac{\Delta}{4} - \frac{c}{2} \\ E\Pi(1) &= \begin{cases} \frac{t}{2} - \frac{c}{4} \text{ for } \rho < 1 \\ \frac{1}{8t}\left(7t^2 + 2t(c - 2\Delta) + (\Delta - c)^2\right) \text{ for } 1 \leq \rho < 3 \\ \frac{1}{4}(\Delta - t) - \frac{c}{2} \text{ for } \rho \geq 3 \end{cases} \end{aligned}$$

We easily obtain that $E\Pi(0) > E\Pi(1)$ whenever $\rho \geq 3$. Hence, differentiation is minimal in equilibrium, with only the high-quality firm being active. For $\rho < 1$, we have

$$E\Pi(0) - E\Pi(1) = \frac{\Delta}{4} - \frac{c}{2} - \left(\frac{t}{2} - \frac{c}{4}\right) = \frac{t}{4}(\rho - 2) < 0$$

so that maximal differentiation prevails. For $1 \leq \rho < 3$, we have

$$E\Pi(0) - E\Pi(1) = \frac{\Delta}{4} - \frac{c}{2} - \frac{1}{8t}\left(7t^2 + 2t(c - 2\Delta) + (\Delta - c)^2\right) = \frac{t}{4}(3\rho - \frac{1}{2}\rho^2 - \frac{7}{2}).$$

This quadratic form in ρ is non positive for $1 \leq \rho < 3 - \sqrt{2}$ and positive for $3 - \sqrt{2} \leq \rho < 3$.

Consequently, we have maximal differentiation ($d = 1$) for $0 \leq \rho < 3 - \sqrt{2}$ and minimal differentiation for $\rho \geq 3 - \sqrt{2}$. This completes the proof.

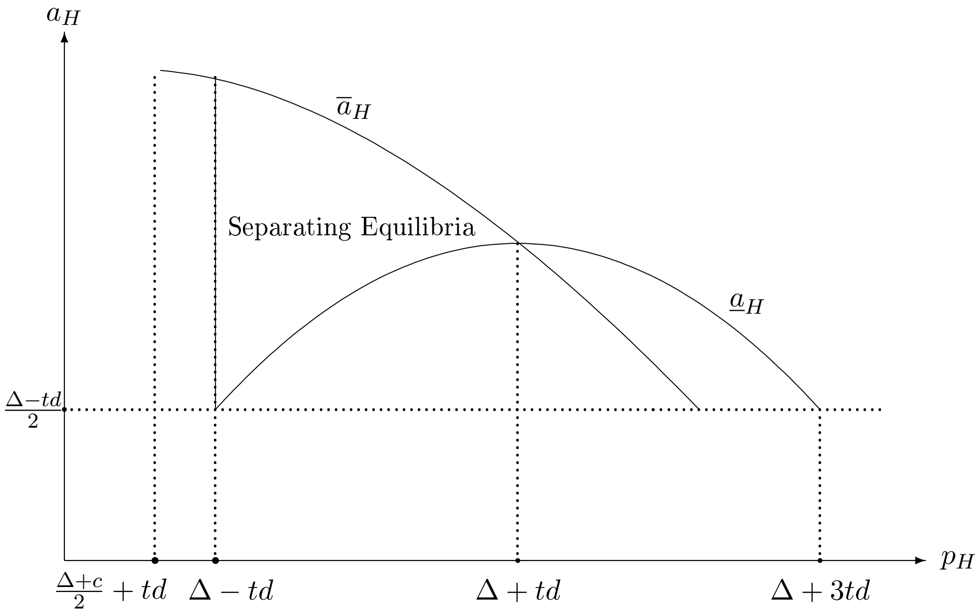

Figure 1: Set Ω when $0 \leq d \leq \frac{\rho}{4}$.

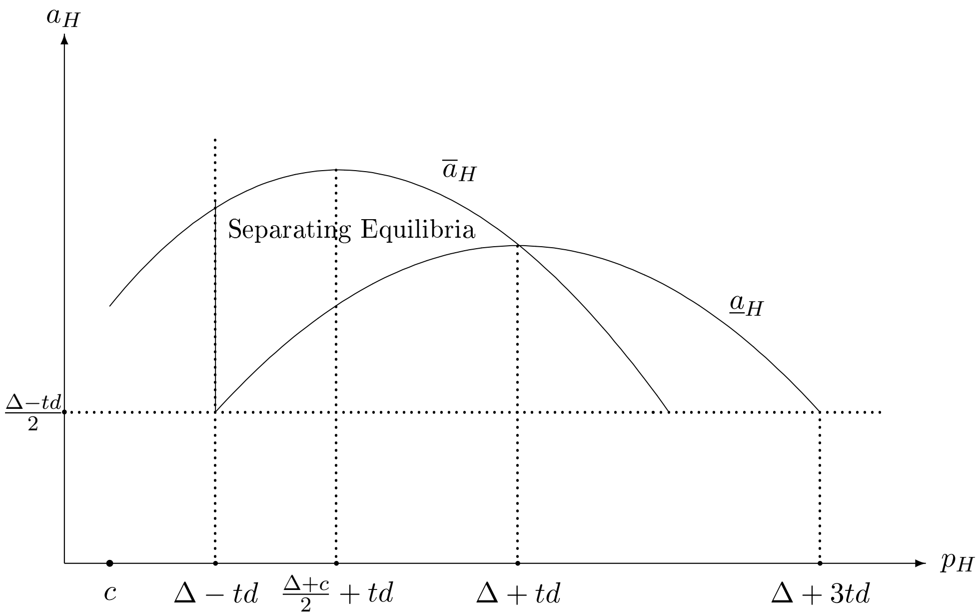

Figure 2: Set Ω when $\frac{\rho}{4} \leq d \leq \rho$.

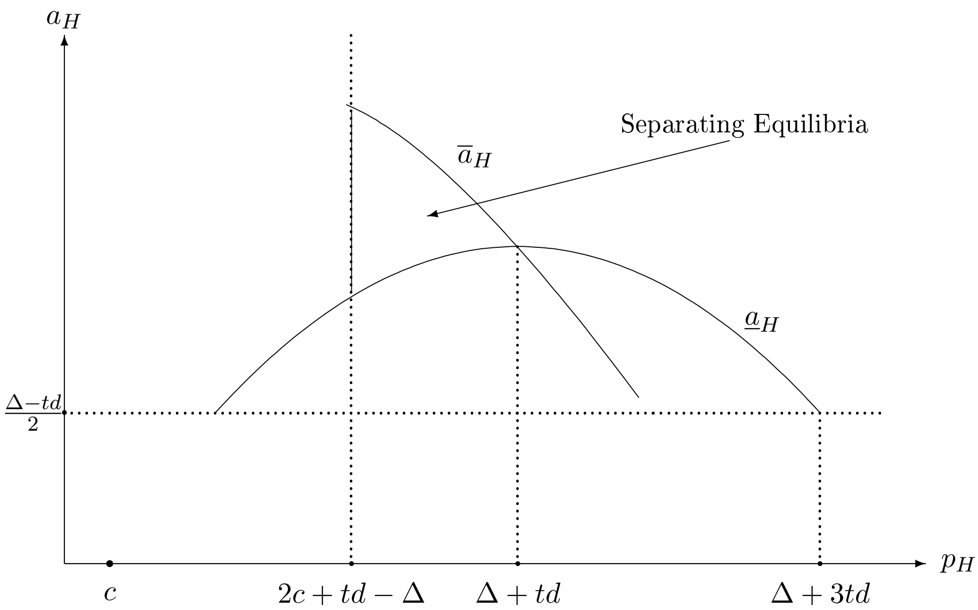

Figure 3: Set Ω when $d \geq \rho$.