The Declining Price Effect in Sequential Auctions : What Theory Does Not Predict*

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Abstract

This paper studies different explanations given for the "price decline anomaly" in sequential auctions, a phenomenon also known as the "afternoon effect". It surveys the dedicated theoretical models and then explores the influence of the institutional or market characteristics (of the sale) on the price trend. Next, it presents different methods used for measuring price trends and analytically identifies the differences between them. Finally, data from wine auctions are used to show that different methods may lead to opposite trends from the same data and that the number of identical objects being sold influences the price trend.

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1 Introduction

Most of the literature on auction has focused on the theory of single object auctions, which is now well established. Although auctions of multiple units are more widespread, they have received less attention. This is because models that try to capture the complexity of multiple auctions rapidly become intractable. Nevertheless, in the last ten years more and more attention has been devoted to multiple unit auctions. This shift has been motivated by the desire to explain price-decline anomaly and by the policy implications and lessons learned from the sale of spectrum rights in the US. Price-decline anomaly is a situation where objects (or lots of objects) are sold at lower prices than those of identical objects (or identical lots of objects) sold earlier within the same sale. Ashenfelter**'s** (1989) findings of a price decline on wine auction data, is the starting point of a new development in the theory of multiple unit auctions.[1]Hence, price-decline anomaly is an example of empirical observations triggering theoretical research.

Since Ashenfelter (1989), many empirical studies have found evidence of declining prices in auctions of wine (McAfee and Vincent, 1993 and Di Vittorio and Ginsburgh, 1994), condominium units (Ashenfelter and Genesove, 1992), stamps (Taylor, 1991), commercial properties (Lusht, 1994), jewellery (Chanel *et al*, 1996), works of art (Pesando and Shum, 1996**,** Beggs and Graddy, 1997) and so forth.

At first glance this price decline appears difficult to explain because theory shows that sequential auctions of identical objects should result on average in identical or rising prices. These results come from Weber (1983), where prices for identical objects follow a random walk in the case of independent

[1]Although Buccola (1982) found statistical evidence of a price decline in cattle auctions by using hedonic regressions.

private values and display an upward drift in the case of general affiliated values.[2] Because of this contradiction between theory and empirical studies, the declining price is considered as an anomaly. Part of the discussion in the literature is whether or not there truly is an anomaly. If prices show a systematic trend it is important to determine if it is an anomaly as postulated by Ashenfelter (1989), or if on the contrary it can be explained by some market characteristics or bidders' preferences and therefore be explained by rational bidders' behaviour. This paper also considers how this price decline has been established in empirical studies and more particularly how it has been computed.

As the methods used in the literature to compute the price trend require a pair of prices, the selection of a pair of lots among a parcel[3] of identical lots is worthy of study. The choice of the pairs of prices in a parcel is found to be crucial. It appears that the methods used to measure the price trend may lead to significantly different results when used on the same sample, and even that the same method can lead to opposite results according to the criterion chosen to select pairs in each parcel.

Price trends raise many issues since the expected revenue of the seller and/or the buyers may depend on the order in which objects are sold (for the seller) or the right time to bid (for the buyers). When the objects are not perfectly identical, the case of two stochastically equivalent objects for example, Bernhardt and Scoones (1994) show that it is optimal for the seller to first auction the object with the greatest variation in valuations. For two different objects, it is better to auction them by declining values (see Beggs and Graddy, 1997). Benoît and Khrisna (1998) reach the same conclusion when there are more than two objects and budget constrained bidders,[4] if

[2] See also Milgrom and Weber (1982a) and (1982b).

[3] A parcel is a set of identical lots of objects.

[4] Bidders who do not have enough resources to purchase all items for sale at their full

there are only two objects the order does not affect the price. Moreover, when there is a price trend, negotiations could occur between the auctioneer and the sellers to determine the optimal place of the object in the order of sale: the auctioneer tries to maximize his revenue from the whole sale and the sellers to maximize the expected sale price(s) of their object(s).

The first question is: what causes a price decrease and what are the theoretical explanations? The second is: does the way this price decline has been established in empirical studies matter ?

The paper is structured as follows. Section 2 presents Weber's model (1983) and surveys the explanations for declining prices based on modifications in one or more of its assumptions. Section 3 then considers the effects of institutional and market characteristics on price trends. Section 4 introduces the main methods used in the literature to measure price trends during sequential auctions. Section 5 focuses on techniques aggregating individual price variations and examines their properties and the analytical spread between them. Finally, in Section 6 the methods are tested on wine auctions data in order to estimate their properties on real data and test several hypotheses through the use of bootstrap techniques. Section 7 concludes the paper.

2 What does the theory say ?

Weber's (1983) sequential auction model with independent private values is used as a reference point. Its main assumptions and results are presented, and the consequences for the price sequences of relaxing them are explored.

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value.

2.1 The theoretical reference model and its variants

Weber's (1983) sequential auction model considers five main hypotheses:

- **H1: The number of bidders is known:** n, the number of bidders is known and exogenous.
- **H2: Risk neutrality:** the n bidders are risk neutral.
- **H3: Private information, Independence and Symmetry:**
  * **(a) Private information:** The values (or types) v_i, $i = 1, \ldots, n$, of the n bidders are private knowledge.
  * **(b) Independence:** Values: $v_1, \ldots, v_n$ are independently distributed, [5]
  * **(c) Symmetry:** Bidders only differ by the realization of their private values, and draw v_i from the same distribution.
- **H4: Unit demand:** Bidders buy at most one unit among the k identical units (or lots) in each parcel.
- **H5: Fixed values:** The v_i's remain unchanged for every new auction.

Weber (1983) shows that under this set of assumptions, a declining price effect cannot occur if bidders are rational. Indeed, two opposite effects coexist so that expected prices for identical goods are constant. On the one hand, when buyers leave the auction, the distributions of the remaining bidders' values move towards the left at each unit sold, so that competition is less acute, and prices have a tendency to decrease. On the other hand, the number of objects being sold decreases, which makes the bidders more competitive

[5] $V_1, \ldots, V_n$ are random variables whose realised values are $v_1, \ldots v_n$. $V_{(1)} \geq V_{(2)} \geq \ldots \geq V_{(n)}$ are the order statistics of the n types.

and leads to an increase in prices. For each unit sold the expected gain — computed for each bidder conditionally on the fact that his valuation is the highest — increases, and the bids go up. These two effects cancel out in equilibrium, and the expected price for each object remains constant.

The strategy for each bidder, as he can buy only one object,[6] consists in discounting his bid by his expected "profit" for future auctions. Equilibrium strategies of the reference model strongly depend on assumption H3. If bidders' valuations are correlated (H3 does not hold), agents not only take future opportunities into account when constructing their strategies but also the information about the object's value released by previous sales.

In a common value model (H3a and H3b do not hold), the value of the object is the same for each bidder ($v_i = v$ for all $i = 1, ..., n$), but is unknown ex ante. Bidders have private (not necessarily independent) signals conditional on the true value of the object. The revealed information induced by previous sales modifies bidders' expected value of the good.

Private and common value models are special cases of the standard affiliated value model introduced by Milgrom and Weber (1982a),[7] where the value of an object is composed of a private part and a common part. In this framework, Milgrom and Weber (1982a) show that information revealed by the seller about the quality of the object should contribute to a price increase (a phenomenon known as the "linkage principle"). This has been established

[6] Under the aforementioned assumptions H1 to H5, the unique perfect symmetric Nash-Bayesian equilibrium for a second price sequential auction is: $b_s(v_i) = E[V_{(k+1)} \,|\, V_{(s+1)} = v_i]$, where $b_s(v_i)$ is the bid submitted by a bidder of type v_i, $i = 1, \ldots, n$, for the auction $s = 1, \ldots, k$, if he has not yet obtained an object, with $k \leq n$. At the last auction, we find the dominant strategy of the Vickrey (1961) model, which consists for a bidder to submit his own valuation. For $k = 2$, for example, the two units would be sold at $E[V_{(3)}]$, a price corresponding to the expected third order statistic.

[7] See Milgrom and Weber (1982a) p.1098 for a definition.

for single auctions, but also holds for sequential auctions since winning bids —known after each auction— also constitute a type of information release about the quality of the object.[8] Consequently, expected prices will follow an increasing pattern for common or affiliated value models as long as information about the quality of the object is released by previous auctions: concerns about the winner's curse which implies prudent bids are balanced out by information about the quality of the object.

2.2 Relaxing the assumptions of Weber's basic model

This section examines the theoretical explanations for declining prices, based on modifications of Weber's basic model (1983).

2.2.1 Stochastically identical (or equivalent) objects (H5 relaxed, H4 and H3b can be relaxed)

A price decrease cannot be justified under the assumptions of Weber's reference model but it can be justified by considering Bernhardt and Scoones' (1994) model. They consider a private value model (H3a) where each bidder draws his value from the same distribution but where he learns his valuation of the second object only after the first object is sold. Bidders do not submit their true values during the first auction, but take into account the expected potential profit they can obtain by participating in the following auction, i.e. the option value of participating in the next auction. All bidders in this stochastically identical object second-price auction model will thus have the same option value, contrary to Weber's model where bidders have different option values. Therefore, in the first auction the bidder with the highest value will submit a higher bid than the one he would have submitted in Weber's framework and as a consequence prices decline.

[8] See Milgrom and Weber (1982a) or Weber (1983).

Within the same line of reasoning Engelbrecht-Wiggans (1994) shows that when H4 is relaxed and if bidders have a marginal decreasing utility, there is also a price decline.[9]

In Menezes and Monteiro (1995), the decision to participate in the next auction is endogenous. Participation fees and the fact that bidders must participate in the first auction in order to participate in the second also lead to a price decline.

2.2.2 Bidders' asymmetries and objectives (H3c relaxed, H4 verified or not)

There are few theoretical explanations justifying the price trends in terms of the buyers' bidding behaviour.

In her experimental study Burns (1985) considers the influence of market characteristics on bidders' strategies. She analyses the behaviour of two types of bidders (students and professionals) during a progressive oral auction of homogeneous goods (wool batches). Participants had to buy a given number of objects (orders) and face decreasing resale values and a two Dollar penalty for orders they were not able to fill. Students adjusted their bids during the auction but a price decline persists for professionals. Burn's explanation is that professionals use a full value bidding rule by bidding up to their full valuation, in order to obtain lots they have been asked for. Milgrom and Weber (1982b) give similar reasons to explain the price decrease observed at Sotheby's November 1981 auctions for leases on RCA satellite-based telecommunications transponders.

Chanel, Gérard-Varet and Vincent (1996) study jewellery sold at judiciary

[9] The extension is trivial since bidders receive a new value from the same distribution for each new auction, and the preceding winner receives a value strictly lower than his previous value.

auctions. They observe the presence of two types of buyers: professionals and amateurs. They also find that price behaviour for “identical homogenized” jewels varies: Prices increase for watches (mainly bought by amateurs), decrease for objects in gold (mainly bought by professionals), and are constant for necklaces, bracelets and rings.

In a second price auction, Vincent (1998) shows that when some bidders, “professionals”, have multi-unit demand and decreasing marginal utility and others, “amateurs”, are interested in only one object, according to the nature of the amateurs’ demand (unit demand or specific unit demand), different price trends emerge in equilibrium. This might suggest an explanation for the different price trends observed on the jewelry market. Therefore, introducing heterogeneity among bidders in an auction model can also induce a price trend.

2.2.3 Risk aversion (H2 is relaxed)

Risk averse bidders maximize their utility and not their expected profit. In this case, the first bids are equal to the future expected auction prices plus a risk premium for not getting the object. Then, the optimal strategy for a risk neutral bidder should be to bid for the last objects in the parcel. Ashenfelter (1989), Lusht (1994) and Pesando and Shum (1996) consider risk aversion as a possible explanation for the observed price decline. However, McAfee and Vincent (1993) in a private values model examine the influence of risk aversion on price patterns, and find that pure strategy equilibrium determination requires nondecreasing absolute risk aversion. This is not compatible with the usual assumptions about individual attitudes towards risk, where absolute risk aversion is often assumed to be decreasing. This leads to a mixed strategy equilibrium, which does not ensure that the bidder with the highest value will get the object. Therefore, ex post efficiency is not guaranteed and resale opportunities are higher.

2.2.4 The number of bidders is unknown (H1 is relaxed)

In an independent private value model, McAfee and McMillan (1987) explore equilibrium strategies in a one-shot auction, where the number of bidders is stochastic, and try to determine whether or not it is to the seller's advantage to release information about this number. They state that for a first-price sealed-bid auction with bidders having constant absolute risk aversion, the seller should conceal his information.[10] They find that information released ex ante about the degree of competition induces a bid-dispersion effect. This increases the variance of bids and tends to lower the selling price. More generally, in a first-price sealed-bid auction with bidders who are risk averse and have affiliated private values, the bid-dispersion effect is associated to the linkage principle, identified by Milgrom and Weber (1982a). Thus, a price increase or a price decrease can be observed depending on whether or not the linkage principle prevails over the bid-dispersion effect.

This bid dispersion effect has been identified in single object auctions, but can be adapted — like the linkage principle — to sequential auctions, which indeed release information during the process, and make it possible for the bidders to revise their knowledge about the number of participants.

[10] In a sealed-bid auction the effect of revealing information is established by comparing a sealed bid auction where no information is revealed and a sealed bid auction where information is revealed. For an English auction the information is revealed by the mechanism itself.

3 How market characteristics influence the price trend

This section considers institutional or market characteristics, which can also explain a price-decline.

3.1 Uncertainty about supply

3.1.1 Endogenous uncertainty about supply: the buyer's option

In some auctions a rule, called the buyer's option, allows the winner of a lot in a parcel to buy any following similar lots of the parcel at the price for which he bought the first one. This option relaxes the unit demand assumption H4. This institutional rule was in force in the auctions underlying the data studied by Ashenfelter (1989), except at Butterfield's San Francisco. Relying on the empirical fact that price decreases are more apparent at Butterfield's, Ashenfelter (1989) suggests that the buyer's option constitutes a way to reduce price decreases.

Black and de Meza (1992) and Burguet and Sakovics (1997) show that prices might decline even if bidders are risk neutral. Burguet and Sakovics (1997) show that the endogenous uncertainty about the number of future auctions generated by the buyer's option leads to a price decrease for second-price auctions and independent private values.

In Black and de Meza (1992), all bidders have decreasing marginal utility and multi-unit demand and the buyer's option still leads to decreasing expected prices. This result can be extended to English auctions. On the contrary, when the buyer's option does not exist prices are likely to increase.

Ginsburgh (1998) notes that this buyer's option is exercised in 37 percent of the cases at wine auctions at Christie's. This suggests that, as the option is not exercised on all the lots, the price paid for the first lot might be higher than the market-clearing price. This option therefore constitutes a good explanation for the price decreases.

3.1.2 Exogenous uncertainty about supply

Exogenous uncertainty about supply can be observed at fish markets. The auction starts with the arrival of the first ship and goes on according to further arrivals. Pezanis-Christou (1997) adds exogenous uncertainty about supply to a Weber (1983) framework and finds a price-decline in first and second price auctions. Burguet and Sakovics (1997) reach the same conclusion when the uncertainty about supply disappears after the first auction in a uniform price auction.

3.2 The sale is organized by decreasing quality

Ordering the items auctioned by decreasing quality might arise from a deliberate choice of the auctioneer and/or the salesroom, or be a consequence of the auction format (pooled auction).

3.2.1 The order of the sale is exogenous

Pesando and Shum (1996) examine the price-decline anomaly by comparing prices of Picasso prints sold during the same sale and assumed to be identical. The test of no significant difference between two prices is always rejected: on the whole sample (101 pairs), the price of the first print is on average 9% higher than the price of the second. However, the authors note that their analysis does not take into account differences in quality and the state of conservation of the prints. Experts from Christie's and Sotheby's admit that, in general, they prefer not to sell two prints of the same work during the same auction, but when this is the case, the better quality print is always sold first. Hence, if buyers are risk neutral, and without quantity constraints, it is quite natural for the price of the first multiple sold to be higher than that of the second.

In an independent private value model, Beggs and Graddy (1997) show that ordering paintings by decreasing value might increase the price-decline: the ratio "hammer-price/presale estimate" is decreasing.

3.2.2 The order of the sale is endogenous: the pooled auction

A particular type of auction can be viewed as a decreasing quality auction: the pooled auction, or right to choose auction. At each step, bidders compete to have the right to choose an object among the objects not yet sold. In the case of objects of different quality, it is not surprising to observe a decrease in price.

Gale and Hausch (1994) compare price trends and efficiency of sealed-bid sequential auctions and right to choose auctions. They consider two bidders with unit demand and two stochastically identical objects. This allows the bidders to have different values for each object. In contrast to Engelbrecht-Wiggans (1994) and Scoones and Bernhardt (1994), bidders are supposed to know their valuations for the two objects before the first auction takes place. Gale and Hausch (1994) show that a second price sealed-bid sequential auction may lead to a non-efficient allocation, whereas a right to choose auction leads to a price decline and an efficient allocation.

3.3 Written bids from absentees

Ginsburgh (1998) shows that in the case of wine auctions the price decline is likely to be caused by a high number of written bids.[11] In fact, he notices a price decline in most of the cases (179 out of 399) and in particular when written bids (registered by the auctioneer before the sale) win all the lots of wine in a parcel. When these reservation values are higher than those of the bidders present at the auction, the auctioneer assigns lots according to the decreasing value of written bids and the corresponding quantities supplied: the decrease is then automatic. In the presence of a high proportion of written bids, the auction mechanism automatically attributes the objects from the highest to the lowest written bids, which explains the price decrease.

The previous theoretical models each relax at least one hypothesis of Weber's reference model (1983). They offer plausible explanations for the price decline observed on some markets. Some specific auction rules may even make the decline inevitable. Nevertheless, the way some goods are sold at public auctions (e.g., wines, paintings, vehicles, collection stamps, jewels, flats, etc.), is far from the reference framework. Since most of the assumptions made in the theoretical models do not hold for the type of objects sold in real auctions, a price trend observed in empirical data cannot be considered as a validation of any theoretical auction model.

Moreover, the way trends are determined in the literature is questionable. The next section presents the three main methods used and focuses on the techniques aggregating individual price variations. Section 5 establishes their relative bias and shows how they may influence the computation of the trend.

[11] The written bids are the reservation values, for one lot or more, by bidders not attending the sale (the absentee bidders).

4 The different measures of the trend

For sequential auctions of multiple lots sold successively, the sequence of lots can be viewed as a sub-sale within the sale. The purpose is to measure the trend of this sequence. The price variations have to be aggregated in order to compute an overall price trend. This exercise is more complex than it seems.

First, the objects are not completely identical. They can often be differentiated: Quality of colors or preservation can differ between two prints of the same work, specific characteristics can explain a price difference between two bottles of the same wine (the "fullness" of a bottle, the colour of the wine, etc.).[12] Information about the quality of the objects is lost unless one attends the auction, but the buyers may take this information into account when bidding. If the objects (although similar) are ranked and auctioned according to their quality (which is frequently the case as mentioned in Section 3.2), the trend is biased.

Second, quantifying the price trend requires aggregation of the price variations of parcels of similar goods. In fact, apart from the (hypothetical) case where the same trend (increase, decrease or stability) occurs for each of the parcels, it is necessary to find a method that sums up these variations statistically, and interpret it. Auction results show that it is possible to observe a 50 % price variation between the prices of two similar goods sold during the same sale. Moreover, price levels of different sub-samples may present a hundred-fold variation! It is important therefore to verify whether the choice of the aggregation mechanism can influence the result.

Third, when there are more than two lots, some authors (McAfee and Vincent, 1993) arbitrarily choose a pair of lots. Section 6 shows the consequences of such a choice on the valuation of the trend.

[12] See Di Vittorio and Ginsburgh (1994) for more details.

4.1 Comparison of prices obtained by two different mechanisms

Ashenfelter and Genesove (1992) compare the prices paid in face-to-face bargaining with the prices fetched for identical condominium units sold at auction. Their line of reasoning is the following: If the price-decline anomaly was truly a result of the auction mechanism, then any relationship between the auction bid price and the order of sale (at the auction) would disappear when the item was resold.

They find that auction prices for identical units are approximately 13% higher than those obtained in face to face bargaining for the unsold units and claim that a price-decline anomaly occurs after correcting for quality differences.

Even though the idea is interesting, the approach of the authors can be questioned. On the one hand, the fact that bidders are not obliged to buy the condominium they bid for might induce a bias in the process leading to an overvaluation of the price fetched by the condominium at the auction. In a pooled auction, this overvaluation is revealed by an earlier order of sale. On the other hand, the authors analyse the trend of the sequence by juxtaposing different pooled auctions, which is also open to criticism. Furthemore, the small numbers of observations of the face to face bargaining (31) might be subject to individual characteristics.

4.2 Comparing individual trends

Ashenfelter (1989) and McAfee and Vincent (1993) compare the prices of identical lots of wines sold at the same sale and observe that prices are twice as likely to decrease than to increase, a phenomenon referred to as the afternoon effect.[13] These two studies implicitly assume that, on average, the number of decreases should be equal to the number of increases.

However, Keser and Olson (1996) notice that this assumption only holds if the price distribution of the objects sold is symmetric, which is not the case when the price distribution relies on the order statistic of buyers' valuations. They show that an asymmetric decline occurs even though the average price is the same. The percentage decline between the first and the second object auctioned varies according to the distribution, the number of bidders and the number of objects. Thus, they conclude that the price trend cannot be computed by simply comparing the number of increases and decreases.

Moreover, counting the number of increases or decreases does not give any information about the amplitude of the price trend.

4.3 Aggregating price variations of similar objects

The objective is to find a statistic (a single number) summing up the variations computed on each parcel of similar lots, in order to test its equality to zero. If equality is accepted, there is no trend on average, otherwise, the sign of the statistic will discriminate between an increase and a decrease.

Index numbers are widely used for constructing price indices. Such index numbers (for example as Laspeyres, Paasche, Fisher, Walsch or Törnqvist indices) aggregate the variations observed between two periods (bilateral index) or chain them in order to study longer periods (multilateral index). These indices can also be adapted to study price movements in auction in a bilateral frame.

[13] Ashenfelter (1989) compare 2370 pairs of objects for Christie's London, 1646 pairs for Sotheby's London, 499 pairs for Christie's Chicago and 100 pairs for Butterfield's San Francisco. McAfee and Vincent (1993) work on 411 pairs.

Consider N parcels i, $i = 1, ..., N$ of similar lots of objects sold at auction. Denote P_i^n the price in parcel i of lot sold at rank n, $n = 1, ..., \bar{n}_i$, where $\bar{n}_i$ is the number of the lots of the parcel i, with $\bar{n}_i \geq 2 \; \forall \; i$.

First, only **pairs** of similar lots are studied. Section 6 will consider different ways of selecting pairs when there are more than two similar lots in a parcel. Let denote P_i^1 the price of the first lot sold of the pair i, and P_i^2 the price of the second lot of the pair i. Define a_i as the ratio P_i^2/P_i^1. Note that the N pairs may come from different sales.

Two types of price indices are usually used for sequential auctions of multiple objects: the Arithmetic Mean of Ratios (AMR) and the Ratios of the Sums of Prices (RSP). Two other methods, the Geometric Mean of Ratios (GMR) and the Fisher Price Index (FPI), that have properties that are more suitable to bilateral indices will also be studied.

The Arithmetic Mean of Ratios, certainly the most widely used in the auction literature, consists in computing the ratio of the second price divided by the first price for each pair of objects, and then computing the arithmetic mean of the ratios. It can be written as:

$$AMR = \left(\frac{1}{N}\right) \sum_{i=1}^{N} \left(\frac{P_i^2}{P_i^1}\right) = \left(\frac{1}{N}\right) \sum_{i=1}^{N} a_i = \overline{a}.$$

This price index has been used by Ashenfelter (1989), and McAfee and Vincent (1993), in studies dealing with similar lots of wine sold at auction. Both found that the mean of the ratios is significantly lower than one for each of the auction houses, and conclude in favour of a price decline.

Despite its common use, a simple counterintuitive example shows an important drawback. Assume two pairs of objects are sold at auction: prices of the first pair are respectively 10 and 20, those of the second pair 20 and

10. The arithmetic mean of the ratios is 1.25, an increase of 25%, whereas a priori no trend exists in this sample.[14]

The Ratio of the Sums of Prices is the sum of the second prices divided by the sum of the first prices. It **independently** aggregates first and second prices, and can be written as:

$$RSP = \frac{\sum_{i=1}^{N} P_i^2}{\sum_{i=1}^{N} P_i^1} = \frac{\sum_{i=1}^{N} a_i P_i^1}{\sum_{i=1}^{N} P_i^1}.$$

Pesando and Shum (1996) use the RSP method and find a price decline in a study on Picasso prints. This method weights individual ratios a_i differently according to the level of price P_i^1, which is not the case for the three other methods.

The two other indices do not seem to have been used to aggregate price movements at auction but seem to be as promising.

The Geometric Mean Ratios can be written as:

$$GMR = \left[\prod_{i=1}^{N}\left(\frac{P_i^2}{P_i^1}\right)\right]^{\frac{1}{N}} = \left[\prod_{i=1}^{N} a_i\right]^{\frac{1}{N}},$$

with P_i^1 and P_i^2 strictly positive.

This method requires every price to be strictly positive, which is clearly the case with auction prices.

The Fisher Price Index often appears in the index number literature as an ideal choice. It is computed through the Harmonic Mean Ratio[15] (HMR) and the AMR, and can be written as:

$$FPI = [AMR \times HMR]^{\frac{1}{2}} = \left[\frac{\sum_{i=1}^{N} a_i}{\sum_{i=1}^{N} \frac{1}{a_i}}\right]^{\frac{1}{2}}.$$

[14] This drawback is known in the literature as "computation formula bias" (see Diewert, 1998 and Reinsdorf, 1998).

[15] $HMR = N/\sum_{i=1}^{N} 1/a_i$.

5 How the choice of a method influences the results

The purpose of this section is to establish how the choice of a price index to measure the mean variation of the prices of similar goods within a sale (or a set of sales) may change the results, depending on the distribution of individual price variations. First, note that AMR, GMR and FPI mask the price levels with the use of ratios, as opposed to RSP where price levels appear in the formula. Second, the methods suitable for pairs are ARM and FPI, because others aggregate or can be rewritten as aggregating the first and the second prices independently. Third, the fact that GMR $<$ AMR constitutes a structural source of bias between the two methods especially when price movements within the pairs vary significantly.

The axiomatic approach to index number theory gives further indications about the properties of the different indices (see Diewert, 1987 for a complete presentation of eleven properties). Special consideration is given to properties relevant for bilateral indices (for example, quantities of each object are fixed to one so that all properties concerning quantities are not considered).

Two indices do not fulfil all properties:

- AMR does not satisfy the "time reversal" property assuming a symmetric treatment of time, which certainly constitutes a serious flaw for a price index:

$$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{P_i^2}{P_i^1}\right) \neq 1/\left[\left(\frac{1}{N}\right)\sum_{i=1}^{N}\left(\frac{P_i^1}{P_i^2}\right)\right]$$

- RSP does not satisfy the "invariance to change in units of measurement" property, because price levels appear in the computations. This does not constitute a major drawback within our particular frame. The RSP method computes price variations in a way closely related to the calculation of the yield of a portfolio of shares, where the number of each type of share is weighted by its individual yield.

The different formula biases between price indices are established hereafter (see appendix for proofs). The different relations do not depend on the distributions of a_i, P_i^1 or P_i^2 at all, but only require that $var(a)$, the variance of a_i, exists.

Using a second-order series approximation, the difference between GMR and AMR appears as:

$$GMR - AMR = AMR \times \exp\left(\frac{-0.5var\,(a_i)}{AMR^2} - 1\right) + h.o.t.$$

$h.o.t.$ stands for higher order term.

The relation between AMR and FPI also results from a second order series approximation:

$$FPI - AMR = \frac{0.5 \times var(a)}{(AMR)^2} + h.o.t.$$

An expression of the bias between FPI and GMR is given by:

$$FPI - GMR = AMR\left[1 - \frac{0.5 \times var(a)}{(AMR)^2} - \exp\frac{(-0.5var(a)}{(AMR)^2}\right] + h.o.t.$$

The bias between RSP and AMR depends on the covariance and the mean of the first prices:

$$RSP - AMR \simeq \frac{cov\,(a, P^1)}{\overline{P^1}},$$

Where $cov\,(a, P^1)$ is the covariance between a_i and P_i^1, and $\overline{P^1}$ is the average of the P_i^1.

Notice that:

1) If $var(a) \rightarrow 0$ then $[FPI - AMR] \rightarrow 0$, $[GMR - AMR] \rightarrow 0$ and $[FPI - GMR] \rightarrow 0$.

2) If $cov\left(a, P^1\right) \rightarrow 0$, then $[RSP - AMR] \rightarrow 0$.

The biases are computed by using formulae presented above, with different values for $var(a)$ and $\overline{a}$. Figures 1 to 3 represent the spread between AMR, GMR and FIP computed for values of $var(a)$, between 0 and 0.02 (stepsize 0.005) and for values of AMR (or $\overline{a}$) between 0.97 and 1.03, i.e. -3% and 3% (stepsize 0.005).

[Figure 1 about here]

Figure 1 shows that differences between GMR and AMR are positive (which is not a surprise), almost linear in AMR, and approximately worth half the value of $var(a)$.

[Figure 2 about here] [Figure 3 about here]

This is also the case with the differences between FPI and AMR, plotted in Figure 2. Figure 3 shows the differences between FPI and GMR and tells quite **a** different story: the spread is not monotonic in $\overline{a}$ nor in $var(a)$, but is very small. To sum up GMR and FPI are very close and AMR yields results very different from those of GMR and FPI. When $var(a)$ is sufficiently large, these differences between the estimators might lead to different conclusions when applied to real data.

To clearly show the differences between RSP and AMR, some simple algebraic reformulation is useful:

$$\begin{aligned} \frac{cov\left(a, P^1\right)}{\overline{P^1}} &= corr\left(a, P^1\right) \times \frac{\sqrt{var\left(a\right) \times var\left(P^1\right)}}{\overline{P^1}} \\ &= corr\left(a, P^1\right) \times \sqrt{var\left(a\right)} \times \left[cv\left(P^1\right)\right], \end{aligned}$$

where *corr* stands for the coefficient of correlation and *cv* stands for the coefficient of variation.

[Figure 4 about here]

In Figure 4 the spread between RSP and AMR is represented as a function of $corr\,(a, P^1)$ and $var(a)$, fixing cv for P^1 to one.[16] Note that the difference increases with the magnitude of the correlation, but with the opposite sign, and increases with $var(a)$. When $corr\,(a, P^1) = 0$, AMR $=$ RSP, as expected.

Therefore, when $corr(a, P^1)$ is close to zero, AMR and RSP are very close and larger than GMR and FPI. When $corr(a, P^1)$ increases in absolute value, RSP differs from AMR, and is lower than GMR and FPI for correlation higher than 0.15.

6 Empirical evidence from wine auctions

The data analysed are 2,947 lots of wine sold at Christie's London between December 1995 and February 1996.[17] The selection criteria used by Ashenfelter (1989) and McAfee and Vincent (1993) are strictly respected: the two last lots of each parcel of wine of same vintage, same capacity, same lot size (2, 6, 12, 24 bottles, etc.). Hence, there are 1.160 lots of wine belonging to 427 parcels whose size lies between 2 and 7 lots. For now, only the two last lots of each parcel are selected. Section 6.2 studies how price trends vary with the choice of two lots from a parcel. Section 6.3 and 6.4, respectively, study the importance of the size of the parcel and of the number of lots still to be sold.

[16] A different value for $cv(P^1)$ only implies an homothetic variation of RSP-AMR. Note for example that auction data for lots of wine (section 6) show a cv of 1.15.

[17] We are very grateful to Victor Ginsburgh for allowing us to use his data.

6.1 Comparing the four estimators

Simple statistics show that prices do not change in 282 cases (66%), decrease in 127 cases (29.7%) and increase in 18 cases (4.3%). The four indices, when computed for the 427 pairs of prices, confirm these first findings, in the sense that each of them is negative, between -1.53% for AMR and -1.84% for RSP. They are noted θ_1 in Table 1.

The 427 observed price pairs can be seen as being drawn from a pair of unknown distribution functions,[18] which will be used to study the empirical distributions and test the difference between the indices with bootstrap procedures.

427 pairs of prices are drawn with replacement among the 427 observed pairs. Then the four indices θ_b^* (AMR, GMR, RSP and FPI) are computed for these pairs. This is executed B times, and results in B values $\theta_b^* = \theta_1^*, \ldots, \theta_B^*$ for each of the indices. The empirical distribution for each θ^*, its mean $\overline{\theta}^*$ and its variance $(\sigma^*)^2$ are also computed. It is then possible to observe their distribution and to test the biases between methods. Bickel and Freedman (1981) have shown, using Mallows' measure, that the θ^* distribution is a convergent and unbiased estimator of the true distribution θ, if both the number of replications B and the size n of the original data are large.

The number of replications B is set to 9.999, which is large enough to allow standard error and confidence intervals computations according to Efron and Tibshirani (1993). Results can be found in Table 1 for each of the four indice estimators (bootstrap sample 1): the mean $\overline{\theta}_1^*$, the standard error σ_1^* and the 95% confidence interval of the mean based on the percentile distribution.

[18] A joint normality test for each of the distributions points to log-normality at the 2.5% significance level. The mean and standard error for the first price distribution are 420.5 and 484, and for the second distribution, 412.8 and 474.

The underlying distributions are represented in Figure 5. RSP exhibits higher standard error than the others.

[Figure 5 about here]

Equality tests between $\overline{\theta}_1^*$ and θ_0 are conducted on the simulated percentile distribution for each index.[19] Indeed, constructing confidence intervals from the computed standard errors σ^* is inadequate when the estimator distribution is not normal, and more generally, when it is not symmetric.

Let α be the significance level, and define $\theta^*_{\ell o}$ the $100 \times (\alpha/2)$ percentile of the θ^* distribution, and θ^*_{up} the $100 \times \{1 - (\alpha/2)\}$ percentile. For each estimator, the interval $[\theta^*_{\ell o}, \theta^*_{up}]$ is constructed.

Bilateral equality tests between $\overline{\theta}^*$ and θ_0 can be written as:

H_0: $\overline{\theta}^* = \theta_0$: the $\overline{\theta}^*$ estimator is equal to θ_0

H_1: $\overline{\theta}^* \neq \theta_0$: the $\overline{\theta}^*$ estimator significantly differs from θ_0.

The decision rule is:

If $\theta_0 \in [\theta^*_{\ell o}, \theta^*_{up}]$, the null hypothesis is accepted at the α significance level,

If not, the null hypothesis is rejected at the α significance level.

The marginal significance level (p-value) is computed through the bootstrap distribution. It represents the probability, if the null hypothesis is true, of observing a test statistic no less extreme than the one observed in the sample. The null hypothesis is rejected if the p-value is inferior to a significance level generally chosen between 1% and 10%.

The corresponding p-values (see Table 1) indicate that no method is significantly biased.

[19] θ_0 is a given value of the parameter, corresponding to the null hypothesis.

A bilateral equality test $\overline{\theta}_1^*$= 0, for each of the methods shows a strongly significant decline (p-values close to zero). Ginsburgh (1998) emphasizes that the price decline may mainly be explained by the buyer's option and the existence of written bids. These possible explanations are ignored in order to be comparable to the Ashenfelter (1989) and MacAfee and Vincent (1993) approaches, and the emphasis is put on computational aspects.

The results obtained for each method are coherent and there is no doubt about the existence of a price decline in this sample, but it is interesting to check if the four methods yield significantly different results, by testing equality between trends shown by the methods taken two by two (6 tests), using the percentiles of their difference as estimator of standard errors. Equality is rejected at the 10% significance level only for RSP-AMR, with a p-value of 0.067. These two methods are widely used to compute price trends in sequential auctions and the fact they could lead to significantly different estimations of the trend suggests that one should be cautious when the decrease is not well-established.

6.2 The importance of choosing pairs

When a parcel contains more than two lots, different choices are possible to select a pair of prices. In order to study the sensitivity of the results to this choice, a second sample is constructed. The sample compares the price of first and last lots of each of the 427 parcels of similar wines instead of the two last lots, i.e. P_i^1 and $P_i^{\bar{n}_i}$, where $\bar{n}_i$ is the size of parcel i. Note that only the first price changes in each pair compared to the previous sample, and that the pairs are equivalent for parcel i with $\bar{n}_i = 2$.

The bootstrap distribution of the estimators is computed for these new pairs. The bottom of Table 1 ("bootstrap sample 2") shows, for each price index, the mean $\overline{\theta}_2^*$, the standard error σ_2^* and the 95% confidence interval of the mean. The decline is more important than before and significant for every method (p-value = 0 for all $\overline{\theta}_2^* = 0$ tests). The difference between bootstrap estimators $\overline{\theta}_1^*$ and $\overline{\theta}_2^*$ — based on the confidence intervals constructed from the $\overline{\theta}_2^*$ distribution percentile — is significant for every method since the marginal significance levels (p-values on the last line of Table 1) are lower than 0.02. It is then possible to obtain significantly different price trends for the same original sample using the same method.

It is possible to go one step further in looking for data leading to different conclusions on the direction of the price trend. Although such samples are not easy to find, the previous wine data are suitable. Suppose that the distributions $\overline{\theta}_1^*$ and $\overline{\theta}_2^*$ of each of the four estimators are shifted to the right[20]. Unilateral equality tests $\overline{\theta}_1^* > 0$ and $\overline{\theta}_2^* < 0$ based on these new distributions percentiles are both accepted at a significance level depending on the estimators: 0.045 for AMR, 0.065 for GMR, 0.07 for FPI and 0.15 for RSP due to its higher variance. Hence, it is possible to draw opposite conclusions on the same sample with the same method but with different extraction criteria from the parcels.

[Table 1 about here]

6.3 The importance of the size of the parcel

Up to now, the influence of the size of the parcel on the price variations has never been analysed. None of the two criteria previously used to extract pairs from a parcel are coherent when the size effect is studied, because both

[20]In practice, this is done by adding 0.02 to each estimator.

aggregate without accounting for it. The two first rows of Table 2 present the distribution of parcels in the sample according to their size (between 2 and 7 lots).

6.3.1 Does the number of lots already sold in a parcel matter?

The marginal price effect specific to the position of the lots in the parcel is presented at the bottom of Table 2.

[Table 2 about here]

For all the methods, there is a very significant decrease between the prices of the first two objects of each parcel (-1.63 to -1.93 %, p-value close to 0). The additional decrease between the second and third objects is also very significant but less important (-1.26 to 1.56 %). For the objects in third and fourth, and fourth and fifth position, the decrease is significant at the 5% level, but the marginal decrease is about half of the magnitude of the one between second and third objects. These results confirm both the importance of the choice of pairs among parcels when computing price indices and the importance of choosing the right time for the buyer to bid according to the size of the parcel.

6.3.2 Does the number of lots still to be sold in the parcel matter?

This phenomenon has not yet been explored. The underlying idea is that the lower the number of objects remaining in the parcel, the higher the decrease should be. Indeed, competition between bidders is supposed to be fiercer on the last lots of each parcel than between any other lots.

The following variable is constructed: $R_{ij} = P_i^{\bar{n}_i-(j+1)}/P_i^{\bar{n}_i-j}$, where j=0 to 5, corresponds to the remaining number of lots to be sold in parcel i, and $\bar{n}_i$ is the size of parcel i.

The corresponding results are in Table 3. As previously found (see top of Table 1) there is a very significant decrease between the prices of the last lots of each parcel (between -1.53 and -1.84 %, p-value close to 0). A weaker but still significant decrease is observed when the number of lots still to be sold (j) increases. R_4 and R_5 (last column of Table 3) however constitute an exception with a strong decrease (between -2.13% and -2.47%). The fact that these pairs are more likely to also be the first pairs of the considered parcels may constitute an explanation.

[Table 3 about here]

In order to clarify this point, a dummy variable FP is created: $FP_i = 1$ for the first pair of parcel i, and 0 otherwise. Then least squares regressions for a_i on R_j and FP (AMR estimator), and for $\ln(a_i)$ on R_j and FP (GMR estimator)[21] are computed. A significant decline is found between the two first lots of each parcel (-0.6%, p-value lower than 5%). The very significant decrease found in Table 3 between the prices of the two last lots of each parcel is confirmed (-1.16 % and -1.31 %, p-value close to 0). The decrease between the lots in the penultimate pairs remains significant at the 5% level but less important (-0.75 % and 0.83 %). No significant effects are found for $j \geq 2$.

Both the size of a parcel and the position of a lot in a parcel therefore constitute relevant information when studying price trends in sequential auctions. The higher price decrease observed between the two first lots of a parcel and the two last lots certainly results from fiercer competition between bidders for the first and the penultimate lots.

[21] The computations for RSP and FPI are much less trivial and have not been done.

7 Conclusion

The main results of this study of price decline in sequential auctions are as follows:

Theoretical predictions are sometimes compared to empirical results although they do not fulfill the same set of assumptions. This is mainly due to the absence of a unified theory. Many phenomena may explain the observation of a price decrease, but are not necessarily applicable to every auction. Some are specific to the bidders preferences (non-unit demand, heterogeneity of bidders) or to the organization of the auction (pooled auction, presence of buyer's option). The specific conditions of each auction must be well-defined and correctly taken into account when one tries to understand and explain price movements observed in empirical data, and efforts are required to integrate them in new theoretical models.

The choice of a method to measure the price trend is another problem. In fact, different methods may lead to important differences when used on simulated data, and to different statistics when applied to real auction prices, in particular with slightly pronounced trends. In order to limit specific bias, it is certainly of interest to compute three index numbers: AMR, RSP and either GMR or FPI (considering the very small spread found between them) to determine and quantify price trends.

The choice of the pairs of objects, the size of the parcels and the position of the lots in each parcel also influence the results on the trends.

Last, bearing in mind that similar objects are not strictly identical and that some differences may only be observed by bidders attending the auction and influence their valuations, conventional approaches used to study price movements for homogeneous goods might not be correct. When these differences are known, a better approach may be, to run hedonic regressions explaining prices with objective characteristics of the objects (including quality differences) in a first step, then to compute the different measures of the price trend corrected for quality differences.

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APPENDIX

Consider a sample $a_i, \ldots, a_N$ an *iid* $\widetilde{a}$ where $\widetilde{a}$ is a positive random variable with finite variance.

Let us define the following quantities:

$$AMR = \frac{1}{N}\sum_{i=1}^{N} a_i = \overline{a} \simeq E\left[a_i\right], \; i = 1, \ldots, N, \tag{1}$$

by the law of large numbers.

$$HMR = N\sum_{i=1}^{N} a_i^{-1}, \tag{2}$$

$$GMR = \left[\prod_{i=1}^{N} a_i\right]^{\frac{1}{N}}, \tag{3}$$

$$FPI = \left[AMR \times HMR\right]^{\frac{1}{2}}, \tag{4}$$

$$RSP = \frac{\sum_{i=1}^{N} P_i^2}{\sum_{i=1}^{N} P_i^1} \simeq \frac{E\left[P_i^2\right]}{E\left[P_i^1\right]}, \; i = 1, \ldots, N, \tag{5}$$

Remember that a second order Taylor series approximation of $f\left(a_i\right)$ around $E\left[a_i\right]$ is:

$$f\left(a_i\right) \simeq f\left[E\left(a_i\right)\right] + f^{'}\left[E\left(a_i\right)\right] \times \left[a_i - E\left(a_i\right)\right] + \frac{1}{2}f^{''}\left[E\left(a_i\right)\right] \times \left[a_i - E\left(a_i\right)\right]^2.$$

Hence, $E\left[f\left(a_i\right)\right]$ is equal to:

$$E\left[f\left(a_i\right)\right] \simeq f\left[E\left(a_i\right)\right] + \frac{1}{2}f^{''}\left[E\left(a_i\right)\right] \times var\left(a_i\right), \tag{6}$$

where $var\left(a_i\right)$ is the variance of a_i, $i = 1, \ldots, N$.

Approximation of the bias between AMR and GMR

Note that $\ln(GMR) = (1/N)\sum_{i=1}^{N}\ln(a_i)$ and that $\ln(AMR) = \ln\left[(1/N)\sum_{i=1}^{N}a_i\right]$ the logarithm function being concave, Jensens's inequality yields:

$$\ln(AMR) \geq \ln(GMR).$$

Hence,

$$\ln(AMR) - \ln(GMR) \geq 0.$$

Taking a second order Taylor series approximation of $\ln(a_i)$ around $E(a_i)$, and from (6), we obtain:

$$E[\ln(a_i)] \simeq \ln[E(a_i)] - \frac{1}{2} \times \frac{var(a_i)}{[E(a_i)]^2}.$$

Hence,

$$\ln(AMR) - \ln(GMR) \simeq \frac{1}{2} \times \frac{var(a_i)}{[E(a_i)]^2}.$$

From there, the exponential leads to:

$$GMR \simeq AMR \times \exp\left(\frac{-0.5var(a_i)}{AMR^2}\right) \tag{7}$$

Approximation of the bias between FPI and AMR

The approximation of the substitution bias formula for the CPI (see Diewert, 1998) is followed and adapted to bilateral framework.

Let ε be the random variable defined by the values:

$$\varepsilon_i \equiv \frac{a_i}{E(a_i)} \quad i = 1, \ldots, N \tag{8}$$

Note that

$$E(\varepsilon) = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i \equiv 1.$$

Moreover, according to (2), HMR may be written as:

$$HMR = E(a_i) \times \left[\frac{1}{N}\sum_{i=1}^{N}\varepsilon_i^{-1}\right]^{-1}.$$

Taking a second order Taylor series approximation of ε_i^{-1} around $\varepsilon_i = 1$, and according to (6):

$$E\left(\varepsilon_i^{-1}\right) \simeq 1 + var\left(\varepsilon_i\right).$$

In order to obtain the inverse development for $E\left(\varepsilon_i^{-1}\right)$, the coefficients in the identity $E\left(\varepsilon_i^{-1}\right) \times \left[E\left(\varepsilon_i^{-1}\right)\right]^{-1} \equiv 1$ are characterized and after some computation, it follows that:

$$\left[E\left(\varepsilon_i^{-1}\right)\right]^{-1} \simeq 1 - var\left(\varepsilon_i\right).$$

Substituting HMR in (4), leads to:

$$FPI \simeq E\left(a_i\right) \times \left[1 - var\left(\varepsilon_i\right)\right]^{\frac{1}{2}}.$$

Taking the second order Taylor series approximation of $var\left(\varepsilon_i\right)$ around 0, yields:

$$FPI \simeq E\left(a_i\right) \times \left[1 - 0.5var\left(\varepsilon_i\right)\right].$$

Hence, FROM (8) and (1):

$$FPI \simeq AMR \times \left[1 - \frac{0.5var\left(a_i\right)}{AMR^2}\right]. \tag{9}$$

Approximation of the bias between FPI and GMR

Using (7) and (9) :

$$FPI \simeq GMR \times \exp\left[\frac{0.5var\left(a_i\right)}{AMR^2}\right] \times \left[1 - \frac{0.5var\left(a_i\right)}{AMR^2}\right]$$

Approximation of the bias between AMR and RSP

Rewrite $E\left(P_i^2\right)$ as:

$$E\left(P_i^2\right) = E\left(a_i P_i^1\right) = E\left(a_i\right) E\left(P_i^1\right) + cov\left(a_i, P_i^1\right) \;\; i = 1, \ldots, N,$$

where $cov\left(\cdot\right)$ stands for the covariance.

Hence, by (5):

$$E\left(a_i\right) + \frac{cov\left(a_i, P_i^1\right)}{E\left(P_i^1\right)} = \frac{E\left(P_i^2\right)}{E\left(P_i^1\right)} \simeq RSP \;\; i = 1, \ldots, N.$$

Q.E.D.

TABLES

Table 1: Statistics of the four estimators for observed auction prices

		AMR	GMR	RSP	FPI
Original Sample 1	θ_1 (in %)	-1.532	-1.646	-1.844	-1.667
Bootstrap 1	$\overline{\theta}_1^*$ (in %)	-1.536	-1.649	-1.846	-1.670
sample	σ_1^* ($\times 10^{-2}$)	0.201	0.245	0.315	0.259
2 last	95% conf. interval	(-1.96,-1.17)	(-2.17,-1.22)	(-2.51,-1.28)	(-2.23,-1.22)
lots of	P-value for $\overline{\theta}_1^* = \theta_1$	0.973	0.951	0.967	0.942
the parcel	P-value for $\overline{\theta}_1^*$ =0	0	0	0	0
Original Sample 2	θ_2 (in %)	-2.320	-2.475	-2.697	-2.506
Bootstrap 2	$\overline{\theta}_2^*$ (in %)	-2.319	-2.473	-2.698	-2.504
sample	σ_2^*($\times 10^{-2}$)	0.234	0.285	0.427	0.305
First and	95% conf. interval	(-2.80,-1.89)	(-3.08,-1.97)	(-3.60,-1.92)	(-3.16,-1.97)
last lots of	P-value for $\overline{\theta}_2^* = 0$	0	0	0	0
the parcel	P-value for $\overline{\theta}_1^* = \overline{\theta}_2^*$	0	0.0003	0.012	0.0004

Table 2: Influence of the number of lots already sold in each parcel[22]

No. of lots in the parcel	2	3	4	5	6 and 7
Frequency by size (in %)	57.4	23.9	11.9	3.7	3.1
Pairs	(P_i^1, P_i^2)	(P_i^2, P_i^3)	(P_i^3, P_i^4)	(P_i^4, P_i^5)	(P_i^5, P_i^6) and $(P_i^6, P$
No. of pairs	427	182	80	29	15
AMR (in %)	-1.632	-1.255	-0.674	-0.577	0.160
P-value of nullity test	0	0	0.029	0.012	0.762
GMR (in %)	-1.694	-1.416	-0.726	-0.585	0.147
P-value of nullity test	0	0	0.023	0.012	0.764
RSP in %	-1.927	-1.563	-0.858	-0.443	0.343
P-value of nullity test	0	0	0.002	0.012	0.554
FPI in %	-1.696	-1.461	-0.728	-0.585	0.147
P-value of nullity test	0	0	0.023	0.012	0.764

Table 3: Influence of the number of lots still to be sold in each parcel[23]

Variable	R_0	R_1	R_2	R_3	R_4 and R_5
No. of pairs	427	182	80	29	15
AMR (in %)	-1.532	-1.121	-0.95	-0.892	-2.237
P-value of nullity test	0	0	0	0.008	0
GMR (in %)	-1.646	-1.166	-0.97	-0.926	-2.133
P-value of nullity test	0	0	0	0.001	0
RSP in %	-1.844	-1.471	-0.735	-1.218	-2.469
P-value of nullity test	0	0	0	0.016	0
FPI in %	-1.667	-1.166	-0.097	-0.927	-2.333
P-value of nullity test	0	0	0	0.007	0

[22] Marginal significance levels (p-values) are computed from the bootstrap distributions.

[23] Marginal significance levels (p-values) are computed from the bootstrap distributions.

FIGURES

Figure 1: Differences between FPI and AMR

Figure 2: Differences between FPI and GMR

Figure 3: Differences between RSP and AMR

Figure 4: Differences between GMR and AMR

Figure 5: Histograms for the four estimators, 9.999 replications (sample 1)