Centre for Efficiency and Productivity Analysis

Working Paper Series
No. WP04/2010

Estimating State-allocable Production Technologies When there are two States of Nature and State Allocations of Inputs are Unobserved

C.J. O'Donnell & S.Shankar

Date: November 2010

School of Economics
University of Queensland
St. Lucia, Qld. 4072
Australia

ISSN No. 1932 - 4398

ESTIMATING STATE-ALLOCABLE PRODUCTION TECHNOLOGIES WHEN THERE ARE TWO STATES OF NATURE AND STATE ALLOCATIONS OF INPUTS ARE UNOBSERVED[1]

by

C.J. O'Donnell and S.Shankar

The University of Queensland
Centre for Efficiency and Productivity Analysis
Brisbane 4072, Australia

Abstract: Chambers and Quiggin (2000) have used state-contingent production theory to establish important results concerning economic behaviour in the presence of uncertainty, including problems of consumer choice, the theory of the firm, and principal-agent relationships. Empirical application of the state contingent approach has proved difficult, not least because most of the data needed for applying standard econometric methods are lost in unrealized states of the world. O'Donnell and Griffiths (2006) show how a restrictive type of state-contingent technology can be estimated in a finite mixtures framework. This paper shows how Bayesian methodology can be used to estimate more flexible types of state-contingent technologies.

[1] Paper presented at the 2009 Annual Conference of the Australian Agricultural and Resource Economics Society (AARES), Cairns. The project was completed while O'Donnell was visiting the Universitat Autonoma Barcelona. Financial support was provided by the Generalitat de Catalonia and the Australian Research Council.

1. INTRODUCTION

State-contingent production theory allows economists to apply the analytical tools of modern microeconomics in a stochastic production setting, provided *ex ante* preferences and production technologies are properly defined. Chambers and Quiggin (2000) have used the theory to establish important results concerning problems of choice under uncertainty, including the problems of moral hazard, incentive regulation and portfolio choice. Unfortunately, empirical implementation of the theory in a production context has proven difficult, not least because the *ex ante* production choices of firms are only partially observed.

O'Donnell and Griffiths (2006) have shown how to empirically estimate *output-cubical* state-contingent technologies in a finite mixtures framework. Unfortunately, output-cubical technologies are inconsistent with important stylized facts concerning behaviour in the presence of risk. The purpose of this paper is to show how the observed inputs and outputs of firms can be used to econometrically estimate more flexible *state-allocable* state-contingent technologies.

The structure of the paper is as follows. Sections 2 and 3 describe some of the important characteristics of stochastic technologies and the producer optimization problem in the presence of risk. Section 4 develops an econometric model for recovering the parameters of a two-state stochastic technology when allocations of inputs to diffierent states of Nature are unobserved. Section 5 uses noiseless simulated data to demonstrate that the methodology can be used to recover unknown parameters and other economic quantities of interest without error. Section 6 successfully applies the methodology to a real-world data set and recovers estimates of the (risk-neutral) probabilities farmers assign to different states. The paper is concluded in Section 7.

2. STOCHASTIC TECHNOLOGIES

We begin by considering a firm that uses a single non-stochastic input to produce a single stochastic output. We assume production activities take place over two time periods: in period 0 the producer chooses the input in the face of uncertainty; in period 1, Nature resolves uncertainty by choosing from a set of states $\Omega = \{1,...,S\}$. If Nature chooses $s \in \Omega$ then the ex post realization of stochastic output is given by the state-*s* production function

(2.1) $$z_s = f(x_s, \beta_s)$$

where β_s is a vector of parameters permitted to vary across states, and $x_s \geq 0$ is the amount of input allocated to production in state *s*. We assume the function *f* is everywhere continuous and satisfies standard regularity properties, including monotonicity and quasi-concavity. Chambers and Quiggin (2000) call such a technology *state-allocable*.

To illustrate the concept of state-allocability, Chambers and Quiggin (2000) provide a simplified cropping example in which a producer makes a pre-season allocation of a fixed amount of effort to the development of irrigation infrastructure and/or flood-control facilities. If the producer allocates more pre-season effort to irrigation than to flood control then output will be relatively high if realized rainfall is low, and relatively low if rainfall is high. Thus, different allocations of pre-season effort imply a trade-off between output realized in a low-rainfall state and output realized in a high-rainfall state. Indeed, we can think of the producer as allocating the input to production in different states, and of reallocating the input between states in order to effect a substitution between state-contingent outputs.

Figure 1 depicts a two-state technology where the total amount of the input used in the production process has been fixed at x^*. Rightward movements along the horizontal axis in panel (a) in Figure 1 correspond to a re-allocation of this fixed amount of input from production in state 1 to production in state 2. The downward-sloping line in this panel shows how output in state 1 decreases as the amount of the input allocated to that state decreases; the upward-sloping function shows how output in state 2 increases as the amount of the input allocated to that state increases. Panel (b) simply depicts the associated production possibilities frontier in state-contingent output space. Observe that by allocating x_1^A units of the input to state 1 and $x_2^A = x^* - x_1^A$ units to state 2 the firm can eliminate risk ($z_1 = z_2$ at point *A*). However, any other allocation of x^* involves risk. For example, if the input is equally-allocated between states the firm will obtain a higher output in state 2 than in state 1 ($z_2 > z_1$ at point *B*). The bisector in panel (b) gives the locus of all riskless state-contingent output pairs. The line passing through point *C* is a fair-odds line that will be discussed later in the paper.

Associated with (2.1) is the state-specific input requirement function $x_s = f^{-1}(z_s, \beta_s)$. It follows that production[2] of the state-contingent output vector $z = (z_1, ..., z_S)'$ requires an input commitment of

(2.2) $$x \geq \sum_{s \in \Omega} f^{-1}(z_s, \beta_s).$$

The input distance function is[3]

(2.3) $$D_I(x, z, \beta) = \frac{x}{\sum_{s \in \Omega} f^{-1}(z_s, \beta_s)}$$

where β contains the distinct elements of $\beta_1, ..., \beta_S$. This functional representation of the technology is the inverse of a Farrell (1957) measure of technical efficiency. Other standard representations of the production technology are also available, including cost and output distance functions[4]. In each of these alternative representations, the vector of state-contingent outputs is treated in the same way as we treat vectors of multiple outputs when production is non-stochastic.

3. FIRM BEHAVIOUR

Given a normalized input price of $w > 0$, the net return in state of Nature *s* is $y_s \equiv z_s - wx$. We assume the firm seeks to maximise a general welfare function that is non-decreasing in state-contingent net returns. Then its optimization problem can be written

(3.1) $$\max_z \{W(y) : D_I(x, z, \beta) \geq 1\}$$

where $y = (y_1, ..., y_S)'$ and *W* is a welfare function with the property $W_s(y) \equiv \partial W(y) / \partial y_s \geq 0$. The first-order conditions for efficient firm behaviour are[5]

---

[2] Strictly speaking, the firm does not produce *z*. Rather, it commits the input in such a way that z_s is produced if Nature chooses *s* from Ω.

[3] The input distance function is defined as $D_I(x, z, \beta) = \max\{\rho : x/\rho \text{ can produce } z\}$. Let ρ^* be the maximum factor by which a firm can contract its input vector and still produce the same output vector. That is $g(z, \beta) - x/\rho^* = 0$. It follows that $D_I(x, z, \beta) = x / g(z, \beta)$.

[4] Given a normalized input price of $w > 0$, the cost function is $c(w, z, \beta) = wg(z, \beta)$. To derive the output distance function, let δ be the largest factor by which a firm can expand its output vector while holding its input vector fixed. Then $g(\delta z, \beta) - x = 0$. If *f* is homogeneous of degree b^{-1} then *g* is homogeneous of degree *b*, so that $\delta = g(z, \beta)^{-1/b} x^{1/b}$. The output distance function gives the *inverse* of the largest factor by which a firm can expand its output vector while holding its input vector fixed. Thus, $D_O(x, z, \beta) = 1/\delta = g(z, \beta)^{1/b} x^{-1/b}$.

[5] See Chiang (1984, pp. 201-202). The partial total derivative of $W(y)$ with respect to $z_s \geq 0$ is

$$\sum_{m \in \Omega} W_m(y) \frac{\partial y_m}{\partial z_s} = \sum_{m \in \Omega} W_m(y) \frac{\partial (z_m - wg(z, \beta))}{\partial z_s} = W_s(y) - w \sum_{m \in \Omega} W_m(y) \frac{\partial g(z, \beta)}{\partial z_s}.$$

The inequality in (3.2) is due to the non-negativity restrictions $z_s \geq 0$.

(3.2) $\pi_s - wm(z_s, \beta_s) \le 0$ for all $s \in \Omega$

where $m(z_s, \beta_s) \equiv \partial f^{-1}(z_s, \beta_s) / \partial z_s > 0$ and

(3.3) $$\pi_s \equiv \frac{W_s(y)}{\sum_{s \in \Omega} W_s(y)} \in (0,1).$$

Because the π_s terms lie in the unit interval and sum to one, they can be interpreted as risk-neutral probabilities – the subjective probabilities a risk-neutral firm would need to have if it were to select the same production plan as a rational firm with preferences $W(y)$. Equation (3.2) implies that any efficient choice for a rational firm with an objective function defined over net-returns can be viewed as though it were generated by a risk-neutral firm with subjective probabilities given by $(\pi_1, ..., \pi_S)'$. Thus, without loss of generality, we can restrict our attention to the risk-neutral case.

Before solving the first-order conditions (3.2) for a specific stochastic technology, it is useful to consider an efficient risk-neutral firm seeking to solve the optimization problem (3.1) subject to the additional constraint that the input level is fixed at x^*. The constrained optimization problem can be written

(3.4) $$\max_{x_1, ..., x_S} \left\{ \sum_{s \in \Omega} \pi_s z_s : z_s = f(x_s, \beta_s) \text{ for all } s; \sum_{s \in \Omega} x_s = x^* \right\}$$

and has an interior solution that satisfies[6]

(3.5) $$\frac{\partial z_s}{\partial z_m} = -\frac{\pi_m}{\pi_s},$$

for all $s, m \in \Omega$. Thus, x^* is optimally allocated (i.e., expected output is maximized) when negative odds ratios are equated to marginal rates of substitution between state-contingent outputs. Panel (b) of Figure 1 depicts the

---

[6] The Lagrangean is

$$L = \sum_{s \in \Omega} \pi_s f(x_s, \beta_s) - \psi \left[x^* - \sum_{s \in \Omega} x_s \right]$$

The first-order conditions are

(1) $$\frac{\partial L}{\partial \psi} = x^* - \sum_{s \in \Omega} x_s = 0 \quad \text{and}$$

(2) $$\frac{\partial L}{\partial x_s} = \pi_s \frac{\partial f(x_s, \beta_s)}{\partial x_s} + \psi = 0$$

From (2):

(3) $$\frac{\partial f(x_s, \beta_s)}{\partial f(x_m, \beta_m)} \frac{\partial x_m}{\partial x_s} = \frac{\pi_m}{\pi_s}$$

and from (1):

$$x_m = x^* - \sum_{s \neq m} x_s.$$

Thus, $\partial x_m / \partial x_s = -1$ and equation (3) collapses to equation (3.5).

case where an optimal allocation of x^* places the efficient firm at point *C* on the efficient frontier. The straight line passing through point *C* is the locus of all points with the same expected output. It has slope $-\pi_1 / \pi_2$ and is known as the *fair-odds line*. Pictorially, optimization involves choosing that fair-odds line that is furthest from the origin and shares a point in common with the production possibilities set.

Finally, Figure 1 allows us to illustrate the importance of properly defining the stochastic technology. Suppose the amount of input allocated to state $s \in \{1,2\}$ is fixed at $0.5x^*$. Then the efficient firm is operating at point *B* in panel (b) of Figure 1. Free disposability of state contingent outputs, together with the fact that the firm has no capacity to reallocate the input between states, means the production possibilities frontier is the rectangle with vertices at the origin and point *B*. For this technology, the fair-odds line that solves the firm's optimization problem will always pass through point *B*, implying the firm will not (cannot) alter the mix of state-contingent outputs in response to changes in the risk-neutral probabilities. Even when the firm believes that state *s* is a near-certainty, it will not (cannot) re-allocate inputs to the production of output in that state. This is implausible. Restrictive technologies of this type (i.e., ones that do not allow substitution between state-contingent outputs) are said to be *output-cubical*. This term derives from the fact that when *S* = 3 the production possibilities set can be represented as a cube in state-contingent output space.

4. ESTIMATION IN THE TWO STATE CASE

In some empirical applications, input allocations to states of Nature and realized states of Nature are both readily observed. For example, O'Donnell, Chambers and Quiggin (2006) describe a sugar-cane production system in which producers plant different varieties of sugar cane (either high-yielding but disease-susceptible, or lower-yielding and disease-resistant) in the face of uncertainty about the incidence of sugar-cane smut disease. Acreages planted to different varieties of cane (input allocations) and levels of disease infestation (realized states) can both be observed *ex post*. In these cases, conventional estimation methods, including data envelopment analysis (DEA) and stochastic frontier analysis (SFA), can be used to recover the parameters of the production technology. In some other empirical contexts, only the input allocations are observed. For example, medical researchers can usually observe the different types of influenza vaccins administered by medical practitioners (input allocations), but cannot observe the numbers of patients exposed to different strains of influenza virus (realized states). In these cases, if the technology is output-cubical, the parameters of the production technology can be estimated within the finite mixtures framework developed by O'Donnell and Griffiths (2006). This paper develops methodology for estimating the parameters of the production technology in a third empirical context, namely when there are two observable states of Nature but input allocations to these states are unobserved.

Underpinning our estimation methodology is the assumption that firms are rational and technically efficient in production. The efficiency assumption, which can be easily relaxed, means that the relationship between total input usage and state contingent outputs is of the form

$$x - \sum_{s \in \Omega} f^{-1}(z_s, \beta_s) = 0. \tag{4.1}$$

The rationality assumption means that an interior solution to the firm's optimisation problem is given by

$$\pi_s = wm(z_s, \beta_s) \qquad \text{for all } s \in \Omega. \tag{4.2}$$

Equation (4.2) is especially important for two reasons. First, if the inverse of $m(z_s, \beta_s)$ exists then we can express state-contingent outputs as a function of normalised input prices and risk-neutral probabilities:

$$z_s = m^{-1}\left(w^{-1}\pi_s, \beta_s\right) \quad \text{for all } s \in \Omega. \tag{4.3}$$

Second, in the two-state case, equation (4.2) allows us to express risk-neutral probabilities as functions of normalised input prices, realized states of Nature, and *observed* outputs:

(4.4) $$\pi_1 = e_1\left[wm(q,\beta_1)\right] + e_2\left[1 - wm(q,\beta_2)\right]$$ and

(4.5) $$\pi_2 = e_2\left[wm(q,\beta_2)\right] + e_1\left[1 - wm(q,\beta_1)\right]$$

where $e_s = 1$ if $s = 1$ (and 0 otherwise). Equations (4.4) and (4.5) can be substituted into equation (4.3), and the result can then be substituted into equation (4.1). This yields a possibly nonlinear relationship between total inputs, normalized input prices, realized states of Nature, observed outputs, as well as the unknown parameters of the production technology. Estimation involves embedding this relationship in a stochastic framework and applying an appropriate econometric estimator, such as nonlinear least squares (NLS). Importantly, equation (4.2) cannot be used on its own to recover the parameters of the technology. To see this, simply note that for any (z_s, β_s) pair there exists a π_s that will satisfy (4.2) exactly. This means that the parameters and risk-neutral probabilities cannot be separately identified unless additional information is introduced into the estimation process. In this paper, this additional information comes in the form of equation (4.1).

5. EXAMPLE – SIMULATED DATA

O'Donnell, et al. (2006) demonstrate that conventional approaches to efficiency measurement may be systematically and seriously biased in the presence of uncertainty. For illustrative purposes, they consider a state-allocable state-contingent production function of the Cobb-Douglas type:

(5.1) $$z_s = x_s^{1/b} a_s^{-1/b}$$

where $b \geq 1$ and $a_s \geq 0$ for $s \in \Omega = \{1,2\}$. In terms of the quantities introduced in Sections 2 to 4:

(5.2) $$f(x_s, \beta_s) = x_s^{1/b} a_s^{-1/b}$$

(5.3) $$f^{-1}(z_s, \beta_s) = a_s z_s^b$$

(5.4) $$m(z_s, \beta_s) = \partial f^{-1}(z_s, \beta_s)/\partial z_s = b a_s z_s^{b-1}$$

(5.5) $$m^{-1}\left(w^{-1}\pi_s, \beta_s\right) = \left(\frac{\pi_s}{bwa_s}\right)^{\frac{1}{b-1}}$$

(5.6) $$\pi_1 = e_1\left(wba_1 q^{b-1}\right) + e_2\left(1 - wba_2 q^{b-1}\right)$$ and

(5.7) $$\pi_2 = e_2\left(wba_2 q^{b-1}\right) + e_1\left(1 - wba_1 q^{b-1}\right)$$

where $\beta_s = (a_s, b)'$. For this technology, the relationship between total inputs, normalized input prices, realized states of Nature and observed outputs is of the form:

.

(5.8) $$x - a_1\left(\frac{e_1\left(wba_1 q^{b-1}\right) + e_2\left(1 - wba_2 q^{b-1}\right)}{bwa_1}\right)^{\frac{b}{b-1}} - a_2\left(\frac{e_2\left(wba_2 q^{b-1}\right) + e_1\left(1 - wba_1 q^{b-1}\right)}{bwa_2}\right)^{\frac{b}{b-1}} = 0$$

An associated econometric estimating equation is:

$$x_{nt} = a_1 \left(\frac{e_{1nt}\left(w_{nt} b a_1 q_{nt}^{b-1}\right) + e_{2nt}\left(1 - w_{nt} b a_2 q_{nt}^{b-1}\right)}{b w_{nt} a_1} \right)^{\frac{b}{b-1}} + a_2 \left(\frac{e_{2nt}\left(w_{nt} b a_2 q_{nt}^{b-1}\right) + e_{1nt}\left(1 - w_{nt} b a_1 q_{nt}^{b-1}\right)}{b w_{nt} a_2} \right)^{\frac{b}{b-1}} + \upsilon_{nt} \quad (5.9)$$

where the subscripts *n* and *t* represent firms and time periods respectively $(n = 1,...,N; t = 1...,T)$, and υ_{nt} is a random variable representing statistical noise. We have used NLS to estimate this conditional input demand function using the simulated data reported in Table 4 of O'Donnell, et al. (2006). The values of the unknown parameters used to generate that table were $b = 2$, $a_1 = 1.5$ and $a_2 = 0.5$. Our NLS estimates of these parameters were $\hat{b} = 2$, $\hat{a}_1 = 1.5$ and $\hat{a}_2 = 0.5$ with standard errors of zero. The associated risk-neutral probabilities and unobserved state-contingent outputs were also recovered without error.

Implementing an NLS algorithm involves choosing starting values for the parameters of the technology that are compatible with risk-neutral probabilities lying in the unit interval. Indeed, this requirement also needs to be met on each iteration of the algorithm. Our experience with the simulated data was that the NLS algorithm failed if we chose starting values that were too far from the true values. This is likely to a problem in real-world situations where the true values are, of course, unknown. In the following section we overcome the problem by estimating the model in a Bayesian framework.

6. EXAMPLE – RICE DATA

O'Donnell and Griffiths (2006) use rice data to estimate an output-cubical state-contingent production frontier. The data consists of more than 300 observations on the inputs and rice outputs of farmers in the Tarlac region of the Philippines. The descriptive statistics reported in Table 1 reveal a large amount of variation in the data set. The sample farmers have no access to irrigation, so at least some of the variation in the output variable can be attributed to variations in rainfall. Observe from Table 1 that data on rice inputs has been aggregated into a single input index. This is convenient because it allows us to work with the following trivial generalization of the technology given by (5.1):

$$z_s = c + x_s^{1/b} a_s^{-1/b} \quad (6.1)$$

where $c \geq 0$, $b \geq 1$ and $a_s \geq 0$ for $s \in \Omega = \{1,2\}$. The associated econometric estimating equation is identical to (5.9) except that q_{nt} is replaced by $q_{nt} - c$. The dummy variable e_{1nt} in (5.9) was set to one if rainfall was observed to fall below the first sample quartile (865 mm).

We begin by writing the full set of *NT* equations represented by (5.9) in the more compact form:

$$x = g(q, w, \beta) + \upsilon \quad (6.2)$$

where $x = (x_{11}, x_{12}, ..., x_{NT})'$, $\beta = (c, a_1, a_2, b)'$ and the remaining definitions are obvious. The errors are assumed to be independent and identically distributed as $N(0, h^{-1})$. Thus, the likelihood function is

$$p(x \mid \beta, h) = f_N(x \mid g(q,w,\beta), h^{-1} I_{NT}) \propto h^{NT/2} \exp\left\{ -\frac{h}{2} \left[x - g(q,w,\beta)\right]' \left[x - g(q,w,\beta)\right] \right\} \quad (6.3)$$

where I_{NT} is an identity matrix of order NT and $f_N(a \mid b, C)$ denotes the probability density function (pdf) of a multivariate normal random vector with mean *b* and covariance matrix *C*. We use the following improper prior:

$$p(\beta, h) \propto h^{-1} \times I(\beta \in R) \times I(h \geq 0) \quad (6.4)$$

where $I(.)$ is an indicator function that takes the value 1 if the argument is true and 0 otherwise, and R is the region of the parameters space where the restrictions discussed in Section 5 are satisfied. That is, R is the region where $c \geq 0, a_1 \geq 0,\ a_2 \geq 0, b > 1$ and all three parameters are such that the risk-neutral probabilities (defined by equations 5.6 and 5.7, but with q replaced by $q - c$) lie in the unit interval. Thus, the posterior pdf is

$$(6.5) \qquad p(\beta, h \mid x) = h^{-1} \times f_N(x \mid g(q, w, \beta), h^{-1} I_{NT}) \times I(\beta \in R) \times I(h \geq 0)$$

Conditional posterior pdfs that can be used within a Gibbs Sampler are:

$$(6.6) \qquad p(\beta \mid x, h) \propto \exp\left\{-\frac{h}{2}\left[x - g(q, w, \beta)\right]' \left[x - g(q, w, \beta)\right]\right\} \times I(\beta \in R) \qquad \text{and}$$

$$(6.7) \qquad p(h \mid x, \beta) = f_G(h \mid \bar{h}, NT)$$

where

$$(6.8) \qquad \bar{h} = \frac{NT}{\left[x - g(q, w, \beta)\right]' \left[x - g(q, w, \beta)\right]}$$

and $f_G(a \mid b, c)$ denotes the pdf of a gamma random variable with mean b and degrees of freedom c. Simulating from the gamma density (6.7) is straightforward using random number generators available in most statistical software packages. However, simulating from (6.6) is slightly more complicated because it is a truncated pdf. To simulate from (6.6) we used a random-walk Metropolis-Hastings algorithm with a multivariate normal proposal density. For details concerning this algorithm, see Koop (2003). During the transition, or burn-in, phase of the algorithm, the covariance matrix of the proposal density was set to a scalar multiplied by an identity matrix. The scalar was set by trial and error to yield an acceptance rate in the range 0.3-0.5. After the burn-in, to improve the efficiency of the algorithm, we used the covariance matrix of the burn-in observations as the covariance matrix in the proposal density. In this paper, we simulated 120,000 observations from the conditional posteriors (6.6) and (6.7) and discarded the first 20,000 draws as a burn-in. Figure 2 presents convergence plots for each of the elements of β and h. We did not use statistical tests to confirm convergence of the MCMC chains because the convergence plots are quite conclusive insofar as they show absolutely no signs of non-stationarity.

Estimates of the unknown parameters are presented in Table 2. The point estimates are the means of the MCMC samples and are optimal Bayesian point estimates under quadratic loss. The inequality restrictions in the prior (6.4) ensure that all the estimates in Table 2 are theoretically plausible. The standard errors are the standard errors of the MCMC samples and suggest that only b has been estimated with any reliability. However, estimated standard errors can be misleading. A more complete picture of the level of uncertainty surrounding the unknown parameters is presented in Figure 3. This figure presents estimated marginal posterior pdfs for each of the parameters. A feature of these pdfs is that the estimated pdfs for a_1, a_2 and b are asymmetric. This is a direct result of the inequality information contained in the prior. A second remarkable feature is that the estimated pdf for b has no support beyond 1.5, indicating a high degree of substitutability between state-contingent outputs. Third, the estimated pdf for c is rectangular. This parameter has only been constrained to be non-negative, so it is somewhat surprising that the estimated density function has been upper-truncated at 0.35. This upper truncation is quite possibly a consequence of constraining the risk-neutral probabilities to the unit interval.

Finally, the estimated parameters can be used to recover estimates of the latent variables in the model, including unrealized state-contingent outputs, input allocations to different states of Nature, and the risk-neutral probabilities assigned to different states of Nature by individual firms. For example, Table 3 presents estimates of π_1 for Firms 1 to 10 in Years 1, 3, 6 and 8. The estimates presented in this table reveal that all rice farmers plausibly tend to assign similar (risk-neutral) probabilities to states of Nature in any given year (e.g., in year 1, we estimate that the first 10 farmers all assessed π_1 in the range 0.66 to 0.81). Furthermore, farmers may attach very different probabilities to future states of Nature from one year to the next (e.g., in year 8, we estimate that

Firms 1 to 10 assessed π_1 in the range 0.09 to 0.19). Importantly, risk-neutral probabilities are utility-deflated probabilities, so variations in these probabilities reflect variations in the probabilities attached to different states of Nature as well as variations in attitudes towards risk.

7. CONCLUSION

Empirical estimation of flexible state-contingent production technologies is complicated by the fact that data on state-contingent outputs and allocations of inputs to different states of Nature are often unobserved. This paper shows how to overcome the problem of lack of data in the two-state case. In theory, the econometric model developed in the paper can be estimated using either sampling theory or Bayesian methodology. In an application to Philippines rice data, the sampling theory approach broke down due to an inability to dynamically control a nonlinear least squares optimisation algorithm. Estimating the model in a Bayesian framework proved much more straightforward and yielded plausible estimates of economic quantities of interest.

REFERENCES

Chambers, R.G. and Quiggin, J. (2000) *Uncertainty, Production, Choice and Agency: The State-Contingent Approach*, Cambridge, UK: Cambridge University Press.

Chiang, A.C. (1984) *Fundamental Methods of Mathematical Economics*, 3rd edn., Blacklick, OH: McGraw-Hill.

Farrell, M.J. (1957) 'The Measurement of Productive Efficiency', *Journal of the Royal Statistical Society, Series A (General)*, 120 (3): 253-290.

Koop, G. (2003) *Bayesian Econometrics*, Chichester: John Wiley and Sons.

O'Donnell, C.J., Chambers, R.G. and Quiggin, J. (2006) 'Efficiency Analysis in the Presence of Uncertainty', *Risk and Uncertainty Program Working Papers*, University of Queensland.

O'Donnell, C.J. and Griffiths, W.E. (2006) 'Estimating State-contingent Production Frontiers', *American Journal of Agricultural Economics*, 88 (1): 249-266.

Table 1. DESCRIPTIVE STATISTICS

	Mean	SD	Min	Max
Q	6.543	5.104	0.3700	31.10
X	2.199e+004	1.660e+004	1889.	9.251e+004
W	1.085	0.3871	0.4253	3.363
E1	0.2500	0.4336	0.0000	1.000

Table 2. ESTIMATED PARAMETERS

Coef	Mean	St.Dev	5th Pctile	95th Pctile
c	0.183	0.106	0.0179	0.350
a1	0.0585	0.0428	0.00403	0.139
a2	0.143	0.0713	0.0327	0.264
b	1.20	0.149	1.02	1.51
h	1.32e-009	1.01e-010	1.16e-009	1.49e-009

Table 3. ESTIMATED (RISK-NEUTRAL) PROBABILITIES ASSIGNED TO STATE 1

Obs	Year	Firm	P(s=1)	St.Dev	5th Pctile	95th Pctile
1	1	1	0.7558	0.09776	0.6079	0.9267
2	1	2	0.7776	0.08983	0.6423	0.9335
3	1	3	0.7501	0.1008	0.5983	0.9253
4	1	4	0.7628	0.09538	0.6144	0.9294
5	1	5	0.7756	0.09006	0.6396	0.9327
6	1	6	0.8127	0.08077	0.6832	0.9478
7	1	7	0.6659	0.1336	0.4629	0.8996
8	1	8	0.7597	0.09604	0.6130	0.9280
9	1	9	0.7854	0.08627	0.6554	0.9356
10	1	10	0.7955	0.08224	0.6715	0.9387
:	:	:	:	:	:	:
87	3	1	0.7638	0.09482	0.6208	0.9291
88	3	2	0.7022	0.1223	0.5154	0.9126
89	3	3	0.8189	0.07318	0.7086	0.9458
90	3	4	0.7134	0.1147	0.5374	0.9145
91	3	5	0.7382	0.1055	0.5793	0.9216
92	3	6	0.8176	0.07933	0.6902	0.9497
93	3	7	0.6821	0.1272	0.4894	0.9045
94	3	8	0.8430	0.06325	0.7442	0.9532
95	3	9	0.7358	0.1065	0.5754	0.9209
96	3	10	0.7574	0.09896	0.6070	0.9283
:	:	:	:	:	:	:
216	6	1	0.8789	0.04880	0.8027	0.9639
217	6	2	0.5949	0.1698	0.3287	0.8824
218	6	3	0.7077	0.1180	0.5301	0.9128
219	6	4	0.8015	0.08013	0.6760	0.9409
220	6	5	0.7706	0.09213	0.6316	0.9311
221	6	6	0.8205	0.07939	0.6924	0.9514
222	6	7	0.7208	0.1145	0.5464	0.9179
223	6	8	0.8142	0.07465	0.6983	0.9448
224	6	9	0.7969	0.08130	0.6739	0.9390
225	6	10	0.6524	0.1406	0.4403	0.8962
:	:	:	:	:	:	:
302	8	1	0.1233	0.08779	0.009265	0.2894
303	8	2	0.1839	0.1360	0.01342	0.4487
304	8	3	0.1306	0.09493	0.009653	0.3131
305	8	4	0.09419	0.06604	0.007067	0.2186
306	8	5	0.1301	0.09459	0.009602	0.3120
307	8	6	0.08734	0.06144	0.006454	0.2029
308	8	7	0.1945	0.1393	0.01455	0.4582
309	8	8	0.1416	0.1005	0.01068	0.3309
310	8	9	0.1368	0.09816	0.01023	0.3232
311	8	10	0.1823	0.1337	0.01344	0.4402

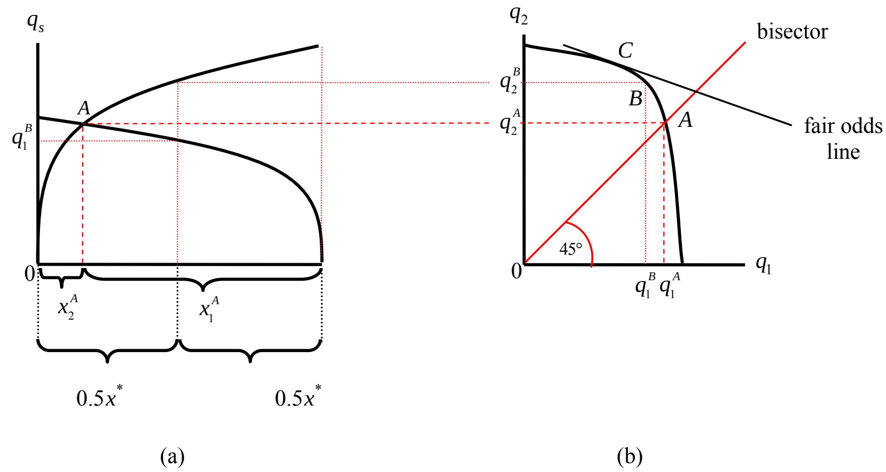

Figure 1. A State-Allocable State-Contingent Technology

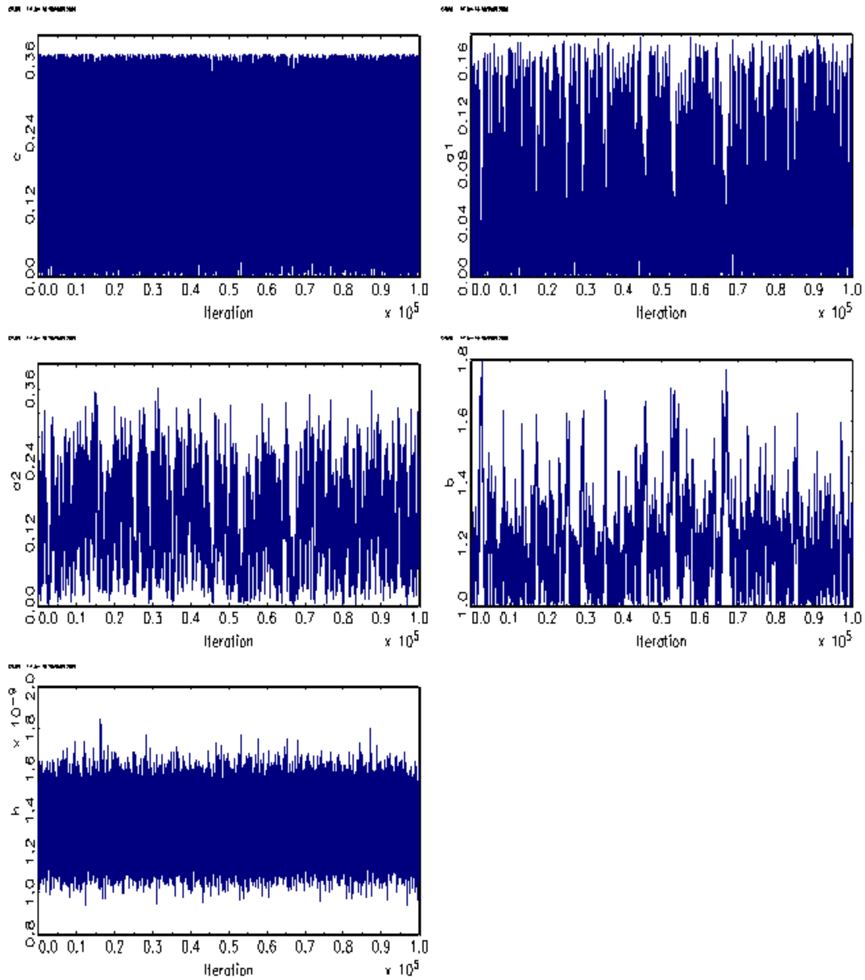

Figure 2. Convergence Plots

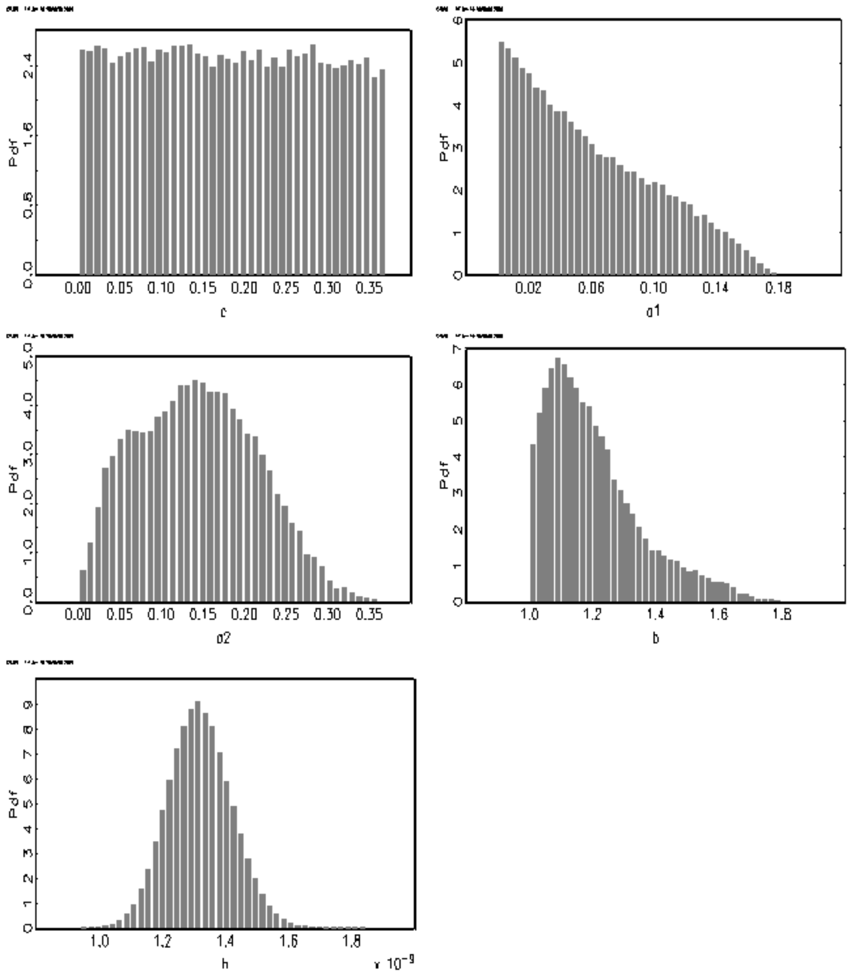

Figure 3. Estimated Posterior Pdfs