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Authors
Antonio Peyrache and Tim Coelli

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School of Economics
University of Queensland
St. Lucia, Qld. 4072
Australia

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Testing procedures for detection of linear dependencies in efficiency models

Antonio Peyrache
Department of Economic Sciences, "La Sapienza" University, Roma, Italy, apeyrache@msn.com

Tim Coelli
School of Economics, University of Queensland, Brisbane, Australia

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Abstract: The validity of many efficiency measurement methods rely upon the assumption that variables such as input quantities and output mixes are independent of (or uncorrelated with) technical efficiency, however few studies have attempted to test these assumptions. In a recent paper, Wilson (2003) investigates a number of independence tests and finds that they have poor size properties and low power in moderate sample sizes. In this study we discuss the implications of these assumptions in three situations: (i) bootstrapping non-parametric efficiency models; (ii) estimating stochastic frontier models and (iii) obtaining aggregate measures of industry efficiency. We propose a semi-parametric Hausman-type asymptotic test for linear independence (uncorrelation), and use a Monte Carlo experiment to show that it has good size and power properties in finite samples. We also describe how the test can be generalized in order to detect higher order dependencies, such as heteroscedasticity, so that the test can be used to test for (full) independence when the efficiency distribution has a finite number of moments. Finally, an empirical illustration is provided using data on US electric power generation.

Key words: correlation; independence; technical efficiency; hypothesis test; aggregation

1. Introduction

The measurement of technical efficiency has been the subject of many studies since the pioneering work of Farrell (1957). Most of these studies have made the implicit assumption that the degree of technical inefficiency of a firm is independent of the inputs (and output mixes) of the firm.[1] However, there are various reasons why this assumption may be incorrect. For example, Wilson (2003) notes that in some instances big firms may have access to better managers and hence are more likely to perform better. Furthermore, Schmidt & Sickles (1984) argue that if a firm knows its level of technical inefficiency this should affect its input choices, creating a potential dependence between the input vector and the efficiency term.

Wilson (2003) surveys a number of the independence tests that could be used to test the independence hypothesis in the context of efficiency measurement. His motivation essentially relates to the fact that if independence can be assumed, one can implement a much simpler bootstrapping methodology to construct confidence intervals for efficiency estimates derived using data envelopment analysis (DEA). He conducts a Monte Carlo experiment to investigate the small sample properties of four independence testing procedures (two bootstrap-based tests and two rank-based tests) and finds that they all have incorrect size properties and poor power properties when the sample size is not large (n=70) and the degree of correlation (ρ) is moderate ($\rho \leq 0.4$), with the rank-based tests not performing as well as the bootstrap tests.

In this study we deviate from the Wilson (2003) study two important ways. First, we discuss two additional situations in which independence information is valuable – namely stochastic frontier models and aggregation of efficiency scores. Secondly, we focus our attention on the hypothesis of uncorrelation (no linear dependence) as opposed to independence. The advantage of testing this weaker condition is that we can produce testing procedures which are easy to implement, and (as we show in our Monte Carlo

[1] This statement assumes output oriented technical efficiency measures are being estimated. In the event that one is alternatively estimating input oriented technical efficiency measures, the output levels and the input mixes are the relevant variables. This is explained further in the discussion below.

experiment) have correct size and much stronger power relative to the independence tests. Of course the downside is that the uncorrelation test cannot identify non-linear relationships. However, in the event that the null hypothesis of uncorrelation is rejected, one can also conclude that the null hypothesis of independence is also rejected. Thus providing a valuable pre-test procedure if independence is the hypothesis of interest.

In this study we discuss three important contexts in which these properties play a fundamental role. *First*, in Stochastic Frontier Models (SFM) an uncorrelation assumption is needed for one to conclude that the corrected ordinary least squares (COLS) estimator provides consistent estimates of the slope parameters (Kumbhakar & Lovell 2000). If correlation between the efficiency term and the regressors arise, we have an endogeneity problem. Furthermore, maximum likelihood estimation (MLE) cannot be used when correlation exists because the increased number of parameters in the model gives rise to identification problems. *Second*, in the aggregation of Farrell type efficiency measures (for example, see Färe and Grosskopf 2005, Fox 2004) the monotonicity property[2] of the aggregate industry efficiency indexes holds if and only if the uncorrelation assumption is satisfied. The failure of the monotonicity property gives rise to the so called Fox Paradox, where one can find that individual efficiency scores can all increase but the measure of overall industry efficiency decreases. Therefore this paradox can be interpreted as an example of the failure of the uncorrelation assumption. *Third*, the uncorrelation assumption is a necessary condition for independence and this last one is used in non-parametric frameworks to justify the use of univariate kernel methods for the estimation of the efficiency distribution (Wilson 2003, Daraio & Simar 2005). If independence fails one has to estimate a multi-dimensional density function, leading to the well known curse of dimensionality problem (Efron & Tibshirani 1993).

The remainder of this paper is organized into sections. In section 2 we define the production technology and introduce formal definitions of independence and uncorrelation. Some aggregation issues and the relations between the uncorrelation assumption and the monotonicity property are discussed in

[2] Given a vector of individual values and an aggregate index based on this individual values, the monotonicity property states that if all the individual values increase also the aggregate index have to increase (Balk 1995).

section 3. In section 4 the impact of the failure of the uncorrelation assumption on stochastic frontier models is explicitly discussed. In section 5 we introduce some statistical procedures to test for uncorrelation and homoscedasticity. Finally, in section 6 we conduct a Monte Carlo experiment and provide an empirical illustration of the problems discussed using data on the US electricity power generation industry. Some concluding remarks are then provided in the final section.

2. The Technology plus some Definitions

2.1. Stochastic Representation of Technology

Consider the density function $f(\mathbf{x},\mathbf{y}) \geq 0$, where $\mathbf{x} \in R^k$, $\mathbf{y} \in R^m$ are the input and the output vectors and $\int_{R^{k+m}} f(\mathbf{x},\mathbf{y})d\mathbf{x}d\mathbf{y} = 1$, where **x** and **y** assume non-negative values. We define the support of the density function as

$$T = \{(\mathbf{x},\mathbf{y}) \in R^{k+m} : f(\mathbf{x},\mathbf{y}) > 0\}$$

and its boundary as an intersection between sets

$$\vartheta T = \{[T \cap cl(\overline{T})] \cup [cl(T) \cap \overline{T}]\}$$

where $\overline{T}$ is the compliment of T and $cl(\cdot)$ is the closure operator. In production economics we refer to the first set as the *Production Set* and to the boundary as the *Production Frontier*. The following regularity conditions (Kumbhakar & Lovell 2000, Fare & Grosskopf 1994) are commonly used in production economics:

A1. *no free lunch*: if $f(\mathbf{x},\mathbf{0}) > 0$ and $f(\mathbf{0},\mathbf{y}) > 0$ then $\mathbf{y} = \mathbf{0}$;

A2. the *Production Set is Closed*: for a succession of points $(\mathbf{x}_n,\mathbf{y}_n) \to (\mathbf{x},\mathbf{y})$, if $f(\mathbf{x}_n,\mathbf{y}_n) > 0 \; \forall n \in N$ then $f(\mathbf{x},\mathbf{y}) > 0$; (in essence, this states that the frontier belong to the production set);[3]

3 The closure of the production set (A2) can be also stated (see Daraio & Simar 2005, Wilson 2003) in terms of *Positiveness*: the density function is strictly positive on the boundary and is continuous in any direction toward the interior (i.e., the density function is discontinuous on the boundary).

A3. the *Production Set is bounded*: for each $\mathbf{x} \in R_{+}^{k}$ exist $\mathbf{y}: f(\mathbf{x},\mathbf{y}) = 0$;

A4. *strong disposability*: if $f(\mathbf{x}_0,\mathbf{y}_0) > 0$ then $f(\mathbf{x}_1,\mathbf{y}_1) > 0$ for each $(-\mathbf{x}_1,\mathbf{y}_1) \le (-\mathbf{x}_0,\mathbf{y}_0)$;

A5. *convexity*: if $f(\mathbf{x}_1,\mathbf{y}_1) > 0$ *and* $f(\mathbf{x}_2,\mathbf{y}_2) > 0$ then $f[\alpha\mathbf{x}_1 + (1-\alpha)\mathbf{x}_2, \alpha\mathbf{y}_1 + (1-\alpha)\mathbf{y}_2] > 0$ $\forall 0 \le \alpha \le 1$;

These are pure statistical restrictions on a stochastic Data Generating Process (DGP) represented by the density function $f(\mathbf{x},\mathbf{y})$. In what follows we assume that assumptions A1 to A4 hold. In addition, we assume the following regularity condition on the DGP (Daraio & Simar 2005):

- *Random Sample*: the sample observations $(\mathbf{x}_i,\mathbf{y}_i)$, $i = 1,\ldots,n$ are realizations of identically and independently distributed random variables $(\mathbf{X},\mathbf{Y})$ which have probability density function $f(\mathbf{x},\mathbf{y})$.

2.2. Average Technical Efficiency, Independence, Uncorrelation and Homoscedasticity

Let's consider the output oriented radial measure of efficiency $\theta = \min\left\{\theta : f\left(\mathbf{x}, \frac{\mathbf{y}}{\theta}\right) > 0\right\}$. Before we proceed, it is useful to explicitly show that it is possible to calculate the efficiency distribution from the original joint density function $f(\mathbf{x},\mathbf{y})$. An easy way to calculate the marginal distribution of efficiency is via the method of cylindrical coordinates (Simar & Wilson 2000). The cylindrical coordinates of a point $(\mathbf{x},\mathbf{y})$ are $(\tau,\boldsymbol{\eta},\mathbf{x})$ where $\tau = \sqrt{\mathbf{y}'\mathbf{y}}$ and $\tan\eta_j = \frac{y_j}{y_1}$ $\forall j = 1,\ldots,m$. The distance between $(\mathbf{x},\mathbf{y})$ and its efficient radial projection on the frontier can be stated in cylindrical coordinates as $\theta = \frac{\tau(\mathbf{y})}{\tau\left(\mathbf{y}/\theta\right)}$.

Since a point $(\mathbf{x},\mathbf{y})$ is fully represented in cylindrical coordinates $(\tau,\boldsymbol{\eta},\mathbf{x})$ and we have a biunivocal correspondence between τ and θ, we can write it as $(\theta,\boldsymbol{\eta},\mathbf{x})$. Then the density function can be written as:

$$f(\mathbf{x},\mathbf{y}) = f(\theta,\boldsymbol{\eta},\mathbf{x}) = f(\theta \mid \boldsymbol{\eta},\mathbf{x}) f(\boldsymbol{\eta} \mid \mathbf{x}) f(\mathbf{x}) \qquad (1)$$

The marginal efficiency distribution can be calculated by integrating the density function (1) with respect to **x** and **η**:

$$f_\theta(\theta) = \int\int f(\theta, \boldsymbol{\eta}, \mathbf{y}) d\boldsymbol{\eta} d\mathbf{x} \tag{2}$$

The knowledge of the density function (2) allows one to aggregate efficiency, or in fact to determine all the moments of its distribution.

We now provide three useful definitions.

DEFINITION 1 *(Independence)*. The efficiency distribution is *fully independent* if and only if $f(\theta \mid \boldsymbol{\eta}, \mathbf{x}) = f(\theta)$. Efficiency is independent from output composition (or *output composition* independence) if and only if $f(\theta \mid \boldsymbol{\eta}, \mathbf{x}) = f(\theta \mid \mathbf{x})$. Furthermore, efficiency is independent from the input set (or *input set* independence) if and only if $f(\theta \mid \boldsymbol{\eta}, \mathbf{x}) = f(\theta \mid \boldsymbol{\eta})$.

DEFINITION 2 *(Uncorrelation or Linear Independence)*. The efficiency distribution is *fully uncorrelated* if and only if $E(\theta \mid \boldsymbol{\eta}, \mathbf{x}) = E(\theta)$. Efficiency is uncorrelated with output composition (or *output composition* uncorrelation) if and only if $E(\theta \mid \boldsymbol{\eta}, \mathbf{x}) = E(\theta \mid \mathbf{x})$. Furthermore, efficiency is uncorrelated with the input set (or *input set* uncorrelation) if and only if $E(\theta \mid \boldsymbol{\eta}, \mathbf{x}) = E(\theta \mid \boldsymbol{\eta})$.

DEFINITION 3 (*Homoscedasticity*). The efficiency distribution is homoscedastic if and only if $Var(\theta_i) = Var(\theta)$ or if its variance is constant across observations.

Since $Var(\theta) = E_2(\theta) + E_1^2(\theta)$, homoscedasticity can be rewritten as

$$E_2(\theta \mid \boldsymbol{\eta}, \mathbf{x}) + E_1^2(\theta \mid \boldsymbol{\eta}, \mathbf{x}) = E_2(\theta) + E_1^2(\theta) \tag{3}$$

From equation (3) it is easy to see that a violation of the uncorrelation assumption implies (excluding some minor cases) a violation of the homoscedasticity assumption.

It is evident that independence implies uncorrelation but the reversal need not be true. This is important in discussing testing procedures since if a test accepts the null hypothesis of independence, we know that the data are also statistically uncorrelated; but if the test rejects independence we cannot say anything on correlation. On the other hand if a test rejects the null hypothesis of zero correlation we know that the independence assumption fails too; but if it accepts the null hypothesis we cannot say anything on independence.

Many of the proposed testing procedures to detect independence show a low power in rejecting the null hypothesis of linear independence. Wilson (2003) shows some Monte Carlo results where data are generated from a multivariate normal distribution with a non-diagonal covariance matrix. The powers of the tests there discussed are lower (in samples of moderate size) in comparison to the results that we will show for the testing procedure here proposed. Since these tests show low power in detecting linear dependencies we have a good reason to introduce a testing procedure which is better able to identify them. If the test accepts the uncorrelation hypothesis other types of dependencies could be present in the data and other types of tests can be used in order to detect them. Anyway, if a zero-correlation testing procedure is available we can use it to exclude linear dependence and this is a pre-condition for any independence test.

Moreover the testing procedure we are introducing can be used to also to detect the presence of heteroscedasticity in the distribution of the efficiency term. Even if heteroscedasticity is one of the more well known violations of the independence assumption it is less aggressive than correlation. Some statistical properties of our models are based on the uncorrelation assumption and not on the homoscedasticity assumption.

2.3. Weighted average estimators

It is of some value to discuss the properties of a special class of estimators that can be labelled Weighted Average Estimators. Let's consider the inference problem of estimating average efficiency from a random sample of n realizations. We know that the sample average is a consistent estimator of average efficiency and satisfies good asymptotic properties (see Greene 1997, page 118). In this section we will show that there is a class of statistics that are consistent estimators of average efficiency only if the uncorrelation assumption holds. Consider the following statistic:

$$\phi = \sum_{i=1}^{n} w_i \theta_i \ , \quad \sum_{i=1}^{n} w_i = 1 \tag{4}$$

where θ_i is the efficiency of observation i and the shares w_i are random weights defined as function of the observed vectors $(\mathbf{x}_i, \mathbf{y}_i)$: $w_i(\mathbf{x}_i, \mathbf{y}_i) = \frac{g(\mathbf{x}_i, \mathbf{y}_i)}{\sum_{i=1}^{n} g(\mathbf{x}_i, \mathbf{y}_i)}$ and $g : R^{k+m} \to R$ is a generic function.[4]

Equation (4) becomes

$$\phi = \sum_{i=1}^{n} \left[\frac{g(\mathbf{x}_i, \mathbf{y}_i)}{\sum_{i=1}^{n} g(\mathbf{x}_i, \mathbf{y}_i)} \theta_i \right] \tag{5}$$

It is worth noting that the statistic (5) can be written in terms of sample averages as

$$\phi = \frac{\sum_i g_i \theta_i \big/ n}{\sum_i g_i \big/ n} = \frac{\overline{g\theta}}{\overline{g}} \tag{6}$$

where we omitted the dependence of g on (x_i, y_i) to simplify the notation. Let's define the population means as $\mu_{g\theta} = E(g\theta)$ and $\mu_g = E(g)$ where $\overline{g\theta} \xrightarrow{p} \mu_{g\theta}$ and $\overline{\theta} \xrightarrow{p} \mu_\theta$ for the consistency of the sample mean. The statistic (5) is a linear aggregator function of the efficiency scores with weights that sum up to one. In this sense we can interpret almost all aggregation procedures for efficiency scores as particular cases of expression (5). For example if we use shares of a particular input, say labour, we are aggregating using labour shares. Again, the use of cost shares in the aggregation procedure can be derived as a particular case of expression (5) where we use prices to weight the inputs.

[4] The simple average is the particular case in which we set $w_i = \frac{1}{n}, \forall i = 1, \ldots, n$, which is a degenerate random variable.

PROPOSITION 1 (*Consistency*). The statistic (5) satisfies the consistency property $\phi \xrightarrow{P} E(\theta)$ only if efficiency is statistically uncorrelated from the arguments of the function that defines the aggregation shares.

Proof: The estimator can be rewritten as

$$\phi = \frac{\sum_i g_i \theta_i}{\sum_i g_i} = \frac{\sum_i g_i \theta_i / n}{\sum_i g_i / n} \xrightarrow{P} \frac{E(g\theta)}{E(g)} = E(\theta)$$

where the limit is a consequence of the consistency of the sample mean and Slutsky's theorem (see, for example, Greene 1997, pp. 118-119) and the last equality derives from the uncorrelation assumption since $E(g\theta) = E(g)E(\theta)$.

□

PROPOSITION 2 (*Asymptotic Normality*): The statistic (5) is asymptotically normally distributed with mean $\frac{\mu_{g\theta}}{\mu_g}$ and variance $Var(\phi) = \frac{\mu_g^2 \sigma_{g\theta}^2 + \mu_{g\theta}^2 \sigma_g^2 + 2\mu_g \mu_{g\theta} Cov(\theta, g)}{n\mu_g}$.

Proof: The statistic (6) is the ratio between two dependent random variables. The following identity holds:

$$\frac{\overline{g\theta}}{\overline{g}} - \frac{\mu_{g\theta}}{\mu_g} = \frac{\mu_g(\overline{g\theta} - \mu_{g\theta}) - \mu_{g\theta}(\overline{g} - \mu_g)}{\overline{g}\mu_g} \tag{7}$$

Let's consider the following transformation of identity (7) numerator: $W_i = \mu_g g_i \theta_i - \mu_{g\theta} g_i$. W_i has zero mean and variance $\sigma_W^2 = \mu_g^2 \sigma_{g\theta}^2 + \mu_{g\theta}^2 \sigma_g^2 + 2\mu_g \mu_{g\theta} Cov(\theta, g)$. If we define $Z_n = \frac{1}{\sqrt{n}\sigma_W} \sum_i W_i$ it is possible to apply the Lindeberg-Lévy central limit theorem to establish:

$$Z_n \to N(0,1)$$

Thus we can write (making use of Slutsky's theorem)

$$\frac{\sqrt{n}}{\sigma_W}\left(\frac{\overline{g\theta}}{\overline{g}}-\frac{\mu_{g\theta}}{\mu_g}\right)=\frac{Z_n}{\overline{g}\mu_g}\xrightarrow{d}N(0,\mu_g^{-4})$$

and we can conclude

$$\frac{\overline{g\theta}}{\overline{\theta}}\overset{a}{\approx}N\left(\frac{\mu_{g\theta}}{\mu_g},\frac{\sigma_W^2}{n\mu_g^4}\right)$$

□

These are interesting results for two reasons. First, we have an estimator of average efficiency that is consistent only if uncorrelation holds and this fact allows us to construct tests for uncorrelation based on Wald statistics. Second, we can consider efficiency indexes constructed using price information (e.g., total revenue) as particular cases of expression (5). This last point is particularly useful in illustrating some results relating to the aggregation of efficiency scores into measures of industry efficiency.

3. Aggregation Issues

Before we outline our the semi-parametric asymptotic testing procedure, it is important to first review the efficiency aggregation debate (see for example Fare & Grosskopf 2005, Zelenyuk 2004, Fox 2004, Soriano, Rao & Coelli 2003). The debate can be summarized as the search for weighting vectors for efficiency scores which give rise to aggregate indexes that respect some properties considered important in production economics. It is worth emphasizing that in this debate the asymptotic properties of aggregate indexes have not previously been discussed. The consideration of the asymptotic properties of the estimators throws new light on various aspects of the aggregation debate, such as the Fox paradox and the related monotonicity property. The monotonicity property states that the aggregate index has to increase if all its arguments increase. We will show that the monotonicity property holds (in statistical terms) only if the uncorrelation assumption is satisfied. The indexes that have been discussed in the literature don't satisfy the uncorrelation assumption, therefore we have a structural problem of a lack of monotonicity due to the correlation between efficiency and the vector of weights. Fortunately, it is always possible to measure this bias as a deviation from average efficiency and give it an economic interpretation.

3.1. The Aggregation "Problem"

In order to provide an illustration of the key issues in the aggregation debate we start with Koopmans theorem in its revenue version (Fare & Grosskopf 2005), and assume that information on the vector of output prices (**p**) is available. The theorem states that the total maximum revenue (i.e., the revenue function) of an industry composed of n firms is equal to the summation of the individual firm-level revenue functions (given the assumption that reallocation of inputs among firms is not permitted). Formally, if we define the revenue function as $R(\mathbf{p},\mathbf{x}) = \max_{\mathbf{y}} \{\mathbf{py} : f(\mathbf{x},\mathbf{y}) > 0\}$, Koopmans theorem states:[5]

$$R_I = \sum_i R_i$$

where R_I is the industry revenue function and R_i is the firm *i* revenue function. This is the starting point for aggregation. Koopmans theorem holds under very general assumptions on the technology (Mas-Colell 1995) and this is the main reason why in aggregating efficiency scores it is recommended to choose aggregator functions that satisfy this relation. We can rewrite the Koopmans relation as

$$E = \sum_i s_i E_i \tag{8}$$

where $E = \frac{R}{\mathbf{py}}$ is the industry-level economic efficiency, $E_i = \frac{R_i}{\mathbf{py}_i}$ are the individual firm-level economic efficiencies and $s_i = \frac{\mathbf{py}_i}{\mathbf{py}}$ is the observed revenue share of the *i*-th firm. The weighted average of individual economic efficiency indexes is equal to the industry efficiency index. Thus, the industry-level economic efficiency index has to be equal to the weighted average of the individual indexes if we want the Koopmans relation to hold.

We know from Farrell (1957) that the following decomposition holds at the individual firm level:

$$E_i = A_i T_i, \quad \forall i = 1,\ldots,n \tag{9}$$

[5] The notation **py** relates to the dot product of the two vectors.

where $T_i = \dfrac{1}{\theta_i}$ is the technical efficiency index and $A_i = \dfrac{E_i}{T_i}$ is the allocative efficiency index (defined in a residual way). We want that a similar relation holds at industry level:

$$E = AT$$

The aggregation problem reduces to finding aggregation procedures for the individual technical and allocative components (9) such that their product is equal to the industry-level economic efficiency (8). Although in theory we can consider very general aggregator functions, in the literature the attention has focused on linear aggregator functions with weights that sum up to one. One advantage of this choice is that these aggregator functions are consistent in aggregation (Blackorby & Russell 1999, Diewert 1978).

In formal terms we are searching vectors of weights (α, β) for the two linear aggregator functions

$$T = \sum_i \alpha_i T_i , \quad \sum_i \alpha_i = 1$$

$$A = \sum_i \beta_i A_i , \quad \sum_i \beta_i = 1 ,$$

such that $E = AT$. With some algebra we can restate the aggregation problem as the search for a solution to the following problem:

$$\sum_i s_i A_i T_i = \sum_i \alpha_i T_i \cdot \sum_i \beta_i A_i , \qquad (10)$$

where (α, β) are the unknowns. It is interesting to note that the aggregator functions we are searching for have to be particular cases of equation (5), then they satisfy all the properties investigated in section (2.3.). Fare & Grosskopf (2005) note that if the allocative efficiency component across firms is constant, the previous formula becomes $\sum_i s_i T_i = \sum_i \alpha_i T_i$ which implies that good weights for technical efficiency are given by the observed revenue shares:

$$T = \sum_i s_i T_i \qquad (11)$$

In this way the original problem (10) is constrained and we can find a solution for the vector β. Zelenyuk (2004) used potential revenue weights to close the system and aggregate the allocative component:

$$A = \sum_i \hat{s}_i A_i \ , \quad \hat{s}_i = \frac{\theta_i \mathbf{py}_i}{\sum_i \theta_i \mathbf{py}_i}$$

It can be shown that these aggregate indexes need not satisfy the monotonicity property.[6] Since this has been an interesting problem in the literature, it is useful to explicitly discuss the so-called Fox paradox (Fox 2004) in relation to the uncorrelation assumption.

3.2. Fox paradox and Uncorrelation

The industry-level efficiency measure is defined as the ratio of actual revenue over potential revenue, as specified in equation (8). Thus, given that AE=1, we write

$$IE = \sum_i s_i T_i = \frac{R_{pot}}{R} = \frac{\sum_i T_i \mathbf{py}_i}{\mathbf{py}} \tag{12}$$

Since industry-level efficiency can also be stated in terms of simple averages as

$$IE = \frac{\overline{R}_{pot}}{\overline{R}} = \frac{\overline{T_i \mathbf{py}_i}}{\overline{\mathbf{py}}}$$

equation (12) can be rewritten as[7]

$$IE = \overline{T} + \frac{\sum_i (T_i - \overline{T})(R_i - \overline{R}) \big/ n}{\overline{R}} \tag{13}$$

The meaning of expression (13) is that industry efficiency is equal to average efficiency plus a term that depends on the covariance between actual revenue and efficiency. The Fox paradox arises when we have inconsistent changes between individual efficiencies and industry efficiency, that is: monotonicity doesn't hold. Since average efficiency respects the monotonicity property, the violation of this property has to be ascribed to the covariance term. Consider the limit of equation (13)

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[6] It easily to produce numerical example that violate the monotonicity property. See, for example, Fox (2004) where a similar paradox is discussed.

[7] From the definition of covariance we have

$$Cov(x,y) = E\{[x-E(x)][y-E(y)]\} = E(xy) - E(x)E(y)$$

The sample counterpart of this expression is:

$$\frac{1}{n}\sum_i (x_i - \bar{x})(y_i - \bar{y}) = \frac{\sum_i x_i y_i}{n} - \bar{x} \cdot \bar{y}$$

$$\lim_{n\to\infty}(IE) = \frac{E(\theta R)}{E(R)} = E(\theta) + \frac{Cov(\theta, R)}{E(R)} \tag{14}$$

We know that if uncorrelation holds the second term in expression (14) vanishes and industry efficiency becomes equal to average efficiency. In other words, uncorrelation is a sufficient condition in order to assure monotonicity of the industry efficiency index: *if the uncorrelation assumption holds the Fox paradox cannot arise*. Obviously in finite samples the covariance term could be different from zero, but this difference is not statistically significant if uncorrelation holds. Therefore, the main result of this section is that if uncorrelation doesn't hold the monotonicity property is violated and Fox-type phenomena can arise.

4. Stochastic Frontier Models

In stochastic frontier models we require the uncorrelation assumption to be confident that COLS provides consistent estimates of the slope parameters of the frontier function. We also require the independence assumption to identify MLE. In this section we explicitly discuss these issues.

Consider a stochastic frontier production model involving a Cobb-Douglas functional form

$$y_i = \beta_0 + \mathbf{x}\boldsymbol{\beta} + v_i - u_i, \tag{15}$$

where the efficiency term is linked to the distance function $u = e^{\theta}$, y_i is the log of the scalar output quantity, $\mathbf{x}$ is a $k\times1$ vector of logged input quantities, β_0 is the intercept parameter, $\boldsymbol{\beta}$ is a $k\times1$ vector of slope parameters, u_i is the inefficiency error term and v_i is a white noise error term.

4.1. OLS estimation

If cross sectional dataset are used one generally assumes independence both between the two error components (u and v) and between these components and the regressors (see Kumbhakar & Lovell 2000, pp. 74). If the efficiency term is correlated with the input matrix, the OLS estimation of $\boldsymbol{\beta}$ is inconsistent. That is

$$E(\mathbf{b}) = E[(\mathbf{x}'\mathbf{x})^{-1}\mathbf{x}'\mathbf{y}] = \boldsymbol{\beta} + \boldsymbol{\alpha}$$

where $\boldsymbol{\alpha} = E[(\mathbf{x}'\mathbf{x})^{-1}\mathbf{x}'\mathbf{u}]$ is the expectation of the vector of coefficients of the regression of $\boldsymbol{u}$ on $\boldsymbol{x}$. It is clear that this vector is zero only if the regressors are uncorrelated with the efficiency term.

4.1. Maximum Likelihood Estimation

In model (15) we assume uncorrelation: $E(\mathbf{u} \mid \mathbf{x}) = E(\mathbf{u})$. Suppose that the error term $\varepsilon_i = v_i - u_i$ has a distribution given by a density function with parameters **δ**: $f(\varepsilon, \boldsymbol{\delta})$. In this way the dataset can be considered a random sample (given **x**) from this probability distribution. The Likelihood function is the product of *n* identical density functions:

$$L(\boldsymbol{\delta}, \boldsymbol{\beta}, \beta_0 \mid y, \mathbf{x}) = \prod_{i=1}^{n} f(\varepsilon_i, \boldsymbol{\delta}, \boldsymbol{\beta}, \beta_0)$$

From the first order conditions we obtain the expression for the maximum likelihood estimator. In general, since we have *n* observations, we can estimate only a number of parameters that is less than *n*. For example, if the error is normally distributed we have to estimate *k* coefficients plus the variance parameters of the two error terms.[8] If a correlation between the error term and the regressors arise, the regressors are informative on the mean of the error term: $E(\varepsilon \mid \mathbf{x}) \neq E(\varepsilon)$. In this case the likelihood is the product of *n* density functions that differ in their mean values. The parameters of the density function now depend on the observation: $f(\varepsilon_i, \boldsymbol{\delta}_i, \boldsymbol{\beta}, \beta_0)$. The likelihood function therefore is

$$L(\boldsymbol{\delta}_i, \boldsymbol{\beta} \mid y, \mathbf{x}) = \prod_{i=1}^{n} f(\varepsilon_i, \boldsymbol{\delta}_i, \boldsymbol{\beta}, \beta_0).$$

In this case we have to estimate *k*+*n*+3 parameters (**β**, the *n* means, the intercept parameter and the two variance parameters), and thus an identification problem arises.

In some cases this problem can be solved by explicitly introducing a functional relationship between the mean of the error term and a set of regressors in order to reduce the number of parameters. So, for example, Battese & Coelli (1995) propose a model where the efficiency component is a linear function of *r*

[8] Assuming a one-parameter distribution such as the half-normal is chosen for the inefficiency error term.

environmental variables (the environmental variables may or may not correlated with the regressors). In this way we have *k*+r+3 parameter and the model can be identified if *n* is large enough.

5. Testing Uncorrelation, Homoscedasticity and other forms of dependence

Another interesting implication of our earlier discussion of weighted average estimators is that it can provide a method testing for uncorrelation. The sample mean is always (assuming a random sample) a consistent estimator of average efficiency (whether uncorrelation holds or not). However, the weighted sample mean (i.e., industry efficiency) is a consistent estimator only if uncorrelation holds. These results can be used to construct a test for uncorrelation that makes use of the logic of Hausman (1978). We propose a testing strategy in the vein of Wilson (2003), where non-parametric enveloping techniques are used in a first step to obtain consistent estimates of the efficiency scores.

4.1. A general framework

Consider two random vectors $\mathbf{u} \in R^p$, $\mathbf{z} \in R^k$ and n realizations of these random vectors $(\mathbf{u}_i, \mathbf{z}_i)\ \forall i = 1, \ldots, n$ that compose an observation matrix $\mathbf{Q} = [\mathbf{u}_i, \mathbf{z}_i]$ of dimension $n\ x\ (p+k)$. Suppose that we are interested in testing the uncorrelation between **u** and **z**. The testing problem can be stated as:

$$\begin{cases} H_0: & E(\mathbf{u} \mid \mathbf{z}) = E(\mathbf{u}) \\ H_1: & E(\mathbf{u} \mid \mathbf{z}) \neq E(\mathbf{u}) \end{cases} \tag{16}$$

From the central limit theorem the sample mean of the *j*-th component of u ($\bar{u}_j = \frac{1}{n}\sum_i u_{ij}$) is a consistent estimator of $E(u_j)$ and is asymptotically normally distributed:

$$\bar{u}_j \overset{p}{\rightarrow} E(u_j)$$

$$\bar{u}_j \overset{a}{\approx} N\left[E(u_j), \frac{Var(u_j)}{n}\right]$$

The following statistics based on equation (4) (section 2.3.) are interesting in order to test uncorrelation:

$$\phi_{hj} = \frac{\sum_i z_{ih} u_{ij}}{\sum_i z_{ih}} = \frac{\overline{z_h u_j}}{\bar{z}_h}, \quad i = 1,\dots,n,\ j = 1,\dots,p,\ h = 1,\dots,k$$

These statistics are simply an averaging procedure where the *h*-th component of vector **z** is used as a weighting scheme for the j-th component of vector **u**. Therefore we have *pk* of these statistics. From proposition 2 (section 2.3.), we know that they are asymptotically normally distributed. Moreover under the null hypothesis of uncorrelation, $\frac{E(z_h u_j)}{E(z_h)} = E(u_j)$, hence ϕ_{hj} is a consistent estimator of $E(u_j)$. Thus it follows that the difference $(\phi_{hj} - \bar{u}_j)$ converges in probability to zero if and only if the null hypothesis holds.

We can restate the test problem (16) by introducing the (pk×1) difference vector

$$\mathbf{d} = [d_{hj}] = \left[\phi_{hj} - \bar{u}_j\right] = \left[\frac{E(z_h u_j)}{E(z_h)} - E(u_j)\right].$$

Since under the null hypothesis this difference vector is zero, the test problem reduces to a test for the following *pk* restrictions:

$$\begin{cases} H_0: & E(\mathbf{d}) = \mathbf{0} \\ H_1: & E(\mathbf{d}) \neq \mathbf{0} \end{cases}$$

It is possible to use the following Wald statistic to test uncorrelation:

$$W = \mathbf{d}'\left[\widehat{\mathrm{Var}}(\mathbf{d})\right]^{-1}\mathbf{d}. \tag{17}$$

The Wald statistic (17) is asymptotically distributed as a Chi-Square with pk degrees of freedom and is used in the standard manner to test the null hypothesis.

4.2. A three step semi-parametric testing procedure for Uncorrelation

The result of the last sub-section can be used to test uncorrelation between efficiency (θ) and the vector composed by the inputs **x** and the output compositions **η**. In this context **u** is a scalar random variable (output oriented efficiency measure θ) and **z** is the random vector $(\boldsymbol{\eta}, \mathbf{x})$ of dimension $k + m - 1$ (the number of inputs and the number of output compositions), so the Wald statistic (17) is distributed as a Chi-Square with $k+m-1$ degrees of freedom.

Two problems arise in applying the previous test: the first one is that we do not have data on efficiency; the second one is the estimation of the covariance matrix of the Wald statistic, $\hat{\mathrm{V}}\mathrm{ar}(\mathbf{d})$, in expression (17). These problems are solved with a three step procedure. The first step involves estimating individual efficiencies via non-parametric Data Enveloping techniques: both DEA and FDH are consistent estimators of the "true" efficiency (Wilson 2003), although they give rise to low rates of convergence (curse of dimensionality). In the second step, we estimate the covariance matrix $\mathrm{Var}(\mathbf{d})$ using bootstrap methods (Efron & Tibshirani 1993) without deriving an analytical expression for it. Finally, in the third step, we compute the value of the Wald statistic in (17) and compare it with the χ^2_{k+m-1} table value.

Since we are using non-parametric estimation of efficiency in the first step, we have to expect that the testing procedure shows lower convergence rates than a full-parametric version, but a full-parametric version cannot be used since the estimation of efficiency (in a parametric framework) is based on the hypothesis we are testing for. Moreover the estimation via non-parametric techniques in the first step is also used in other testing strategies (see Wilson 2003) proposed in the literature.

Testing Homoscedasticity

The previous testing procedure can be easily adapted to test for homoscedasticity. We have only to consider the second moment of the efficiency distribution. Now, we have to test uncorrelation between the vector $\left[\theta,\theta^2\right]$ and the vector composed of output compositions and inputs. If the test accepts the null hypothesis, then both the uncorrelation and the homoscedasticity assumptions holds. If the test rejects the null hypothesis one of the two assumptions is violated.

It is noted that if we assume a truncated normal parametric family for the efficiency distribution, then testing for both uncorrelation and homoscedasticity is equivalent to testing for independence. In the same way, when assuming a half-normal or exponential distribution, it is sufficient to test for uncorrelation in order to investigate independence.

Our test can be further generalized if we consider some function of the efficiency term $h(\theta)$. We can test for the dependence of the skewness on the input/output composition vector considering the matrix $\left[\theta,\theta^2,\theta^3\right]$. Or we can test for the dependence of the kurtosis on the input vector considering $\left[\theta,\theta^4\right]$. Obviously the dependence between the efficiency term and the input/output composition vector can take many different forms, so in general terms we have to specify a function and then test for the uncorrelation of the vector $h(\theta)$ from the input set. Considering the moment functions, at each step we are excluding a particular form of dependence of the efficiency distribution from the input/output composition vector. For example, it is possible to test the uncorrelation between $\left[\theta,\theta^2,\theta^3,\theta^4\right]$ and $(\boldsymbol{\eta},\mathbf{x})$; if the test accepts the null hypothesis, we are excluding linear dependence, heteroscedasticity, skewness dependence and kurtosis dependence from the input/output composition vector.

The basic forms of dependence (linear dependence and heteroscedasticity) are arguably the most aggressive forms of dependence. They create a lot of problems and impact in stronger ways relative to other types of dependence (such as skewness dependence or other forms of non-linear dependence). As the test is generalised to include higher order moments the degrees of freedom of the test increase and hence more data may be required. For example, if we test for uncorrelation we have (m+k-1) degrees of freedom, but if we test both for uncorrelation and homoscedasticity we double this number, and so on if we introduce other types of dependence.

6. Monte Carlo experiment and empirical illustration

We are interested in illustrating two key results from the previous discussion. First, the statistical test we have introduced is asymptotic, so we investigate its finite sample behaviour with some Monte Carlo simulations. Second, we explicitly measure the bias of the industry efficiency indexes showing the size of the covariance term and reaching a full decomposition of the industry efficiency indexes. This is illustrated using a dataset on the US electric power generation (Christensen & Greene 1976).

6.1. Monte Carlo experiment

In order to investigate the finite sample properties of the test, we produce results that are comparable with the Monte Carlo experiment performed by Wilson (2003), emphasizing the power of the test in detecting linear dependencies in comparison to the nonparametric tests surveyed by Wilson. Moreover, drawing from a multivariate normal distribution, the rejection rate of the test is investigated under different assumptions regarding the parameter values (correlation coefficient, sample size and the number of variables). Following Wilson (2003), we assume a seven-dimensional (i.e., m+k-1=7) multivariate normal distribution with a covariance matrix that presents ones on the diagonal and the same correlation coefficient outside the diagonal. Bootstrap methods have been used to estimate the covariance matrix of the Wald statistic (17).

We expect rejection rates that are near the size of the test when the correlation coefficient is zero. Moreover, for larger values of the correlation coefficient, we expect a higher power of the test. Of course, testing uncorrelation give us less information than testing for independence and this is the main reason why the power of the test is increased: we are trading the strong-ness of the hypothesis with the power of the test. In Table 1 the results of the first Monte Carlo experiment (involving 1,00 replications) are summarized. In the first row ($\rho = 0$) we can see that the test correctly shows a power close to the size of the test. The power of the test then increases sharply with the value of the correlation coefficient, which is as one would wish.

The values in brackets reported in Table 1 are the results obtained in Wilson (2003) for his bootstrap test ($\hat{T}_{4n}$), which is the best performing test for independence in his experiment. It is evident that under the null hypothesis this test has too high an acceptance rate, indicating incorrect size. Moreover, the test for independence show a rejection rate that increases quite slowly with increasing values of the correlation coefficient (linear dependence), whereas our test shows a sharply increasing rejection rate in the presence of linear dependence in the data. This fact suggests that it is useful to use this test for uncorrelation as a pre-test procedure in order to exclude linear dependencies in the data in a more general strategy oriented in testing the independence assumption.

In Table 2 we report the results of other Monte Carlo simulations where we varied sample size (N), correlation coefficient (ρ) and the number of variables. The rejection rate increases sharply both with the sample size and the correlation coefficient. As a rule of thumb we can expect that for an efficiency model with three inputs, one output and less than a hundred of observations, the test performs quite well in detecting linear dependencies in the data.

6.2. Empirical illustration

To illustrate the empirical use of these tools, we analyse data on the US electric power generation. Christensen & Greene (1976) used two cross-sectional datasets in order to assess the performance of the US electricity sector during the period 1955-1970. In this study wee concentrate on the 1970 dataset that contains 158 observations. Data are available on total cost, total output, wage rate, cost share for labour, capital price index, cost share for capital, fuel price and cost share for fuel. Using these data, we calculated implicit labour, capital and fuel quantities. Thus the final dataset is a collection of data on three input quantities and one output quantity for each observation.

We estimated an output oriented efficiency measure using both Constant Return to Scale (CRS) and Variable Return to Scale (VRS) technologies in a DEA framework. The differences between the variable and constant return to scale results suggest the presence of regions of non-constant return to scale, as was found in Christensen & Greene (1976) when using parametric cost function methods. In order to summarize the results and to discuss the aggregation problems introduced in section 3, we constructed four different industry efficiency indexes, using output, labour, capital and total cost shares as the weights.

In the CRS results reported in Table 3 we can see that there is a significant difference between these indexes and the simple average aggregation procedure. The p-value of our test statistic is reported with each aggregate index. We conducted a test of uncorrelation both between each input quantity and the efficiency term and then between the full set of input quantities and the efficiency term. A p-value less than the size of the test (usually $\alpha = 0.05$) is interpreted as a rejection of the null hypothesis of uncorrelation.

The same exercise has been done with the VRS efficiency scores and the results are reported in Table 4. Moreover, in both tables we report the value of the covariance term which captures the differences between simple average efficiency and industry efficiency indexes (as shown in equation (13)). As can be seen the industry indexes constructed using the VRS technology are very different from simple averaging. Following the suggestion of Fare & Grosskopf (2005), we also constructed an industry efficiency index using output share weights. The value of this index is 0.867 versus the (unweighted) average value of 0.771.

The failure of the uncorrelation assumption indicates that a weighted aggregator should be used in estimating industry efficiency. On the contrary, if the uncorrelation assumption holds aggregation is not sensitive to the choice of the aggregator weights. In this case the aggregation problem is unlikely to be considerable, since all the industry efficiency indexes converge to the simple mean. The results obtained from this dataset, however, suggest that the efficiency term is strongly correlated with the inputs and hence the use of an unweighted average to estimate industry efficiency could be misleading due to the lack of monotonicity ascribed to correlation.

7. Conclusions

Independence and/or uncorrelation assumptions are important in many aspects of efficiency analysis. We make note of three particular cases in this study: (i) bootstrapping non-parametric efficiency models; (ii) estimating stochastic frontier analysis (SFA) and (iii) obtaining aggregate measures of industry efficiency. The first case, involving bootstrapping DEA models, has been discussed in some detail in Wilson (2003). In the case of SFA, we note that an uncorrelation assumption is required for COLS estimators to be consistent, while an independence assumption is needed for ML estimation to be feasible. Finally, for the case of aggregation we show that uncorrelation is needed for an (unweighted) average efficiency measure to be a consistent estimator of industry efficiency.

Our discussion of alternative weighted average measures of industry efficiency lead us to propose a semi-parametric Hausman-type asymptotic test for linear independence (uncorrelation) between technical efficiency and variables such as input quantities and output mixes. Wilson (2003) has previously investigated a number of (full) independence tests and found that they had poor size properties and low power in moderate sample sizes. We provide a Monte Carlo experiment which indicates that our test for uncorrelation has superior size and power properties in finite samples, relative to these independence tests.

Obviously, since independence implies uncorrelation but not the converse, our test is not as useful in situations where a test for independence is required (e.g., in bootstrapping DEA models). However, it can still be useful to some extent. For example, if one finds that uncorrelation is rejected then there is no need for one to conduct the independence test (which is more involved and has lower power). Secondly, we have shown how the test can be generalized in order to detect higher order dependencies, such as heteroscedasticity. Thus, the test could be used to test for (full) independence in situations where one is willing to assume that the efficiency distribution has a finite number of moments. For example, when one believes an exponential or truncated normal distribution provides a suitable approximation to the true efficiency distribution.

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References

Aigner D, Lovell C.A.K., Schmidt P. (1977), "Formulation and estimation of stochastic frontier production function models", *Journal of Econometrics*, 6, 21-37

Alvarez A., Amsler C., Orea L., Schmidt P. (2006), "Interpreting and testing the scaling property in models where inefficiency depends on firms characteristics", *Journal of Productivity Analysis*, 25:201-212

Amemiya T. (1973), "Regression analysis when the dependent variable is truncated normal", *Econometrica*, Vol. 41, No 6, pp. 997-1016

Amemiya T. (1994), "Introduction to Statistics and Econometrics", Harvard University Press

Balk B.M. (1995), "Axiomatic price index theory: a survey", *International Statistical Review*, Vol. 63, No 1, pp. 69-93

Battese G.E., Coelli T.J. (1995), "A model for technical inefficiency effects in a stochastic frontier production function for panel data", *Empirical Economics*, 20:325-332

Blackorby C., Russell R. R. (1999), "Aggregation of Efficiency Indices", *Journal of Productivity Analysis*, 12, 5-20

Christensen L.R., Greene W.H. (1976), "Economies of scale in US electric power generation", *The Journal of Political Economy*, Vol. 84, No 4, pp. 655-676

Coelli T.J., Prasada Rao D.S., Battese G.E. (2005), "An Introduction to Efficiency and Productivity Analysis", Kluwer Academic Publishers

Coelli T.J. (1995), "Estimators and Hypothesis tests for a stochastic frontier function: a Monte Carlo analysis", *Journal of Productivity Analysis*, 6, 247-268

Daraio C., Simar L. (2005), "Introducing Environmental Variables in Nonparameric Frontier Models: A Probabilistic Approach", *Journal of Productivity Analysis*, 24, 93-121

Daraio C., Simar W. (2007), "Advanced robust and nonparametric methods in efficiency analysis", Kluwer Academic Publishers

Diewert W.E. (1978), "Superlative Index Numbers and Consistency in Aggregation", *Econometrica*, Vol. 46, No. 4

Efron B., Tibshirani R. J. (1993), "An Introduction to the Bootstrap", Chapman & Hall

Fare R., Primont D. (2003), "Luenberger Productivity Indicators: Aggregation Across Firms", *Journal of Productivity Analysis*, 20, 425-435

Fare R., Grosskopf S., Knox Lovell C.A. (1994), "Production Frontiers", Cambridge University Press

Fare R., Grosskopf S., Zelenyuk V. (2004), "Aggregation Bias and its Bound in Measuring Technical Efficiency", *Applied Economics Letters*, 11, 657-660

Farrell M.J. (1957), "The Measurement of Productive Efficiency", *Journal of the Royal Statistical Society*, Vol. 120, No. 3, 253-290

Fox K.J. (2004), Problems with (Dis)Aggregating Productivity and another Productivity Paradox, The University of New South Wales

Greene W.H. (1997), "Econometric Analysis", Prentice Hall

Grosskopf S. (2003), "Some Remarks on Productivity and its Decomposition", *Journal of Productivity Analysis*, 20, 459-474

Hausmann J.A. (1978), "Specification tests in econometrics", *Econometrica,* Vol 46, No 6, pp 1251-1271

Kumbhakar S. C., Knox Lovell C. A. (2000), "Stochastic Frontier Analysis", Cambridge University Press

Mas-Colell A., Whinston M.D. (1995), "Microeconomic Theory", Oxford University Press

Schmidt P., Sickles R.C. (1984), Production frontiers and panel data, *Journal of Business & Economic Statistics*, Vol 2, No. 4

Simar L., Wilson P.W. (1998), "Sensitivity analysis of efficiency scores: how to bootstrap in nonparametric frontier models", *Management Science*, Vol 44, No 1, pp. 49-61

Simar L., Wilson P.W. (2000), "Statistical inference in nonparametric frontier models: the state of the art", *Journal of Productivity Analysis*, 13, 49-68

Soriano F.A., Prasada Rao D.S., Coelli T. (2003), "On the Aggregation of Total Factor Productivity Measures", *Economic Measurement Group Workshop*, University of New South Wales

Wand M. P., Jones M. C. (1995), "Kernel Smoothing", Chapman & Hall

Wilson P.W. (2003), "Testing Independence in Models of Productive Efficiency", *Journal of Productivity Analysis*, 20, 361-390

Zelenyuk V. (2004), "Aggregation of Malmquist Productivity Indexes", Insitut de Statistique, Université Catholique de Louvain

Appendix – Tables

Table 1: Rejection rates for the uncorrelation test*

REJECTION RATE		Size of Test (α)		
		0.1	0.05	0.01
Correlation (ρ)	x	0.111 (0.007)	0.047 (0.029)	0.01 (0.004)
	0.1	0.432 (0.186)	0.320 (0.112)	0.152 (0.029)
	0.2	0.901 (0.431)	0.856 (0.301)	0.730 (0.102)
	0.3	0.990 (0.739)	0.988 (0.584)	0.970 (0.301)
	0.4	1.000 (0.930)	1.000 (0.856)	1.000 (0.594)
	0.5	1.000 (0.991)	1.000 (0.969)	1.000 (0.871)

* Sample size is n=70, number of variables is (*m*+*k*-1)=7 and Wilson independence test results are in brackets.

Table 2 – Effect of sample size and number of variables on rejection rates

	ρ				
	0.025	0.05	0.1	0.2	0.4
N	3 Variables				
25	0.065	0.080	0.112	0.256	0.722
50	0.053	0.073	0.150	0.423	0.939
100	0.054	0.087	0.223	0.709	0.999
200	0.074	0.126	0.436	0.951	1.000
400	0.118	0.226	0.724	1.000	1.000
800	0.116	0.420	0.953	1.000	1.000
	5 Variables				
25	0.076	0.086	0.122	0.333	0.854
50	0.076	0.078	0.191	0.539	0.983
100	0.058	0.114	0.330	0.860	1.000
200	0.084	0.192	0.584	0.985	1.000
400	0.104	0.322	0.879	1.000	1.000
800	0.177	0.634	0.995	1.000	1.000
	7 Variables				
25	0.073	0.086	0.158	0.375	0.893
50	0.055	0.082	0.216	0.668	0.998
100	0.078	0.126	0.414	0.939	1.000
200	0.093	0.225	0.695	0.999	1.000
400	0.138	0.421	0.945	1.000	1.000
800	0.225	0.739	0.999	1.000	1.000
	10 Variables				
25	0.088	0.091	0.173	0.490	0.928
50	0.061	0.105	0.297	0.767	0.999
100	0.084	0.138	0.483	0.964	1.000
200	0.097	0.256	0.792	1.000	1.000
400	0.159	0.526	0.979	1.000	1.000
800	0.273	0.817	1.000	1.000	1.000

TABLE 3 – CRS-DEA, Aggregate Indexes (n=158)

Mean	Cost	Product	Labour	Capital	Fuel
0.723	0.768	0.785	0.749	0.769	0.780
	(0.0012)	(0.0003)	(0.2003)	(0.0167)	(0.0025)
Covariance Term	0.045	0.062	0.026	0.046	0.057

TABLE 4 – VRS-DEA, Aggregate Indexes (n=158)

Mean	Cost	Product	Labour	Capital	Fuel
0.771	0.859	0.867	0.844	0.856	0.862
	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0000)
Covariance Term	0.088	0.096	0.073	0.085	0.091