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Abstract

In this paper we consider the question of measuring *aggregate* economic growth and its sources. We derive a theoretically justified solution for aggregating (across firms, industries, countries, etc.) growth rates and their sources within the framework of Solow's (1957) growth accounting method. The resulting aggregation scheme turns out to be quite intuitive and, in fact, the one that is sometimes, in an *ad hoc* way, used in practice, and so the main value of our work is that our formal derivations clearly show under what conditions this scheme is theoretically justified. We also provide a small empirical illustration of our method on the real data set and show how different the conclusions can be depending on the aggregation scheme used.

Key Words: Growth Accounting, Productivity, Aggregation.

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Introduction

Nowadays, it became very popular to estimate, analyze and forecast the economic growth rates of not only particular decision making units (firms, industries, countries, etc.), hereafter DMUs, but also of certain groups of them. On the firm level, for example, a researcher may be interested in grouping of plants, owned by the same firm, by geographical location or by the type of technology used or by other criteria and then estimating aggregate growth rates of these particular groups as well as aggregating them into the growth rate of the entire firm. Similar grouping could be of interest on the industry level, or one could be interested in grouping into public and private firms, foreign and local firm, etc. and then seek for the aggregate growth rates of these groups as well as an aggregate of them, i.e., the aggregate growth rate of the entire industry. In the macroeconomic context, one also can find many examples of groupings of DMUs. The investment banking community, the International Monetary Fund (IMF) and the World Bank, for example, often divide the world into such groups as developed and developing economies, the emerging and the frontier markets, the OECD countries, the EU, the CEE (Central and Eastern Europe), the BRIC (Brazil, Russia, India and China), and so on. The aggregate growth rates of such groups are then estimated, forecasted and presented for the sake of understanding of how the world develops, which groups are lagging and which are leading in terms of economic growth. To serve this purpose, some studies (e.g., Kumar and Russell (2002)) just aggregated the individual growth rates with the *simple* (i.e., equally-weighted) average. The strength of this aggregation is that it is simple and, more importantly, possesses the well known statistical properties (consistency, normality, etc.), under quite general assumptions. A weakness of this aggregation is that it ignores an ‘economic weight’ of each DMU in the sample. For this reason, other researchers are using the weighted averages, and the weights are often chosen to be the total output or revenue (e.g., GDP, in case of countries) shares of each DMU in the group.¹ The problem with this approach is that, besides a common sense intuition, it is not theoretically clear why the weights must be the output shares and not

¹ E.g., see World Economic Outlook of International Monetary Fund. Notably, often researchers are not explicit in whether they use weighted or non-weighted aggregation and what weights they use.

something else. In particular, when the output growth is decomposed into various contributions, e.g., such as contributions from the capital change and the labor change, a natural question is why the weights must be the ‘output shares’ and not the ‘capital shares’ and the ‘labor shares’, respectively?

To put it differently, the weights in the aggregation of the output growth rates are currently chosen in an *ad hoc* way—just because they are intuitive or because others used it. The goal of this paper is to provide a clear theoretical justification for why and when these or other weights can or cannot be chosen, i.e., under what assumptions they are theoretically appropriate and when they are not.

A clear theoretical foundation for choosing weights is especially needed because results and conclusions of it may heavily depend on the chosen system of weights. Preferably, the weighting system must be derived from some economic principles with clarity on what assumptions are needed—and this is what we do in this paper. Specifically, we derive a theoretically justified system of weights for aggregating (across firms, countries, etc.) growth rates as well as its sources estimated using the growth accounting (Solow, 1957) method. The results are based on the revenue aggregation theorem from Färe and Zelenyuk (2003), Zelenyuk (2006) and Simar and Zelenyuk (2007), which in turn are the revenue analogues and extensions of Koopmans (1957) theorem for aggregating profit functions. In the final section of this paper, we also provide a small empirical illustration of our method on the real data set and show how different the conclusions can be in our weighted aggregation relative to the simple average aggregation.

The paper is structured as follows. In section 1 we define characterization of technology and of output growth on individual level, while in section 2 we do the same on the aggregate level and arrive to the main aggregation result there. In section 3 we discuss the intuition and meaning of the results from section 2 and of assumptions that yielded them. In section 4 we extend our results to aggregation of sub-group aggregates into a larger group aggregates. In section 5 we provide small empirical illustration of the estimation and in section 6 we conclude.

1. Characterization of Individual Technologies and Measurement of Growth

Consider a group-wise heterogeneous population of n decision making units (firms, industries, countries), hereafter DMUs, indexed by $k = 1, 2, \dots, n$. Assume they can be grouped into L distinct (non-overlapping) sub-groups with n_l ($l = 1, \dots, L$) DMUs in each of the sub-groups. Assume also that in any period t , a DMU k in a group l has an endowment of N resources $x_t^{l,k} = (x_{t,1}^{l,k}, \dots, x_{t,N}^{l,k})' \in \mathfrak{R}_+^N$ to produce a vector of M outputs, denoted by $y_t^{l,k} = (y_{t,1}^{l,k}, \dots, y_{t,M}^{l,k})' \in \mathfrak{R}_+^M$. We assume that technology of a DMU k (in a group l) in a period t can be characterized by the output sets

$$P_t^{l,k}(x_t^{l,k}) \equiv \{y_t : \text{all } y_t \in \mathfrak{R}_+^M \text{ producible from } x_t^{l,k} \in \mathfrak{R}_+^N\}, \quad x_t^{l,k} \in \mathfrak{R}_+^N, \quad (1.1)$$

that satisfy the standard regularity conditions of production theory, by which we would mean that each output set satisfies *free disposability of outputs*, i.e., $y^o \in P_t^{l,k}(x) \Rightarrow y \in P_t^{l,k}(x), \forall y \leq y^o$ and is compact, i.e., closed and bounded (for all $x \in \mathfrak{R}_+^N$ and for all k and l). We also assume that $y \notin P_t^{l,k}(\mathbf{0}_N), \quad \forall y \geq \mathbf{0}_M$ (“*no free lunch*”) and $\mathbf{0}_M \in P_t^{l,k}(x), \quad x \in \mathfrak{R}_+^N$ (“*producing nothing is possible*”). To involve duality results, we also assume that $P_t^{l,k}(x)$ is convex for all $x \in \mathfrak{R}_+^N$ and for all k and l .²

Note that if we were to have only one output, i.e., $M = 1$, then the computation of growth rates in this output and its decomposition into sources and aggregation would be a fairly simple issue. In the multi-output context, however, one needs to use some suitable aggregator of the M outputs into a single quantity to be operated with. One of the popular tools for this is the Shephard’s distance function, which completely characterizes technology (under some regularity conditions).

Strictly speaking, the Shephard’s distance function and the measures of productivity based on it, e.g., such as Malmquist Productivity Indexes (MPIs), Hicks-Moorsteen Productivity Indexes (HMPI), etc., require the original data to be in physical units, while in practice it is often available only in value-weighted aggregated form, e.g., as revenues of firms or industries of particular types of products or just

as total revenue or as GDP.³ In this sense, a more suitable tool for characterizing technology might be the *revenue* function, defined as

$$q_t^{l,k} \equiv f_t^{l,k}(x_t^{l,k} | p_t) \equiv \max_y \{p_t y : y \in P_t^{l,k}(x_t^{l,k})\}, \quad (1.2)$$

where $p_t = (p_{t,1}, \dots, p_{t,M}) \in \mathfrak{R}_+^M$ denotes the vector of some nonnegative output prices in period t , which we assume to be the same across all DMUs (e.g., representing the world prices in the macroeconomic context or the equilibrium prices in the microeconomic context). Given our regularity conditions and sue to duality theory in economics (Shephard's, 1970 and Färe and Primont, 1995), the revenue function (1.2) is a complete characterization of $P_t^{l,k}(x_t^{l,k})$.

Note that the revenue function $f_t^{l,k}$ can also be understood as the ‘aggregate production function’ of DMU k in group l in period t —where the (optimal, in terms of revenue optimization) outputs are aggregated into one quantity, with prices being the weights of the aggregation. When the DMU is a firm, then $f_t^{l,k}$ would stand for the maximal revenue, valued at current prices, $p_t = (p_{t,1}, \dots, p_{t,M}) \in \mathfrak{R}_+^M$, that can be generated by the firm k with technology given by $P_t^{l,k}(x_t^{l,k})$. In the macroeconomic context, i.e., when the DMU is a country and $y_t^{l,k} = (y_{t,1}^{l,k}, \dots, y_{t,M}^{l,k})' \in \mathfrak{R}_+^M$ represents all the *final* goods and services produced within this country in time period t , and $p_t = (p_{t,1}, \dots, p_{t,M}) \in \mathfrak{R}_+^M$ are their corresponding market prices, then $f_t^{l,k}$ can be interpreted as the function yielding the value of maximal GDP for period t of country k in a sub-group l . Whether it is micro or macro context, we will refer to the values yielded by $f_t^{l,k}$ as the values of *maximal total output*, or to simplify the wording—as ‘the total output’, denoting it with $q_t^{l,k}$.

² E.g., see Färe and Primont (1995) for details on these assumptions and their implications.

³ In practice, however, the MPI and HMPI are often used when data is available in value-weighted form as well (e.g., see Färe et al. (1994), Kumar and Russell (2002), Henderson and Russell (2005), Henderson and Zelenyuk (2006) as well as references cited therein).

From the growth accounting paradigm (Solow, 1957), given appropriate differentiability of (1.2), the growth rate of the total output, hereafter denoted as $g(q_t^{l,k})$, is given by

$$g(q_t^{l,k}) \equiv \frac{dq_t^{l,k} / dt}{q_t^{l,k}} = \frac{df_t^{l,k}(x_t^{l,k} | p_t) / dt}{q_t^{l,k}} = \sum_{i=1}^N e_{i,t}^{l,k} \cdot g(x_{i,t}^{l,k}) + g(f_t^{l,k}) \quad (1.3)$$

where $e_{i,t}^{l,k} \equiv (\partial f_t^{l,k}(x_t^{l,k} | p_t) / \partial x_{i,t}^{l,k})(x_{i,t}^{l,k} / q_t^{l,k})$ is the (dual) partial scale elasticity with respect to input i (or *factor i 's share* in the total output), while $g(x_{i,t}^{l,k}) \equiv (dx_{i,t}^{l,k} / dt) / x_{i,t}^{l,k}$ is the growth rate of this input i , and $g(f_t^{l,k}) \equiv (\partial f_t^{l,k} / \partial t) / f_t^{l,k}$ is the growth rate of the function $f_t^{l,k}$. The latter component is often referred to as the total factor productivity (TFP) change, or the Solow's residual, which, in the absence of inefficiency (the case we consider in this paper) serves as a measure of the technological change.⁴

Overall, in words, the expression (1.3) says that the growth rate of the total output is the weighted average of the growth rates in each input $x_{i,t}^{l,k}$ weighted by the corresponding partial scale elasticities, plus the growth rate in the technology. Also note that (under additional assumption of constant returns to scale), equation (1.3) can be rearranged to give decomposition of an input (usually, the labor) *productivity* growth equation.

To summarize this section, it might be worth noting that while the Solow's growth accounting method is not the only method of computing economic growth (in total output or in productivity) and decomposing it into various sources, it is certainly one of the major methods used in practice. Perhaps the most popular alternative to this method is the one based on MPIS, originally proposed by Caves et al. (1982). The MPI method also allows for various decompositions into sources of growth, the most popular of which seem to be the one proposed by Färe et al. (1994), which decomposes productivity growth into technology change (an analogue to the Solow's residual) and efficiency change. These authors also seem to be among the first who noted relationship of MPI to the Solow's growth

⁴In case of Hicks-neutral technical change, $f_t^{l,k}(x_t^{l,k} | p_t) \equiv A(t)f_t^{l,k}(x_t^{l,k} | p_t)$, we obtain $g(f_t^{l,k}) \equiv (\partial A(t) / \partial t) / A(t)$.

accounting method. This relationship was recently elaborated even more in the work of Kumar and Russell (2002), who decomposed the labor productivity index (total output normalized by total labor) into three sources: capital deepness, technology change and efficiency change. While one of these methods has advantages over the other in some circumstances and some disadvantages in others, in this paper we are not in a position to judge or compare the methods, but to develop the aggregation theory framework for one of them, the growth accounting method, as we do in the next section, while for the MPI method such framework has been developed to some extent earlier, by Zelenyuk (2006).

2. The Aggregate Technologies and the Aggregate Growth

The goal of this section is to define similar notions to those outlined in the previous section, but now on the *aggregate (sub-group)* level. For this, let us denote the input allocation among countries *within* a sub-group $l \in \{1, \dots, L\}$ by $X_t^l = (x_t^{l,1}, \dots, x_t^{l,n_l})$.

A critical step is to define a *sub-group technology*—the *aggregate* technology of *all* DMUs within a sub-group. One natural way would be to assume the *additive structure of aggregation* for the *output sets*, i.e.,

$$\bar{P}_t^l(X_t^l) \equiv \bar{P}_t^l(x_t^{l,1}, \dots, x_t^{l,n_l}) \equiv \sum_{k=1}^{n_l} P_t^{l,k}(x_t^{l,k}), \quad l \in \{1, \dots, L\}. \quad (2.1)$$

Thus, the *output set* of the sub-group l of DMUs, $\bar{P}_t^l(X_t^l)$, is the sum of the individual *output sets* of all DMUs in this sub-group. The properties of such aggregate technology $\bar{P}_t^l(X_t^l)$ would depend on the properties of technologies of each DMU in the sub-group $P_t^{l,k}(x_t^{l,k})$. For example, the satisfaction of the standard regularity conditions of production theory, which we listed below (1.1), by each individual output set would ensure satisfaction of these conditions by the aggregate output set defined in (2.1). On the other hand, if at least one of the conditions is violated for at least one of the individual output sets (e.g., if one of them is not closed or not convex) then the aggregate output set defined in (2.1) is not guaranteed to satisfy that particular condition (e.g., to be closed or convex).

Given the sub-group technology (2.1), the l^{th} *sub-group revenue* function can be defined in a fashion similar to definition on disaggregated level, but now defined on the sub-group technology, i.e.,

$$Q_t^l \equiv F_t^l(X_t^l | p_t) \equiv \max_y \{p_t y : y \in \bar{P}_t^l(X_t^l)\}, \quad l \in \{1, \dots, L\}. \quad (2.2)$$

Now, F_t^l can be understood as the ‘aggregate production function’ for the *sub-group* l in period t .

Similarly, the growth rate of the total output of a sub-group $l \in \{1, \dots, L\}$, denoted as $G(Q_t^l)$, is given by

$$G(Q_t^l) \equiv \frac{dF_t^l(X_t^l | p_t) / dt}{Q_t^l} = \sum_{k=1}^{n_t} \sum_{i=1}^N E_{i,t}^{l,k} \cdot g(x_{i,t}^{l,k}) + \frac{\partial F_t^l / \partial t}{Q_t^l} \equiv \sum_{i=1}^N G_{i,t}^l + G(F_t^l). \quad (2.3)$$

where $G_{i,t}^l \equiv \sum_{k=1}^{n_t} E_{i,t}^{l,k} \cdot g(x_{i,t}^{l,k})$ is the contribution of change in input i occurring in all DMUs in sub-group $l \in \{1, \dots, L\}$, while $E_{i,t}^{l,k} \equiv (\partial F_t^l(X_t^l | p_t) / \partial x_{i,t}^{l,k})(x_{i,t}^{l,k} / Q_t^l)$ is the (dual) partial scale elasticity with respect to input i of the sub-group technology, $g(x_{i,t}^{l,k}) \equiv (dx_{i,t}^{l,k} / dt) / x_{i,t}^{l,k}$ is the growth rate of this input i , and finally, $G(F_t^l) \equiv (\partial F_t^l / \partial t) / Q_t^l$ is the growth rate of technology of *sub-group* $l \in \{1, \dots, L\}$.

We now want to establish a relationship between the *aggregate* and *disaggregate* approaches to growth accounting. In particular, we are interested in how to obtain the aggregate or sub-group growth rate and its sources from their disaggregate analogues. In practice, researchers usually used the non-weighted arithmetic average for this purpose. With the derivations that follow, we will justify the use of *weighted* arithmetic averages, where the weights (and the aggregation function) are derived from the economic optimization behavior and specific assumptions on aggregation of technologies and on prices.

The fundamental for our study result is the fact that, for each $l \in \{1, \dots, L\}$, we have

$$F_t^l(X_t^l | p_t) = \sum_{k=1}^{n_t} f_t^{l,k}(x_t^{l,k} | p_t), \quad x_t^{l,k} \in \mathfrak{R}_+^N, \quad p_t \in \mathfrak{R}_+^M. \quad (2.4)$$

The economic intuition of this result is straightforward: the sum of the total outputs of *individual* revenue-maximizing DMUs in a given sub-group is the same as the total output obtained by revenue-maximizing sub-group, as one entity, whose technology is defined in (2.1), given that the output price vector is the same for all DMUs. This result is an intertemporal extension of Färe and Zelenyuk (2003) and Simar and Zelenyuk (2007), also found in Zelenyuk (2006), which in turn are elaborations of the theorem from Koopmans (1957) for aggregating the profit functions. (For the sake of completeness, the proof of this results is provided in Appendix A).

Rearranging (1.3), (2.3), (2.4) yields a simple and intuitive, yet very important theoretical result

$$G(Q_t^l) = \sum_{k=1}^{n_l} g(q_t^{l,k}) S_t^{l,k}, \quad S_t^{l,k} \equiv \frac{q_t^{l,k}}{Q_t^{l,k}}, \quad l \in \{1, \dots, L\}. \quad (2.5)$$

In words, the growth rate of total output of a sub-group l , *under certain assumptions*, is equal to the weighted average of the growth rates of total output of all DMUs in this sub-group, where the weights are the shares of total output of each DMU in the sub-group in the same period t .

The result (2.5) can be extended to the aggregation of the distinct *sources* of growth across DMUs—so that we can obtain each component of aggregate decomposition expression (2.3) by properly aggregating their disaggregate analogues given in (1.3). In particular, again using the key expression (2.4), we obtain

$$G_{i,t}^l = \sum_{k=1}^{n_l} e_{i,t}^{l,k} g(x_{i,t}^{l,k}) S_t^{l,k}, \quad S_t^{l,k} \equiv \frac{q_t^{l,k}}{Q_t^{l,k}}, \quad l \in \{1, \dots, L\}. \quad (2.6)$$

Thus, in words, the contribution of change in input i in all DMUs in sub-group l to the growth in the total output of the sub-group l can be obtained as the *weighted* average of contributions of this same input to the growth of the total output of each DMU in the sub-group, where the weights are, again, the shares of the total output of each DMU to the aggregate total output of the entire sub-group.

Similarly, again using (2.4), we also obtain

$$G_{f,t}^l = \sum_{k=1}^{n_l} g(f_t^{l,k}) S_t^{l,k}, \quad l \in \{1, \dots, L\}. \quad (2.7)$$

i.e., the contribution of change in technology in all DMUs in sub-group l onto the growth in the total output of the sub-group l can be obtained as the *weighted* average of contributions of change in each DMU's (in the sub-group) technology to the growth of their total outputs, where the weights are also the shares of the total output of each DMU to the total output of the entire sub-group.

Thus, we again receive a theoretical justification for the simple way of obtaining the aggregate *sub-group* contributions of each source of growth onto the growth of the total output of this sub-group—through the weighted sum of contributions of the corresponding sources from each DMU, with weights derived from economic optimization and certain assumptions on technology and prices.

3. Intuition of Results and of Underling Assumptions

Before going further with theoretical derivations of aggregation results, it is worth to pause and think of some intuition as well as some practical implications of the results obtained in the previous section. We will briefly summarize our thoughts in the following remarks.

Remark 1. Simplicity.

The results we obtained above are quite simple, after being established. It also might look like the aggregation result (2.5) can be obtained even in a simpler way: without assumption on equal prices and without revenue optimization—just by differentiating and summing up the individual production functions of each DMU. This is misleading, however, and would work only for a *single*-output case. In a multi-output case, the question of allocation of outputs arises and so the output prices must enter the equations and optimization with respect to them must be involved. (In fact, even in a single output case, the revenue optimization is also imposed implicitly, because each DMU is assumed to be on its frontier, i.e., technologically efficient.)

Remark 2. Ad hoc vs. theory-justified aggregation.

As we mentioned above, in some empirical studies, researchers summarize their results on cross-DMU growth accounting by presenting the simple (equally-weighted) average of the growth rates. This statistic is certainly useful as an estimator of the population mean of the distribution of growth rates. However, because the growth rates are normalized quantities, such that they ignore the size of the unit that generated it, the simple average might give distorted picture of the reality, captured better by the weighted average. For this reason other studies used the weighted average, with weights being the total output (e.g., GDP) shares, yet they were chosen in an ad hoc way, without providing any theoretical justification for why those weights and not others are used. The derivations above, yielding equation (2.5), now clearly outline what assumptions underlie or/and theoretically justify these ‘common sense’ weights and this is one of the major value added points of this work.

Remark 3. Key Assumptions Driving the Result.

So what are these assumptions that justify the ‘natural’ aggregation scheme? The first assumption is that *the aggregate technology structure must satisfy (2.1)*. This structure shall not be confused with the union of the production sets, and shall be interpreted as the summation of production possibilities of different DMUs *without* allowing for reallocation of inputs across the DMUs in the group (e.g., no labor migration or capital flow is allowed). Such structure is often used in international trade models. The second assumption is that *all DMUs face the same output prices* (e.g., world or equilibrium prices). The third assumption is that each DMU individually and as a group of them exhibit *revenue maximizing behavior*. Thus, anyone who would use the aggregation scheme (2.5), however intuitive it may sound, must explicitly or implicitly accept these three assumptions, whether they like them or not, or show that other assumptions would imply this or other aggregation scheme to be used.

Remark 4. Domar-weights

Interestingly, similar weighting scheme has been suggested by Domar (1961)—for the context of aggregation across industries, however it was derived under different and stricter assumptions, as well as

using different methods than ours. In particular, our approach does not impose any assumption of the form of the production or on the revenue function.

Remark 5. The Law of One Price

The assumption on equal prices across DMUs might be viewed as fairly strict, and we absolutely agree with it. Yet, it is the necessary assumption to achieve (2.5) and there is no way around it. Perhaps a balm on a wound can become the fact that it is not the first time that *positive* aggregation results in economics requires some additional, often strong and perhaps undesirable assumptions (e.g., the reader can recall assumptions needed for the aggregation of demands over consumers or over goods). Although strict, this type of ‘Law of One Price’ assumption is fairly common (although debated) in empirical cross-DMU analysis.⁵ One could think of this assumption here as an attempt to get a hypothetical (e.g., based on average prices that approximate equilibrium prices) benchmark, to be able to use a justifiable aggregation scheme that accounts for economic importance of each DMU in the sample. In the macroeconomic context, this result also justifies the use of the purchasing-power-parity (PPP) adjustment for the GDP data across countries, which is often used in cross-country empirical growth studies already.

Remark 6. Continuous vs. Discrete timing I.

Note that the derivations of the key growth accounting expressions (1.3) and (2.4) are done conditioned on prices fixed in the period t . While such conditioning might be reasonable for the infinitesimal changes in time, practical implementation would normally involve discrete data, e.g., with 1-year change, and so validity of such assumption might become questionable. A way out in such a situation might be to resort to the index number theory to obtain relevant real quantities, in the spirit of Laspeyres, Paaschee, or the average of the two, such as Fisher, Tornqvist or Walsh approaches.

Remark 7. Continuous vs. Discrete timing II.

⁵ E.g., see Kuosmanen et al. (2006) for more detailed discussion on the issue of ‘Law of One Price’ in economics.

Note that the aggregation weights we arrived at in (2.5) also refer to the particular time period t . This result was derived in the continuous time framework, while in practice, when the actual estimation of the total output growth and its contributions is made, researchers take the discrete time framework and then one should choose the weights in t or the weights in $t-1$ and they might differ substantially. Similarly as in the Remark 6 above, as a reconciliation step, one might consider taking the average of period t and $t-1$ weights, but certain consistency aggregation property (as discussed below) will be lost.

Remark 8. Additivity of Output Sets

As mentioned in the Remark 3, the assumption (2.1) on the aggregate technology, implying that no factor mobility is allowed between the DMUs, is a critical assumption to arrive to the results presented in this paper. Aggregation scheme based on this assumption (2.1), in general, implies that it would not give the same growth rate as the growth rate of aggregated unit (i.e., of a hypothetical DMU constructed as the sum of all individual DMUs in the group)—because the latter presumes possibility of reallocation across DMUs. If assumption (2.1) is relaxed to allow some or any reallocation between DMUs then a different aggregation structure (function or/and weights) might emerge. For example, one might define the aggregate or group technology as the summation of individual technology sets (defined as a set of (x,y) such that x can produce y) rather than the summation of individual output sets. Following this route, Nesterenko and Zelenyuk (2007), using results from Li and Ng (1995), relaxed the no-reallocation assumption in the context of aggregation of efficiency scores and arrived to harmonic aggregation structure. It is possible that similar result would emerge in the context of aggregating growth rates, but we have not reached solutions here yet and leave it for further work.

Remark 9. Freedom in Functional Assumptions

Observe that the aggregation solution derived here not only does not impose a particular functional form for the production or revenue function but also allows for totally different technologies underlying those functions. In particular, it allows for the (partial) scale elasticities of each input to be different for each DMU (firm, country, etc.) to be aggregated over.

Remark 10. Relationship to Malmquist Productivity index

Although we leave it for future work to relate and compare the aggregation frameworks developed here and the one developed for the MPIs by Zelenyuk (2006), a few hints might be in order. First, observe that the aggregation for MPIs was developed also using similar (Koopmans-type) theorem, yet it then involved somewhat different algebra. Specifically, aggregation solution for MPI proposed by Zelenyuk (2006) was component-wise, with aggregating individual revenue functions and decomposing them into distance functions and allocative inefficiency, within each component and with no differentiation involved. Thus, the final aggregation result for MPI had an additive aggregation structure placed inside the multiplicative structure of the MPI. As a result, one could hardly establish a direct link of the aggregate MPI to the aggregate growth accounting results obtained here, yet a first order approximation relationship, under certain conditions on technology, could be possible and we leave it for explorations in another paper.

Remark 11. Weighting the Components

It might be also worth looking closer again at equation (2.6). Specifically, note that (2.6) makes it clear that the contribution to change due to each input also must be weighted by the weights based on total output shares, rather than, for example, by weights based on the respective input shares, which some researchers might be tempted to use.⁶ Similar emphasis can be made about (2.7).

4. Aggregation across Sub-groups into a Larger Group

We now consider the case of aggregation of growth contributions *across* all (or several) *sub-groups* into a larger group. Let $X_t = (x_t^1, \dots, x_t^n)$ and assume that the aggregate output set of the entire group is the sum of output sets of all of its sub-groups (or all of its DMUs, due to (2.1)), i.e.,

$$\bar{P}_t(X_t) \equiv \bar{P}_t(X_t^1, \dots, X_t^L) \equiv \sum_{l=1}^L \bar{P}_t^l(X_t^l) = \sum_{k=1}^n P_t^k(x_t^k), \quad (4.1)$$

Then, we can obtain (for the proof see Appendix A) an extension of (2.4) saying that,

⁶ We thank anonymous referee for noting about the importance of this result.

$$F_t(X_t | p_t) = \sum_{l=1}^L F_t^l(X_t^l | p_t). \quad (4.2)$$

Hence, the growth rate of the total output of the *entire* (or larger) group, denoted with $G(Q_t)$, is

$$G(Q_t) \equiv \frac{dF_t(X_t | p_t) / dt}{Q_t} = \sum_{k=1}^n \sum_{i=1}^N E_{i,t}^k \cdot g(x_{i,t}^k) + \frac{\partial F_t / \partial t}{Q_t} \equiv \sum_{i=1}^N G_{i,t} + G(F_t) \quad (4.3)$$

and it can now be obtained from the L sub-group estimates, as

$$G(Q_t) = \sum_{l=1}^L G(Q_t^l) S_t^l, \quad S_t^l \equiv \frac{Q_t^l}{Q_t}, \quad l \in \{1, \dots, L\}. \quad (4.4)$$

Similarly, it follows that contributions from each input and technology changes to the growth of the total output of the entire group can be obtained by aggregating the corresponding L sub-group analogues, as

$$G_{i,t} = \sum_{l=1}^L G_{i,t}^l S_t^l, \quad \text{and} \quad G_{f,t} = \sum_{l=1}^L G_{f,t}^l S_t^l, \quad l \in \{1, \dots, L\}. \quad (4.5)$$

Intuitively, the growth rate of the entire group (and its sources) can be obtained via the weighted sum of the growth rates of *all sub-groups* within this entire group, where the weights are the total output shares of each sub-group in the entire group, derived from the economic optimization behavior.

Thus, from expressions (4.3)-(4.5) we see that we have internally consistent aggregation: across smaller units into distinct sub-groups of them and then across these sub-groups into larger distinct groups and so on. This is a very desirable property of aggregation. One could think of it as aggregation of growth rates of firms into growth rates of sub-industries, then further aggregation into growth rates of industries, and further into growth rate of GDP of a country and so on. Importantly, note that this property will be lost if one takes the average of period t and $t-1$ (or $t+1$) weights (e.g., for the sake of ‘smoothing’ the discreteness of data) instead of t -period weights. In other words, to keep the aggregation consistency property, it might be desirable to rest on the t -period weights, as derived in all the theoretical results above. Therefore, because the data on the real total output often comes as

deflated by the Laspeyres-type price index, whose dual is the Paasche quantity index, it might be better to choose the *current* (i.e., Paasche) time period for weighting of the (quantity) growth rates.

5. Empirical Illustration

The goal of this section is to make a small illustration of how our method works for a real data set, to contrast its results to those obtained from the simple average approach, and to illustrate how big the differences can be not only quantitatively, but, more importantly, in qualitative conclusions.

While the aggregation methods we considered in this paper can be adapted to different levels of economic activity—firm level, industry level, country level, etc.—in this section we present an illustration for a macroeconomic level data set, as it might be interesting for a wide audience, and an application for other levels, e.g., some particular industries, would be similar. In particular, we use the data from the recent, yet frequently cited study of Kumar and Russell (2002) that contains information on capital, labor and GDP of 57 countries in the world for 1965 and 1990 (originally extracted from Penn World Tables). In addition, we also split the sample and obtain results for two groups: OECD and non-OECD countries, as it was by 1965.

Table 1. Results of Simple vs. Weighted Aggregation in Growth Accounting

Categories	Aggregation method	Estimate of Aggregate Growth and its Sources			
		Overall GDP	due to Labor	due to Capital	due to TFP
All	Simple	0.960	0.319	0.446	0.195
	Weighted	0.934	0.244	0.464	0.225
Non-OECD	Simple	1.016	0.405	0.456	0.156
	Weighted	1.204	0.377	0.517	0.310
OECD	Simple	0.863	0.172	0.429	0.261
	Weighted	0.847	0.201	0.447	0.198

Notes: Based on calculation of author using data from Kumar and Russell (2002).

In general, the factor shares of each input (or partial scale elasticities) needed to compute the sources of GDP growth are likely to vary across countries, depending on the level of development, countries' endowments, their institutions, etc. In practice, such information is rarely available for many countries, and so we follow one common practice for empirical growth accounting literature by taking these shares as 1/3 for capital and 2/3 for labor, for all the countries (e.g., see Mankiw et al, 1992). As the main scenario (presented here) we used the t -period (1990) weights, to secure aggregation consistency between the subgroups. Table 1 summarizes the results, while detailed estimates are given in Appendix B. (We also tried the average weights and the results were qualitatively the same and, in fact, very similar quantitatively, yet they lacked the aggregation consistency property, as we discussed earlier, in the Remark 7.)

The general look at the Table 1 tells us that the difference between the two methods can indeed be substantial, implying conclusions that can be different both quantitatively and qualitatively. For example, the World aggregate growth in GDP due to change in labor is estimated to be 32% according to the simple average, while it is only 24% according to the weighted aggregation. Also note that the simple average also underestimated the GDP growth due to capital change and due to change in TFP.

Looking at the non-OECD countries we see that its aggregate GDP growth over 1965-1990 period was about 102% according to the simple average. On the other hand, when we account for economic weights of each country in its group, this number rises to 120%. Interestingly, when we use the simple average, then it appears that most of the GDP growth is due to change in capital and change in labor (45% and 40%, respectively), while only 16% is left for the TFP change. However, under the weighted aggregation, the picture we get is quite different: the capital change had much more pronounced impact on aggregate growth of GDP of non-OECD countries than the labor change (52% vs. 38%). Moreover, much (twice!) bigger impact is reported for the TFP growth (31% instead of 16%) of the OECD countries. Note that these two conclusions of the weighted aggregation are very important and quite logical from the economic perspective, since most of the non-OECD countries

were labor abundant, while short on capital and on advanced technologies, before 1965, and those who managed to increase their capital stock and improved technologies had the biggest contribution to their economic growth. The highlights of such examples were Hong Kong, South Korea and Taiwan. Such facts must be adequately accounted in the aggregate growth rates and simple averaging that puts equal weights to all members of the group, misses this point. For example, in the weighted aggregation, the economic weight of South Korea was 7.8% in its group, while in the simple average it was only 2.8% and so its enormous growth in GDP due to capital deepening (92%) and due to technology improvement (94%) were quite undervalued in the simple averaging of the growth rates for its group (see details on estimates for this and other countries in Appendix B).

Furthermore, for the OECD group, the simple averaging overestimated the GDP growth due to change in TFP (26%) relative to what our weighted aggregation suggests (20%) accounting for economic weight of each member of the group. Notably, for the OECD group, both the simple and the weighted aggregations showed similar results for the GDP growth due to changes in labor and capital stocks. This is likely to be due to the fact that OECD group is more homogeneous than the non-OECD countries, especially in terms of growth over the studied period.

Another very interesting observation from Table 1 is that if we rely on the simple averaging, then we conclude that the economic growth of the non-OECD group was larger than that of the OECD group by about 16 percentage points. However, if we rely on the aggregation that accounts for the economic weight of each country, then we see that the difference is about 36 percentage points. This is quite a substantial difference.

We also see qualitatively different judgments when compare contributions of TFP (technology change) onto the economic growth of the two groups: the simple averaging suggests that for non-OECD group it was only 16%, which is substantially smaller than that of the OECD group, as much as 26%. On the other hand, the weighted aggregation suggests exactly the opposite conclusion—that it

was substantially lower for the OECD group (20%) than it was for the non-OECD group (31%). Thus, we again get not only quantitatively large difference, but also qualitatively opposite conclusions.

In summary, this small empirical illustration showed that if one accounts for the economic weight of each DMU in estimating the aggregate rates of the total output growth and of its sources, then researchers might be led into conclusions that are not only quantitatively but also qualitatively very different from conclusions based on the commonly used simple average. Such different measurement results can naturally lead to very different policy implications and recommendations. This is not to say that the simple mean should not be used—it should, as an estimator of the first moment of the distribution. Yet, we strongly suggest that, along with the use of the simple mean, it is certainly worth presenting the weighted aggregation results, with weights that account for economic importance of each disaggregated unit in the sample, especially for non-homogeneous samples.

Importantly, because results can radically depend on the chosen weights, one should try to choose weights that have some theoretical grounds or justification as well as know what they mean. We think that the weighting scheme, for which we provided theoretical justification, is a good candidate—because it is not *ad hoc* but derived based on clear theoretical foundation involving optimization behavior of economic agents, production relationships and assuming a reasonable structure on aggregate technology and prices.

6. Concluding Remarks

In this paper we obtained a simple but important result. We have derived theoretical grounds for aggregation across DMUs (firms, industries, countries, group of countries, etc.) of growth rates and their decompositions, within the growth accounting framework. The derivation is based on assuming optimization behavior and additive aggregation structure on technologies.

Using a real data set in empirical application of our method, we illustrated how dramatically different could be both quantitative and qualitative conclusions obtained from the simple averaging approach vs. the weighted aggregation approach for which we justified theoretical grounds.

This paper is just the first layer of theoretical foundation for analyzing aggregate growth rates. The next layer shall be the statistical foundation. Indeed, besides presenting an average of the data, researchers are also interested in some measures of spread of the sample, such as the standard deviation, coefficient of variation, interquartile range, etc., as well as in the possibility to use some statistical testing procedures for inferring on various hypotheses. A hypothesis of an interest, for example, might be whether the true growth rate in the total output (or of any of its contributions) for a particular *sub-group* is different from zero or not. Another hypothesis of interest might be whether the growth rates in the total output (or of any of its contributions) are equal *across* different sub-groups or not, or across time for the same *sub-groups*, etc. For the commonly used non-weighted sample mean this issue is somewhat standard and this is perhaps one of the key reasons why it is so popular in practice. On the other hand, estimating characteristics of the sampling distribution of a statistic for a *weighted* mean and related testing is a more tricky issue. One potential solution is to adapt the bootstrapping techniques proposed by Efron (1979). This is a subject in itself, in some way similar to the recent work of Simar and Zelenyuk (2007) for bootstrapping the aggregate efficiency scores obtained via the data envelopment analysis method (Charnes et al., (1978)), and we leave it for further research.

Another natural extension to the present work would be to allow for factor mobility between DMUs in the technology aggregation structure and one of the ways to do this is to adopt approach of Nesterenko and Zelenyuk (2007) to the case of economic growth accounting.

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APPENDIX A.

Proof of (2.4).⁷ The entire group of DMUs is partitioned into L (non-intersecting) *sub-groups* (indexed by $l = 1, \dots, L$) using some exogenous criterion with number of DMUs in each group l equal to a positive integer n_l . Recall that $x_t^{l,k} = (x_{t,1}^{l,k}, \dots, x_{t,N}^{l,k})' \in \mathfrak{R}_+^N$ and $y_t^{l,k} = (y_{t,1}^{l,k}, \dots, y_{t,M}^{l,k})' \in \mathfrak{R}_+^M$ are input and output vectors, respectively, of a particular DMU k ($k = 1, \dots, n_l$) in sub-group l (in period t). Input allocation over the entire group of n DMUs is denoted with a (N by n) matrix $X_t = (x_t^1, \dots, x_t^n)$. The input allocation among DMUs *within* a group l will be denoted by a (N by n_l) matrix $X_t^l = (x_t^{l,1}, \dots, x_t^{l,n_l})$. In general, technology of a particular DMU k ($k = 1, \dots, n_l$) within a group l is assumed to be characterized by the output sets

$$P_t^{l,k}(x_t^{l,k}) \equiv \{y_t : \text{all } y_t \in \mathfrak{R}_+^M \text{ producible from } x_t^{l,k} \in \mathfrak{R}_+^N\}, \quad x_t^{l,k} \in \mathfrak{R}_+^N. \quad (\text{A1})$$

Technology of a particular *sub-group* l is assumed to be related to technologies of its DMUs as

$$\bar{P}_t^l(X_t^l) \equiv \bar{P}_t^l(x_t^{l,1}, \dots, x_t^{l,n_l}) \equiv \sum_{k=1}^{n_l} P_t^{l,k}(x_t^{l,k}), \quad l = 1, \dots, L. \quad (\text{A2})$$

which yields one of the main aggregation results we stated in the text as (2.4):

$$F_t^l(X_t^l | p_t) = \sum_{k=1}^{n_l} f_t^{l,k}(x_t^{l,k} | p_t), \quad x_t^{l,k} \in \mathfrak{R}_+^N, \quad p_t \in \mathfrak{R}_+^M \quad (\text{A3})$$

where

$$f_t^{l,k}(x_t^{l,k} | p_t) \equiv \max_y \{p_t y : y \in P_t^{l,k}(x_t^{l,k})\} \quad (\text{A4})$$

and

$$Q_t^l \equiv F_t^l(X_t^l | p_t) \equiv \max_y \{p_t y : y \in \bar{P}_t^l(X_t^l)\} \quad (\text{A5})$$

To prove this, for each $k = 1, \dots, n_l$, take $y_t^{l,k}$ to be an arbitrary vector in $P_t^{l,k}(x_t^{l,k})$ and use them to define $\bar{Y}_t^l \equiv \sum_{k=1}^{n_l} y_t^{l,k}$. Because of (A2) we have $\bar{Y}_t^l \in \bar{P}_t^l(X_t^l)$, and due to (A5), we obtain

$$p_t \bar{Y}_t^l \leq F_t^l(X_t^l | p_t) \quad (\text{A6})$$

Since $y_t^{l,k}$ is an *arbitrary* vector in $P_t^{l,k}(x_t^{l,k})$, it implies that (A6) also holds for those $y_t^{l,k}$ that solve (A4), call them $\tilde{y}_t^{l,k}$. Then, let $\tilde{Y}_t^l \equiv \sum_{k=1}^{n_l} \tilde{y}_t^{l,k}$, in which case we would have

$$p_t \tilde{Y}_t^l \equiv \sum_{k=1}^{n_l} p_t \tilde{y}_t^{l,k} = \sum_{k=1}^{n_l} f_t^{l,k}(x_t^{l,k} | p_t) \leq F_t^l(X_t^l | p_t) \quad (\text{A7})$$

⁷ This proof is adapted from Zelenyuk (2006) and is an extension of proofs from Fare and Zelenyuk (2003) and Simar and Zelenyuk (2007), which are in turn an elaboration of theorem from Koopmans (1957).

On the other hand, let \bar{Y}_t^l be an arbitrary vector in $\bar{P}_t^l(X_t^l)$, then due to (A2) there exist $y_t^{l,k} \in P_t^{l,k}(x_t^{l,k})$, for each $k = 1, \dots, n_t$, such that $\bar{Y}_t^l = \sum_{k=1}^{n_t} y_t^{l,k}$. Therefore, due to (A4) we have

$$p_t \bar{Y}_t^{l,k} \equiv \sum_{k=1}^{n_t} p_t y_t^{l,k} \leq \sum_{k=1}^{n_t} f_t^{l,k}(x_t^{l,k} | p_t), \quad (\text{A8})$$

and since \bar{Y}_t^l is an *arbitrary* vector in $\bar{P}_t^l(X_t^l)$, expression (A8) is also true for those \bar{Y}_t^l that solve (A5), call them \tilde{Y}_t^l , in which case we get

$$p_t \tilde{Y}_t^l \equiv F_t^l(X_t^l | p_t) \leq \sum_{k=1}^{n_t} f_t^{l,k}(x_t^{l,k} | p_t) \quad (\text{A9})$$

Clearly, expressions (A7) and (A9) can simultaneously hold if and only if

$$F_t^l(X_t^l | p_t) = \sum_{k=1}^{n_t} f_t^{l,k}(x_t^{l,k} | p_t). \quad \text{Q.E.D.}$$

One immediate implication from this conclusion is that if $L = 1$, i.e., the subgroup is the entire group (thus indexing with l can be dropped, e.g., $n_t = n$), then

$$F_t(X_t | p_t) = \sum_{k=1}^n f_t^k(x_t^k | p_t), \quad (\text{A10})$$

where

$$F_t(X_t | p_t) \equiv \max_y \{p_t y : y \in \bar{P}_t(X_t)\} \quad (\text{A11})$$

and

$$\bar{P}_t(X_t) = \sum_{l=1}^L \sum_{k=1}^{n_t} P_t^{l,k}(x_t^{l,k}) \quad (\text{A12})$$

Another important implication is about the relationship between the maximal revenues of the subgroups to the maximal revenue of the entire group. In particular, since

$$\sum_{l=1}^L F_t^l(X_t^l | p_t) = \sum_{l=1}^L \sum_{k=1}^{n_t} f_t^{l,k}(x_t^{l,k} | p_t) \quad (\text{A13})$$

thus along with (A10) we get the result that we use in (4.2), namely that

$$F(X_t | p_t) = \sum_{l=1}^L F_t^l(X_t^l | p_t). \quad (\text{A14})$$

which insures 'internally consistent' aggregation *within* and *between* the subgroups.

APPENDIX B: Data for Empirical Illustration

Table B1. Estimates from Growth Accounting for Kumar and Russell (2002) data

COUNTRY	In OECD by 65	GDP Growth	Due to Growth In Labor	Due to Growth In Capital	Due to Growth in TFP
ARGENTINA	0	0,31	0,18	0,32	-0,19
AUSTRALIA	0	0,90	0,36	0,39	0,15
AUSTRIA	1	0,79	0,08	0,52	0,18
BELGIUM	1	0,73	0,10	0,34	0,29
BOLIVIA	0	0,83	0,37	0,44	0,03
CANADA	1	0,98	0,36	0,46	0,15
CHILE	0	0,71	0,37	0,40	-0,06
COLOMBIA	0	1,16	0,42	0,42	0,32
DENMARK	1	0,58	0,17	0,37	0,05
DOMINICAN REP.	0	1,15	0,49	0,68	-0,02
ECUADOR	0	1,24	0,43	0,55	0,26
FINLAND	0	0,86	0,12	0,39	0,34
FRANCE	1	0,80	0,14	0,43	0,22
GERMANY, WEST	1	0,67	0,09	0,44	0,14
GREECE	1	0,96	0,09	0,47	0,41
GUATEMALA	0	0,89	0,42	0,39	0,07
HONDURAS	0	1,01	0,53	0,41	0,07
HONG KONG	0	1,91	0,44	0,43	1,04
ICELAND	1	1,04	0,35	0,52	0,16
INDIA	0	1,08	0,32	0,46	0,29
IRELAND	1	1,04	0,13	0,44	0,47
ISRAEL	0	1,29	0,45	0,41	0,43
ITALY	1	0,89	0,08	0,36	0,46
IVORY COAST	0	0,81	0,45	0,66	-0,29
JAMAICA	0	0,49	0,35	0,17	-0,03
JAPAN	1	1,38	0,17	0,73	0,48
KENYA	0	1,32	0,68	0,28	0,36
KOREA, REP. S.	0	2,25	0,40	0,92	0,94
LUXEMBOURG	1	0,78	0,13	0,28	0,37
MADAGASCAR	0	0,17	0,35	0,23	-0,41
MALAWI	0	1,00	0,42	0,69	-0,11
MAURITIUS	0	0,99	0,36	0,40	0,23
MEXICO	0	1,15	0,51	0,50	0,14
MOROCCO	0	1,18	0,50	0,35	0,33
NETHERLANDS	1	0,76	0,23	0,38	0,15
NEW ZEALAND	0	0,49	0,28	0,34	-0,13
NIGERIA	0	0,96	0,42	0,54	0,01
NORWAY	1	0,89	0,24	0,20	0,45
PANAMA	0	0,96	0,45	0,52	-0,01
PARAGUAY	0	1,23	0,50	0,75	-0,01
PERU	0	0,49	0,44	0,32	-0,28
PHILIPPINES	0	0,99	0,42	0,33	0,24
PORTUGAL	1	1,21	0,15	0,49	0,57
SIERRA LEONE	0	0,30	0,24	0,52	-0,46
SPAIN	1	0,93	0,12	0,57	0,24
SRI LANKA	0	1,00	0,30	0,41	0,29
SWEDEN	1	0,55	0,16	0,39	0,00

SWITZERLAND	1	0,53	0,13	0,36	0,04
SYRIA	0	1,48	0,50	0,40	0,59
TAIWAN	0	2,06	0,42	0,93	0,71
THAILAND	0	1,75	0,45	0,76	0,54
TURKEY	1	1,31	0,32	0,55	0,44
U.K.	1	0,60	0,08	0,34	0,18
U.S.A.	1	0,69	0,28	0,37	0,04
YUGOSLAVIA	0	0,84	0,14	0,49	0,22
ZAMBIA	0	0,33	0,49	0,06	-0,23
ZIMBABWE	0	1,03	0,62	0,18	0,23

Notes: This estimation was done on data from Kumar and Russell (2002), Henderson and Russell (2005) and Henderson and Zelenyuk (2006), originally extracted from Penn World Tables, and generously given to the author by Daniel Henderson.