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A Method to Construct World Tables of Purchasing Power Parities and Real Incomes Based on Multiple Benchmarks and Auxiliary Information: Analytical and Empirical Results

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An Econometric Approach to Construct World Tables of Purchasing Power Parities and Real Incomes: Analytical Properties and Tables for 1970-2005

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Abstract

The paper presents a new method for the construction of a consistent panel of Purchasing Power Parities (*PPPs*), and real incomes, using a single step econometric framework that combines all the available information of *PPPs* for countries and over time. The method improves upon the current practice used in the construction of the Penn World Tables, PWT, and similar tables produced by the World Bank. Like its predecessors, it combines *PPPs* for benchmark years constructed by the International Comparison Program (ICP) with *PPP* predictions from a model of the national price level (or exchange rate deviation index) for all countries and years. The method also uses data on price movements available from national sources. The approach ensures the model's prediction of the *PPP* series for the reference country is identically equal to one in all time periods and predictions are invariant to the choice of reference country. The smoothed *PPP* predictions (and standard errors) obtained through the state-space representation of the model are produced for both ICP- participating and non-participating countries and non-benchmark years. A number of analytical results to highlight some of the properties and flexibility of the method are presented. The empirical illustration shows the general model can produce variants that: a) result in *PPP* predictions that accurately track the available ICP's *PPPs* (benchmarks); or b) preserve the growth rates in price levels implicit in individual countries' national accounts data. A data set for 141 countries for the period 1970 to 2005 is used to illustrate the flexibility of the method and to compare its performance to PWT6.2.

Keywords Purchasing Power parities, Penn World Tables, State-space models, Spatial autocorrelation, Kalman Filter

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1 Introduction

Over the last four decades, there has been a consensus that market exchange rates are not suitable for converting economic aggregate data from different countries expressed in respective national currency units[1]. Instead, purchasing power parities (*PPPs*) of currencies which measure price level differences across countries are widely used for purposes of converting nominal aggregates into real terms.[2] *PPP*-converted real per capita incomes are used in influential publications like the World Development Indicators of the World Bank ([Worrs]) and the Human Development Report ([UND06]) which publishes values of the Human Development Index (HDI) for all countries in the world. The *PPPs* are also used in a variety of areas including: the study of global and regional inequality ([Mil02]); measurement of regional and global poverty using international poverty lines like \$1/day and \$2/day (regularly published in the World Development Indicators, World Bank); the study of convergence and issues surrounding carbon emissions and climate change ([MS05]; [CH03]); and in the study of catch-up and convergence in real incomes ([BS04]; [DJT05]; [Sal02]).

What are the main sources of *PPP* data? The only source for *PPPs* for the economy as a whole is the International comparison Program (ICP). The *PPP* data are compiled under the ICP which began as a major research project by Kravis and his associates at the University of Pennsylvania in 1968 and in more recent years has been conducted under the auspices of the UN Statistical Commission. Due to the complex nature of the project and the underlying resource requirements, it has been conducted roughly every five years since 1970. ICP comparisons are known as *benchmarks* and thus the term is used subsequently without further explanation. The latest round of the ICP for the 2005 benchmark year was released in early 2008. The final results are available on the World Bank website: `http://siteresources.worldbank.org/ICPINT/Resources/ICP_final-results.pdf`. In the more recent years, beginning from early 1990's, the OECD and EUROSTAT have been compiling *PPPs* roughly every three years.

The country coverage of the ICP in the past benchmarks has been limited with 64 countries participating in the 1996 benchmark comparisons. However this coverage has increased dramatically to 147 for the 2005 benchmark year[3]. Details of the history of the ICP and its coverage are well documented in the recent report of the Asian Development Bank ([ADB05]). However, international organizations such as the World Bank and the United Nations, as well as economists and researchers, seek *PPP* data for countries not covered by the ICP and also for the non-benchmark years. For most analytical and policy purposes, there is a need for *PPPs* covering all the countries and a three to four-decade period[4]. The Penn World Tables has been the main source of such data. Summers and Heston are pioneers in this field, and [SH91] provides a clear description of the construction of the earlier versions of the Penn World Tables. The most recent version, PWT 6.2, available on `http://pwt.econ.upenn.edu`, covers 170 countries and a period in excess of five decades starting from 1950. In addition to the PWT, there is the real gross domestic product (GDP) series constructed by Angus Maddison ([Mad95, Mad07]). The Maddison series, available on the Groningen Growth and Development Centre website: `www.ggdc.net/dseries/totecon.html`, makes use of a single benchmark and national growth rates to construct panel data of real GDP and no estimates are available for non-ICP participating countries (the term non-benchmark countries is also used). The World Bank also constructs *PPPs* series that are available in various issues of World Development Indicators publication. The World Bank series are based on the methodology described in [Ahm96] and the construction of the series makes use of a single

[1]For a detailed discussion of the issues relating to the use of exchange rates, the reader is referred to [KSH82] as well as the ICP Handbook available on the World Bank website. In addition the most recent publication from the Asian Development Bank on the 2005 comparisons in the Asia Pacific ([ADB05]) also provides an in-depth discussion on the use of exchange rates and purchasing power parities.

[2]Nominal values refer to aggregates expressed in national currency units, and, in contrast, real aggregates are obtained by converting nominal values using *PPPs*. These are termed "real' since the use of *PPPs* eliminates price level differences.

[3]It covers the People's Republic of China for the first time and India participated in 2005 after its last participation in 1985.

[4]For example, the Human Development Index is computed and published on an annual basis. Similarly, the World Development Indicators publication provides *PPP* converted real per capita incomes for all the countries in the world for every year.

benchmark year for which extrapolations to non-benchmark countries are derived using a regression-based approach. The benchmark and non-benchmark *PPPs* are extrapolated using national growth rates in national prices[5].

The construction of the PWT essentially uses a two-step method: (i) extrapolation of to non-benchmark countries in an ICP benchmark year using ICP benchmark *PPP* data (normally from the most recent available exercise) and national level data through the use of cross-sectional regressions; and (ii) extrapolation to non-benchmark years. The second step combines information from step (i) with GDP deflators from national accounts data, to produce the tables. Details of the PWT methodology can be found in [SH91] and [HSA06].[6]

There are several important issues associated with the PWT methodology. First and foremost is the problem of time-space consistency of the data produced from different benchmarks. It is quite clear that a set of time-space comparisons can be derived using *PPPs* from just one benchmark year and that such comparisons are not invariant to this choice. For example, the use of 1990 benchmark data may result in one set of tables and the use of 1996 or 1999 may result in a very different set of tables of *PPPs*, real incomes and other aggregates. In solving this problem, the *PPP* data from the most recent benchmark comparison from the ICP is taken as the preferred starting point and the extrapolations across space and over time are derived using country-specific growth rates. This choice of a single benchmark to construct PWT means that a large body of data from other benchmarks are not utilised[7]. Even when attempts are made to make use of the information from several benchmarks, no clear methodology for combining information from different benchmarks is currently available which results in a related problem associated with the use of PWT and other available series, ie the absence of any measures of reliability such as standard errors. Most researchers using PWT data consider them to be similar to data from national accounts or other national or international sources. There is no general recognition that the data presented in the Penn World Tables are indeed based on predictions from regression models and that they are also projections over time. Thus, the PWT data should be treated and used as predictions with appropriate standard errors. Though the PWT data provide an indication of the quality of data for different countries, there are no quantitative indicators of reliability in terms of confidence intervals for predictions.

The main objective of the paper is to propose a new method that adequately addresses problems associated with the PWT and other sources of extrapolated *PPPs*. In particular, the method allows the use of data on *PPPs* from all the past benchmarks along with data available from national sources on price movements in the form of national price deflators and socio-economic indicators available through international sources. The new method is designed to make efficient use of all the available information in obtaining optimal predictors of *PPPs* for all the countries and time periods. In addition, standard errors associated with the extrapolated *PPPs* can be derived using the approach. The econometric model and the state-space formulation used are designed to generate predictions of *PPPs* over time and across countries that are broadly consistent with the benchmark data on *PPPs* and the observed country-specific temporal movements in prices. The method is flexible enough for emphasis to be placed on either tracking benchmarks or tracking the observed national price movements accurately. We present formal proofs of some of the properties of the method.

The structure of the paper is as follows. Section 2 discusses the underlying economic theory and associated measurements used to form the econometric model. Section 3 presents an econometric formulation of the problem. Section 4 states the special properties and features of the proposed methodology. This section demonstrates the flexibility and generality of the method proposed in the paper. Section 5 outlines the estimation procedure and the Kalman filter/smoother used in producing the predictions of *PPPs*. Section 6 presents selected results from the empirical implementation of the methodology proposed to 141 countries for the period 1971 - 2005. The paper is concluded with some remarks in Section 7. A set of appendices showing mathematical proofs of some of the

[5]We define "growth rates in national prices" in the next section.

[6]A description of the early attempts to construct consistent panels of *PPPs* can be found in [SH88].

[7]Use is made of data from the earlier benchmark years for countries which are not in the latest benchmark but have participated in earlier benchmark comparisons.

analytical properties discussed in Section 4 are also included.

2 The economic model and sources of measurement

The econometric methodology proposed in the paper is designed to make optimal use of all the information available for the purpose of constructing a panel of *PPPs*. The variable of interest will be denoted by $p_{it} = ln(PPP_{it})$ for country $i = 1, ..., N$ and time $t = 1, ..., T$ where PPP_{it} represents the purchasing power parity of the currency of country i with respect to a reference country currency. Although it is directly unobservable, we can identify four noisy sources of information that can be combined to obtain an optimal prediction[8], p^*_{it}. They are: theory of national price levels used in predicting *PPPs*, derived growth rates in national prices that can be used in updating *PPP* information, *PPPs* from ICP benchmark exercises, and a constraint used for the reference country identification. We discuss each source in turn and formally develop an econometric model in the next section.

2.1 A model derived using the theory of national price levels

There is considerable literature focusing on the problem of explaining the national price levels. If ER_{it} denotes the exchange rate of currency of country i at time t, then the national price level for country i (also referred to as the exchange rate deviation index) is defined as the ratio:

$$R_{it} = \frac{PPP_{it}}{ER_{it}} \tag{1}$$

For example, if the *PPP* and *ER* for Japan, with respect to one US dollar, are 155 and 80 yen respectively, then the price level in Japan is 1.94 indicating that prices in Japan are roughly double those in the United States. A value of this ratio greater than one implies national price levels in excess of international levels and vice versa.

Most of the explanations of price levels are based on productivity differences in traded and non-traded goods across developed and developing countries. Much of the early literature explaining national price levels ([KL83]; [HSA06]) has relied on the structural characteristics of countries such as the level of economic development, resource endowments, foreign trade ratios, education levels. More recent literature has focused on measures like openness of the economy, size of the service sector reflecting the size of the non-tradable sector and on the nature and extent of any barriers to free trade ([Ahm96]; [Ber91, Ber96]; [Cla88]).

It has been found that for most developed countries the price levels are around unity and for most developing countries these ratios are usually well below unity. In general it is possible to identify a vector of regressor variables and postulate a regression relationship:

$$r_{it} = \beta_{0t} + \boldsymbol{x}'_{it}\boldsymbol{\beta}_s + u_{it} \tag{2}$$

where,

$r_{it} = ln(PPP_{it}/ER_{it})$

$\boldsymbol{x}'_{it}$ is a set of conditioning variables

β_{0t} intercept parameter

$\boldsymbol{\beta}_s$ a vector of slope parameters

u_{it} a random disturbance with specific distributional characteristics.

Equation (2) can be made more general by allowing heterogeneity in the slope parameters, although we do not pursue this route in this paper.

[8] We return to the optimality of the prediction in Section 3.2

Provided estimates of β_{0it} and $\boldsymbol{\beta}_s$ are available, model (2) can provide a prediction of the variable of interest consistent with price level theory.

$$\hat{p}_{it} = \hat{\beta}_{0t} + \boldsymbol{x}'_{it}\hat{\boldsymbol{\beta}}_s + ln(ER_{it}) \tag{3}$$

where,
$\hat{p}_{it}$ is a prediction
$\hat{\beta}_{0t}$ and $\hat{\boldsymbol{\beta}}_s$ are estimates.
We will return to the estimation of β_{0it} and $\boldsymbol{\beta}_{sit}$ in Section 5.

2.2 The derived growth rates of PPPs

The movements in national price level, PPP_{it}/ER_{it}, can be measured through the gross domestic product deflator (or the GDP deflator) for period t relative to period $t-1$ and through exchange rate movements. This is due to the fact that *PPPs* from the ICP refer to the whole GDP. GDP deflators are used to measure changes in *PPP* and the national price level. If the US dollar is used as the reference currency to measure *PPPs* and exchange rates, the *PPP* of country i in period t can be expressed as:

$$PPP_{i,t} = PPP_{i,t-1} \times \frac{GDPDef_{i,[t-1,t]}}{GDPDef_{US,[t-1,t]}} \tag{4}$$

Equation (4) defines the growth rate of PPP_{it}.[9] GDP deflators are computed from national accounts. The availability of resources to national statistical offices is likely to be positively related to the level of resources (technical and human) available in individual countries. Thus, we assume growth rates are measured with error. Taking the logarithm of (4) and accounting for the measurement error:

$$p_{it} = p_{i,t-1} + c_{it} + \eta_{it} \tag{5}$$

where,
$c_{it} = ln\left(\frac{GDPDef_{i,[t-1,t]}}{GDPDef_{US,[t-1,t]}}\right)$
η_{it} is a random error accounting for measurement error in the growth rates

2.3 *PPPs* computed by the ICP for each benchmark year.

Due to the complexity in the design and collection of the ICP benchmark data (see Chapters 4-6 of the ICP Handbook which can be found on the World Bank ICP website:`www.worldbank.org/data/ICP`), the observed *PPPs* are likely to be contaminated with some measurement error. As the surveys for these benchmark exercises are conducted by national statistical offices, the argument made above in relation to measurement errors applies here also. Thus, ICP benchmark observations are given by

$$\tilde{p}_{it} = p_{it} + \xi_{it} \tag{6}$$

where,
$\tilde{p}_{it}$ is the ICP benchmark observation for participating country i at time t
ξ_{it} is a random error accounting for measurement error and $E(\eta_{it}\xi_{it}) = 0$

[9] Equation (4) simply updates *PPPs* using movements in the GDP deflator of the country concerned. Equation (4) would be a simple identity if *PPPs* were based on the price of a single commodity. However in the case of *PPPs* at the GDP level, the same argument holds if GDP is treated as a composite commodity.

2.4 Reference Country Definition

The definition of *PPP* requires a choice of reference country. The reference country is defined to have a *PPP* of one for all time periods.[10] Thus, we know the value of the variable of interest for the reference country for all time periods. As the USA is taken as the reference country, it then follows that for all t

$$p_{US,t} = 0 \tag{7}$$

In the next section we provide an econometric model that is designed to take into account all the information described in this section.

3 Econometric formulation of the problem

The objective is to produce a panel of predictions of p_{it} (denoted by p_{it}^*) which optimally uses all relevant available data accompanied by standard errors, and is internally consistent in a sense to be defined subsequently.

As a matter of notation, for any quantity a_{it} we define the $N-$vector $\boldsymbol{a}_t$ as $\boldsymbol{a}_t = (a_{1t}, a_{2t}, \ldots, a_{Nt})'$. This notation will be used throughout without further definition. Matrices will be defined in upper case and bold face.

3.1 Assumptions

1. The errors u_{it} in the regression relationship (2) are assumed to be spatially correlated. We assume an error structure of the form

$$\mathbf{u}_t = \phi \mathbf{W}_t \mathbf{u}_t + \mathbf{e}_t \tag{8}$$

   where $\phi < 1$ and $\mathbf{W}_t$ $(N \times N)$ is a spatial weights matrix. That is, its rows add up to one and the diagonal elements are zero. The term spatial in the present context refers to economic distance rather than the traditional geographical distance[11]. It follows that $\mathrm{E}(\mathbf{u}_t\mathbf{u}_t')$ is proportional to $\boldsymbol{\Omega}$, where $\boldsymbol{\Omega}= (\mathbf{I}-\phi\mathbf{W}_t)^{-1} (\mathbf{I}-\phi\mathbf{W}_t)^{-1\,\prime}$.

2. The measurement errors in the observation of $ln(PPP_{it})$ during benchmark years, equation (6), are assumed spatially uncorrelated, but might be heteroskedastic. Thus, if ξ_{it} is a measurement error associated with country i at time t,

   then

$$\begin{aligned} E(\xi_{it}) &= 0 \\ E(\xi_{it}^2) &= \sigma_\xi^2 V_{it} \end{aligned} \tag{9}$$

   where σ_ξ^2 is a constant of proportionality and $\boldsymbol{V}_t$ is defined below.

3. The measurement error in the growth rates are assumed spatially uncorrelated, but might be heteroskedastic. Thus, η_{it} in (5) is assumed

$$\begin{aligned} E(\eta_{it}) &= 0 \\ E\left(\eta_{it}^2\right) &= \sigma_\eta^2 V_{it} \end{aligned} \tag{10}$$

[10] The benchmark *PPPs* between currencies of two countries are invariant to the choice of the base country. In the current study, we use the US dollar as the reference currency which, in turn, gives equation (7). The method proposed here is invariant to the choice of the reference currency (see Section 4.5).

[11] In the empirical section we test for cross-sectional dependence and specify a model of economic distance to obtain the weights.

where σ_η^2 is a constant of proportionality and $\boldsymbol{V}_t$ is defined below.

4. The measurement error variance-covariance is of the form

$$\boldsymbol{V}_t = \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \sigma_{1t}^2 \boldsymbol{j}\boldsymbol{j}' + diag(\sigma_{2t}^2, ..., \sigma_{Nt}^2) \end{bmatrix} \tag{11}$$

where, σ_{it}^2 is the variance of country i at time t, which we measure as the inverse of the a country's degree of development,[12] and σ_{1t}^2 is the variance of the reference country.

This form of the covariance was derived from theory (see [RRD09]) and it is sufficient for the invariance of the method to the choice of reference country (see Section 4 and Appendix 3 for details).

3.2 The Econometric Model and a State Space Representation

The econometric problem is one of signal extraction. That is, we need to combine all sources of "noisy" information and extract the signal from the noise. A state-space (SS) is a suitable representation for this type of problem. We start by extending equation (5) to define the 'transition equation' of the SS:

$$\mathbf{p}_t = \mathbf{p}_{t-1} + \mathbf{c}_t + \boldsymbol{\eta}_t \tag{12}$$

where,
$\boldsymbol{c}_t$ is the observed growth rate of $\boldsymbol{p}_t$ (see equation (2) in Section 2.2)
$\boldsymbol{\eta}_t$ is an error with $E(\boldsymbol{\eta}_t) = 0$ and $E\left(\boldsymbol{\eta}_t \boldsymbol{\eta}_t'\right) \equiv \mathbf{Q}_t = \sigma_\eta^2 \mathbf{V}_t$

Equation (12) simply updates period $t-1$ *PPPs* using the observed price changes over the period represented by $\boldsymbol{c}_t$.

As previously discussed, noisy observations of $\mathbf{p}_t$ are given by (3), a prediction from the regression model, $\hat{\mathbf{p}}_t$, and (6) a measurement by the ICP, $\tilde{\mathbf{p}}_t$. Equations (2) and (3) relate the conditioning variables, $\boldsymbol{X}_t$, to the price level ratio. Since the form of the observation equation of a SS model relates the observations ($\hat{\mathbf{p}}_t$, $\tilde{\mathbf{p}}_t$) to the state vector $\mathbf{p}_t$, it is convenient to re-write equation (2):

$$p_{it} = \beta_{0it} + \boldsymbol{x}_{it}' \beta_{sit} + ln(ER_{it}) + u_{it}$$

and subtracting equation (3) we obtain:

$$\hat{p}_{it} = p_{it} + (\hat{\beta}_{0it} - \beta_{0it}) + \boldsymbol{x}_{it}'(\hat{\beta}_{sit} - \beta_{sit}) - u_{it} \tag{13}$$

Throughout the paper we will reserve the symbol $\boldsymbol{\theta}$ to represent the error in a current estimate of a parameter $\boldsymbol{\beta}$.

Thus,

$$\hat{\theta}_{0it} = \hat{\beta}_{0it} - \beta_{0it} \text{ and } \hat{\theta}_{sit} = \hat{\beta}_{sit} \text{-} \beta_{sit} \tag{14}$$

It is always possible to write equation (13) in the form

[12] The reader is referred to [RRD09] for the theoretical derivation of this covariance strucutre. In the empirical implementation we model σ_{it}^2 as inversely related to GDP_{it} per capita measured in \$US (exchange rates adjusted). This means that reliability of an observed *PPP* or growth rate is lower for low-income countries.

$$\hat{\mathbf{p}}_t = \mathbf{p}_t + \mathbf{X}_t\boldsymbol{\theta} + \mathbf{v}_t \tag{15}$$

where,

$\boldsymbol{\theta} = [\theta_1', ..., \theta_T']'$

$v_{it} = -u_{it}$

Because the explicit form of $\mathbf{X}_t$ depends on the particular identifying restrictions imposed on β_{0it} and $\boldsymbol{\beta}_{sit}$, we will define it later in the context of a particular case. Finally, in order to express these different types of observations (viz, those given by (6) and (15)) as a single equation, it is convenient to define four 'selection matrices',

$\mathbf{S}_1 = [1, \mathbf{0}'_{N-1}]$ (selects the reference country $i = 1$)[13]

$\mathbf{S}_p$ is a known matrix which selects the N_t participating countries (excluding the reference country) in the benchmark year t.

$\mathbf{S}_{np}$ is a known matrix which selects $(N - 1 - N_t)$ non-participating countries in the benchmark year t.

We are now able to consolidate these sources of information into a single equation on an 'observation vector' $\mathbf{y}_t$, viz

$$\mathbf{y}_t = \mathbf{Z}_t\mathbf{p}_t + \mathbf{B}_t\mathbf{X}_t\boldsymbol{\theta} + \boldsymbol{\zeta}_t \tag{16}$$

with variables defined as follows:

i) Non-benchmark years:

$$\mathbf{y}_t = \begin{bmatrix} 0 \\ \mathbf{S}_{\mathrm{np}}\hat{\mathbf{p}}_t \end{bmatrix}, \mathbf{Z}_t = \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{S}_{\mathrm{np}} \end{bmatrix}, \mathbf{B}_t = \begin{bmatrix} 0 \\ \mathbf{S}_{\mathrm{np}} \end{bmatrix}, \boldsymbol{\zeta}_t = \begin{bmatrix} 0 \\ \mathbf{S}_{\mathrm{np}}\mathbf{v}_t \end{bmatrix} \tag{17}$$

$$\mathrm{E}(\boldsymbol{\zeta}_t\boldsymbol{\zeta}_t') \equiv \mathbf{H}_t = \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \sigma_u^2 \boldsymbol{S}_{np}\boldsymbol{\Omega}\boldsymbol{S}'_{np} \end{bmatrix} \tag{18}$$

with σ_u^2 a constant of proportionality, and in (18) the countries are ordered so that the reference country is the first row[14]

ii) Benchmark years

$$\mathbf{y}_t = \begin{bmatrix} 0 \\ \mathbf{S}_{np}\hat{\mathbf{p}}_t \\ \tilde{\mathbf{p}}_t \end{bmatrix}, \mathbf{Z}_t = \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{S}_{np} \\ \mathbf{S}_p \end{bmatrix}, \mathbf{B}_t = \begin{bmatrix} 0 \\ \mathbf{S}_{np} \\ \mathbf{0} \end{bmatrix}, \boldsymbol{\zeta}_t = \begin{bmatrix} 0 \\ \mathbf{S}_{np}\mathbf{v}_t \\ \mathbf{S}_p\xi_t \end{bmatrix} \tag{19}$$

$$E\left(\boldsymbol{\zeta}_t\boldsymbol{\zeta}_t'\right) \equiv \mathbf{H}_t = \begin{bmatrix} 0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sigma_u^2\boldsymbol{S}_{np}\boldsymbol{\Omega}\boldsymbol{S}'_{np} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \sigma_\xi^2\boldsymbol{S}_p\boldsymbol{V}_t\boldsymbol{S}'_p \end{bmatrix} \tag{20}$$

$\tilde{\mathbf{p}}_t$ is an $N_t \times 1$ vector of benchmark observations.

$\hat{\mathbf{p}}_t$ is an $N \times 1$ vector of regression predictions for all countries[15].

Again, σ_{u}^2 and σ_ξ^2 are constants of proportionality and the first row is the reference country.

Equations (12) and (16), together with the matrix definitions (17) to (20), constitute the 'transition' and 'observation' equations, respectively of a state space model for the unobservable 'state vector', $\mathbf{p}_t$.

[13]Without loss of generality country 1 is the reference country.

[14]The inclusion of the reference country constraint is a necessary condition for invariance of the results to the chosen reference country.

[15]For invariance to hold it is necessary that the observation for participating countries in benchmark years be the ICP benchmark observations. The estimation of $\boldsymbol{\theta}$, to produce $\hat{\boldsymbol{p}}_t$, is based on all N countries in the sample. See Appendixes 3 and 4 for details.

Given the unknown parameters, $\boldsymbol{\theta}$, ϕ, σ_u^2, σ_η^2, σ_ξ^2 and the distribution of the initial vector, $\mathbf{p}_0$, under Gaussian assumptions, the Kalman filter computes the conditional (on the information available at time t) mean $\check{\boldsymbol{p}}_t$, and covariance matrix, $\boldsymbol{\Psi}_t$, of the distribution of $\mathbf{p}_t$. Further, $\check{\boldsymbol{p}}_t$ is a minimum mean square estimator (MMSE) of the state vector, $\mathbf{p}_t$. When Gaussian assumptions are dropped, the Kalman filter is still the optimal estimator in the sense that it minimizes the mean square error within the class of all linear estimators (see [Har89], pp. 100-12, [DK01] Sections 4.2 and 4.3).

4 Special Features and Properties of the Method

The state-space model formulated in Section 3 is a flexible model that can easily accommodate a number of common approaches to the production of $PPPs$. We demonstrate how the model can be specialized to ensure that the predicted $PPPs$ equal the observed ICP $PPPs$ for the benchmark years or that the movements in the implicit GDP deflator are preserved. We also provide analytical results that show the constructed series are invariant to the reference country and they are weighted averages of previous benchmark observations.

4.1 Constraining the model to track benchmark $PPPs$

As $PPPs$ for currencies of the ICP participating countries are determined using price data collected from extensive price surveys, one may consider it necessary that the predicted $PPPs$ from the state-space model described above track these benchmark $PPPs$ accurately. This can be achieved simply by setting $\sigma_\xi^2 = 0$ in (20). The last line in (19) then becomes a constraint, guaranteeing that predicted $PPPs$ are identical to the corresponding benchmark observations. This particular property of Kalman filter predictions follows from the results presented in [**?**].

4.2 Constraining the model to preserve movements in the implicit GDP deflator

A standard requirement considered in international comparisons of prices is that $PPPs$ in different years preserve the movements in national price levels as measured by the implicit GDP deflators. As the GDP deflator data are provided by the countries and such deflators are compiled using extensive country-specific data, it is considered important that the estimated $PPPs$ preserve the observed growth rates implicit in the GDP deflator[16]. This essential feature can be achieved by setting $\sigma_\eta^2 = 0$ in (12) (see also Section 2.2). This result is proved in Appendix 1 for the Kalman Smoother and its application is then demonstrated in the empirical section.

4.3 Flexibility in the use of regression predictions

An important feature of the model is that the information provided by relevant socio-economic variables can be utilized in all time periods, both benchmark and non-benchmark through the regressors $\mathbf{x}'_{it}$ in (2). If we wish to produce estimates that use only growth rates between benchmark years, the second line of equation (17) is removed. The algorithm will then automatically update predictions between benchmarks using only growth rates in deflators. We present an illustration of the results obtained under this simplified model in Section 6.

4.4 Kalman Filter predictions as 'weighted averages' of benchmark year only predictions

As mentioned earlier, current methodology for the estimation of a panel of $PPPs$ is a two step procedure. First, in a benchmark year, observations on participating countries are obtained and then used to extrapolate to non-

[16]Preserving movements in the implicit deflator will ensure that the growth rates in GDP at constant prices (real) and growth in per capita income reported and used at the country level are preserved in the international comparisons.

participating countries through regression relationships. Thus, in benchmark years predictions for the whole cross-section are obtained. The second step consists of completing the panels by using growth rates obtainable from national accounts.

If there are $M+1$ benchmark years $(j=0,\ldots,M)$[17], applying growth rates to benchmark $PPPs$ will produce $M+1$ different panels of PPP estimates. Faced with the dilemma of which panel to use, two possible approaches (of many) would be to: (a) use the panel based on the most recent benchmark year; or (b) to take some sort of average of the $M+1$ different panels.

An important property of our method is that in the case that benchmark year estimates and growth rates are used, but no information is introduced for years in between benchmark years, the panel of PPP estimates produced is a 'weighted average' of the $M+1$ panels discussed above. More specifically, suppose $\overleftrightarrow{\boldsymbol{p}}_{t,j}$ is the vector of PPP estimates in year t obtained by applying growth rates to the jth benchmark. Then, denoting the corresponding Kalman Filter estimates by $\breve{\boldsymbol{p}}_t$, we have

$$\breve{\boldsymbol{p}}_t = \sum_{j=0}^{M} \Upsilon_j^{(M)} \overleftrightarrow{\boldsymbol{p}}_{t,j} \tag{21}$$

where the $\Upsilon_j^{(M)}$ are positive definite matrices, and

$$\sum_{j=0}^{M} \Upsilon_j^{(M)} = \mathbf{I}_N \tag{22}$$

It is in this sense the prediction in (21) is considered as a 'weighted average' although it is not generally true that the elements of $\breve{\boldsymbol{p}}_t$ are a weighted average of those of the $\overleftrightarrow{\boldsymbol{p}}_{t,j}$. The elements of $\breve{\boldsymbol{p}}_t$ are a weighted average of the corresponding elements of the $M+1$ 'benchmark only' panels if the measurement errors in growth rates and benchmark $PPPs$ are uncorrelated across countries. Then, it can be shown that

$$\breve{\boldsymbol{p}}_t = \sum_{j=0}^{M} \upsilon_{ii,j}^{(M)} \overleftrightarrow{\boldsymbol{p}}_{t,j} \tag{23}$$

where, $\upsilon_{ii,j}^{(M)} \cdots i^{th} \cdots \upsilon_j^{(M)}$, $\upsilon_{ii,j}^{(M)} > 0$ and $\sum\limits_{j=0}^{M} \upsilon_{ii,j}^{(M)} = 1$.

The above result demonstrates that the Kalman filter estimates are a weighted average of all the corresponding elements of the $M+1$ panels. Furthermore, the weights are not chosen in some arbitrary way, but derived from the covariance properties of the model. Details of the derivation of the above property appear in the appendix of [RRD09][18]

4.5 Invariance of the Estimated PPPs to the Choice of Reference country

An important property of our method is that it is invariant to the choice of which country is used as the reference country. That is, if we denote by $\breve{\boldsymbol{p}}_t^{(1)}$ the Kalman Filter estimates when the reference country is $i=1$ (e.g. the USA), and by $\breve{\boldsymbol{p}}_t^{(2)}$ the Kalman Filter estimates when the reference country is $i=2$ (e.g. the UK); then

$$\breve{p}_{it}^{(2)} = \breve{p}_{it}^{(1)} - \breve{p}_{2t}^{(1)} \tag{24}$$

The proof is presented in Appendix 3.

[17] It will be convenient for the algebraic derivations presented shortly to set the number of benchmarks to $M+1$.

[18] The proof is repeated for reference in Appendix 2.

5 Estimation

In order for the Kalman filter to deliver a predictor of the state vector and its covariance matrix, we require estimates of the unknown parameters and a distribution of the initial state vector. The estimation of the parameters of a state-space system can be handled with likelihood based methods ([Har89], pp. 125-46) or Bayesian methods (see for instance [DK01], [KvD00], and [HTvD05]). The results presented in this paper are obtained using likelihood based methods. The distribution of the initial state vector, $\mathbf{p}_o$, is assumed to be centered at zero and its covariance has been derived as follows.

5.1 Distribution of the Initial State Vector

For this specification we can derive a non-diffuse covariance for the initial state vector, $\mathbf{p}_o$, by making use of equation (3). Suppose at $t = 0$ we have socio-economic data, $\mathbf{X}_o$. Then we can define,

$$\mathbf{p}_o = \mathbf{X}_o\boldsymbol{\beta} + ln(\mathbf{ER}) + \mathbf{u}_o \tag{25}$$

where,

$\boldsymbol{\beta} = [\beta_{oo}\boldsymbol{\beta}'_{so}]$

$\mathbf{p}_o = \begin{bmatrix} \mathbf{p}_o^{(1)} \\ \mathbf{p}_o^{(2)} \end{bmatrix}$

$\mathbf{X}_o = \begin{bmatrix} \mathbf{X}_o^{(1)} \\ \mathbf{X}_o^{(2)} \end{bmatrix}$

$\mathbf{X}_o^{(1)}$ and $\mathbf{p}_o^{(1)}$ represent the partition containing the observations from participating countries.

Then a prediction of $\mathbf{p}_o$ and its associated covariance are given by

$$\hat{\mathbf{p}}_o = \mathbf{X}_o\hat{\boldsymbol{\beta}} + ln(\mathbf{ER}) \tag{26}$$

$$\mathrm{cov}(\hat{\mathbf{p}}_o) = \boldsymbol{\Psi}_o = \hat{\sigma}^2\mathbf{X}_o(\mathbf{X}_o^{(1)\prime}\mathbf{X}_o^{(1)})^{-1}\mathbf{X}'_o \tag{27}$$

We use the expression in (27) to obtain an estimate of the covariance of the initial state vector in the empirical section. We note that under normality of the disturbances, the conditional distribution of the observation vector $\mathbf{y}_t$ is given directly by the Kalman filter[19] (we refer the reader to [Har89] for details).

5.2 Algorithm

There are two types of parameters to be estimated in the state-space, namely, hyperparameters, and coefficients associated with explanatory variables. Hyperparameters are those associated with the covariance structure. In our case these are:ϕ, σ_u^2 , σ_η^2 , σ_ξ^2. These parameters must be estimated by numerical maximization of the likelihood function (in a likelihood based estimation). The other parameters, $\boldsymbol{\theta}$, in our case, can be estimated by a generalised least squares procedure (GLS) in conjunction with the numerical maximization of the likelihood function, which we denote by KF/GLS as it involves running the Kalman filter through both $\mathbf{y}_t$ and the columns of $\mathbf{X}_t$ (see [Har89], pp. 130-3 and Appendix 4 for details)[20].

The complete algorithm consists of an estimation component and a smoothing component as follows:

Estimation Algorithm:

[19] Therefore, by writing the log likelihood in prediction error decomposition form a pass through the KF allows the computation of a value of the likelihood function.

[20] The code for the empirical estimations was written by the authors in GAUSS and includes a procedure to evaluate the likelihood function when some of the parameters are obtained by the KF/GLS approach (see Appendix 4 for details).

Step 1: Obtain an initial estimate of $\boldsymbol{\beta}$, $\hat{\boldsymbol{\beta}}^0$, by regressing $\boldsymbol{r}_t$ on $\boldsymbol{X}_t$, see equation (2), and construct an initial prediction, $\hat{\boldsymbol{p}}_{it}^0$, using equation (3). These initial predictions are based on an OLS estimation and do not take into account the spatial structure of the errors.

Step 2: Given starting values for ϕ, σ_u^2, σ_η^2, σ_ξ^2, a Newton-Raphson iterative procedure is used to maximise the likelihood function. The KF/GLS procedure is built into the computation of the likelihood function, so that at each iteration all parameters are updated. This procedure uses data for all countries including the reference country which insures invariance results hold (see Appendix 3). Upon convergence, a set of MLE estimates of ϕ, σ_u^2 , σ_η^2, σ_ξ^2, and $\boldsymbol{\theta}$ are obtained. These updated estimates account for the spatial correlation structure of the errors through the KF/GLS estimation of $\boldsymbol{\theta}$.

Step 3: Use updated estimates, $\hat{\boldsymbol{\theta}}$, to find $\hat{\beta}_{0t} = \hat{\beta}_{0t}^0 - \hat{\theta}_0$ $\hat{\boldsymbol{\beta}}_s = \hat{\boldsymbol{\beta}}_t^0 - \hat{\theta}$ and obtain an updated $\hat{p}_{it} = \hat{\beta}_{ot} + \mathbf{x}_{it}'\hat{\boldsymbol{\beta}}_s + \ln ER_{it} + \hat{u}_{it}^*$, where $\hat{\mathbf{u}}_t^* = \hat{\phi}\mathbf{W}_t\hat{\mathbf{u}}_t$. For invariance to hold the predictions require adjustment by subtracting the base country's prediction, $\hat{p}_{it}^{adjusted} = \hat{p}_{it} - \hat{p}_{1t}$ (see Appendix 3, Section A3.2 for details).

Step 4: Repeat 2 and 3 until the change in the estimates of $\hat{\beta}_{0t}$ and $\hat{\boldsymbol{\beta}}_s$ between iterations are sufficiently close to zero.

Smoothing Algorithm:

Given the parameter estimates obtained from Steps 1 to 4, the sample is run through the equations of the Kalman Filter and Kalman smoother to obtain the model's predicted p_{it} (for all i and t), p_{it}^* , and associated standard errors.

A prediction of PPP_{it} is given by:

$$P\hat{P}P_{it} = e^{p_{it}^*} \tag{28}$$

where,

p_{it}^* is the corresponding Kalman smoothed element.

The standard errors for the predicted *PPPs* are computed as follows[21]:

$$se(P\hat{P}P_{it}) = \sqrt{e^{2p_{it}^*}e^{\psi_{ii,t}^*}(e^{\psi_{ii,t}^*} - 1)} \tag{29}$$

where,

$\psi_{ii,t}^*$ is the *ith* diagonal element of the estimated smoothed covariance of the state vector, $\boldsymbol{\Psi}_t^*$.

6 Empirical Results

In this section we present different alternative estimates obtained by constraining some of the parameters of the model. We also present the estimates obtained from the model when 2005 is assumed not to be a benchmark year. This allows us to compare our estimates to those of PWT 6.2 which is based on benchmarks up to the 2002 OECD/EUROSTAT comparison. The aim is to illustrate the flexibility of the method as stated in Sections 4.1 - 4.1 as well as provide an empirical comparison of our method to the well estabished PWT.

6.1 Data compilation and data construction

This section describes the data set used in this study. The data set covers 141 countries over the years 1970 to 2005. Detailed description of the data used are available in [RRD08] as follows: Appendix Table DA.1 lists the 141 countries included in the study. This table also lists the currency of each country and the years each country has participated in the ICP Benchmark comparisons. The empirical analysis in this paper includes ICP *PPP* data

[21]The standard errors are computed under the assumption of the log-normality of the predictions.

from the 2005 round. Out of the 141 included countries, 110 are amongst the 147 countries that participated in the 2005 ICP round. That is, there are 31 countries in our data set that did not participate in the 2005 ICP. Appendix Table DA.2 gives definitions and sources of the variables used in the study, while Table DA.3 provides some basic descriptive statistics of the variables. The dimensions of the data set were largely determined by data availability. That is, a number of countries were excluded because of missing data (see the notes for Table DA.1), and the time frame 1970-2005 was likewise chosen because of poor data availability prior to 1970. Many variables which were initially considered for the analysis were also excluded due to data unavailability.

6.1.1 PPP Data

The state variable in the state space model is $ln(PPP_{it})$, and observed values (which define the dependent variable in the measurement equation) are obtained from all the benchmarks conducted so far. Thus PPP data are drawn from the early benchmarks of 1975, 1980 and 1985 as well as from more recent benchmark information for the years 1990, 1993, 1996, 1999, 2002 and 2005. Several features of the *PPP* data are noteworthy. The first benchmark covered 13 countries. The 1980, 1985 and the recent 2005, benchmarks represent truly global comparisons with *PPPs* computed using data for all the participating countries. For the years beginning from 1990 to 2002, data are essentially from the OECD and EU comparisons with the exception of 1996[22]. The 1996 benchmark year again is a global comparison with $PPPs$ for countries from all the regions of the world. However, the 1996 benchmark may be considered weaker than the 1980, 1985 and 2005 benchmark comparisons as no systematic linking of regional $PPPs$ was undertaken. In terms of reliability, one would consider the 1996 benchmark *PPPs* to be less reliable. Another related point of interest is the fact that *PPPs* for all the benchmarks prior to 1990 were based on the Geary-Khamis method and *PPPs* for the more recent years are all based on the EKS method of aggregation.[23]In the current empirical analysis, we have not made any adjustments to the *PPP* data but making the series comparable through the use of the same aggregation methodology is part of our ongoing research programme.

6.1.2 Socio-Economic Variables included in the Regression

Table DA.2 includes a description of the socio-economic variables that are included in the regressions in the study. The reader is refereed to [RRD09] for a more detailed discussion on the specification of the price level regression model used. The variables used come under two categories. We use a set of variables that are essentially dummy variables designed to capture country-specific episodes that may influence the exchange rates or *PPPs* or both as well as time dummies. The second set of variables are more of a structural nature commonly discussed in the works of [KL83], [HSA06], [Cla88], [Ber91, Ber96] and [Ahm96].

6.1.3 Covariance Variables

Measuring spatial correlation:

The spatial weights matrix, $\boldsymbol{W}_t$, used in modeling the spatial error structure (see equation (8)) is derived from a measure of economic distance constructed for this project. The measure is constructed by extracting a common factor (through principal components analysis) for each country, using a model that combines trade closeness, geographical proximity, and cultural closeness. We present a brief description of its construction next. The reader is refereed to [RRG09b] and [RRG09a] for a comparison and sensitivity of the results to three alternative spatial model specifications. The measure used in this paper is that with the lowest in- and out-of-sample prediction error.

[22]We are indebted to Ms Francette Koechlin (OECD) for providing ICP benchmark data for these years. *PPPs* for those countries which joined in the Euro zone, the pre-Euro domestic currencies were converted using the 1999 Irrevocable Conversion Rates (Source: `http://www.ecb.int/press/date/1998/html/pr981231_2.en.html`). The irrevocable conversion rate of the drachma vis a vis the euro was set at GRD 340.750. Source: `http://www.bankofgreece.gr/en/euro`.

[23]This was brought to our attention by Steve Dowrick who attended a seminar on the topic presented at the Australian National University in October 2007.

Variables included in the measure of economic distance

- *Trade closeness* is measured as the percentage of bilateral trade between each country and all others in the sample (compiled using data from [Ros04] and IMF Trade Directions).
- *Geographical proximity* is measured by a series of dummies for *border* (both land and sea proximity), and *regional membership* (such us Asia pacific region, Europe, south America, north and central America, sub Saharan Africa, middle east). The data were constructed using Atlas, CIA factbook and individual country references.
- *Cultural and colonial closeness* dummies are used for *common language* and *common colonial history.* The data were constructed from the CIA factbook and individual country references.

Construction of the distance score

The objective is to measure "an economic distance" between pairs of countries. The steps involved in the estimation can be summarised as follows:

1) A separate principal components (PC) model was estimated for each country to measure the distance between the respective country and each of the other countries in the sample. Therefore, for each time period 141 models are estimated. The analysis was conducted for the years 1970, 1975, 1980, 1985, 1990, 1995, 2000 and 2005 to account for the changing paterns in bilateral trade over time.

2) After the PC are extracted for a particular country and time period the first PC is retained since the number of variables is small. This is the estimated *common factor* for each country and time period.

3) A factor score is computed using the estimated factor loadings and the data. These scores are not bounded; therefore, they are rescaled to prepare the a *proximity matrix* using the formula:

$$S_{jg} = \left[\frac{f_{jg} - f_{\min}}{f_{\max} - f_{\min}} \right]$$

where, $f_{\min}$, $f_{\max}$ and f_{jg} are respectively the minimum value, maximum value and factor score of country g in relation to j. These rescaled factor scores are in the range of 0 to 1, and if country g and j are the same (e.g. $g = j = 1$), the rescaled value is zero.

The distance or proximity score is assumed to be constant within the five yearly intervals (e.g. from 1970 to 1974, 1975 to 1979, and so on).

Construction of the Weights Matrix

The proximity matrix is transformed into a row stochastic matrix $\boldsymbol{W}_t$ (i.e. rows add up to one) by simply dividing each proximity score within a row (which represents a country) by the sum of that row, and thus creating weights.

The relative perfomance of the above specification of $\boldsymbol{W}_t$ against other alternative spatial weight matrices (including no spatial errors) within the context of this model has been studied by [RRG09a]. The reader is also referred to this work for a more detailed exposition of the construction of the proximity matrix.

Accuracy of benchmarks and national accounts' growth rates:

The specification allows for the modeling of the accuracy of benchmark $PPPs$ and national growth rates (equations (5) and (6)). We assume that the measurement errors in both cases have variances that are inversely proportional to the per capita GDP expressed in US dollars. This means that countries with higher per capita incomes are expected to have more reliable data, as reflected by lower variances associated with them.[24]

[24] We make use of exchange rate converted per capita incomes to overcome the problem of possible endogeniety arising out of the use

6.2 Empirical Evidence

In this section we present the testing for cross-sectional dependence as well as alternative estimates and *PPPs* predictions obtained by constraining some of the parameters of the model. Estimates obtained from the model when 2005 is assumed not to be a benchmark year are also presented. The later allows the comparision of our predictions to those of PWT 6.2 which is based on benchmarks up to the 2002 OECD/EUROSTAT comparison. We first present the computed test for the residuals of the price level regression and estimated models obtained following the estimation algorithm outlined in Section 5.1. A tableau of *PPPs* is obtained by runing the smoothing algorithm (see Section 5.1) covering all 141 countries and the period 1971-2005. The *PPP* series for five countries in the sample are presented in detail to illustrate the method.

6.2.1 Cross-Section Dependence Testing and Parameter Estimates

The price level model of equation (2) is an unbalanced panel with fixed time effects. The available data to test the residuals of this model correspond to those years when there has been either an ICP or OECD/EUROSTAT benchmark comparison (that is, 1975, 1980, 1985, 1990, 1993, 1996, 1999, 2002 and 2005 in our sample). The sample is very unbalanced as the number of countries participating in 1975 was very small (thirteen), there are only three global comparisons (1980, 1985 and 2005), and most countries in the world participated for the first time in the 2005 round. Testing for spatial dependence requires the specification of a spatial model (that is $H_o : \phi = 0$ in eq. (8)), and therefore it is dependent on the specification of the spatial weights matrix, $\boldsymbol{W}_t$. An alternative strategy is to use a robust test for cross-sectional dependence, such as that proposed by [Pes04] which does not require the specification of a spatial model. The test is based on simple averages of all pair-wise correlation coefficients of the OLS residuals from the individual regressions in the panel. For the case of unbalanced panels the CD test takes the following form (the reader is referred to Section 9 of [Pes04] for more details):

$$\hat{\rho}_{ij} = \frac{\sum_{m \in T_i \cap T_j} (\tilde{u}_{im} - \bar{u}_i)(\tilde{u}_{jm} - \bar{u}_j)}{\left[\sum_{m \in T_i \cap T_j} (\tilde{u}_{im} - \bar{u}_i)^2\right]^{1/2} \left[\sum_{m \in T_i \cap T_j} (\tilde{u}_{jm} - \bar{u}_j)^2\right]^{1/2}} \tag{30}$$

where,

$\hat{\rho}_{ij}$ correlation coefficient between country i and j.

T_i set of benchmark years where country i has participated in the ICP.

$\tilde{u}_{im}$OLS residual for country i in benchmark year m.

$\bar{u}_i = \dfrac{\sum_{m \in T_i \cap T_j} \tilde{u}_{im}}{\mathrm{T}_{ij}}$

T_{ij} is the number of common data points in $T_i \cap T_j$

The CD statistic for the unbalanced panel is given by

$$CD = \sqrt{\frac{2}{N(N-1)}} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \sqrt{\mathrm{T}_{ij}} \hat{\rho}_{ij} \right) \tag{31}$$

Under the null hypothesis of no cross sectional dependence, $CD \sim N(0,1)$. The computed value of the CD test is -100.9, and therefore the null hypothesis is rejected at all levels of significance[25].

Table 1 presents a series of estimated models that are then used to construct the *PPPs* series for all 141 countries. Five set of estimates are presented. Panel 1 is the national price level regression model (equation (3)) estimated by least squares assuming non-spatial errors. The estimates include the sample of participating countries for all

of PPP converted exchange rates. These data are drawn from UN sources. Given the systematic nature of the exchange rate deviation index (ratio of PPP to ER), use of exchange rate converted per capita GDP is likely to magnify differences in per capita incomes.

[25] LM tests for spatial correlation were computed for three alternative specifictions of the weight matrix and results can be found in [RRG09b]

available benchmarks since 1975 to 2005, and it includes intercept dummies for each benchmark year. This model is used to produce the initial predictions to start the estimation algorithm (refer to Section 5.1).

Panel 2 are the estimates from the state-space model produced without restrictions to any of the parameters in the model. The only constrain in the system is the reference country constrain. The regression component of the system is assumed to have spatial errors. The estimate of the spatial parameter is 0.59 and it is statistically significant. The covariance proportionality parameters associated with the error in the growth rates, regression predictions and ICP benchmarks are estimated to be 6.6, 4.5, and 0.8, respectively, and they are all statistically significant. Using these estimates, the *PPP* series obtained from the Kalman filter are not constrained to track ICP benchmarks or growth rates. The *PPP* predictions from this model will be labeled with the postfix "UN."

Panel 3 is the state-space model estimates obtained by setting the parameter that controls the error in ICP benchmarks to zero, i.e. $\sigma_{\eta}^{2}=0$. The maximization of the likelihood is subject to this restriction. The spatial parameter as well as the parameters associated with errors in the growth rates and regression predictions are statistically significant. The log-likelihood of this model is lower than that of the model in Panel 2. These can be compared by a likelihood ratio test as the model in Panel 3 is a restriction of the model in Panel 2. The computed LR value is significantly different from zero and therefore the restriction that benchmarks do not suffer from measurement error is rejected. The Kalman filtered series produced by this model are therefore constrained to track ICP benchmarks. In the next section we will show the Kalman smoothed estimates produced from this model. The *PPP* predictions from this model will be labeled with the postfix "CON."

Panel 4 is a simpler form of the general model in Panel 2 in that in non-benchmark years no regression prediction information is used (refer to equations (17) and (18)). The regression predictions are used in benchmark years for non-participating countries (see equations (19) and (20)). For the years between benchmarks the only information included is the temporal movement through the transition equation (12). By this design, the model's estimates are weighted averages of the observed ICP benchmarks for countries that participated in all benchmarks, a weighted average of the combination of the ICP benchmarks and regression predictions for countries that only participated in some of the ICP benchmarks, or a weighted average of the regression predictions from the national price level model for those countries that never participated in an ICP benchmark (see Section 4.3, and Appendix 2). As shown in Appendix 2, the weights decrease inversely with time so that older observations are weighted less. Both benchmarks and growth rates are assumed to be measured with error as in Panel 2. The value of the likelihood functions is higher than that of Panel 2 although the two models are not strictly nested. The smoothed PPPs series produced from this model will be labeled with the postfix "No Reg." However, the smoothed predictions (presented in the next section) have standard errors that are larger than those produced from the model in Panel 2 in most cases.

Panel 5 has been estimated in order to allow a comparison of our predictions to those available from PWT 6.2. As the later were produced before the 2005 round of the ICP had been conducted, we estimate this model for the time period 1971-2005 as before; however, the year 2005 is treated as a non-benchmark year in that the ICP benchmarks are ignored. Identical to the case of Panel 2, all sources of measurement errors are allowed as parameters are not restricted. An equivalent regression to that in Panel 1 is run to obtain starting values although the 110 benchmark values for 2005 are not in the sample. We will label the *PPP* predictions by "No05."

[Table 1]

6.2.2 PPP Predictions

In this section we present an illustration of the predictions of the method obtained from the models presented in the previous section. Two sets of predicted *PPP* series can be computed for each model depending on how the Kalman filtered predictions obtained from the above models are smoothed. Two smoothed *PPPs* series are obtained from each of the alternative model specifications described in Section 6.2.1 for each country. The first set is obtained by smoothing the Kalman filtered predictions using the well known fixed interval Kalman smoother (the equations are shown in Appendix 1). A second set of predictions is obtained under a form of the smoother that insures the resulting series follows the latest available implicit price deflator movements published for each country (see Appendix 1 for derivations). The latter will be distinguished from the first by the postfix "GRC."

The series labelled "CON" are those obtained from the model in Panel 3 and as they are constrained to track the observed benchmarks, the corresponding standard errors for particpating countries in benchmark years are zero. However, standard errors for non-benchmark years are larger than those estimated by the unrestricted version of the model (Panel 2).

Tables 2-6 and Figures 1-5 summarise the results. We have chosen five countries to illustrate, they are: Australia, China, India, Nigeria and Honduras. Australia is an OECD country and has participated in benchmark comparisons since 1985. Results for Australia are representative of results for the case of a developed country that has consistently participated in most of the global as well as OECD comparisons; and, it will illustrate the case when all sources of available information (national accounts and benchmark data) seem to provide a consistent picture. China participated in a benchmark comparison for the first time in 2005. India had participated in earlier benchmarks; however it had not participated since 1985 and has again participated in the 2005 round. Nigeria had participated in the earlier comparisons, 1975, 1980, 1985 and 1996 and has participated in the 2005 round. Honduras had participated in the 1980 comparison and it is one of the countries in the sample that did not participate in the intial 2005 round[26].

Predictions for Australia

Table 2 and Figure 1 present the *PPP* predictions for Australia. To note is the consistency between the series where the movements in the implied price deflator are maintaned (labelled GRC) and the ICP *PPP* benchmarks, specially since 1990, across all estimated models (see Table 2). In Figure 1 it is clear that from 1985 onwards all alternative *PPP* series are within the two-standard errors band generated by the model in Panel 5 (without 2005 benchmark). The standard error for the 2005 prediction is AUD 0.05 and reduces to AUD 0.01 by the inclusion of the 2005 benchmark.

[Table 2 and Figure 1]

Predictions for China

Figures 2 and Table 3 present the predictions for China. A few important points can be made. First, the predictions that have not been smoothed to follow the published GDP Deflator movements (PPP-UN) differ substantially from the series obtained when this is imposed (PPP-UN-GRC) specially before 1990, indicating that internationally available data on socio-economic variables for China, especially for the years before 1990, provide a different picture than that available through the movements in the latest available data on the GDP Deflators. Further, and as expected, the standard error of the estimates generated from Panel 5 (without the 2005 benchmark) is very large, Yuan 1.684. The standard error reduces to Yuan 0.092 when the 2005 data are included (to Yuan 0.103 for the predictions from the model in Panel 4). However, the performance of our method in predicting the 2005 *PPP* value is substantially better than that of PWT 6.2 as our model was predicting the *PPP* for China to be Yuan 3.01 for 2004, while PWT 6.2's prediction was Yuan 2.14. The 2005 ICP benchmark was Yuan 3.45 and our prediction would have been Yuan 3.09. These results illustrate how the analytical consistency of the method translates into much improved predictions. We return to this issue in more detail in Section 6.2.3.

[Table 3 and Figure 2]

[26]The authors understand that a "second 2005 round" was conducted where the Central American countries participated and the ICP is planning to release those results in the near future.

Predictions for India

India's case is different from that of China (refer to Table 4 and Figure 3). India participated in several benchmarks; however, its last participation before 2005 was 1985. The differences between PPP-UN and PPP-UN-GRC are large, as in the case of China, for the earlier part of the sample. The PPP-UN is close to the benchmark observations as expected; however, it is clear that the movements implied by the latest available GDP deflator are inconsistent with earlier benchmarks (see PPP-UN-GRC). For instance, for 1985, the benchmark was Rupee 4.667, while the estimated value when growth rates implied by the most recent revision of the GDP Deflator are maintained is Rupee 5.952. The *PPP* series derived from the model without the 2005 benchmark is closer to the actual observation in the 2005 round (Rupee 14.670) than that of PWT6.2. For example, for the year 2003, the PWT6.2 estimate is Rupee 8.146, while our estimate is Rupee 10.085 (standard error of Rupee 5.331). The large standard error arises because the last available ICP benchmark for India is 1985 and there are some inconsitencies between the information from the socioeconomic variables and the GDP deflator, which introduces the uncertainty shown in the standard errors. The inclusion of the 2005 benchmark reduces the standard error to 0.502 (using the model in Panel 2), .

[Table 4 and Figure 3]

Predictions for Nigeria

Nigeria participated in four benchmarks, 1980, 1985, 1996 and 2005 (Table 5 and Figure 4 present the results). As in the case of India, it is clear that the growth rates implied by the latest GDP deflator is inconsistent with earlier benchmarks. An important point to note from the results is that in Nigeria's case the standard errors of the estimated series derived from the model in Panel 4 (No Reg) are substantially higher, Naira 3.240, than those derived from the model in Panel 2 (UN), Naira 2.894. They are both much lower than those of the series without the 2005 benchmark which was Naira 30.916 indicating an extremely large level of uncertainty likely to arise from the inconsistency between the information contained in socio-economic variables, GDP deflators and earlier benchmarks. The predicted value for 2005 from the model in Panel 5 came to Naira 65.968 which is higher than the ICP 2005 benchmark (Naira 60.00). For the year 2004 PWT6.2 estimate was Naira 58.771 while our model had predicted Naira 54.086. Using the movement in the GDP deflator, the PWT6.2 for 2005 would have come to Naira 68.8 which is even higher than our estimate.

[Table 5 and Figure 4]

Predictions for Honduras

Honduras is one of the countries that did not participate in the 2005 round of the ICP. Since no Central American country participated in this benchmark comparison, the available information for the region is only that from socio-economic variables and GDP deflators. Table 6 and Figure 5 show the results. The estimated series from the model without the 2005 benchmark data predicts the 2005 *PPP* to be Lempira 10.596 (standard error of 5.052) and for 2004 to be Lempira 9.927. The last available estimate from the PWT6.2 for 2004 is Lempira 7.986 which is substantially lower. The predicted series from the model in Panel 2 is Lempira 10.337 for 2005 with a standard error of 4.918. In the case of Honduras the simplified model in Panel 4 (No Reg) predicts a *PPP* for 2005 of Lempira 9.747 with a standard error of Lempira 5.461 which is larger than that produced by the unrestricted model in Panel 2. It is also worth noting that the benchmark constrained model has the largest standard errors, Lempira 6.410 for 2005.

[Table 6 and Figure 5]

6.2.3 Discussion

The new method is a methodological improvement for several reasons. First, it is a one-step method (estimation-smoothing) that insures consistency of the estimates (parameters, regression predictions) used in the smoothing (Kalman filter and smoother). This consistency is given because the method combines benchmark *PPPs*, which are invariant to the reference country by construction, with predicted *PPPs* from the national price levels model which are also invariant by construction. The smoothing component preserves this invariance as the covariance structure of the model, designed to account for measurement error, is also invariant to the reference country (see Appendix 3 for details and proofs). The estimation algorithm outlined in Section 5.2.1 insures that all parameter estimates are obtained in a single step unlike previous methods where the national price level regression is first estimated and used to predict non-participating countries in benchmark years and those benchmark are then extrapolated to non-benchmark years. Second, the method is based on a transparent model where it is analytically clear what the outcome is to be when setting alternative parameters to specific values (for example, $\phi = 0$ allows the errors in the national price level model to be spherical, $\sigma_\xi^2 = 0$ results in a final smoothed series that passes through the observed benchmarks without error, and so on). Finally, it is the first available method that provides standard errors for the constructed panel or tableau of *PPPs* allowing the user to incorporate this uncertainty into their modelling when making use of these *PPPs*.

The results for a handful of countries were used to illustrate in the previous section; however, from the overall empirical results and constructed tableau (available from the author's) we can provide the following summary:

1. For the majority of countries, the *PPP* predictions are improved by the inclusion of regression information both in benchmark and non-benchmark years in that the standard errors are smaller if all the information from regression predictions is used instead of the simplified version which excludes regression information in non-benchmark years. For a small group of developed countries that have consistently participated in the ICP and OECD/Eurostat benchmark comparisons, the inclusion of the regression information does not improve the predictions, as expected, and it might result in slightly larger standard errors when the regression information is included. However, there are only 23 countries in this group.

2. The use of the full state-space model is justified when comparing the predictions from our model without the inclusion of the 2005 benchmark data to those by PWT 6.2 and the actual benchmark values produced by the ICP for 2005. Predictions from our approach when all sources of information (all benchmarks and regression predictions), except for the 2005 benchmark values are included are much closer to those found through the ICP round than those by PWT6.2 for most countries (see China and India, Tables 3 and 4). Furthermore, and as expected, the difference between the predictions of our method, the ICP benchmarks and PWT 6.2 for countries such as Australia are minimal especially after the mid-1990s.

3. The strongest contribution of the 2005 ICP round has come in the form of a reduction in uncertainty, which is very clear by comparing the size of the standard errors for the models with and without 2005 benchmark data included.

7 Conclusions

The econometric methodology suggested in the paper for the construction of a consistent panel of purchasing power parities represents a significant attempt to provide a clear and coherent approach since the first attempt of [?] in 1988. The approach is designed to make use of all the principal and auxiliary information available for the purpose of extrapolation of the International Comparison Program (ICP) benchmarks. The first source used in the study is the data on *PPPs* from all the benchmark comparisons undertaken within the auspices of ICP since 1975 including the latest round for the year 2005. The second source of data used for the purpose of constructing the panel of *PPPs* are the data on implicit price deflators at the GDP level published in all the countries included in the study.

The ICP *PPP* benchmarks and growth rates implied by the national GDP deflators are assumed to suffer from measurement error which is inversely proportional to the development level of each country. In addition to these two sources of data, an analytical constraint that requires the *PPP* of the reference country to be unity is also used as an additional piece of information. The fourth source of information is for the purpose of extrapolating *PPPs* to countries not participating in the benchmark comparisons and to all countries in non-benchmark years. A model of national price level, assumed to have spatially auto-correlated disturbances, fitted with data on a host of socio-economic variables is an integral component of the method in that the estimated parameters of this model as well as predictions of *PPPs* produced maintain the invariance to the reference country and thus provide an internally consistent method.

Existing approaches to the construction of panels of *PPPs* are two-step methods, while the new method is a single step method. The econometric model is expressed as a state-space model as the problem of estimating *PPPs* is one of signal extraction. The paper demonstrates that the new approach is flexible in that it can be used to consider a number of scenarios including restrictions on some variance parameters to generate extrapolations that track the observed ICP *PPPs* in benchmark years; the implied price movements over time for individual countries; and those that track both. An explicit form of the estimator is derived to show the estimates are weighted sums of past information. The estimator is a weighted average of past benchmark *PPPs* under simplified assumptions. Further, this is the first available approach to producing not only a panel of *PPPs,* but also associated standard errors that can be incorporated into any further modelling using these estimates.

The methodology proposed is applied to a large data set covering 141 countries and a thirty-five year period 1970 to 2005 for generating predictions. The results from the empirical estimation are illustrated through the *PPP* series generated for a selected group of countries, including China, India, Australia Nigeria and Hounduras, to examine the plausibility of the extrapolations. The results from the new methodology are contrasted with the published *PPPs* from the Penn World Table Version 6.2. The results are satisfactory and very encouraging. Further analysis and study of the results for all the 141 countries is currently underway and it is expected that the full panel of *PPPs* can be released for public use in the not too distant a future.

Appendix 1: Preserving Movements in Implicit GDP Deflators through the Smoothing Filter

In this appendix we show that using a fixed interval smoother with $\sigma_\eta^2 = 0$, the resulting smoothed estimates of the state vector, $\mathbf{p}^*_{t|\mathrm{T}}$, preserve the movement in the implicit price deflator and the covariance matrix of the smoothed estimate equals the Kalman filter estimate of the covariance at time T for all t.

The equations of a fixed interval smoother are,

$$\mathbf{p}^*_{t|T} = \check{\boldsymbol{p}}_t + \hat{\boldsymbol{\Psi}}_t(\mathbf{p}^*_{t+1|T} - \mathbf{c}_{t+1} - \check{\boldsymbol{p}}_t) \tag{32}$$

$$\boldsymbol{\Psi}^*_{t|T} = \boldsymbol{\Psi}_t + \hat{\boldsymbol{\Psi}}_t(\boldsymbol{\Psi}^*_{t+1|T} - \boldsymbol{\Psi}_{t+1|t})\hat{\boldsymbol{\Psi}}'_t \tag{33}$$

$$\hat{\boldsymbol{\Psi}}_t = \boldsymbol{\Psi}_t\boldsymbol{\Psi}^{-1}_{t+1|t} \tag{34}$$

where,

$\check{\boldsymbol{p}}_t$ is the Kalman filter estimate of the state vector

$\boldsymbol{\Psi}_t$ is the Kalman filter unconditional covariance of the state vector

$\boldsymbol{\Psi}_{t+1|t}$ is the Kalman filter conditional covariance of the state vector

$\mathbf{p}^*_{t|T}$ is the Kalman smoothed estimate of the state vector

$\boldsymbol{\Psi}^*_{t|T}$ is the covariance of $\mathbf{p}^*_{t|T}$.

Now, if $\sigma_\eta^2 = 0$, $\boldsymbol{\Psi}_{t+1|t} = \boldsymbol{\Psi}_t$, which from (34) implies $\hat{\boldsymbol{\Psi}}_t = \mathbf{I}_N$. Therefore, $\mathbf{p}^*_{t|\mathrm{T}} = \mathbf{p}^*_{t+1|\mathrm{T}} - \mathbf{c}_{t+1}$, or

$$\mathbf{p}^*_{t+1|\mathrm{T}} = \mathbf{p}^*_{t|\mathrm{T}} + \mathbf{c}_{t+1} \tag{35}$$

.

That is, smoothed estimates, $\mathbf{p}^*_{t|\mathrm{T}}$ preserve the movement in the implicit price deflator.

Now consider the covariance matrix in (33). Since, $\boldsymbol{\Psi}_{t+1|t} = \boldsymbol{\Psi}_t$ and $\hat{\boldsymbol{\Psi}}_t = \mathbf{I}_N$ we have, $\boldsymbol{\Psi}^*_{t|\mathrm{T}} = \boldsymbol{\Psi}^*_{t+1|\mathrm{T}}$. Thus, $\boldsymbol{\Psi}^*_{t|\mathrm{T}}$ is constant with respect to t and,

$$\boldsymbol{\Psi}^*_{t|\mathrm{T}} = \boldsymbol{\Psi}^*_{T|T} = \boldsymbol{\Psi}_{T|T} \text{ for all } t \tag{36}$$

Appendix 2: Kalman filter predictions with no regression information in non-benchmark years is a weighted sum of observed benchmarks

We present the equations of the Kalman filter to assist the presentation.

$$\check{\boldsymbol{p}}_{t|t-1} = \check{\boldsymbol{p}}_{t-1} + \mathbf{c}_t \tag{37}$$

$$\boldsymbol{\Psi}_{t|t-1} = \boldsymbol{\Psi}_{t-1} + \hat{\mathbf{Q}} \tag{38}$$

$$\check{\boldsymbol{p}}_t = \check{\boldsymbol{p}}_{t|t-1} + \boldsymbol{\Psi}_{t|t-1}\mathbf{Z}_t'\mathbf{F}_t^{-1}(\mathbf{y}_t - \mathbf{B}_t\mathbf{X}_t\hat{\boldsymbol{\theta}} - \mathbf{Z}_t'\check{\boldsymbol{p}}_{t|t-1}) \tag{39}$$

$$\boldsymbol{\Psi}_t = \boldsymbol{\Psi}_{t|t-1} - \boldsymbol{\Psi}_{t|t-1}\mathbf{Z}_t'\mathbf{F}_t^{-1}\boldsymbol{\Psi}_{t|t-1} \tag{40}$$

$$\mathbf{F}_t = \mathbf{Z}_t\boldsymbol{\Psi}_{t|t-1}\mathbf{Z}_t' + \hat{\mathbf{H}}_t \tag{41}$$

where,

$\hat{\mathbf{Q}} = \hat{\sigma}_\eta^2\mathbf{V}_t$, $\hat{\mathbf{H}}_t$, $\hat{\boldsymbol{\theta}}$ are estimates

$\check{\boldsymbol{p}}_t$is the Kalman filter estimate of the state vector

$\boldsymbol{\Psi}_t$ is the Kalman filter estimate of the unconditional covariance of the state vector

$\boldsymbol{\Psi}_{t+1|t}$ is the Kalman filter estimate of the conditional covariance of the state vector.

Suppose there are $M+1$ benchmark years at times $t(0), t(1), ..., t(M)$, where $t(0) = 0$, and no information is added between benchmark years.

Let $\check{\boldsymbol{p}}_T$ be the Kalman filter estimate of $\mathbf{p}_T$ and $\overleftrightarrow{\boldsymbol{p}}_{T,j}$, $j = 0, 1, ..., M$ be the $M+1$ different estimates of $\mathbf{p}_T$ obtained by applying growth rates to the benchmark observations until time $t = T$. Further, we define $\mathbf{G}(i)$, *the Kalman gain*[27] at $t = t(i)$ which in our case takes the form:

$$\mathbf{G}(i) = \begin{cases} \boldsymbol{\Psi}_{t|t-1}\mathbf{F}_t^{-1} & \text{for}\, i > 0 \\ \mathbf{I} & \text{for}\, i = 0 \end{cases} \tag{42}$$

Proposition

The Kalman filter estimate, $\check{\boldsymbol{p}}_T$, is a weighted sum of the $\overleftrightarrow{\boldsymbol{p}}_{T,j}$, $j = 0, 1, ..., M$.

That is,

$$\check{\boldsymbol{p}}_T = \sum_{i=0}^{M} \Upsilon_i^{(M)} \overleftrightarrow{\boldsymbol{p}}_{T,i} \tag{43}$$

where the weights $\Upsilon_i^{(M)}$ are defined as

$$\Upsilon_i^{(M)} = \begin{cases} \left[\prod_{j=1}^{M-i} (\mathbf{I} - \mathbf{G}(M-j+1))\right] \mathbf{G}(i) & i = 0, 1, ..., M-1 \\ \mathbf{G}(i) & i = M \end{cases} \tag{44}$$

Lemma

The $\Upsilon_i^{(M)}$ defined in (44) are the product of positive definite (pd) matrices and

[27] See [Har89], p 110

$$\sum_{i=0}^{M} \Upsilon_i^{(M)} = \mathbf{I}_N \tag{45}$$

Proof of Lemma

In (38)$\mathbf{\Psi}_{t-1}$ is positive semidefinite (psd) or pd and $\mathbf{Q}_t$ is positive definite (pd). Therefore, $\mathbf{\Psi}_{t|t-1}$ is pd for all t. Also, by definition $\mathbf{F}_t$ in (41) must be pd as $\mathbf{H}_t$ is pd. Thus, $\mathbf{G}(i)$ is the product of pd matrices for all i.

Also, post-multiplying (41) by$\mathbf{F}_t^{-1}$, we have

$$\mathbf{I}_N = \mathbf{\Psi}_{t|t-1}\mathbf{F}_t^{-1} + \hat{\mathbf{H}}_t\mathbf{F}_t^{-1}$$

$$= \mathbf{G}(i) + \hat{\mathbf{H}}_t\mathbf{F}_t^{-1}$$

Therefore, $\mathbf{I}_N - \mathbf{G}(i) = \hat{\mathbf{H}}_t\mathbf{F}_t^{-1}$, and is also the product of pd matrices for all i. Thus, it follows that by (44) $\Upsilon_i^{(M)}$ is the product of pd matrices.

We will now establish that for $\Upsilon_i^{(M)}$ defined by (44), (45) holds. The proof will proceed by induction and we note that the form of $\Upsilon_i^{(M)}$ in (44) implies that

$$\Upsilon_i^{(M)} = [\mathbf{I} - \mathbf{G}(M)]\, \Upsilon_i^{(M-1)} \tag{46}$$

We will now assume that (45) is true for M - 1 . That is,

$$\sum_{i=0}^{M-1} \Upsilon_i^{(M-1)} = \mathbf{I}_N \tag{47}$$

Then from (46) and (45)

$$\sum_{i=0}^{M} \Upsilon_i^{(M)} = \sum_{i=0}^{M-1} \Upsilon_i^{(M)} + \Upsilon_M^{(M)}$$

$$= [\mathbf{I} - \mathbf{G}(M)] \sum_{i=0}^{M-1} \Upsilon_i^{(M-1)} + \mathbf{G}(M)$$

and so by the assumption (47)

$$\sum_{i=0}^{M} \Upsilon_i^{M} = \mathbf{I}_N$$

Therefore if (45) is true for $M-1$, it is also true for M.

Now, set $M = 1$

$$\sum_{i=0}^{M} \Upsilon_i^{(M)} = \Upsilon_0^{(1)} + \Upsilon_1^{(1)}$$

From (44) and (42)

$$\Upsilon_o^{(1)} = (\mathbf{I} - \mathbf{G}(1)), \qquad \Upsilon_1^{(1)} = \mathbf{G}(1) \tag{48}$$

Therefore, (45) is true for M = 1 and so, by induction,

$$\sum_{i=0}^{M} \Upsilon_i^{(M)} = \mathbf{I}_N$$

for all M as required.

Proof of Proposition

In order to ease the notational burden, we will prove (43) first for the case $T = t(M)$ and then extend to the case $T > t(M)$

Assume (43) and (44) are true for $T = t(M-1)$.That is,

$$\check{\boldsymbol{p}}_{t(M-1)} = \sum_{i=0}^{M-1} \Upsilon_i^{(M-1)} \overleftrightarrow{\boldsymbol{p}}_{(M-1),i} \tag{49}$$

Now, at $t = t(M)$ a benchmark observation, $\mathbf{y}(M)$, becomes available. By definition $\overleftrightarrow{\boldsymbol{p}}_{t(M),M} = \mathbf{y}(M)$.

The Kalman filter updating formula (see (39)) gives:

$$\check{\boldsymbol{p}}_{t(M)} = (\check{\boldsymbol{p}}_{t(M-1)} + \bar{\mathbf{c}}) + \mathbf{G}(M)[\mathbf{y}(M) - (\check{\boldsymbol{p}}_{t(M-1)} + \bar{\mathbf{c}})] \tag{50}$$

where $\bar{\mathbf{c}}$ is the cumulated growth rates from $t(M-1)$ to $t(M)$.

Thus,

$\check{\boldsymbol{p}}_{t(M)} = [\mathbf{I} - \mathbf{G}(M)] \left[\check{\boldsymbol{p}}_{t(M-1)} + \bar{\mathbf{c}}\right] + \mathbf{G}(M)\check{\boldsymbol{p}}_{t(M),M}$

Now, by assumption (49)

$$\check{\boldsymbol{p}}_{t(M-1)} + \bar{\mathbf{c}} = \sum_{i=0}^{M-1} \Upsilon_i^{(M-1)} \overleftrightarrow{\boldsymbol{p}}_{t(M-1),i} + \bar{\mathbf{c}}$$

$$= \sum_{i=0}^{M-1} \Upsilon_i^{(M-1)} \left(\overleftrightarrow{\boldsymbol{p}}_{t(M-1),i} + \bar{\mathbf{c}}\right) \text{(by 47)}$$

$$= \sum_{i=0}^{M-1} \Upsilon_i^{(M-1)} \overleftrightarrow{\boldsymbol{p}}_{t(M),i}$$

Thus,

$$\check{\boldsymbol{p}}_{t(M)} = \sum_{i=0}^{M-1} [\mathbf{I} - \mathbf{G}(M)] \Upsilon_i^{(M-1)} \overleftrightarrow{\boldsymbol{p}}_{t(M),i} + \mathbf{G}(M)\check{\boldsymbol{p}}_{t(M),M}$$

$$= \sum_{i=0}^{M} \Upsilon_i^{(M)} \overleftrightarrow{\boldsymbol{p}}_{t(M),i}$$

And so if (43) and (44) are true for $t(M-1)$, then they are also true for $t(M)$.

Now set $M = 1$. This implies two benchmark years, at $t(0) = 0$ and $t(1)$. By definition,

$\check{\boldsymbol{p}}_0 = \check{\boldsymbol{p}}_{t(0),0} = \mathbf{y}_0$; $\check{\boldsymbol{p}}_{t(1),1} = \mathbf{y}_1$ and $\check{\boldsymbol{p}}_{t(1),0} = \check{\boldsymbol{p}}_{t(0),0} + \bar{\mathbf{c}} = \check{\boldsymbol{p}}_0 + \bar{\mathbf{c}}$.

Then, using the Kalman updating formula,

$$\check{\boldsymbol{p}}_{t(1)} = [\mathbf{I} - \mathbf{G}(1)] (\check{\boldsymbol{p}}_0 + \bar{\mathbf{c}}) + \mathbf{G}(1)\mathbf{y}(1) = [\mathbf{I} - \mathbf{G}(1)] \overleftrightarrow{\boldsymbol{p}}_{t(1),0} + \mathbf{G}(1) \overleftrightarrow{\boldsymbol{p}}_{t(1),1}$$

$$= \Upsilon_0^{(1)} \overleftrightarrow{\boldsymbol{p}}_{t(1),0} + \Upsilon_1^{(1)} \overleftrightarrow{\boldsymbol{p}}_{t(1),1} (\text{by } (44))$$

Thus (43) and (44) hold for $M = 1$, and hence, by induction, for all M.

We can now easily extend the result for $T > t(M)$. If we denote the cumulated growth rates from $t(M)$ to T by $\bar{\mathbf{c}}$, then

$$\check{\boldsymbol{p}}_T = \check{\boldsymbol{p}}_{t(M)} + \bar{\mathbf{c}}$$

$$= \sum_{i=0}^{M} \Upsilon_i^{(M)} \overleftrightarrow{\boldsymbol{p}}_{t(M),i} + \bar{\mathbf{c}}$$

$$= \sum_{i=0}^{M} \Upsilon_i^{(M)} (\overleftrightarrow{\boldsymbol{p}}_{t(M),i} + \bar{\mathbf{c}})$$

$$\check{\boldsymbol{p}}_T = \sum_{i=0}^{M} \Upsilon_i^{(M)} \overleftrightarrow{\boldsymbol{p}}_{T,i}$$

Special case

If the elements of η_t and ξ_t are contemporaneously uncorrelated (that is, $\mathbf{Q}_t$ and $\mathbf{H}_t$ are diagonal) it is easily shown that the $\Upsilon_i^{(M)}$ are diagonal and positive definite for all $i = 1, ..., M$, provided $\boldsymbol{\Psi}_0 \neq \mathbf{0}$.

Suppose that $\check{\boldsymbol{p}}_{jT}$ and $\overleftrightarrow{\boldsymbol{p}}_{jT,i}$ are the Kalman filter and benchmark estimates (from the ith benchmark) of the PPP of country j at time $t = T > t(M)$. Denote by $\upsilon_{jj,i}^{(M)}$the jth diagonal element of $\Upsilon_i^{(M)}$. It then follows that

$$\check{\boldsymbol{p}}_{jT} = \sum_{i=0}^{M} \upsilon_{jj,i}^{(M)} \overleftrightarrow{\boldsymbol{p}}_{jT,i}$$

Furthermore, because $\Upsilon_i^{(M)}$ is pd, and from (45), it follows that $\upsilon_{jj,i}^{(M)} > 0$ and $\sum_{i=0}^{M} \upsilon_{jj,i}^{(M)} = 1$.

Thus, in this special case the Kalman filter estimate for country j is weighted average of the $M + 1$ "benchmark only" estimates for that country. The weights are not arbitrary, but determined by the fundamental covariance matrices $\mathbf{Q}_t$ and $\mathbf{H}_t$.

Appendix 3. The Invariance of the Kalman Filter Predictions to the Reference Country

A3.1 Notation and Conventions

Without loss of generality we will take two reference countries as countries 1 and 2, and denote the $ln(PPP_t)$ relative to the two bases as $\mathbf{p}_t^{(1)}$ and $\mathbf{p}_t^{(2)}$. Other consequent notation will usually be obvious, making definition unnecessary.

By definition

$$\mathbf{p}_t^{(2)} = \mathbf{p}_t^{(1)} - p_{2t}^{(1)} \tag{51}$$

Also,

$p_{2t}^{(2)} \equiv p_{1t}^{(1)} \equiv 0.$

Because the p_{it} is always zero for the base country, we will remove it from the Kalman filter cycle, and re-define $\mathbf{p}_t^{(1)}$ and $\mathbf{p}_t^{(2)}$ as the $N-1$ vectors $\mathbf{p}_t^{(1)} = [p_{2t}^{(1)}, p_{3t}^{(1)}, ..., p_{Nt}^{(1)}]'$ and $\mathbf{p}_t^{(2)} = [p_{1t}^{(2)}, p_{3t}^{(2)}, ..., p_{Nt}^{(2)}]'$. It follows from (51) that

$$\mathbf{p}_t^{(2)} = \mathbf{A}\mathbf{p}_t^{(1)} \tag{52}$$

where $\boldsymbol{A}$ is a non-stochastic, non-singular $(N-1) \times (N-1)$ matrix given by

$$\mathbf{A} = \begin{bmatrix} -1 & \mathbf{0}'_{N-2} \\ -\boldsymbol{j}_{N-2} & \boldsymbol{I}_{N-2} \end{bmatrix} \tag{53}$$

$\mathbf{j}_{N-2}$ is a vector of ones and $\mathbf{0}'_{N-2}$ a (row) vector of zeros.

Denoting the Kalman filter estimates obtained by using observations relative to the two base countries by $\check{\boldsymbol{p}}_t^{(1)}$and $\check{\boldsymbol{p}}_t^{(2)}$, the invariance property holds if it can be established that

$$\check{\boldsymbol{p}}_t^{(2)} = \boldsymbol{A}\check{\boldsymbol{p}}_t^{(1)} \tag{54}$$

A3.2 Regression Estimates

a) *Benchmark years*

Estimates of β_{ot} and β_s are obtained by regressing benchmark observations $\tilde{\mathbf{p}}_t$ on the conditioning variables $\mathbf{x}_t = [x_{1t}$, x_{2t}, ..., $x_{N_1t}]'$ where we have taken countries $i = 1, 2, \ldots, N_1$, as the participating countries.

Now, by definition,

$$\tilde{\mathbf{p}}_t^{(2)} = \tilde{\mathbf{p}}_t^{(1)} - \tilde{p}_{2t}^{(1)}\mathbf{j}_{N-2} \tag{55}$$

That is, the dependent variable $\tilde{\mathbf{p}}_t^{(2)}$ is obtained by subtracting the same number $\tilde{p}_{2t}^{(1)}$ from each observation in $\tilde{\mathbf{p}}_t^{(1)}$. Because the regressors $\mathbf{X}_t$ do not change when the base country is changed from 1 to 2, by standard regression theory

$$\hat{\beta}_{0t}^{(2)} = \hat{\beta}_{0t}^{(1)} - \tilde{p}_{2t}^{(1)} \tag{56}$$

$$\hat{\boldsymbol{\beta}}_s^{(2)} = \hat{\boldsymbol{\beta}}_s^{(1)} = \hat{\boldsymbol{\beta}}_s$$

That is, intercepts change but slopes are invariant. It follows that for non-participating countries

$$\hat{\mathbf{p}}_t^{(2)} = \hat{\mathbf{p}}_t^{(1)} - \hat{p}_{2t}^{(1)}\mathbf{j}$$

Thus, defining the "observation vector" $\mathbf{y}_t$ by $\mathbf{y}_t = [\tilde{\mathbf{p}}_t, \hat{\mathbf{p}}_t]'$ and discarding the base country observation (as it is always zero) we have

$$\mathbf{y}_t^{(2)} = \mathbf{A}\mathbf{y}_t^{(1)} \tag{57}$$

b) *Non-benchmark years*

Here the observation is the regression prediction $\hat{\beta}_o^{(i)}\mathbf{j}_N + \mathbf{X}_t\hat{\boldsymbol{\beta}}_s$ $(i = 1, 2)$. We now adjust the observation by subtracting the base country prediction from all predictions. This ensures the base country observation is zero, and the value of the intercept is irrelevant.

Then,

$$y_{it}^{(1)} = (x'_{it} - x'_{1t})\hat{\boldsymbol{\beta}}_s$$

$$y_{it}^{(2)} = (x'_{it} - x'_{2t})\hat{\boldsymbol{\beta}}_s$$

$$= (x'_{it} - x'_{1t})\hat{\beta}_s - (x'_{2t} - x'_{1t})\hat{\boldsymbol{\beta}}_s$$

$$= y_{it}^{(1)} - y_{2t}^{(1)}$$

Thus,

$$\mathbf{y}_t^{(2)} = \mathbf{A}\mathbf{y}_t^{(1)} \tag{58}$$

It follows from (57) and (58) that for both benchmark and non-benchmark years, the fundamental transformation $\mathbf{y}_t^{(2)} = \mathbf{A}\mathbf{y}_t^{(1)}$ holds.

A3.3 The covariance of the measurement error

The measurement error in the benchmark *PPPs* and growth rates are assumed to have a covariance proportional to the form:

$$\mathbf{V}_t = \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \sigma_{1t}^2\boldsymbol{jj}' + diag(\sigma_{2t}^2, ..., \sigma_{Nt}^2) \end{bmatrix} \tag{59}$$

where, σ_{it}^2 is the variance of country i at time t and σ_{1t}^2 is the variance of the reference country.

Let $\mathbf{V}_t^{(1)}$ the $(N-1)\times(N-1)$ matrix obtained by ignoring the first row and column of $\mathbf{V}_t$,

$$\mathbf{V}_t^{(1)} = \sigma_\eta^2[\sigma_{1t}^2\mathbf{jj}' + diag(\sigma_{2t}^2, ..., \sigma_{Nt}^2)]$$

Then,

$$\mathbf{A}\mathbf{V}_t^{(1)}\mathbf{A}' = \sigma_\eta^2[\sigma_{2t}^2\mathbf{jj}' + diag(\sigma_{1t}^2, \sigma_{3t}^2, ..., \sigma_{Nt}^2)]$$

$$= \mathbf{V}_t^{(2)}$$

A.3.4 The observation equation

The fundamental observation equation used in the method is

$\mathbf{y}_t = \mathbf{p}_t + \boldsymbol{\zeta}_t, \qquad E(\boldsymbol{\zeta}_t\boldsymbol{\zeta}_t') = \mathbf{H}_t$

where

$\mathbf{y}_t$ is an observation of the unobserved state

$\mathbf{p}_t$ and

$\boldsymbol{\zeta}_t$ is an observation error.

Because $\mathbf{p}_t^{(2)} = \mathbf{A}\mathbf{p}_t^{(1)}$ by definition and $\mathbf{y}_t^{(2)} = \mathbf{A}\mathbf{y}_t^{(1)}$ by regression properties and construction (see previous sections) it follows that

$$\boldsymbol{\zeta}_t^{(2)} = \mathbf{A}\boldsymbol{\zeta}_t^{(1)}$$

And thus because $\boldsymbol{A}$ is non-stochastic,

$$\mathbf{H}_t^{(2)} = \mathbf{A}\mathbf{H}_t^{(1)}\mathbf{A}' \tag{60}$$

This is the fundamental result that enables us to prove invariance.

A3.5 The transition equation

The transition equation used is of the form

$$\mathbf{p}_t = \mathbf{p}_{t-1} + \mathbf{c}_t + \boldsymbol{\eta}_t \tag{61}$$

where,

c_t is the observed growth rate of p_t

$\boldsymbol{\eta}_t$ is an error with $E(\boldsymbol{\eta}_t) = 0$ and $E(\boldsymbol{\eta}_t\boldsymbol{\eta}_t') \equiv \mathbf{Q}_t = \sigma_\eta^2\mathbf{V}_t$

By defining $\mathbf{V}_t$ as in (59), it follows that,

$$\mathbf{Q}_t^{(2)} = \mathbf{A}\mathbf{Q}_t^{(1)}\mathbf{A}' \tag{62}$$

A3.6 Invariance proved

For the reader's reference the Kalman filter equations, are given by

Prediction Equations

$\check{\boldsymbol{p}}_{t|t-1} = \check{\boldsymbol{p}}_{t-1} + \mathbf{c}_t$

$\boldsymbol{\Psi}_{t|t-1} = \boldsymbol{\Psi}_{t-1} + \mathbf{Q}_t$

Updating Equations

$\mathbf{F}_t = \boldsymbol{\Psi}_{t|t-1} + \mathbf{H}_t$

$\check{\boldsymbol{p}}_t = \check{\boldsymbol{p}}_{t|t-1} + \boldsymbol{\Psi}_{t|t-1}\mathbf{F}_t^{-1}(\mathbf{y}_t - \check{\boldsymbol{p}}_{t|t-1})$

$\boldsymbol{\Psi}_t = \boldsymbol{\Psi}_{t|t-1} - \boldsymbol{\Psi}_{t|t-1}\mathbf{F}_t^{-1}\boldsymbol{\Psi}_{t|t-1}$

Assume,

$$\check{\boldsymbol{p}}_{t-1}^{(2)} = \mathbf{A}\check{\boldsymbol{p}}_{t-1}^{(1)} \tag{63}$$

from which it follows (because $\boldsymbol{A}$ is non-stochastic) that

$$\boldsymbol{\Psi}_{t-1}^{(2)} = \mathbf{A}\boldsymbol{\Psi}_{t-1}^{(1)}\mathbf{A}' \tag{64}$$

Following the Kalman filter covariance cycle

$$\mathbf{\Psi}_{t|t-1}^{(2)} = \mathbf{\Psi}_{t-1}^{(2)} + \mathbf{Q}_t^{(2)}$$

$$= \mathbf{A}\mathbf{\Psi}_{t-1}^{(1)}\mathbf{A}' + \mathbf{A}\mathbf{Q}_t^{(1)}\mathbf{A}' \text{ (by(62))}$$

$$= \mathbf{A}\mathbf{\Psi}_{t|t-1}^{(1)}\mathbf{A}' \tag{65}$$

$$\mathbf{F}_t^{(2)} = \mathbf{\Psi}_{t|t-1}^{(2)} + \mathbf{H}_{t|t-1}^{(2)}$$

$$= \mathbf{A}\mathbf{\Psi}_{t|t-1}^{(1)}\mathbf{A}' + \mathbf{A}\mathbf{H}_t^{(1)}\mathbf{A}' \text{ (by (59))}$$

$$\mathbf{F}_t^{(2)} = \mathbf{A}\mathbf{F}_t^{(1)}\mathbf{A}' \tag{66}$$

The updating equation for $\check{\boldsymbol{p}}_t^{(2)}$ is

$$\check{\boldsymbol{p}}_t^{(2)} = \check{\boldsymbol{p}}_{t-1}^{(2)} + \mathbf{\Psi}_{t|t-1}^{(2)} \left(\mathbf{F}_t^{(2)}\right)^{-1} (\mathbf{y}_t^{(2)} - \mathbf{c}_t^{(2)} - \check{\boldsymbol{p}}_{t-1}^{(2)})$$

Substituting using 60, 63, 64 and 57,

$$\check{\boldsymbol{p}}_t^{(2)} = \mathbf{A}\check{\boldsymbol{p}}_{t-1}^{(1)} + \mathbf{A}\mathbf{\Psi}_{t|t-1}^{(1)}\mathbf{A}' \left(\mathbf{A}\mathbf{F}_t^{(1)}\mathbf{A}'\right)^{-1} (\mathbf{A}\mathbf{y}_t^{(1)} - \mathbf{A}\mathbf{c}_t^{(1)} - \mathbf{A}\check{\boldsymbol{p}}_{t-1}^{(1)})$$

$$= \mathbf{A}[\check{\boldsymbol{p}}_{t-1}^{(1)} + \mathbf{\Psi}_{t|t-1}^{(1)} \left(\mathbf{F}_t^{(1)}\right)^{-1} (\mathbf{y}_t^{(1)} - \mathbf{c}_t^{(1)} - \check{\boldsymbol{p}}_{t-1}^{(1)})] \text{ (because } \boldsymbol{A}\text{is non-singular)} \tag{67}$$

From the definition of $\boldsymbol{c}_t$ following equation (5), it is clear that $\boldsymbol{c}_t^{(2)} = \boldsymbol{A}\boldsymbol{c}_t^{(1)}$.

Thus,

$$\check{\boldsymbol{p}}_t^{(2)} = \mathbf{A}\check{\boldsymbol{p}}_t^{(1)} \tag{68}$$

It follows by induction that if the estimation is commenced when (68) holds, invariance will be true for all subsequent years.

Appendix 4 – Estimation of the Regression Parameters in the State-Space Model

A4.1 State Space Equations

These are repeated for convenience

$\mathbf{y}_t = \mathbf{Z}_t \mathbf{p}_t + \mathbf{B}_t \mathbf{X}_t \boldsymbol{\theta} + \boldsymbol{\zeta}_t$

$\mathbf{p}_t = \mathbf{p}_{t-1} + \mathbf{c}_t + \boldsymbol{\eta}_t$

A4.2 Kalman Filter Equations

The forward filter is conceptually composed of two sets of equations (see Section A3.6)

A4.3 Estimation of unknown parameters

There are unknown parameters in the covariance structure as well as the vector of parameters in the mean of the measurement equation (associated with the regression part)

$\mathbf{Q}_t = \sigma_\eta^2 \mathbf{V}_t$

$\mathbf{H}_t = \begin{bmatrix} 0 & \mathbf{0} & 0 \\ 0 & \sigma_u^2 \boldsymbol{S}_{np} \boldsymbol{\Omega} \boldsymbol{S}'_{np} & 0 \\ 0 & 0 & \sigma_\xi^2 \boldsymbol{S}_p \boldsymbol{V}_t \boldsymbol{S}'_p \end{bmatrix}$

$\boldsymbol{\Omega} = (\boldsymbol{I} - \phi \boldsymbol{W}_t)^{-1} (\boldsymbol{I} - \phi \boldsymbol{W}_t)^{-1\prime}$

To obtain estimates we maximize the log Likelihood

$\log L(\phi, \sigma_\eta^2, \sigma_u^2, \sigma_\xi^2, \theta) = -\frac{TN}{2} \log 2\pi - \frac{1}{2} \sum_{t=1}^{T} \log |\mathbf{F}_t^{-1}| - \frac{1}{2} \sum_{t=1}^{T} \hat{\boldsymbol{\zeta}}'_t \mathbf{F}_t^{-1} \hat{\boldsymbol{\zeta}}_t$

We numerically maximize this function over σ_η^2, σ_u^2, σ_ξ^2, and ϕ. At each iteration, a GLS estimate, $\hat{\boldsymbol{\theta}}(\phi, \sigma_\eta^2, \sigma_u^2, \sigma_\xi^2)$, is computed by regressing a set of "innovations" $\mathbf{y}_t^{**}$ on the "innovations" $\mathbf{X}_t^{**}$, where these "innovations" are obtained by running the same Kalman Filter separately for $\mathbf{y}_t$ and each column of $\mathbf{X}_t$ (see [Har89], pp. 130-133). This is a very convenient approach as the parameters in the mean of the observation equation can be estimated at each step but outside the numerical search for the four covariance parameters. This greatly reduces the difficulties associated with maximizing the likelihood function.

To see why this GLS procedure is appropriate we concentrate on the portion of the measurement equation that involves the regressors and the transition equation of the state-space and show that the model can be re-written as a generalized linear model. The following has been adapted from [Har89].

$$\hat{\mathbf{p}}_t = \mathbf{p}_t + \mathbf{X}_t \boldsymbol{\theta} + \mathbf{v}_t \tag{69}$$

$$\mathbf{p}_t = \mathbf{p}_{t-1} + \mathbf{c}_t + \boldsymbol{\eta}_t \tag{70}$$

$E(\boldsymbol{\eta}_t \boldsymbol{\eta}'_t) \equiv \sigma_\eta^2 \mathbf{Q}_t$

$E(\mathbf{v}_t \mathbf{v}'_t) \equiv \mathbf{H}_{1t} = \sigma_u^2 \boldsymbol{\Omega}$

$\mathbf{X}_t$ is non-stochastic matrix of conditioning variables and $\mathbf{p}_0 \sim (\bar{\mathbf{p}}_0, \boldsymbol{\Psi}_0)$.

Re-write equations (69) and (70), using result (3.1.17) from [Har89]

$$\hat{\mathbf{p}}_t = \bar{\mathbf{p}}_t + \mathbf{X}_t \theta + \bar{\mathbf{c}}_t + \mathbf{v}_t \tag{71}$$

$$\bar{\mathbf{p}}_t = \bar{\mathbf{p}}_{t-1} + \boldsymbol{\eta}_t \tag{72}$$

where,
$\mathbf{p}_t = \bar{\mathbf{p}}_t + \bar{\mathbf{c}}_t$
$\bar{\mathbf{c}}_t = \sum_{i=1}^{t} \mathbf{c}_i$
Write (71) and (72) in regression form.

$$\hat{\mathbf{p}}_t^* = \mathbf{X}_t\boldsymbol{\theta} + \mathbf{e}_t \tag{73}$$

$$\mathbf{e}_t = \bar{\mathbf{p}}_t + \mathbf{v}_t \tag{74}$$

where,
$\hat{\mathbf{p}}_t^* = \hat{\mathbf{p}}_t - \bar{\mathbf{c}}_t$
$E(\mathbf{e}_t) = \mathbf{0}$, $Var(\mathbf{e}_t) = \mathbf{\Sigma}$
[Har89](pp. 130) states that under the assumption $\mathbf{p}_0 \sim (\mathbf{0}, \mathbf{\Psi}_0)$,
the GLS estimator of θ is given by:

$$\hat{\boldsymbol{\theta}} = (\mathbf{X}'\mathbf{\Sigma}^{-\mathbf{1}}\mathbf{X})^{-\mathbf{1}}\mathbf{X}'\mathbf{\Sigma}^{-\mathbf{1}}\hat{\mathbf{p}}^* \tag{75}$$

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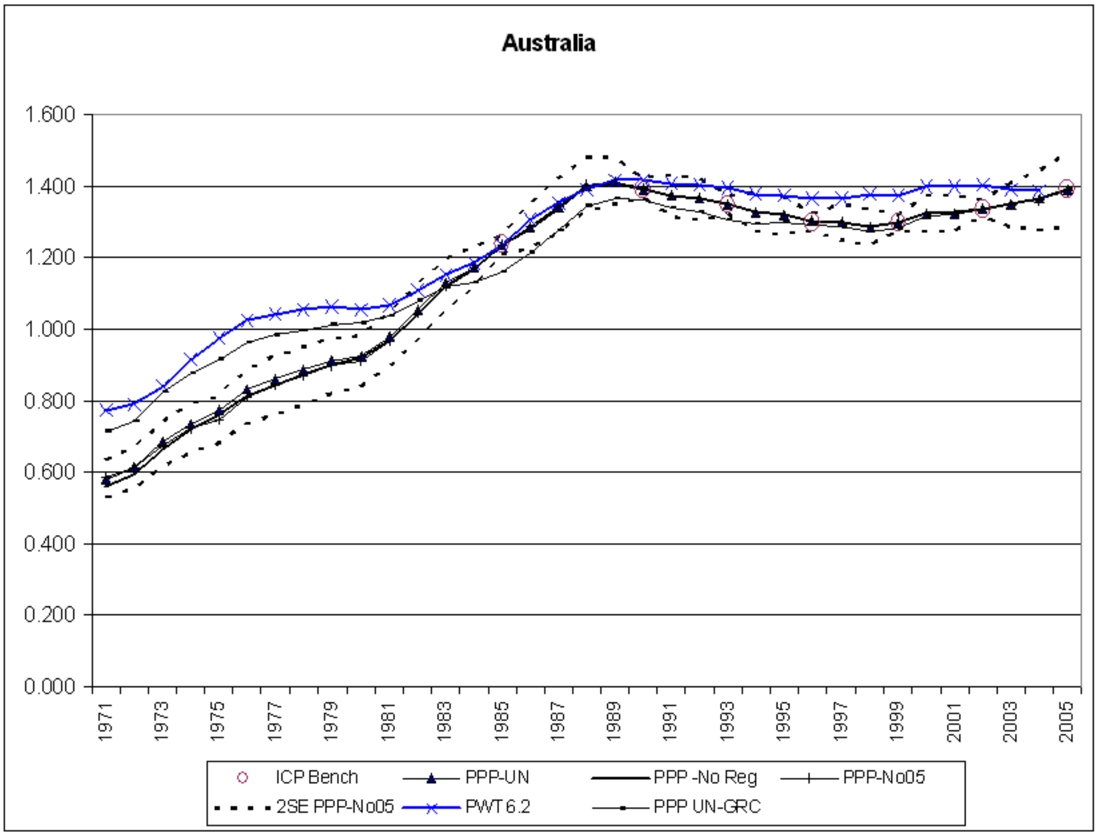

Figure 1: PPP (AUD per USD) Series for Alternative Specifications

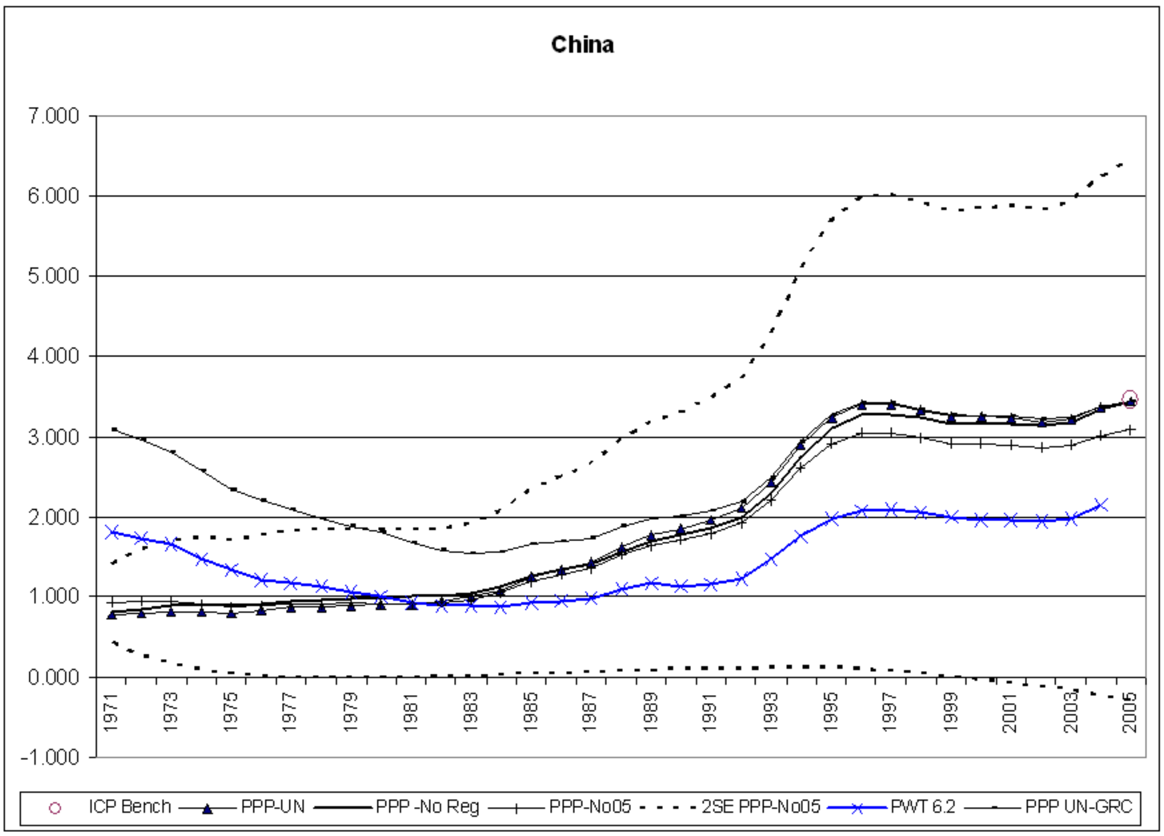

Figure 2: PPP (Yuan per USD) Series for Alternative Specifications

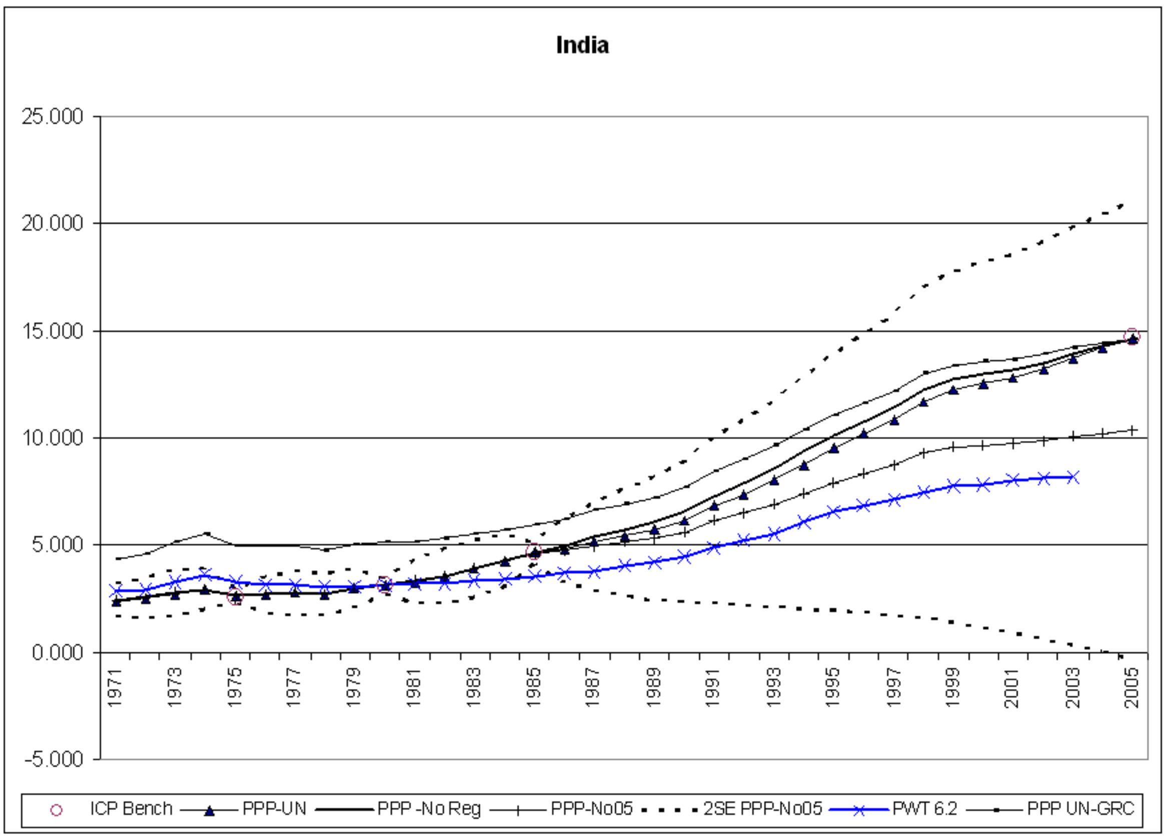

Figure 3: PPP (Rupees per USD) Series for Alternative Specifications

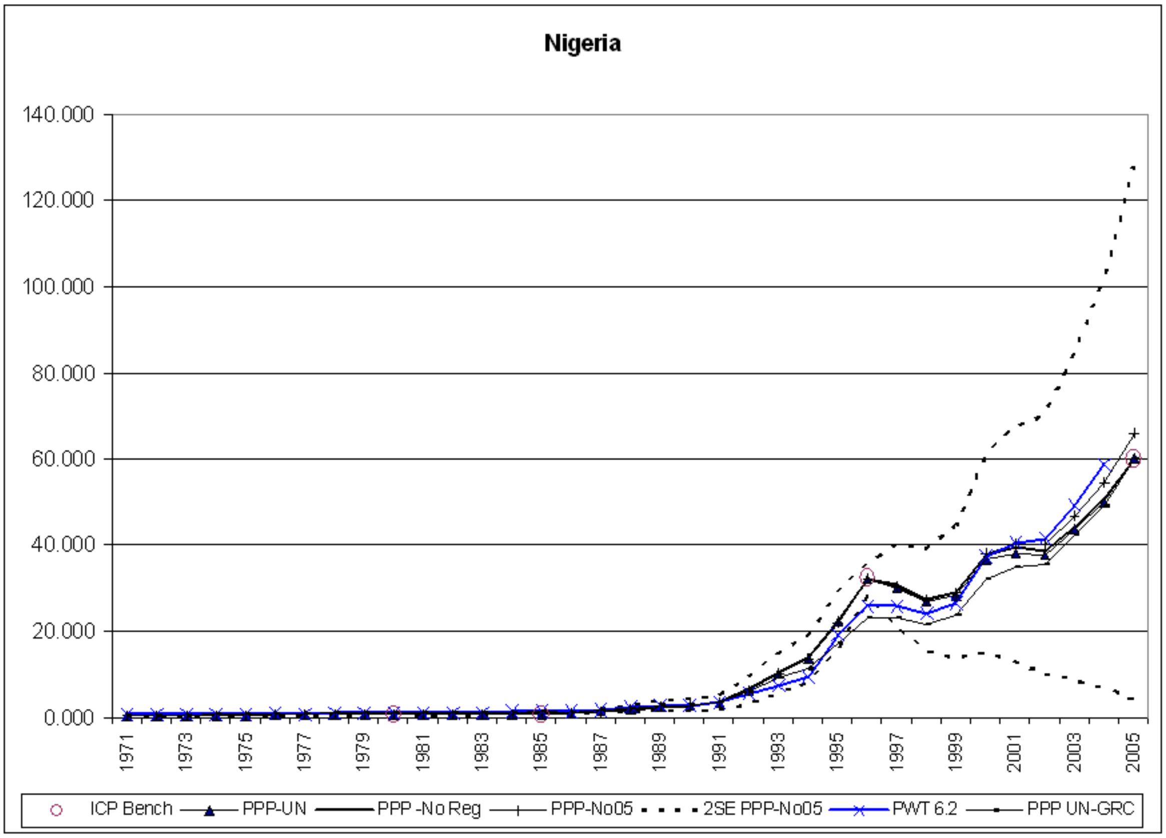

Figure 4: PPP (Naira per USD) Series for Alternative Specifications

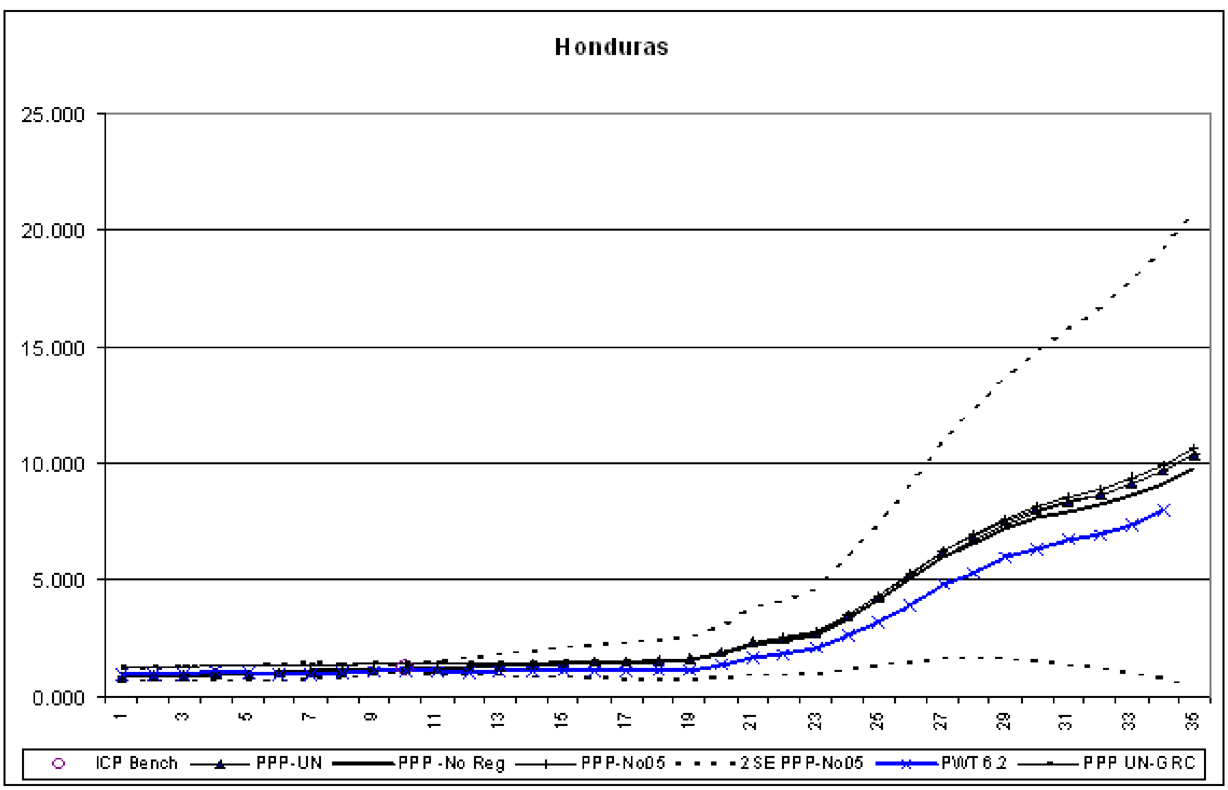

Figure 5: PPP (Lempira per USD) Series for Alternative Specifications

Table 1: Parameter Estimates Under Alternative Specifications

	Regression		State Space Model							
	Without Spatial Errors (Panel1)		Benchmark Unconstrained (Panel 2)		Benchmark Constrained (Panel 3)		No Regression In Non-Benchmark Years (Panel 4)		2005 Not a Benchmark Year (Panel 5)	
Variable	Estimate	S.E.	Estimate	S.E.	Estimate	S.E.	Estimate	S.E.	Estimate	S.E.
Intercept			2.988	1.245	3.959	1.190	4.410	1.128	3.502	1.246
dum75_79	-0.23	0.063	0.393	0.255	0.015	0.199	-0.044	0.179	-0.149	0.259
dum80_84	-0.021	0.231	0.070	0.249	-0.253	0.195	-0.300	0.176	-1.673	0.253
dum85_89	-0.505	0.231	-0.665	0.240	-0.966	0.188	-1.017	0.170	-2.419	0.244
dum90_92	-0.135	0.235	-0.778	0.276	-1.017	0.213	-1.074	0.192	-2.570	0.279
dum93_95	-0.302	0.235	-1.123	0.274	-1.317	0.212	-1.371	0.191	-2.925	0.277
dum96_98	-0.291	0.232	-1.406	0.275	-1.517	0.213	-1.568	0.193	-3.193	0.278
dum99_01	-0.535	0.236	-1.441	0.279	-1.599	0.218	-1.652	0.198	-3.238	0.283
dum02_04	-0.636	0.228	-1.692	0.283	-1.821	0.223	-1.877	0.202	-3.612	0.266
dum05	-0.376	0.226	-3.205	0.425	-3.107	0.323	-3.198	0.291		
D_anz	-0.715	0.219	-0.635	0.380	-0.553	0.382	-0.568	0.362	-0.709	0.379
D_asean	-0.011	0.076	0.180	0.252	0.262	0.257	0.265	0.239	0.147	0.251
D_cac	-0.075	0.153	0.376	0.268	0.489	0.269	0.522	0.251	0.331	0.268
D_euro	0.118	0.044	0.213	0.175	0.253	0.171	0.265	0.164	0.275	0.175
D_mercsr	-0.114	0.076	0.889	0.258	1.169	0.254	1.146	0.238	0.927	0.258
D_nafta	-0.21	0.084	-0.022	0.307	0.093	0.296	0.122	0.282	-0.017	0.307
D_scucar	0.225	0.147	0.432	0.254	0.538	0.258	0.577	0.244	0.410	0.254
D_spr	0.593	0.205	1.097	0.252	1.115	0.259	1.114	0.241	1.016	0.252
D_usd	0.037	0.067	0.583	0.129	0.563	0.130	0.571	0.124	0.537	0.129
Agedep	0.533	0.145	-0.393	0.556	-0.419	0.553	-0.484	0.533	0.103	0.557
Agvagun	-0.01	0.002	-0.024	0.006	-0.025	0.006	-0.026	0.006	-0.021	0.006
Tractorpw	0.093	0.061	0.283	0.248	0.324	0.243	0.347	0.236	0.137	0.250
Labpop	-0.002	0.003	-0.018	0.011	-0.023	0.011	-0.025	0.010	-0.015	0.011
Life	-0.007	0.003	-0.013	0.011	-0.019	0.010	-0.021	0.010	-4.8E-04	0.011
Literate	1.1E-04	1.2E-04	-2.4E-04	3.9E-04	-2.4E-04	3.9E-04	-2.7E-04	3.8E-04	-3.5E-04	3.9E-04
Ntrvag2	-0.004	0.002	-0.010	0.008	-0.010	0.008	-0.010	0.007	-0.011	0.008
Blackind	0.051	0.034	0.315	0.069	0.317	0.068	0.328	0.065	0.302	0.069
Expg	-0.002	0.003	-0.007	0.006	-0.007	0.006	-0.007	0.005	0.001	0.006
Phones	0.001	1.8E-04	0.002	0.001	0.002	0.001	0.003	0.001	0.002	0.001
Radpccn	7.0E-06	7.0-06	-5.7E-05	2.3E-05	-6.4E-05	2.2E-05	-6.7E-05	2.2E-05	-4.0E-05	2.3E-05
Rurpop	-0.005	0.001	-0.005	0.005	-0.006	0.005	-0.006	0.005	-0.006	0.005
Tradegun	1.6E-04	0.002	0.001	0.003	0.001	0.003	0.001	0.003	-0.003	0.003
Manufexp	-0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.001	0.002
Manufimp	0.002	0.001	0.003	0.004	0.002	0.004	0.002	0.004	0.005	0.004
R^2	0.732									
lnL			-1.34E+7		-1.37E+7		-1.24E+7		-1.49E+4	
Benchmark Samples	449		449		449		449		339	
σ^2_η			6.600		9.000		7.000		6.600	
σ^2_u			4.500		4.200		4.000		4.500	
σ^2_ξ			0.800		0.000		1.000		1.000	
ϕ			0.590		0.450		0.400		0.590	

Table 2: Predicted PPP Series for Australia

YEAR	ER	ICP	UN	SE	UN-GRC(a)	CON	SE	No Reg-GRC(b)	No05-GRC(c)	PWT6.2
1971	0.883		**0.581**	0.026	**0.712**	**0.595**	0.031	**0.713**	**0.710**	0.771
1972	0.839		**0.614**	0.030	**0.745**	**0.630**	0.035	**0.745**	**0.742**	0.789
1973	0.703		**0.683**	0.034	**0.821**	**0.699**	0.040	**0.821**	**0.818**	0.838
1974	0.697		**0.736**	0.036	**0.878**	**0.749**	0.042	**0.878**	**0.874**	0.916
1975	0.764		**0.772**	0.036	**0.915**	**0.778**	0.042	**0.915**	**0.911**	0.973
1976	0.818		**0.830**	0.041	**0.964**	**0.843**	0.048	**0.964**	**0.960**	1.026
1977	0.902		**0.861**	0.043	**0.981**	**0.876**	0.050	**0.981**	**0.977**	1.041
1978	0.874		**0.887**	0.043	**0.995**	**0.901**	0.051	**0.995**	**0.991**	1.053
1979	0.895		**0.912**	0.042	**1.011**	**0.922**	0.048	**1.011**	**1.007**	1.062
1980	0.878		**0.925**	0.037	**1.015**	**0.928**	0.043	**1.016**	**1.011**	1.054
1981	0.870		**0.979**	0.040	**1.037**	**0.986**	0.046	**1.037**	**1.033**	1.068
1982	0.986		**1.053**	0.041	**1.077**	**1.064**	0.047	**1.078**	**1.073**	1.108
1983	1.110		**1.128**	0.038	**1.118**	**1.140**	0.044	**1.118**	**1.113**	1.150
1984	1.140		**1.173**	0.031	**1.129**	**1.183**	0.034	**1.130**	**1.125**	1.185
1985	1.430	1.240	**1.235**	0.011	**1.158**	**1.240**	0.000	**1.158**	**1.153**	1.236
1986	1.500		**1.285**	0.031	**1.213**	**1.292**	0.035	**1.214**	**1.208**	1.307
1987	1.430		**1.341**	0.039	**1.277**	**1.348**	0.044	**1.277**	**1.271**	1.353
1988	1.280		**1.400**	0.040	**1.345**	**1.406**	0.046	**1.345**	**1.339**	1.392
1989	1.260		**1.410**	0.034	**1.367**	**1.413**	0.037	**1.367**	**1.361**	1.414
1990	1.280	1.389	**1.391**	0.012	**1.361**	**1.389**	0.000	**1.362**	**1.356**	1.416
1991	1.280		**1.373**	0.030	**1.339**	**1.372**	0.033	**1.340**	**1.334**	1.406
1992	1.360		**1.363**	0.029	**1.325**	**1.364**	0.033	**1.326**	**1.320**	1.402
1993	1.470	1.350	**1.348**	0.011	**1.306**	**1.350**	0.000	**1.307**	**1.301**	1.393
1994	1.370		**1.325**	0.028	**1.295**	**1.326**	0.031	**1.296**	**1.290**	1.378
1995	1.350		**1.317**	0.027	**1.299**	**1.316**	0.030	**1.299**	**1.293**	1.374
1996	1.280	1.299	**1.300**	0.010	**1.291**	**1.299**	0.000	**1.292**	**1.286**	1.363
1997	1.350		**1.296**	0.026	**1.286**	**1.294**	0.029	**1.287**	**1.281**	1.366
1998	1.590		**1.285**	0.025	**1.273**	**1.284**	0.028	**1.274**	**1.268**	1.377
1999	1.550	1.297	**1.296**	0.010	**1.281**	**1.297**	0.000	**1.282**	**1.276**	1.374
2000	1.720		**1.323**	0.026	**1.314**	**1.323**	0.028	**1.315**	**1.309**	1.399
2001	1.930		**1.324**	0.025	**1.320**	**1.324**	0.028	**1.320**	**1.314**	1.398
2002	1.840	1.337	**1.337**	0.010	**1.337**	**1.337**	0.000	**1.337**	**1.331**	1.404
2003	1.540		**1.348**	0.025	**1.349**	**1.347**	0.028	**1.350**	**1.344**	1.389
2004	1.360		**1.365**	0.025	**1.366**	**1.365**	0.028	**1.367**	**1.361**	1.386
2005	1.309	1.390	**1.390**	0.010	**1.390**	**1.390**	0.000	**1.390**	**1.384**	

Growth Rates Preserved SE: *(a)* 0.010 *(b)* 0.011 *(c)* 0.051.

Table 3: Predicted PPP Series for China

year	ER	ICP	UN	SE	UN-GRC(*a*)	CON	SE	No Reg-GRC(*b*)	No05-GRC(*c*)	PWT6.2
1971	2.460		**0.780**	0.206	**3.086**	**0.768**	0.236	**3.083**	**2.765**	1.808
1972	2.250		**0.805**	0.281	**2.961**	**0.779**	0.318	**2.958**	**2.652**	1.733
1973	1.990		**0.824**	0.333	**2.808**	**0.786**	0.370	**2.805**	**2.515**	1.654
1974	1.960		**0.822**	0.363	**2.582**	**0.774**	0.397	**2.579**	**2.312**	1.473
1975	1.860		**0.806**	0.375	**2.331**	**0.750**	0.403	**2.329**	**2.088**	1.347
1976	1.940		**0.836**	0.402	**2.200**	**0.777**	0.430	**2.198**	**1.971**	1.216
1977	1.860		**0.866**	0.423	**2.091**	**0.800**	0.448	**2.089**	**1.873**	1.172
1978	1.680		**0.881**	0.431	**1.980**	**0.807**	0.451	**1.978**	**1.774**	1.134
1979	1.550		**0.899**	0.437	**1.893**	**0.814**	0.450	**1.891**	**1.696**	1.055
1980	1.500		**0.907**	0.434	**1.801**	**0.809**	0.440	**1.800**	**1.613**	0.997
1981	1.700		**0.920**	0.432	**1.685**	**0.818**	0.435	**1.683**	**1.509**	0.937
1982	1.890		**0.942**	0.431	**1.585**	**0.838**	0.434	**1.583**	**1.420**	0.900
1983	1.980		**0.997**	0.443	**1.541**	**0.888**	0.447	**1.539**	**1.380**	0.888
1984	2.320		**1.088**	0.468	**1.558**	**0.969**	0.472	**1.556**	**1.395**	0.878
1985	2.940		**1.250**	0.521	**1.665**	**1.116**	0.526	**1.663**	**1.491**	0.926
1986	3.450		**1.347**	0.543	**1.703**	**1.211**	0.553	**1.702**	**1.526**	0.952
1987	3.720		**1.445**	0.563	**1.741**	**1.308**	0.578	**1.740**	**1.560**	0.981
1988	3.720		**1.630**	0.613	**1.888**	**1.486**	0.633	**1.886**	**1.691**	1.101
1989	3.770		**1.769**	0.638	**1.979**	**1.619**	0.664	**1.977**	**1.773**	1.172
1990	4.780		**1.852**	0.638	**2.013**	**1.702**	0.667	**2.011**	**1.803**	1.142
1991	5.320		**1.958**	0.643	**2.076**	**1.809**	0.676	**2.074**	**1.860**	1.161
1992	5.510		**2.104**	0.657	**2.190**	**1.954**	0.695	**2.188**	**1.962**	1.235
1993	5.760		**2.425**	0.718	**2.491**	**2.262**	0.764	**2.489**	**2.231**	1.475
1994	8.620		**2.892**	0.812	**2.942**	**2.706**	0.868	**2.939**	**2.635**	1.755
1995	8.350		**3.243**	0.860	**3.279**	**3.043**	0.923	**3.276**	**2.938**	1.975
1996	8.310		**3.406**	0.848	**3.425**	**3.211**	0.915	**3.422**	**3.068**	2.074
1997	8.290		**3.401**	0.793	**3.420**	**3.219**	0.860	**3.417**	**3.063**	2.083
1998	8.280		**3.337**	0.721	**3.353**	**3.178**	0.788	**3.350**	**3.004**	2.059
1999	8.280		**3.254**	0.644	**3.264**	**3.122**	0.709	**3.261**	**2.924**	1.997
2000	8.280		**3.245**	0.582	**3.260**	**3.132**	0.645	**3.257**	**2.920**	1.963
2001	8.280		**3.229**	0.513	**3.249**	**3.138**	0.572	**3.246**	**2.910**	1.960
2002	8.280		**3.186**	0.433	**3.212**	**3.119**	0.484	**3.209**	**2.877**	1.941
2003	8.280		**3.215**	0.358	**3.230**	**3.169**	0.399	**3.227**	**2.893**	1.977
2004	8.280		**3.358**	0.270	**3.365**	**3.335**	0.295	**3.362**	**3.014**	2.145
2005	8.194	3.450	**3.448**	0.092	**3.448**	**3.450**	0.000	**3.445**	**3.089**	

Growth Rates Preserved SE: *(a)* 0.092 *(b)* 0.103 *(c)* 1.684.

Table 4: **Predicted PPP Series for India**

YEAR	ER	ICP	UN	SE	UN-GRC *(a)*	CON	SE	No Reg-GRC *(b)*	No05-GRC *(c)*	PWT6.2
1971	7.490		**2.359**	0.378	**4.329**	**2.418**	0.449	**4.330**	**3.069**	2.820
1972	7.590		**2.483**	0.476	**4.605**	**2.541**	0.566	**4.607**	**3.265**	2.933
1973	7.740		**2.743**	0.523	**5.143**	**2.789**	0.613	**5.145**	**3.646**	3.257
1974	8.100		**2.914**	0.454	**5.505**	**2.935**	0.512	**5.506**	**3.902**	3.641
1975	8.380	2.594	**2.607**	0.145	**4.951**	**2.594**	0.000	**4.953**	**3.510**	3.287
1976	8.960		**2.691**	0.415	**4.961**	**2.706**	0.467	**4.963**	**3.517**	3.144
1977	8.740		**2.747**	0.505	**4.925**	**2.775**	0.584	**4.927**	**3.492**	3.119
1978	8.190		**2.701**	0.496	**4.718**	**2.729**	0.574	**4.720**	**3.345**	3.027
1979	8.130		**2.968**	0.451	**5.044**	**2.986**	0.506	**5.046**	**3.576**	3.067
1980	7.860	3.104	**3.116**	0.169	**5.156**	**3.104**	0.000	**5.158**	**3.655**	3.099
1981	8.660		**3.305**	0.488	**5.198**	**3.318**	0.546	**5.199**	**3.684**	3.175
1982	9.460		**3.538**	0.620	**5.278**	**3.569**	0.716	**5.279**	**3.741**	3.166
1983	10.100		**3.904**	0.674	**5.528**	**3.946**	0.779	**5.530**	**3.919**	3.352
1984	11.400		**4.246**	0.600	**5.723**	**4.287**	0.675	**5.725**	**4.057**	3.444
1985	12.400	4.667	**4.640**	0.235	**5.952**	**4.667**	0.000	**5.954**	**4.219**	3.553
1986	12.600		**4.845**	0.721	**6.217**	**4.880**	0.804	**6.219**	**4.407**	3.660
1987	13.000		**5.161**	1.012	**6.607**	**5.200**	1.161	**6.609**	**4.684**	3.789
1988	13.900		**5.429**	1.226	**6.918**	**5.465**	1.416	**6.920**	**4.904**	4.047
1989	16.200		**5.707**	1.404	**7.221**	**5.734**	1.625	**7.223**	**5.119**	4.215
1990	17.500		**6.126**	1.591	**7.685**	**6.137**	1.839	**7.688**	**5.448**	4.435
1991	22.700		**6.823**	1.840	**8.452**	**6.829**	2.127	**8.455**	**5.991**	4.893
1992	25.900		**7.365**	2.024	**8.992**	**7.366**	2.340	**8.995**	**6.375**	5.239
1993	30.500		**8.008**	2.211	**9.624**	**8.005**	2.554	**9.627**	**6.822**	5.489
1994	31.400		**8.748**	2.404	**10.338**	**8.734**	2.774	**10.341**	**7.328**	6.048
1995	32.400		**9.510**	2.571	**11.040**	**9.486**	2.964	**11.044**	**7.826**	6.539
1996	35.400		**10.193**	2.684	**11.618**	**10.169**	3.094	**11.622**	**8.236**	6.868
1997	36.300		**10.807**	2.751	**12.171**	**10.769**	3.167	**12.175**	**8.628**	7.095
1998	41.300		**11.689**	2.841	**12.987**	**11.651**	3.271	**12.991**	**9.206**	7.510
1999	43.100		**12.241**	2.804	**13.398**	**12.221**	3.230	**13.403**	**9.498**	7.734
2000	44.900		**12.555**	2.673	**13.574**	**12.528**	3.076	**13.579**	**9.623**	7.845
2001	47.200		**12.809**	2.476	**13.670**	**12.783**	2.843	**13.674**	**9.690**	8.003
2002	48.600		**13.254**	2.237	**13.957**	**13.239**	2.559	**13.962**	**9.894**	8.116
2003	46.600		**13.730**	1.923	**14.198**	**13.720**	2.183	**14.203**	**10.065**	8.146
2004	45.300		**14.206**	1.453	**14.439**	**14.212**	1.605	**14.444**	**10.236**	
2005	44.272	14.670	**14.637**	0.502	**14.637**	**14.670**	0.000	**14.642**	**10.376**	

Growth Rates Preserved SE: *(a)* 0.502 *(b)* 0.562 *(c)* 5.331.

Table 5: **Predicted PPP Series for Nigeria**

YEAR	ER	ICP	UN	SE	UN-GRC *(a)*	CON	SE	No Reg-GRC *(b)*	No05-GRC *(c)*	PWT6.2
1971	0.713		**0.332**	0.050	**0.503**	**0.349**	0.060	**0.504**	**0.553**	0.551
1972	0.658		**0.329**	0.062	**0.497**	**0.346**	0.076	**0.498**	**0.545**	0.540
1973	0.658		**0.328**	0.069	**0.496**	**0.345**	0.085	**0.497**	**0.544**	0.562
1974	0.630		**0.432**	0.096	**0.655**	**0.451**	0.116	**0.656**	**0.719**	0.651
1975	0.616		**0.487**	0.109	**0.739**	**0.503**	0.131	**0.740**	**0.811**	0.797
1976	0.627		**0.533**	0.117	**0.799**	**0.551**	0.140	**0.800**	**0.877**	0.887
1977	0.645		**0.561**	0.115	**0.831**	**0.579**	0.138	**0.833**	**0.913**	0.863
1978	0.635		**0.601**	0.108	**0.885**	**0.616**	0.127	**0.886**	**0.971**	0.937
1979	0.604		**0.622**	0.086	**0.911**	**0.631**	0.096	**0.912**	**1.000**	1.085
1980	0.547	0.643	**0.644**	0.030	**0.939**	**0.643**	0.000	**0.940**	**1.031**	1.210
1981	0.618		**0.688**	0.097	**0.997**	**0.689**	0.109	**0.999**	**1.095**	1.087
1982	0.673		**0.669**	0.117	**0.965**	**0.671**	0.134	**0.966**	**1.059**	1.086
1983	0.724		**0.753**	0.135	**1.078**	**0.755**	0.155	**1.079**	**1.183**	1.222
1984	0.767		**0.854**	0.127	**1.215**	**0.852**	0.142	**1.217**	**1.333**	1.356
1985	0.894	0.860	**0.866**	0.047	**1.222**	**0.860**	0.000	**1.224**	**1.341**	1.354
1986	1.750		**0.883**	0.140	**1.178**	**0.879**	0.155	**1.179**	**1.293**	1.388
1987	4.020		**1.367**	0.286	**1.720**	**1.361**	0.325	**1.722**	**1.888**	1.891
1988	4.540		**1.696**	0.402	**2.018**	**1.686**	0.461	**2.021**	**2.215**	2.279
1989	7.360		**2.492**	0.628	**2.807**	**2.474**	0.722	**2.812**	**3.082**	2.752
1990	8.040		**2.718**	0.700	**2.896**	**2.695**	0.805	**2.901**	**3.179**	2.916
1991	9.910		**3.349**	0.859	**3.363**	**3.331**	0.990	**3.368**	**3.692**	3.491
1992	17.300		**6.394**	1.579	**6.036**	**6.382**	1.824	**6.046**	**6.626**	5.698
1993	22.100		**10.161**	2.321	**9.006**	**10.182**	2.679	**9.021**	**9.886**	7.220
1994	22.000		**13.556**	2.695	**11.270**	**13.616**	3.091	**11.287**	**12.371**	9.319
1995	21.900		**22.192**	3.352	**17.226**	**22.390**	3.753	**17.253**	**18.909**	19.146
1996	21.900	32.539	**32.029**	1.645	**23.141**	**32.539**	0.000	**23.178**	**25.403**	25.925
1997	21.900		**30.142**	4.489	**23.071**	**30.297**	5.017	**23.107**	**25.325**	25.910
1998	21.900		**26.751**	5.147	**21.551**	**26.712**	5.879	**21.585**	**23.657**	24.183
1999	92.300		**28.309**	6.119	**23.854**	**28.187**	7.032	**23.891**	**26.185**	26.616
2000	102.000		**36.661**	8.296	**32.256**	**36.364**	9.532	**32.306**	**35.407**	37.479
2001	111.000		**38.177**	8.560	**34.879**	**37.819**	9.821	**34.934**	**38.287**	40.613
2002	121.000		**37.696**	7.897	**35.610**	**37.375**	9.030	**35.666**	**39.090**	41.683
2003	129.000		**43.642**	8.003	**42.167**	**43.313**	9.074	**42.233**	**46.288**	49.188
2004	133.000		**50.027**	6.995	**49.272**	**49.758**	7.722	**49.349**	**54.086**	58.771
2005	131.274	60.000	**60.096**	2.894	**60.096**	**60.000**	0.000	**60.190**	**65.968**	

Growth Rates Preserved SE: *(a)* 2.894 *(b)* 3.240 *(c)* 30.916.

Table 6: **Predicted PPP Series for Hondura**

YEAR	ER	ICP	UN	SE	UN-GRC *(a)*	CON	SE	No Reg-GRC *(b)*	No05-GRC *(c)*	PWT6.2
1971	2.000		**0.861**	0.097	**1.274**	**0.910**	0.118	**1.201**	**1.306**	0.991
1972	2.000		**0.882**	0.126	**1.269**	**0.937**	0.156	**1.197**	**1.301**	0.975
1973	2.000		**0.901**	0.144	**1.266**	**0.958**	0.178	**1.194**	**1.298**	0.979
1974	2.000		**0.970**	0.163	**1.333**	**1.025**	0.201	**1.257**	**1.367**	1.064
1975	2.000		**0.961**	0.162	**1.296**	**1.005**	0.197	**1.222**	**1.328**	1.026
1976	2.000		**1.016**	0.166	**1.330**	**1.063**	0.201	**1.254**	**1.363**	0.973
1977	2.000		**1.098**	0.166	**1.403**	**1.143**	0.199	**1.323**	**1.438**	0.983
1978	2.000		**1.099**	0.145	**1.377**	**1.133**	0.171	**1.299**	**1.412**	1.006
1979	2.000		**1.148**	0.117	**1.418**	**1.167**	0.131	**1.337**	**1.453**	1.069
1980	2.000	1.202	**1.206**	0.042	**1.472**	**1.202**	0.000	**1.388**	**1.508**	1.075
1981	2.000		**1.229**	0.129	**1.442**	**1.252**	0.145	**1.359**	**1.478**	1.091
1982	2.000		**1.257**	0.181	**1.419**	**1.307**	0.213	**1.338**	**1.455**	1.053
1983	2.000		**1.341**	0.234	**1.460**	**1.419**	0.281	**1.377**	**1.497**	1.105
1984	2.000		**1.381**	0.273	**1.455**	**1.479**	0.333	**1.372**	**1.491**	1.099
1985	2.000		**1.453**	0.316	**1.485**	**1.571**	0.388	**1.401**	**1.523**	1.132
1986	2.000		**1.493**	0.353	**1.510**	**1.635**	0.439	**1.424**	**1.547**	1.081
1987	2.000		**1.509**	0.382	**1.511**	**1.671**	0.478	**1.424**	**1.549**	1.105
1988	2.000		**1.565**	0.418	**1.555**	**1.748**	0.527	**1.467**	**1.594**	1.120
1989	2.000		**1.626**	0.456	**1.604**	**1.827**	0.575	**1.513**	**1.645**	1.172
1990	4.110		**1.905**	0.558	**1.872**	**2.150**	0.705	**1.765**	**1.919**	1.378
1991	5.320		**2.328**	0.710	**2.280**	**2.642**	0.900	**2.150**	**2.337**	1.677
1992	5.500		**2.490**	0.788	**2.431**	**2.838**	1.001	**2.293**	**2.492**	1.820
1993	6.470		**2.771**	0.906	**2.700**	**3.171**	1.152	**2.546**	**2.768**	2.137
1994	8.410		**3.508**	1.187	**3.408**	**4.024**	1.510	**3.214**	**3.494**	2.673
1995	9.470		**4.304**	1.503	**4.171**	**4.950**	1.915	**3.933**	**4.276**	3.189
1996	11.700		**5.206**	1.874	**5.031**	**6.006**	2.390	**4.744**	**5.157**	3.895
1997	13.000		**6.229**	2.311	**6.051**	**7.191**	2.949	**5.706**	**6.203**	4.815
1998	13.400		**6.850**	2.618	**6.681**	**7.922**	3.345	**6.300**	**6.848**	5.324
1999	14.200		**7.510**	2.959	**7.347**	**8.709**	3.792	**6.928**	**7.531**	5.978
2000	14.800		**8.004**	3.252	**7.884**	**9.280**	4.173	**7.435**	**8.082**	6.302
2001	15.500		**8.384**	3.513	**8.312**	**9.721**	4.516	**7.838**	**8.520**	6.719
2002	16.400		**8.703**	3.760	**8.683**	**10.093**	4.848	**8.187**	**8.901**	6.916
2003	17.300		**9.176**	4.095	**9.162**	**10.633**	5.294	**8.639**	**9.392**	7.337
2004	18.200		**9.693**	4.466	**9.685**	**11.224**	5.794	**9.132**	**9.927**	7.986
2005	19.000		**10.337**	4.918	**10.337**	**11.966**	6.410	**9.747**	**10.596**	

Growth Rates Preserved SE: *(a)* 4.918 *(b)* 5.461 *(c)* 5.052.