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Labor-Market Heterogeneity, Aggregation, and the Lucas Critique

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Labor-Market Heterogeneity, Aggregation, and the Lucas Critique

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Abstract

This paper assesses biases in policy predictions due to the lack of invariance of "structural" parameters in representative-agent models. We simulate data under various fiscal policy regimes from a heterogeneous-agents economy with incomplete asset markets and indivisible labor supply. Imperfect aggregation manifests itself through preference shocks in the estimated representative-agent model. Preference and technology parameter estimates are not invariant with respect to policy changes. As a result, the bias in the representative-agent model's policy predictions is large compared to the length of predictive intervals that reflect parameter uncertainty.

KEY WORDS: Aggregation, Fiscal Policy, Heterogeneous Agents Economy, Lucas Critique, Representative Agent Models.

JEL: C11, C32, E32, E62.

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1 Introduction

The Lucas critique of econometric policy evaluation (1976) argues that if econometric models do not capture the primitive parameters of preferences and technology, their coefficients can be expected to vary with changes in policy regimes. The quantitative work inspired by the Lucas critique has proceeded by replacing econometric models that were parameterized in terms of agents' decision rules with dynamic stochastic general equilibrium (DSGE) models in which parameters characterize the objective functions and constraints faced by representative economic agents. With these "deep" parameters in hand, it is possible to re-derive agents' decision rules under alternative economic policies. In recent years, estimated representative-agent DSGE models have been widely used to study the effects of monetary policy (e.g., Smets and Wouters, 2003, 2007; Christiano, Eichenbaum, and Evans, 2005) and fiscal policy (e.g., Forni, Monteforte, and Sessa, 2009; Leeper, Plante, and Traum, 2010) changes.

The tacit assumption underlying the DSGE model-based policy analysis has been that the parameters that characterize the preferences of a representative agent and the production technologies of a representative firm as well as the exogenous structural shocks are policy invariant. However, to the extent that macroeconomic time series on variables such as output, consumption, investment, and employment are constructed by aggregating across heterogeneous households and firms, the assumption of policy invariance is not self-evident. More than two decades ago, Geweke (1985, p. 206) pointed out that while the treatment of expectations and dynamic optimization has been careful, potential problems due to aggregation have usually been ignored: *"Whenever econometric policy evaluation is undertaken using models estimated with aggregated data, it is implicitly presumed that the aggregator function is structural with respect to the policy intervention."*

The goal of this paper is to assess the quantitative importance of biases in policy predictions due to the potential lack of invariance of preference and technology parameters in representative-agent models. To do so, we simulate data under various fiscal policy regimes from a heterogeneous-agents economy in which households have to insure themselves against idiosyncratic income risks (e.g., Bewley, 1983; Huggett, 1993; Aiyagari, 1994). Following Chang and Kim (2006), our model economy extends Krusell and Smith's (1998) heterogeneous-agents model with incomplete capital markets (Aiyagari, 1994) to indivisible labor supply (Rogerson, 1988).[1] The equilibrium outcomes depend on the cross-sectional distributions of households' wealth and earnings, which in turn depend on the policy regime. Due to the indivisible nature of labor supply at the micro level, aggregate labor supply depends on the shape of the cross-sectional reservation wage distribution rather than the individual households' willingness to substitute leisure across times.

The heterogeneous-agents economy is calibrated to match the cross-sectional distribution of wealth and earnings in the U.S. We simulate this economy with an aggregate productivity shock that is comparable to the time series of measured aggregate TFP in post-war U.S. data. Using aggregate times series on output, consumption, wages, and employment generated from the heterogeneous-agents model, we estimate a representative-agent model with state-of-the-art Bayesian methods (Schorfheide, 2000; An and Schorfheide, 2007) and examine the potential lack of policy invariance of the representative-agent model parameters. It turns out, using Geweke's expression, that the aggregator function is not invariant to policy changes.

More specifically, the quantitative analysis generates the following findings. First, the effects of imperfect aggregation manifest themselves through the presence of preference shocks in the representative-agent model. While it is common to include such preference shocks in the specification of estimable representative agent models, their interpretation is subject to controversy. Some researchers regard them as fundamental aggregate demand shocks that contribute to business cycle fluctuations (e.g., Smets and Wouters, 2007). Other authors view them as wedges in optimality conditions and thus as a sign of model misspecification (e.g., Chari, Kehoe, and McGrattan, 2007). According to a variance decomposition computed based on our estimated representative-agent model, the measured preference shocks explain more than 50% of the fluctuations of hours worked. Second, using the standards of the DSGE model estimation literature, the estimated representative-agent model fits the aggregate time series data from the heterogeneous-agents economy well. A posterior odds comparison with a more flexible vector autoregression (VAR) favors the structural model by a substantial margin.[2] Thus, as long as aggregate preference shocks are regarded as *a*

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[1] Both the theoretical and the empirical importance of incomplete asset markets and indivisible labor supply are by now widely recognized. See, for instance, Krusell and Smith (1998), Chang and Kim (2006), Ljungqvist and Sargent (2007), Nakajima (2007), Krusell, Mukoyama, Rogerson, and Sahin (2008), and Rogerson and Wallenius (2009).

[2] If a similar comparison were done based on actual U.S. data, for most DSGE models the posterior odds

priori plausible, the aggregate time series provide no evidence of model misspecification.

Third, if the representative-agent model is estimated with data from the heterogeneous-agents economy under different policy regimes, several important parameters vary considerably. For instance, the aggregate labor supply elasticity, often recognized as a crucial parameter for fiscal policy analysis (e.g., Auerbach and Kotlikoff, 1987; Judd, 1987; Prescott 2004), depends on the cross-sectional distribution of reservation wages, which in turn is a function of the fiscal policy regime. The average level of total factor productivity is also not policy invariant because fiscal policy affects labor-market participation and thereby the cross-sectional distribution of productivities of the employed workforce.

Finally, to assess the quantitative implications of the lack of policy invariance, we construct predictive distributions for the effects of fiscal policy changes on output, consumption, employment, and aggregate welfare based on the estimated representative-agent model under the benchmark fiscal policy, assuming that the preference and technology parameters are unaffected by the policy shifts. We find that the lack of policy invariance of the aggregator function is sufficiently strong enough to render predictions from the representative-agent model inaccurate. In particular, the prediction bias due to imperfect aggregation is substantially larger than the prediction intervals that reflect parameter estimation uncertainty.

While there exists a fairly extensive body of research on aggregation issues as well as on the Lucas critique, we will only briefly discuss two strands of the literature that are most closely related to this paper. First, calibrated heterogeneous-agents economies similar to the one in this paper have been used to assess equilibrium conditions derived from a representative-agent model in Chang and Kim (2006, 2007) and An, Chang, and Kim (2009). However, none of the three papers considers the (fiscal) policy invariance of the parameters in an estimated representative-agent model. Second, our analysis focuses on the cross-sectional heterogeneity on the household side and fiscal policies that distort households' labor supply and savings decision. Much of the literature on policy analysis with estimated DSGE models, however, focuses on monetary policy analysis in the context of New Keynesian models. For the propagation of monetary policy shifts the heterogeneity on the firm side, in particular with respect to pricing decisions, plays an important role.

Until now the literature on New Keynesian DSGE model has mostly focused on the question of whether the cost of changing nominal prices is invariant to, say, changes in the

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comparison would favor the VAR, e.g., Del Negro, Schorfheide, Smets, and Wouters (2007).

target inflation rate. Fernández-Villaverde and Rubio-Ramiréz (2007) estimate a model in which both monetary policy rule parameters and nominal rigidity parameters are allowed to vary over time. They find that during high inflation episodes, the estimated cost associated with nominal price changes is lower[3] and they interpret the negative correlation between policy and price-adjustment parameters as evidence against policy invariance. Cogley and Yagihashi (2009) conduct the following experiment. They simulate data under two monetary policy regimes from an economy in which firms are heterogeneous with respect to their price setting history and face some menu costs of nominal price adjustments. Based on the simulated data, the authors then use Bayesian methods to estimate an approximating model that assumes that firms are able to re-optimize their nominal prices with a fixed probability in every period as in Calvo (1983). The attractive feature of the Calvo mechanism is that a first-order approximation to the aggregate production function and the law of motion of inflation can be derived analytically. As in our analysis, Cogley and Yagihashi (2009) find that some of the preference and technology parameters of the approximating model are not policy invariant. However, policy recommendations derived from the approximating model still lead to good outcomes under the data-generating economy.

The remainder of the paper is organized as follows. Section 2 lays out the heterogeneous-agents economy that features incomplete capital markets and indivisible labor. We calibrate the model economy to match salient features of the cross-sectional income and wealth distribution in the U.S. as well as some key business cycle properties. Section 3 presents the representative-agent model that is estimated based on simulated data from the heterogeneous-agents economy and used to predict the effect of policy changes. The quantitative results are presented in Section 4. In Section 5 we repeat the quantitative analysis for model economies with divisible labor supply and complete asset markets, in order to distinguish the separate roles played by the two frictions considered in Section 4. Finally, Section 6 concludes. Detailed derivations for the representative-agent model as well data sources can be found in the Appendices.

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[3] In a Calvo model of nominal rigidity this cost can be interpreted as the probability with which a firm is unable to re-optimize its price.

2 Heterogeneous-Agents Economy

We provide a description of the heterogeneous-agents economy that serves as a data-generating mechanism for the quantitative analysis. The model economy is based on Chang and Kim (2007), which extends Krusell and Smith's (1998) heterogeneous-agents model with incomplete capital markets (Aiyagari, 1994) to indivisible labor supply (Rogerson, 1988). Due to the indivisible nature of labor supply the aggregate labor supply depends on the shape of the cross-sectional reservation wage distribution, which in turn is affected by the policy regime.

2.1 Economic Environment

Households: The model economy consists of a continuum (measure one) of worker-households who have identical preferences but different productivities *ex post*. Household-specific idiosyncratic productivity x_t varies exogenously according to a stochastic process with a transition probability distribution function $\pi_x(x'|x) = \Pr(x_{t+1} \leq x'|x_t = x)$. A household maximizes its utility by choosing consumption c_t and hours worked h_t:

$$\begin{aligned} \max \quad & \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \left\{ \ln c_{t+s} - B \frac{h_{t+s}^{1+1/\gamma}}{1+1/\gamma} \right\} \right] \qquad (1) \\ \text{s.t.} \quad & c_t + a_{t+1} = a_t + (1-\tau_H) W_t x_t h_t + (1-\tau_K) R_t a_t + \bar{T} \\ & a_{t+1} \geq \underline{a}, \quad h_t \in \{0,\ \bar{h}\}. \end{aligned}$$

Households trade assets a_t which yield the real rate of return R_t. These assets are either claims to the physical capital stock or IOUs, which are in zero net supply. Both asset types generate the same return R_t, which is subject to the capital tax τ_K.

Households face a borrowing constraint, $a_{t+1} \geq \underline{a}$, and supply their labor in an indivisible manner, that is, h_t either takes the value 0 or $\bar{h}$. We normalize the endowment of time to one and assume $\bar{h} < 1$. If a household supplies $\bar{h}$ units of labor, labor income is $W_t x_t \bar{h}$, where W_t is the aggregate wage rate for an efficiency unit of labor. Labor income is subject to the tax τ_H and $\bar{T}$ denotes lump-sum taxes or transfers. Ex post households differ with respect to their productivity and asset holdings. The joint distribution of productivity, x_t, and asset holdings, a_t, is characterized by the probability measure $\mu_t(a_t, x_t)$.

Firms: A representative firm produces output Y_t according to a constant-returns-to-scale Cobb-Douglas technology in capital, K_t, and efficiency units of labor, L_t.[4]

$$Y_t = F(L_t, K_t, \lambda_t) = \lambda_t L_t^{\alpha} K_t^{1-\alpha}, \tag{2}$$

where λ_t is the aggregate productivity shock with a transition probability distribution function $\pi_\lambda(\lambda'|\lambda) = \Pr(\lambda_{t+1} \leq \lambda'|\lambda_t = \lambda)$. The representative firm's profit function is:

$$\Pi_t = Y_t - W_t L_t - (R_t + \delta)K_t. \tag{3}$$

The first-order conditions for the profit maximization are

$$W_t = \alpha Y_t / L_t \quad \text{and} \quad (R_t + \delta) = (1-\alpha) Y_t / K_t. \tag{4}$$

The return on capital (net of depreciation), R_t, is subject to capital tax. The physical capital stock evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{5}$$

where I_t is aggregate investment and δ is the depreciation rate. The total factor productivity process λ_t is the only aggregate disturbance. While this feature of the model economy does not necessarily reflect our views about the sources of business cycle fluctuations, it makes the quantitative analysis more transparent. Since the aggregation error will show up as a preference shift in the representative-agent model, we intentionally exclude shocks that shift households' preferences, e.g., labor supply shocks, from the heterogeneous-agents economy.

Fiscal Policy: Fiscal policy in the model economy are characterized by labor and capital tax rates (τ_H and τ_K) as well as the level of lump-sum transfers ($\bar{T}$). We assume that transfers are constant over time and the government maintains a balanced budget in each period. The fiscal authority collects the revenue from income tax and spends it on fixed lump-sum transfers to households $\bar{T}$ or purchases of goods for its own consumption G_t:

$$\bar{T} + G_t = \tau_H W_t \int x_t h(a_t, x_t; \lambda_t, \mu_t)\, d\mu_t(a_t, x_t) + \tau_K R_t \int a_t d\mu_t(a_t, x_t). \tag{6}$$

In order to obtain total tax revenues we have to integrate over the distribution of household types over the measure $\mu_t(a_t, x_t)$. For simplicity, we assumed that government purchases

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[4] This implicitly assumes that workers are perfect substitutes for each other. While this assumption abstracts from reality, it greatly simplifies the labor-market equilibrium because we only need to clear the labor market through the total efficiency units of labor.

G_t do not affect the household's marginal utility from private consumption or leisure nor the production function of the representative firm. Due to the additional assumption that the lump-sum transfers are a constant fraction χ of the steady state tax revenue, that is,

$$\bar{T} = \chi \left(\tau_H \bar{W} \int x h(a, x; \bar{\lambda}, \bar{\mu}) \; d\bar{\mu}(a, x) + \tau_K \bar{R} \int a \, d\bar{\mu}(a, x) \right), \tag{7}$$

the time-varying level of government expenditures does not affect the decisions of households and firms, which greatly simplifies the solution of the model.

Further Equilibrium Conditions and Model Solution: Since IOUs are in zero net supply, the net supply of assets has to equal the capital stock. Moreover, in equilibrium the labor hired by the firms has to equal the total supply of efficiency units by the households:

$$K_t = \int a_t d\mu_t(a_t, x_t), \quad L_t = \int x_t h(a_t, x_t; \lambda_t, \mu_t) d\mu_t(a_t, x_t). \tag{8}$$

The aggregate resource constraint can be expressed as

$$Y_t = \int c(a_t, x_t; \lambda_t, \mu_t) d\mu_t(a_t, x_t) + I_t + G_t. \tag{9}$$

To solve for the competitive equilibrium fluctuation of the model economy, it is useful to express the households' optimization problem in recursive form. The state variables for the households' decision problem are its asset holdings a_t, its idiosyncratic productivity x_t, aggregate productivity λ_t, as well as the joint distribution of asset holdings and idiosyncratic productivities in the economy, $\mu_t(a_t, x_t)$. It is convenient to drop the time subscripts and use variables with a prime ($'$) to denote the next period's values. The value function for an employed household, denoted by V^E, is given by

$$\begin{aligned} V^E(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}} \Bigg\{ & \frac{c^{1-\sigma} - 1}{1 - \sigma} - B \frac{\bar{h}^{1+1/\gamma}}{1 + 1/\gamma} \\ & + \beta E \Big[\max \left\{ V^E(a', x'; \lambda', \mu'), V^N(a', x'; \lambda', \mu') \right\} \big| x, \lambda \Big] \Bigg\} \end{aligned} \tag{10}$$

subject to the constraints

$$c + a' = a + (1 - \tau_H) W x \bar{h} + (1 - \tau_K) R a, \quad a' \geq \underline{a},$$

$$\mu' = \mathbb{T}(\lambda, \mu),$$

where $\mathbb{T}(\cdot)$ denotes a transition operator that defines the law of motion for the distribution of household types $\mu(a, x)$. The value function for a not-employed household, denoted by

$V^N(a, x; \lambda, \mu)$, is defined similarly with $h = 0$. Then, the labor-supply decision is characterized by:

$$V(a, x; \lambda, \mu) = \max_{h \in \{0, \bar{h}\}} \left\{ V^E(a, x; \lambda, \mu), V^N(a, x; \lambda, \mu) \right\}.$$

The households' decision rules for consumption $c(\cdot)$, asset holdings $a(\cdot)$, and labor supply $h(\cdot)$ are functions of the individual-specific state variables a and x and the aggregate states λ and μ.

To reduce the dimensionality of the state-space we use the "bounded rationality" method developed by Krusell and Smith (1998). We replace the distribution $\mu(a, x)$ by a finite set of moments, assuming that agents make use of a finite set of moments of μ in forecasting aggregate prices. We also assume that agents forecast these moments $\mathbb{T}$ using a log linear form law of motion. As in Krusell and Smith (1998), we achieve a fairly precise forecast when we use the first moment of aggregate capital, K. A detailed description of the computational procedure can be found in Chang and Kim (2007).

2.2 Calibration

In order to simulate data from the heterogeneous-agents economy we have to specify values for the preference and technology parameters as well as the fiscal policy parameters. A summary is provided in Table 1. The unit of time is a quarter.

Firm Parameters: On the production side of the economy, we let capital depreciate at the rate $\delta = 0.025$ and set the capital share parameter $\alpha = 0.64$ to generate a labor share that is consistent with post-war U.S. data. The aggregate productivity shock, λ_t, is a discrete approximation of a continuous AR(1) process:

$$\ln \lambda_t = \rho_\lambda \ln \lambda_{t-1} + \sigma_\lambda \epsilon_{\lambda,t}, \quad \epsilon_{\lambda,t} \sim \mathcal{N}(0, 1). \tag{11}$$

We set $\rho_\lambda = 0.95$ and $\sigma_\lambda = 0.007$. These parameter values are obtained by fitting an AR(1) process to a de-trended measured TFP (e.g., Kydland and Prescott, 1982).

Household Parameters: On the household side, we assume that the idiosyncratic productivity x_t follows an AR(1) process:

$$\ln x_t = \rho_x \ln x_{t-1} + \sigma_x \epsilon_{x,t}, \quad \epsilon_{x,t} \sim N(0, 1). \tag{12}$$

The values of $\rho_x = 0.939$ and $\sigma_x = 0.287$ reflect the persistence and standard deviation of innovation to individual wages estimated from the PSID (see Chang and Kim, 2007).[5] According to the Michigan Time-Use survey, a working individual spends one-third of his discretionary time $\bar{h} = 1/3$. We set the intertemporal substitution elasticity of hours worked equal to $\gamma = 0.4$. Given all other parameters, we set the preference parameter B such that the steady-state employment rate is 60%, the average employment in our sample period. The discount factor β is chosen so that the quarterly rate of return to capital is 1% in the steady state. Finally, we let the borrowing constraint $\underline{a} = -2$. In our model this corresponds to half of the annual earnings of the household with average productivity, which is consistent with the average unsecured credit-limit-to-income ratio of U.S. households – 28% in 1992 and 47.5% in 1998 – reported by Narajabad (2010) based on data from Survey of Consumer Finances.

Fiscal Policies: Figure 1 depicts U.S. labor and capital tax rates, obtained from Chen, Imrohoroglu, and Imrohoroglu (2007). The capital tax rate fell from 45% to roughly 32% over the period from 1950 to 2003. Over the same time span the labor tax rate rose from about 22% to 30%. The ratio of transfer in total government expenditure, $\chi = T/(T+G)$, has shown a strong trend in the last half century. It rose from 22% in 1960 to 47% in 2010.[6] For our benchmark economy we choose fiscal policy in 1984, the midpoint of our sample ($\tau_H = 0.29$, $\tau_K = 0.35$, $\chi = 0.36$).

In addition to our benchmark fiscal policy, we consider 5 alternative fiscal policy regimes in Section 4: (i) low labor income tax ($\tau_H = 0.22$), (ii) high capital income tax ($\tau_K = 0.47$), (iii) higher ratio of lump-sum transfer in government expenditure ($\chi = 0.5$), (iv) the 1960 fiscal policy ($\tau_H = 0.229$, $\tau_K = 0.443$, $\chi = 0.224$), and (v) the 2004 fiscal policy ($\tau_H = 0.269$, $\tau_K = 0.327$, $\chi = 0.417$). These values, respectively, correspond to the lower or upper bound, or the beginning or end point of U.S. fiscal policy during the sample period.

Implications: Since the goal of our analysis is to determine the magnitude of aggregation

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[5] Chang and Kim (2007) restrict the household sample to those with a household head between 35 and 55 years of age with a high school education to avoid the fixed effect in wages. With this restricted sample, the estimates are $\rho_x = 0.929$ and $\sigma_x = 0.227$. Here, however, we use the whole sample of PSID, ages 18 to 65, to encompass the overall distribution of wages and obtain a larger shock for idiosyncratic productivity.

[6] We compute this ratio based on the government consumption (NIPA3.1 Line 16) and net government social benefits to persons (NIPA3.1 Line 19 - Line 13) with the caveat that in reality the government transfer payments are not made in a lump-sum fashion and distributed equally to all households.

biases in policy predictions, it is desirable for the model economy to possess a realistic amount of heterogeneity, similar to that in the U.S. data. We compare the cross-sectional distributions of earnings and wealth – two important observable dimensions of heterogeneity in the labor market – found in the model and in the data. Table 2 summarizes both the PSID (1984 survey) and the model's detailed information on wealth and earnings. Family wealth in the PSID reflects the net worth of houses, other real estate, vehicles, farms and businesses owned, stocks, bonds, cash accounts, and other assets. For each quintile group of the wealth distribution, we calculate the wealth share, ratio of group average to economy-wide average, and the earnings share.

In both the data and the model, the poorest 20% of families in terms of wealth distribution were found to own virtually nothing. In fact their share of wealth is negative, indicating that they are net borrowers, potentially constrained in their consumption. The PSID found that households in the 2nd, 3rd, 4th, and 5th quintiles own 0.50, 5.06, 18.74, and 76.22% of total wealth, respectively, while, according to the model, they own 3.27, 11.38, 24.74, and 62.17%, respectively. The average wealth of those in the 2nd, 3rd, 4th, and 5th quintiles is, respectively, 0.03, 0.25, 0.93, and 3.81 times larger than that of a typical household, according to the PSID. These ratios are 0.16, 0.57, 1.24, and 3.11 according to our model. Households in the 2nd, 3rd, 4th, and 5th quintiles of the wealth distribution earn, respectively, 11.31, 18.72, 24.21, and 38.23% of total earnings, according to the PSID. The corresponding groups earn 15.76, 19.97, 23.72, and 30.81%, respectively, in the model. While the model economy cannot generate an extreme concentration of wealth observed in the data,[7] we deduce that it possesses a reasonable degree of heterogeneity, thus making it possible to study the effects of aggregation in the labor market.

Table 3 reports the second moments (standard deviation and correlation) of aggregate output, consumption, and hours generated from the heterogeneous-agents economy and post-war U.S. economy. Data definitions for the U.S. time series are provided in Appendix B. Since the representative-agent model (presented in the next section) accommodates a deterministic balanced-growth path, we remove a linear trend from log output and consumption. Since the model economy allows for an aggregate productivity shock only, the aggregate output of the model exhibits only about three-quarters of the volatility of actual

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[7] For example, in the PSID, the top 5% of households own 46% of total wealth, whereas in our model that group owns 25.5% of total wealth.

output. Consumption is as volatile as that in the data. A striking difference is the standard deviation of hours. It is three times more volatile in the actual data than it is in the simulated data. This is in part due to the low-frequency movement in labor supply, not captured in the model economy. In fact, the volatility of hours in the model-generated data is about half as volatile as the standard deviation of actual Hodrick-Prescott-filtered hours, which removes the low frequency variation. Output, consumption, and hours are all positively correlated. The correlations between output and hours as well as between consumption and hours are slightly stronger in the simulated data than they are in the U.S. data. While the main purpose of our analysis is not to match the business cycle statistics of aggregate time series, the heterogeneous-agents economy is successful in replicating some salient business cycle features.

3 A Representative-Agent Model

In this section we describe a representative-agent model through which we will interpret the equilibrium outcome of the heterogeneous-agents economy. We will estimate the fundamental preference and technology parameters of the representative-agent model using the aggregate time series generated from the heterogeneous-agents economy. In other words, we look for the parameters that best approximate the underlying heterogenous-agents economy. We then use the estimated representative-agent model to predict the effects of alternative fiscal policies and compare those with the actual equilibrium outcome from the heterogeneous-agents economies under those policies.

3.1 Model Specification

The households in the model economy specified in Section 2 have identical preferences but exhibit *ex post* heterogeneity with respect to idiosyncratic productivity and asset holdings. We now replace the heterogeneous, borrowing-constrained households with a stand-in representative household that solves the following problem:

$$\begin{aligned} \max \quad & \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^{t+s} Z_{t+s} \left\{ \ln C_{t+s} - \frac{(H_{t+s}/B_{t+s})^{1+1/\nu}}{1+1/\nu} \right\} \right] \qquad (13) \\ \text{s.t.} \quad & C_t + K_{t+1} = K_t + (1-\tau_H) W_t H_t + (1-\tau_K) R_t K_t + \bar{T}. \end{aligned}$$

Because of incomplete capital markets and the indivisible nature of the labor supply, households' preferences in the heterogeneous-agents economy do not aggregate exactly to (13). Chang and Kim (2007) document that the lack of exact aggregation leads to a wedge between the marginal product of labor and the marginal rate of substitution. This labor-market wedge is also well documented in the U.S. data, e.g., Hall (1997), and often interpreted as an intratemporal aggregate labor supply shock, which we denote as B_t in (13). Scheinkman and Weiss (1986), Krüger and Lustig (2007), and Liu, Waggoner, and Zha (2008) show that capital market incompleteness can lead to a stochastic term in aggregate preferences that affects the intertemporal first-order condition of the stand-in representative household. Thus, we introduce a second preference "shock" Z_t in (13).

As is common in the literature on estimated DSGE models (e.g., Smets and Wouters 2003, 2007), we assume that both preference shifters follow independent autoregressive processes:

$$\begin{aligned} \ln(B_t/\bar{B}) &= \rho_B \ln(B_{t-1}/\bar{B}) + \sigma_B \epsilon_{B,t}, \quad \epsilon_{B,t} \sim N(0,1) \\ \ln Z_t &= \rho_Z \ln Z_{t-1} + \sigma_Z \epsilon_{Z,t}, \quad \epsilon_{Z,t} \sim N(0,1). \end{aligned} \tag{14}$$

It is important to note that the law of motion in (14) is not derived from the underlying aggregation problem, but rather reflects a commonly made assumption in the empirical literature. We also anticipate that the aggregate labor supply elasticity, denoted by ν, can be different from the micro elasticity of household labor supply γ, that appears in (1). The representative household owns the capital stock and its budget constraint resembles that of the households at the micro-level. As in Section 2, the return R_t is defined in excess of the depreciation rate δ and the evolution of the capital stock is given by (5).

The production technology in the representative-agent model is of the Cobb-Douglas form, identical to the one used in the heterogeneous-agents economy:

$$Y_t = A_t H_t^{\alpha} K_t^{1-\alpha}, \tag{15}$$

where technology evolves according to the AR(1) process

$$\ln(A_t/\bar{A}) = \rho_A \ln(A_{t-1}/\bar{A}) + \sigma_A \epsilon_{A,t}, \quad \epsilon_{A,t} \sim N(0,1). \tag{16}$$

The first-order conditions for the firm's static profit maximization are identical to (4) except that L_t needs to be replaced by H_t. The produced output is either consumed by the representative household, invested to accumulate capital, or consumed by the government. Thus, the aggregate resource constraint takes the form

$$Y_t = C_t + I_t + G_t \tag{17}$$

and resembles (9). Finally, as in the heterogeneous-agents economy the government uses its tax revenues for transfers $\bar{T}$ and purchases G_t, maintaining a balanced budget:

$$\bar{T} + G_t = \tau_H W_t H_t + \tau_K R_t K_t. \tag{18}$$

To construct an approximate solution to the representative-agent model, we log-linearize the equilibrium conditions around the deterministic steady state and apply a standard solution method for a linear rational expectations model.

3.2 Econometric Analysis

We will use Bayesian techniques developed in Schorfheide (2000) and surveyed in An and Schorfheide (2007) in Section 4 to estimate the representative-agent model based on aggregated data from the heterogeneous-agents economy. As observables we use log levels of output Y_t, consumption C_t, and employment E_t, where

$$C_t = \int c(a_t, x_t; \lambda_t, \mu_t) d\mu_t(a_t, x_t), \quad E_t = (1/\bar{h}) \int h(a_t, x_t; \lambda_t, \mu_t) d\mu_t(a_t, x_t).$$

Since α and δ, are easily identifiable based on long-run averages of the labor share, and the investment-capital ratio, we fix these parameters in the estimation using the "true" values reported in Table 1.[8] Moreover, we assume that the econometrician knows the "true" fiscal policy parameters (τ_H, τ_K, and χ). Finally, we also fix the autocorrelation of the inter-temporal preference shock process (ρ_Z) to 0.99.[9]

Bayesian inference combines a prior distribution with a likelihood function to obtain a posterior distribution of the model parameters. Marginal prior distributions for the remaining parameters of the representative-agent model are provided in Table 4. Our prior

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[8] We conducted the Bayesian inference based on non-dogmatic priors elicited from beliefs about steady-state relationships as in Del Negro and Schorfheide (2008). The results were essentially the same as the ones reported below.

[9] A preliminary analysis generated estimates of this parameter that were always very close to one. Fixing the parameter improved the numerical performance of the Bayesian computations.

is diffuse with respect to the coefficients determining the law of motion of the exogenous shocks and assigns a high probability to the event that the annualized real interest rate lies between 0 and 8% and the aggregate labor supply elasticity falls into the interval from 0 to 2. The joint prior distribution for all DSGE model parameters is obtained simply by taking the product of the marginals.

4 Quantitative Results

We consider three main questions in our quantitative analysis. First, we estimate the representative-agent model that best approximates the aggregate times series generated from the calibrated heterogeneous-agents economy. As is explained above, we anticipate that aggregation error manifests itself through preference shocks of a representative household. We also compare the estimates based on simulated data to those from the U.S. economy. Second, we study to what extent the parameters of the representative-agent model are invariant to changes in fiscal policy. We do so by re-estimating the representative-agent model using the data generated from the heterogeneous-agents model under different policy regimes. Finally, we use the representative-agent model parameter estimates to predict the effect of new policies assuming that taste and technology parameters are policy-invariant. We assess the accuracy of these predictions by comparing them to the true equilibrium outcomes from the heterogeneous-agents models.

4.1 Benchmark Estimates of the Representative-Agent Model

We begin by fitting the representative-agent model using the aggregate output, consumption, and employment generated from the heterogeneous-agents economy under the benchmark fiscal policy. Posterior estimates based on 200 and 2,500 aggregate observations are reported in Table 5. The sample size of 200 observations would correspond to 50 years of quarterly observations. While the larger sample size of 2,500 is unrealistic, the consistency property of Bayes estimators implies that the resulting parameter estimates are very close to the pseudo-true representative-agent model parameters that minimize the information-theoretic Kullback-Leibler distance between the approximating representative-agent model and the data-generating heterogeneous-agents economy. In addition, we also report estimates based on actual U.S. aggregate data from 1964:I to 2006:IV.

Our discussion of the estimation will highlight the following four findings: (i) the estimation of the representative-agent model detects sizeable preference shocks. (ii) With these preference shocks the estimated representative-agent model fits the aggregate output, consumption, and employment data well in comparison with a VAR. (iii) The estimated aggregate labor supply elasticities are related to the slope of the reservation wage distribution in the heterogeneous-agents economy. (iv) Due to a composition effect of the labor force, measured total factor productivity A_t in the representative-agent model differs from the underlying technology shock λ_t in the heterogeneous-agents economy. Findings (iii) and (iv) will be very important for understanding the outcomes of the subsequent policy experiments.

Preference Shocks: Although there are no aggregate preference shocks in the underlying heterogeneous-agents economy, the representative-agent model estimation detects both intratemporal (B_t) and intertemporal (Z_t) preference shocks. For example, for the sample of 2,500 observations the estimated $\ln B_t$ has an autocorrelation coefficient of 0.92 with a standard deviation of innovation of 0.3%. The overall volatility (unconditional standard deviation) is 0.77%, about one-third of the volatility of aggregate consumption in the heterogeneous-agents economy. The estimated overall volatility of $\ln Z_t$ is 2.13%, similar to that of aggregate consumption. A variance decomposition of the observables, based on the *a priori* assumption of uncorrelated shocks, is provided in Table 6. Jointly, the two preference shocks account for about 10% of the variation in output and consumption, and more than 38% of the variation in hours worked. While we estimated the representative-agent model subject to the assumption that all three shock processes are uncorrelated at all leads and lags, it turns out that *ex post* the correlation between the technology process and the intratemporal (intertemporal) preference shocks is 0.30 (0.2).[10]

The variance decomposition based on actual U.S. data assigns even more importance to the intratemporal preference shock, since it explains almost 50% of the fluctuations in output and consumption and almost all of the variation in employment. To the extent that U.S. business cycles are driven by other demand shocks, it is probably not surprising that the preference shock plays a larger role in the actual data. The overall role of the intertemporal

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[10] Our estimates complement the findings in Chang and Kim (2007), who construct a time series for $\ln B_t$ directly as the wedge between the marginal product of labor and the marginal rate of substitution and then study its cyclical properties.

shock Z_t, estimated based on simulated data, appears to be much smaller than those based on U.S. data (or those in the literature). The intertemporal preference shock, Z_t, captures mis-specification of the consumption Euler equation. The fact that our estimation excludes the use of asset returns might explain the muted role of this shock. While it is difficult to make direct comparisons with the literature that estimates richer DSGE models or employs alternative empirical methods, a substantial variation of preference shocks for employment or hours worked seems broadly in line with recent studies by Hall (1997) and Chari, Kehoe, McGrattan (2007), and Justiniano, Primiceri, and Tambalotti (2010). In sum, our results indicate the need for caution in interpreting the measured preference shocks: the preference shocks often measured from the aggregate time series analysis may reflect the specification error (e.g., aggregate error) rather than a fundamental driving force behind business cycles.

Time Series Fit: For the subsequent policy experiments with the representative-agent model to be plausible, it is desirable that it be able to track the simulated aggregate time series. We therefore compute the posterior odds of the estimated representative-agent model relative to a VAR(4) with Minnesota prior. Such comparisons are commonly used to assess the overall time series fit of estimated DSGE models, as discussed in Del Negro, Schorfheide, Smets, and Wouters (2007). We find that the posterior odds based on the sample of 200 observations favor the structural representative-agent model over the VAR by e^{23}. This indicates that the cross-equation restrictions embodied in the representative-agent model are well specified in view of the more flexible but less parsimonious VAR. If the same calculation is repeated for actual U.S. data, the odds shift to e^{46} in favor of the VAR. More generally, unlike most DSGE models that have been estimated in the literature based on actual data, the representative-agent model fits the aggregate time series simulated from the heterogeneous-agents model very well, at least if one regards aggregate preference as *a priori* plausible.

Aggregate Labor Supply Elasticity: The 90% credible interval for the aggregate labor supply elasticity of a representative household (ν) ranges from 1.57 to 1.86 for the sample of 200 observations and from 2.01 to 2.26 for the sample of 2,500 observations. Thus, it is quite different from $\gamma = 0.4$ that we assumed for the individual households in (1). This point has been stressed in Chang and Kim (2006). In our heterogeneous-agents model, which features indivisible labor and incomplete markets, the aggregate elasticity is determined by the shape of reservation wage distribution, which we describe later, instead of the willingness of intertemporal substitution of leisure by individual households. It turns out that the estimated aggregate labor supply elasticity ($\hat{\nu} = 0.34$) based on U.S. data is much smaller than the estimates obtained from the simulated data.[11] Two salient features of the aggregate labor market of the U.S. economy are a high volatility of quantities (hours) relative to prices (productivity) and a lack of systematic correlation between hours and productivity. These features lead to estimates that imply a low aggregate labor supply elasticity and fairly large preference shocks. According to Table 6, the U.S. estimates imply that almost all of the variation in hours worked is due to preference shocks.

Composition Effect: The point estimates of ρ_A and σ_A imply that the (unconditional) standard deviation of the aggregate technology process in the representative-agent model is about 1.2%. The standard deviation of the productivity process λ_t in the heterogeneous-agents model, on the other hand, is about 2.2%. The discrepancy arises because of a composition effect in the workforce. In the heterogeneous-agents economy, newly hired workers during the expansion are, on average, less productive than existing workers, lowering the average productivity of the workforce. Vice versa, it is the low-productivity workers who leave the workforce during the recession. This composition effect of the workforce makes the measured aggregate productivity less volatile than the true aggregate technology. It also contributes to a larger estimate of aggregate labor supply because the measured hours worked (e.g., employment) exhibit a larger volatility than the total labor input in efficiency units. The composition effect is also well documented for actual U.S. data. For instance, Bils (1985) estimates, based on PSID data, that the average wage of newly hired workers is 19% lower than the average wage of existing workers.

4.2 Policy (In)variance of Model Parameters

We now investigate whether the parameters of the representative-agent model are invariant with respect to policy changes. To do so, the heterogeneous-agents economy is simulated under the alternative fiscal policies listed in Table 1. In these simulations the sequences of aggregate and idiosyncratic shocks are kept identical to those used for the benchmark analysis. Finally, the representative-agent model is re-estimated based on the new data sets.

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[11] A more detailed empirical analysis based on post-war U.S. data can be found in Rios-Rull, Schorfheide, Fuentes-Albero, Kryshko, and Santaeulalia-Llopis (2009).

If the representative-agent parameters were truly "structural," the parameter estimates should be the same (up to some estimation uncertainty), regardless of the policy regime.

The resulting posterior mean parameter estimates and 90% credible intervals for samples of 2,500 observations are reported in the top panel of Table 7. In order to understand how policy changes affect the parameter estimates, two pieces of information are useful. First, in the bottom panel of Table 7, we show how the steady states react to the policy changes. Second, in Figure 2 we plot pseudo aggregate labor supply schedules based on the steady-state reservation wage distribution, i.e., the inverse function of the cumulative wage distribution, for the various fiscal policy regimes. Each curve represents the employment rate (on the x-axis) at a given wage rate (y-axis). The vertical line denotes the steady-state level of employment under each policy regime. The reservation-wage schedules shed light on why changes in fiscal policy may affect aggregate labor supply estimates.

A few observations stand out. First, while the estimate of the preference parameter $\ln \bar{B}$ is fairly stable across policy regimes, there is considerable variation – 90% credible intervals do not overlap – in the estimates of average log productivity, $\ln \bar{A}$, the aggregate labor supply elasticity ν, and the implicit steady-state interest rate r_A, which determines the discount factor β. Second, the estimated aggregate shock processes (not shown in the table) are also sensitive to the policy regime. Third, the second panel of Table 7 documents that the average labor productivity in the heterogeneous-agents economy falls – due to the compositional effect in the workforce discussed above – whenever total employment increases. Fourth, Figure 2 indicates that the aggregate labor supply schedule in the heterogeneous-agents economy becomes steeper toward the full employment level, as the economy moves toward the right tail of the reservation wage distribution. This pattern is mirrored in the labor-supply elasticity estimates generated with the representative-agent model.[12] We now consider the labor tax cut, the rise in the capital tax rate, and the increase in transfers in more detail. The 1960 and the 2004 policy lead to a combination of the effects described subsequently.

Labor Tax Cut: When the labor income tax rate is lowered (to $\tau_H = 0.22$), the employ-

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[12] The representative-agent-based estimate of the labor supply elasticity is not identical to the slope of the reservation wage distribution in the heterogeneous-agents economy. The calculation based on the slope of the reservation wage distribution assumes that the entire wealth-earnings distribution remains unchanged, whereas the aggregate productivity shock shifts the wealth-earnings distribution over time.

ment rate increases by almost 7% (from 60% to 63.8%). Because of the tax cut, the total tax revenue decreases (not reported). Given the fixed proportion of lump-sum transfers ($\chi = 0.36$), each household receives fewer lump-sum transfers and increases the precautionary savings motive. Higher labor input reinforces the accumulation of capital given the complementarity between capital and labor. As a result, the aggregate capital stock rises by 6% (from 15.17 to 16.07), lowering an equilibrium annual interest rate from 4% to 3.68%. Aggregate output increases about 4% (from 1.48 to 1.53). The measured average labor productivity decreases by 3% (from 2.46 to 2.39) due to compositional effect. Finally, the employment increase raises the slope of the reservation in the neighborhood of the steady state and thereby lowers the implicit labor supply elasticity. In order for the representative agent model to capture the composition effect and precautionary savings, the estimates of the discount rate, r_A, and average productivity $\ln \bar{A}$ have to fall, as in the second column of Table 7. As the average employment rate rises with the labor tax cut, the economy moves toward a thinner part of the reservation wage distribution, requiring the labor supply elasticity ν, of the representative agent model to decrease.

Rise in Capital Tax Rate: When we increase the capital income tax rate from $\tau_K = 0.35$ to $\tau_K = 0.47$, the equilibrium employment rate remains essentially unchanged. Thus, the workforce composition effect is not operational. A high capital tax, however, decreases savings and results in a decrease in the capital stock of 8% (from 15.2 to 14.0), raising the equilibrium interest rate from 4% to 4.76%. Unlike in the case of the labor tax cut, the estimates of the representative-agent parameters are more or less unaffected by the policy change. The basic channels through which the capital tax increase operates are well captured by the steady-state relationships in the representative-agent model.

More Transfers: The increase in the ratio of lump-sum transfers in government expenditures from $\chi = 0.36$ to 0.5 generates a negative income effect on the labor supply, decreasing the employment rate to 57%. A larger transfer also discourages the precautionary motive of savings, decreasing aggregate capital stock by 3% (from 15.2 to 14.76), and slightly raises the equilibrium interest rate. Labor productivity, however, increases as the employment rate decreases because less-productive workers retreat from the labor market. Finally, the heterogeneous-agents economy moves to a point in the reservation wage distribution with a flatter slope, implying a larger labor supply elasticity. Again, these changes (such as the composition effect in the workforce and the change in the precautionary savings motive) are captured in the representative-agent model by bigger estimates of r_A, ν, and $\ln \bar{A}$.

4.3 Accuracy of Policy Predictions

In order to assess the quantitative importance of the policy dependence of the parameters of the representative-agent model, we now examine the accuracy of the policy predictions that the representative-agent model delivers under the assumption that its parameters are unaffected by policy interventions. To do so, we construct posterior predictive distributions for the effects of the policy changes based on the model estimated under the benchmark fiscal policy. We consider the percentage change in long-run aggregate output, consumption, and employment as well as the overall welfare effect induced by the policy change. Before discussing the quantitative results, some more details regarding our measure of welfare are provided.

Welfare Measure: An important advantage of DSGE models over reduced-form models, such as vector autoregressions, is the welfare analysis. Following Aiyagari and McGrattan (1998), we define the social welfare as:[13]

$$\mathcal{W} = \int V(a,x) d\mu(a,x), \tag{19}$$

where $\mu(a,x)$ is the steady-state joint distribution of asset holdings and idiosyncratic productivity and $V(a,x)$ is the value function associated with the optimal decisions, i.e.,

$$V(a,x) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c(a_t,x_t) - B\frac{h(a_t,x_t)^{1+1/\gamma}}{1+1/\gamma} \right\}. \tag{20}$$

$c(a,x)$ and $h(a,x)$ are the optimal decision rules for an individual whose asset holdings are a and idiosyncratic productivity is x. This is a utilitarian social welfare function that measures the *ex ante* welfare in the steady state—i.e., the welfare of an individual before the realization of initial assets and productivity, which is drawn from the steady-state distribution $\mu(a,x)$. We measure the welfare gain or loss due to a policy change by the constant percentage change in consumption each period for all individuals which is required

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[13] This measure of social welfare or its variants have been widely used in the literature. Examples include Domeij and Heathcote (2004), Young (2004), Pijoan-Mas (2005), Heathcote, Storesletten and Violante (2008) and Rogerson (2009). Detailed justifications for this welfare measure are provided in Aiyagari and McGrattan (1998).

to equate the social welfare before and after the policy change. Specifically, we compute Δ that solves

$$\begin{aligned}\int \left\{ \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \ln\big((1+\Delta) c_0(a_t, x_t)\big) - B \frac{h_0(a_t, x_t)^{1+1/\gamma}}{1+1/\gamma} \right\} \right] \right\} d\mu_0(a_t, x_t) \\ = \int \left\{ \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \ln c_1(a_t, x_t) - B \frac{h_1(a_t, x_t)^{1+1/\gamma}}{1+1/\gamma} \right\} \right] \right\} d\mu_1(a_t, x_t)\end{aligned} \tag{21}$$

where c_0, h_0, and μ_0 are consumption, labor supply, and steady-state distribution before the policy change and c_1, h_1, and μ_1 are those after the policy change. A positive Δ implies that average welfare improves upon a policy change. With the logarithmic utility, the welfare gain Δ can be expressed as

$$\Delta = \exp\big((\mathcal{W}_1 - \mathcal{W}_0)(1-\beta)\big) - 1, \tag{22}$$

where $\mathcal{W}_0$ and $\mathcal{W}_1$ represent social welfare before and after the policy change, respectively. In the representative-agent model the distribution $\mu(a, x)$ is degenerate and the computation of the welfare effect simplifies considerably:

$$\Delta = \exp\left(\ln\big(\bar{C}_1/\bar{C}_0\big) - B \frac{\bar{H}_1^{1+1/\gamma} - \bar{H}_0^{1+1/\gamma}}{1+1/\gamma} \right) - 1, \tag{23}$$

where $\bar{C}_0$ and $\bar{H}_0$ are the steady-state values of consumption and labor supply in the benchmark economy, while $\bar{C}_1$ and $\bar{H}_1$ are those in an economy with a different policy. One caveat is needed before we discuss the welfare comparison across policies. Our welfare analysis does not consider transition effects. We compare the welfare measures based on the steady-state ergodic distributions. The equilibrium of the representative-agent model is approximated with a first-order log-linearization, which is known to be fairly accurate for the stochastic growth model considered in this paper. Under this approximation the mean levels of output, consumption, and hours are identical to the steady-state levels. As described above, the welfare effect is calculated directly from the steady-state levels of consumption and hours.

Quantitative Findings: The quantitative results for the policy predictions are summarized in Table 8. The entries in the table refer to percentage changes relative to the benchmark values. The "true" policy effect is computed based on the new ergodic distribution of the heterogeneous-agents economy. The "90% interval" entries correspond to 90% predictive intervals computed based on the posterior distribution of the parameters of the representative-agent model obtained from 200 observations under the benchmark fiscal policy. These intervals reflect the uncertainty with respect to the "structural" parameters of the representative-agent model. With the widespread adoption of Bayesian methods in empirical macroeconomics, such predictive distributions are frequently used to conduct policy analysis under parameter (and model uncertainty) as, for instance, in Levin, Onatski, Williams, and Williams (2006). Moreover, the use of the predictive distribution allows us to relate the magnitude of the prediction biases due to lack of parameter invariance to the overall level of uncertainty associated with the predictions. Finally, we report "p-values," which indicate how far in the tails of a Gaussian approximation of the predictive distribution the realization of the policy effect lies.

Across the entries in Table 8 we find that the "true" effects of the policies, both with respect to the average level of output, consumption, and hours as well as with respect to households' welfare, lie almost always far outside the 90% intervals. Almost all of the p-values are essentially zero. Among the three "single-instrument" policy changes, the prediction of the effect of a capital tax increase is the most accurate. This is consistent with our previous finding that the parameter estimates under the high-capital-tax regime are very close to the ones under the benchmark fiscal regime. If the representative-agent model is used to rank the five alternative policies, its welfare predictions imply that the labor tax cut is the most beneficial and the capital tax increase is the worst policy. The welfare ranking based on the actual effects in the heterogeneous-agents economy, however, is quite different. The most favorable policy is the increase in transfers. The "1960 policy" is the worst, leading to a larger welfare loss than the pure capital tax increase. Thus, using Geweke's terminology, the lack of invariance of the aggregator function is sufficiently strong to render predictions from the representative-agent model inaccurate and the predicted rankings of policies incorrect.[14]

We now consider the prediction of the effects of an increase in government transfers from 36% to 50% of government spending in more detail. Due to the income effect, total hours worked decrease by 5.25% in the heterogeneous-agents economy. However, the representative-agent model predicts, under the assumption of policy-invariant preference and technology parameters, a decrease of only 3.04%, to 3.22%, with 90% probability. The representative-agent model under-predicts the employment effect because hours measured

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[14] This result is not affected by the estimates based on the large sample size (T=2500).

in efficiency units tend to move less than hours measured in physical units.

While aggregate consumption increases by 3.09% in the heterogeneous-agents economy, the representative-agent model only predicts a rise between 1.79% and 1.98%. The increased transfer tends to have a stronger effect on consumption of households near the borrowing constraint in the heterogeneous-agents model (less need for precautionary savings). Aggregate output decreases by 2.17% in the heterogeneous-agents economy, whereas, according to the representative-agent model, it is predicted to fall by 3.04% to 3.22%. The representative-agent model overpredicts the fall in output for two reasons. First, due to the composition effect, labor productivity rises in the heterogeneous-agents economy. Second, the aggregate capital stock decreases by more than the representative-agent model predicts, due to less need for precautionary savings.

Finally, the average welfare of households increases by 5.81% in the heterogeneous-agents economy. The welfare effect predicted by the representative-agent model, on the other hand, is much smaller. The 90% predictive interval ranges from 3.10% to 3.18%. The representative-agent model under-predicts the welfare gains because the increased transfer provides an additional insurance and reduces the need for precautionary savings in the heterogeneous-agents economy.

Remedies: The first-best approach to addressing the prediction inaccuracy is to work with a better model. In practice, of course, "true" models remain elusive and the best response is to try to model and measure the policy-relevant mechanisms and trade-offs as well as possible. For instance, while for the assessment of the capital tax change a careful modeling of labor-market heterogeneity was not particularly important, its inability to capture the effects of labor-market heterogeneity rendered the predictions of the representative-agent model with regard to labor tax and transfer changes grossly misleading.

From the perspective of a policy maker who has to make decisions based on imperfect models, our analysis indicates that the parameter uncertainty reflected in the formal Bayesian estimation of the representative-agent model captures only a small aspect of the policy maker's "risk." The results in Table 8 could be interpreted as the predictive intervals being too small because the possibility that preference and technology parameters may shift in response to a policy change is not being entertained. Let $\theta_{(np)}$ denote the non-policy-related preference and technology parameters of the representative-agent model. Moreover, let $\Delta\theta_{(np)}$ denote an intervention-induced shift in the preference and technology parameters. In order to account for the possibility of a parameter change, a policy maker could specify a conditional distribution of $\Delta\theta_{(np)}$ given $\theta_{(np)}$.

For concreteness, assume this distribution is independent normal and the standard deviations for our parameters r_A, ν, $\ln \bar{A}$, and $\ln \bar{B}$ are 0.09, 0.09, 0.002, and 0.003, respectively. These numbers correspond to he posterior standard deviations of the four parameters associated with the $T = 200$ estimates in Table 5. In Table 9 we compare the predictive intervals obtained with and without accounting for the possibility of a parameter shift. While the mean predictions do not change much, the predictive intervals become a lot wider in the latter case and encompass the "true" effects on consumption and output. The p-values now range from 0.06 to 0.32. To the extent that an econometrician has access to observations from different policy regimes, statistical techniques, such as the estimation of time-varying coefficients or regime-switching models, could be used to quantify the magnitude of potential parameter shifts. However, providing an operational procedure is beyond the scope of this paper and we leave it as a future research topic.

5 Alternative Model Economies and Measurements

According to our heterogeneous-agents economy, the interaction of two frictions – indivisible labor and incomplete capital markets – prevents the aggregation of individual households' optimality conditions to the optimality condition that arises in our representative-agent model. In order to examine the separate role of the two frictions, we repeat the analysis in Section 4 for an economy with incomplete capital markets but divisible labor and for an economy in which asset markets are complete but labor is indivisible. The third modification of our benchmark empirical analysis consists of using efficiency-adjusted aggregate hours instead of employment (or raw hours worked) as an observable when estimating the representative-agent model based on data from the economy with indivisible labor and incomplete asset markets.

5.1 Divisible Labor

The first alternative model economy we consider allows for divisible labor supply, but capital markets remain incomplete. This is essentially the same specification as in Krusell and

Smith (1998) with endogenous hours choice. The equilibrium of this economy can be defined similarly to that of the benchmark model with the worker's value function with divisible labor, $V^D(a, x; \lambda, \mu)$:

$$V^D(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}, h \in (0,1)} \left\{ \ln c - B\frac{h^{1+1/\gamma}}{1+1/\gamma} + \beta E\Big[V^D(a', x'; \lambda', \mu')\big|x, \lambda\Big] \right\}$$

subject to

$$c = (1-\tau_H)w(\lambda, \mu)xh + (1 + (1-\tau_K)r(\lambda, \mu))a + \bar{T} - a', \quad a' \geq \bar{a}, \quad \mu' = \mathbf{T}(\lambda, \mu).$$

We again estimate a representative-agent model that best approximates the aggregate time series generated from the heterogeneous-agents model. As Table 10 shows, the aggregate labor supply elasticity ν of a stand-in household is 0.37, very close to the elasticity of individual households, $\gamma = 0.4$. Moreover, the estimated standard deviations for the two preference shocks (not reported in the table) are very close to zero. This is consistent with a "near perfect" aggregation result by Krusell and Smith (1998) – a representative-agent model is a good approximation of the heterogeneous-agents economy with incomplete markets. A comparison of the entries in Table 10 and Table 5 indicates that for the divisible-labor economy the parameter estimates are much less sensitive to the tax policy than in our benchmark economy.[15] For instance, the estimate of $\ln \bar{A}$ is not at all affected by the policy regime, indicating that the divisibility of labor essentially eliminated the labor-force composition effect.

5.2 Complete Asset Markets

Our second auxiliary model economy has complete capital markets but labor supply is indivisible. Due to perfect risk sharing, agents enjoy the same level of consumption regardless of their employment status, productivity, or asset holdings.[16] The equilibrium of this economy is identical to the allocation made by a social planner who maximizes the equally weighted

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[15] Across model economies, the estimates of $\ln \bar{A}$ and $\ln \bar{B}$ reported in Table 10 differ across models. This is because we use different measures of labor input, namely, the number of actual hours worked for the divisible labor economy, the employment rate for the economy with indivisible labor, and the efficiency units of hours for the last economy.

[16] The distribution of workers is no longer a state variable in the individual optimization problem. Moreover, because of the ergodicity of the stochastic process for idiosyncratic productivity, the cross-sectional distribution of workers is always stationary.

utility of the population. The planner chooses the sequence of consumption $\{C_t\}_{t=0}^{\infty}$ and the cut-off productivity $\{x_t^*\}_{t=0}^{\infty}$ for labor-market participation. To ensure an efficient allocation, the planner assigns workers who have a comparative advantage in the market (more productive workers) to work. If a worker's productivity is above x_t^*, he supplies $\bar{h}$ hours of labor.

The planner's value function in the complete market, denoted by $V^C(K,\lambda)$, and the decision rules for consumption, $C(K,\lambda)$, and cut-off productivity, $x^*(K,\lambda)$, satisfy the following Bellman equation:

$$V^C(K,\lambda) = \max_{C,x^*}\left\{\ln C - B\frac{\bar{h}^{1+1/\gamma}}{1+1/\gamma}\int_{x_t^*}^{\infty}\phi(x)dx + \beta E\Big[V^C(K',\lambda')\big|\lambda\Big]\right\}$$

subject to aggregate resource constraint and tax policy parameters. The aggregate effective unit of labor is denoted by L where $L = \bar{h}\int_{x^*}^{\infty} x\phi(x)dx$ and $\phi(x)$ is the productivity distribution of workers. The planner chooses the cut-off productivity x^* so that:

$$\frac{1}{C}(1-\tau_H)F_L(K,L,\lambda)\bar{h}x^*\phi(x^*) = B\frac{\bar{h}^{1+1/\gamma}}{1+1/\gamma}\phi(x^*). \tag{24}$$

The left-hand side is the (society's) utility gain from assigning the marginal worker to production. There are $\phi(x^*)$ number of workers with productivity x^* in the economy. Each of them supplies $\bar{h}x^*$ units of effective labor, and the marginal product of labor is F_L. The right-hand side represents the disutility incurred by these workers. The key point here is that, under complete markets, the first-order condition for the choice between hours and consumption is *exactly* defined in terms of effective units of labor and wages at the aggregate level.

In theory, our estimation of a representative agent should reveal the preference of a social planner. However, since we use the employment rate (instead of efficiency unit of labor) in our estimation, the estimated parameters are still subject to a measurement error (mostly due to composition effects of the heterogeneous workforce). According to Table 10 the aggregate labor supply elasticity (ν) is estimated to be 1.42. First, as (24) illustrates, the aggregate elasticity of a social planner has nothing to do with individual households' intertemporal substitution elasticity of leisure, γ. Second, we estimate the labor supply elasticity in terms of employment, not in efficiency units of labor. Nevertheless, at the estimated parameters the intratemporal and intertemporal first-order conditions of the representative household hold almost exactly, and the estimated standard deviations of the preference shocks are very close to zero. In response to a policy change, the estimates of ν and $\ln \bar{A}$ change, but not as drastically as under the benchmark economy. The change of the $\ln \bar{A}$ estimate from -0.24 to -0.23 and -0.25, respectively, indicates the presence of the labor-force composition effect. The labor tax cut and the increase in transfer induce a change in the measure of total factor productivity of about 1%.

5.3 Efficiency Units of Hours

When we replace the employment rate by effective units of labor in the estimation of the representative-agent model, yet retain the indivisibility of labor and the market incompleteness in the underlying heterogeneous-agents economy, the estimates of ν, and $\ln \bar{A}$ become more stable across policies compared to the benchmark analysis. For example, with efficiency units of labor, the estimate for the aggregate elasticity of labor supply ($\hat{\nu}$) ranges between 0.4 and 0.62, implying a smaller aggregate labor supply elasticity.[17] The average level of aggregate productivity, $\ln \bar{A}$, remains close to zero across policies. We are now using actual labor input in the estimation and therefore measure the average level of "true" total factor productivity $\ln \lambda_t$, which is zero. Given the approximate invariance of parameters, the use of efficiency units might appear a promising alternative. However, in practice, it is extremely difficult, if not impossible, to obtain the efficiency unit measures of quantity (hours) and prices because it is almost impossible to capture *all* the heterogeneity using observed characteristics – a typical cross-sectional wage regression barely reaches R^2 of 0.4.

6 Conclusion

Representative-agent dynamic stochastic general equilibrium models are widely used for economic policy analysis. A key assumption in policy experiments is that fundamental parameters of the model such as taste and technology are invariant with respect to policy changes. We demonstrate that this is not always the case. We construct a heterogeneous-agents economy in which equilibrium outcomes depend on the distributions of wealth and earnings, which in turn depend on the policy regime. We estimate a representative-agent

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[17] Hansen (1993) obtains a similar result. He finds that hours in efficiency units (measured by demographic variables of the households from the CPS) move less than actual hours over the business cycle.

models that best approximates the aggregate times series generated from the heterogeneous-agents model. We find that (i) the aggregation error manifests itself as a preference shift of a representative household; (ii) taste and technology parameters in the representative-agent model are not policy invariant; and (iii) fiscal policy predictions from the representative-agent model are often inaccurate.

We demonstrate that the representative agent model that abstracts from cross-sectional heterogeneity can potentially mislead fiscal policy predictions. While it may not always be feasible to model the various types of heterogeneity explicitly, it is important to account for the possibility that preference and technology parameters of an estimated model may shift in response to policy changes. To the extent that an econometrician has access to observations from different policy regimes, statistical techniques, such as the estimation of time-varying coefficients or regime-switching models, could be useful to quantify the magnitude of potential parameter shifts.

References

Aiyagari, S. Rao (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, **109(3)**, 659-684.

Aiyagari, S. Rao, and Ellen McGrattan (1998): "The Optimum Quantity of Debt," *Journal of Monetary Economics*, **42**, 447-469.

An, Sungbae, Yongsung Chang, and Sun-Bin Kim (2009): "Can a Representative Agent Model Represent a Heterogeneous Agent Economy?" *American Economic Journal - Macroeconomics*, **1(2)**, 29-54.

An, Sungbae, and Frank Schorfheide (2007): "Bayesian Analysis of DSGE Models," *Econometric Reviews*, **26(2-4)**, 113-172.

Auerbach, Alan, and Lawrence Kotlikoff (1987): *Dynamic Fiscal Policy*, Cambridge U.K., Cambridge University Press.

Bewley, Truman (1983): "A Difficulty with the Optimum Quantity of Money," *Econometrica*, **51**, 1485-1504.

Bils, Mark. (1985): "Real Wages over the Business Cycle: Evidence from Panel Data," *Journal of Political Economy*, **93(4)**, 666-689.

Calvo, Guillermo (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, **12(3)**, 383-398.

Chang, Yongsung, and Sun-Bin Kim (2006): "From Individual to Aggregate Labor Supply: A Quantitative Analysis Based on a Heterogeneous Agent Macroeconomy," *International Economic Review*, **47(1)**, 1-27.

Chang, Yongsung, and Sun-Bin Kim (2007): "Heterogeneity and Aggregation: Implications for Labor-Market Fluctuations," *American Economic Review*, **97(5)**, 1939-1956.

Chari, V.V., Patrick Kehoe, and Ellen McGrattan (2007): "Business Cycle Accounting," *Econometrica*, **75(3)**, 781-836.

Chen, Kaiji, Ayse Imrohoroglu, and Selahattin Imrohoroglu (2007): "A Quantitative Assessment of the Decline in the U.S. Saving Rate," *Manuscript*, University of Southern California.

Cho, Jang-Ok (1995): "Ex-post Heterogeneity and the Business Cycle," *Journal of Economic Dynamics and Control*, **19(3)**, 533-551.

Christiano, Lawrence, Martin Eichenbaum, and Charles Evans (2005): "Nominal Rigidities and the Dynamic Effect of a Shock to Monetary Policy," *Journal of Political Economy*, **113**, 1-45.

Cogley, Timothy, and Takeshi Yagihashi (2009): "Are DSGE Approximating Models Invariant to Shifts in Policy?," *Manuscript*, New York University.

Del Negro, Marco, and Frank Schorfheide (2008): "Forming Priors for DSGE Models (and How It Affects the Assessment of Nominal Rigidities)," *Journal of Monetary Economics*, **55(7)**, 1191-1208.

Del Negro, Marco, Frank Schorfheide, Frank Smets, and Rafael Wouters (2007): "On the Fit of New Keynesian Models," *Journal of Business & Economics Statistics*, **25(2)**, 123-162.

Domeij, David, and Jonathan Heathcote (2004): "On the Distributional Effects of Reducing Capital Taxes," *International Economic Review*, bf 45(2), 523-554.

Fernández-Villaverde, Jesús, and Juan F. Rubio-Ramiréz (2007): "How Structural Are Structural Parameters?," *NBER Macroeconomics Annual*, MIT Press, 83-138.

Forni, Lorenzo, Libero Monteforte, and Luca Sessa (2009): "The General Equilibrium Effects of Fiscal Policy: Estimates for the Euro Area," *Journal of Public Economics*, **93(3-4)**, 559-585.

Geweke, John (1985): "Macroeconomic Modelling and the Theory of the Representative Agent," *American Economic Review: Papers and Proceedings*, **75(2)**, 206-210.

Hall, Robert (1997): "Macroeconomic Fluctuations and the Allocation of Time," *Journal of Labor Economics*, **15**, S223-S250.

Hansen, Gary (1993): "The Cyclical and Secular Behaviour of the Labour Input: Comparing Efficiency Units and Hours Worked," *Journal of Applied Econometrics*, **8(1)**, 71-80.

Huggett, Mark (1993): "The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control*, **17(5-6)**, 953-969.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante (2008): "Insurance and Opportunities: A Welfare Analysis of Labor Market Risk," *Journal of Monetary Economics*, **55**, 501-525.

Judd Kenneth (1987): "Welfare Cost of Factor Taxation in a Perfect Foresight Model," *Journal of Political Economy*, **95**, 67-709.

Justiniano, Alejandro, Giorgio Primiceri, and Andrea Tambalotti (2010): "Investment Shocks and Business Cycles," *Journal of Monetary Economics*, forthcoming.

King, Robert., Charles Plosser, and Sergio Rebelo (1988): "Production, Growth and Business Cycles: I. The Basic Neoclssical Model," *Journal of Monetary Economics*, **21(2-3)**, 195-232.

Krüger, Dirk, and Hanno Lustig (2007): "When Is Market Incompleteness Irrelevant for the Price of Aggregate Risk?" *Manuscript*, University of Pennsylvania.

Krusell, Per, Toshihiko Mukoyama, Richard Rogerson, Aysegul Sahin (2008): "The Aggregate Implications of Indivisible Labor, Incomplete Markets and Search Frictions," *Journal of Monetary Economics*, **55(5)**, 961-979.

Krusell, Per, and Anthony Smith (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, **106(5)**, 867-896.

Kydland, Finn, and Edward Prescott (1982): "Time to Build and Aggregate Fluctuations," *Econometrica*, **50(6)**, 1345-1370.

Leeper, Eric, Michael. Plante, and Nora Traum (2010): "Dynamics of Fiscal Financing in the United States," *Journal of Econometrics*, **156(2)**, 304-321.

Levin, Andrew, Alexei Onatski, John Williams, and Noah Williams (2006): "Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models," in *NBER Macroeconomics Annual 2005*, ed. by Mark Gertler and Kenneth Rogoff, **20**, 229-287. MIT Press, Cambridge.

Liu, Zheng, Daniel Waggoner, and Tao Zha (2008): "Asymmetric Expectation Effects of Regime Switches in Monetary Policy," *Manuscript*, Federal Reserve Bank of Atlanta.

Ljungqvist, Lars, and Thomas Sargent (2007): "Indivisible Labor, Human Capital, Lotteries, and Personal Savings: Do Taxes Explain European Employment?" *2006 NBER Macroeconomics Annual*, MIT Press, 181-224.

Lucas, Robert (1976): "Econometric Policy Evaluation: A Critique," *Carnegie-Rochester Conference on Public Policy*, **1**, 19-46.

Nakajima, Makoto (2007) "Business Cycles in the Equilibrium Model of Labor Search and Self-Insurance," *Manuscript*, Federal Reserve Bank of Philadelphia.

Narajabad, Borghan (2010) "Information Technology and the Rise of Household Bankruptcy," *Manuscript*, Rice University.

Pijoan-Mas, Josep (2005): "Precautionary Savings or Working Longer Hours?" *Review of Economic Dynamics*, **9**, 326-352.

Prescott, Edward C., (2004) "Why do Americans Work So Much More Than Europeans?," *Quarterly Review*, Federal Reserve Bank of Minneapolis, July, 2-13.

Rios-Rull, Jose-Victor, Frank Schorfheide, Cristina Fuentes-Albero, Maxym Kryshko, and Raul Santaeulalia-Llopis (2009): "Methods versus Substance: Measuring the Effects of Technology Shocks," *NBER Working Paper*, **15375**.

Rogerson, Richard (1988): "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics*, **21(1)**, 3-16.

Rogerson, Richard (2009): "Taxes, Transfers and Employment in an Incomplete Markets Model," *Manuscript*, Arizona State University.

Rogerson, Richard, and Johanna Wallenius (2009): "Micro and Macro Elasticities in a Life Cycle Model with Taxes," *Journal of Economic Theory*, forthcoming.

Scheinkman, Jose, and Laurence Weiss (1986): "Borrowing Constraints and Aggregate Economic Activity," *Econometrica*, **54**, 23-45.

Schorfheide, Frank (2000): "Loss Function Based Evaluation of DSGE Models," *Journal of Applied Econometrics*, **15(6)**, 645-670.

Smets, Frank, and Rafael Wouters (2003): "An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, **1**, 1123-1175.

Smets, Frank, and Rafael Wouters (2007): "Shocks and Frictions in U.S. Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, **97(3)**, 586-606.

Young, Eric (2004), "Unemployment Insurance and Capital Accumulation," *Journal of Monetary Economics*, **51**, 1683-1710.

Table 1: PARAMETERIZATION OF THE HETEROGENEOUS-AGENTS ECONOMY

Preference and Technology Parameters

Parameter	Description
$\beta = 0.98332$	Discount factor
$\gamma = 0.4$	Intertemporal substitution elasticity of leisure
$B = 101$	Utility parameter
$\overline{h} = 1/3$	Labor supply if working
$\underline{a} = -2.0$	Borrowing constraint
$\rho_x = 0.939$	Persistence of idiosyncratic productivity shock
$\sigma_x = 0.287$	Standard deviation of innovation to idiosyncratic productivity
$\alpha = 0.64$	Labor share in production function
$\delta = 0.025$	Capital depreciation rate
$\rho_\lambda = 0.95$	Persistence of aggregate productivity shock
$\sigma_\lambda = 0.007$	Standard deviation of innovation to aggregate productivity

Fiscal Policy Parameters

	Benchmark	Labor Tax Cut	Capital Tax Raise	More Transfers	1960 Policy	2004 Policy
τ_H	0.29	0.22			.229	.269
τ_K	0.35		0.47		.443	.327
χ	0.36			0.50	.224	.417

Table 2: CHARACTERISTICS OF WEALTH DISTRIBUTION

	Quintile					
	1st	2nd	3rd	4th	5th	Total
PSID						
Share of wealth	-.52	.50	5.06	18.74	76.22	100
Group average/population average	-.02	.03	.25	.93	3.81	1
Share of earnings	7.51	11.31	18.72	24.21	38.23	100
Benchmark Model						
Share of wealth	-1.56	3.27	11.38	24.74	62.17	100
Group average/population average	-.08	.16	.57	1.24	3.11	1
Share of earnings	9.74	15.76	19.97	23.72	30.81	100

Notes: The PSID statistics reflect the family wealth and earnings levels published in the 1984 survey. Family wealth in the PSID reflects the net worth of houses, other real estate, vehicles, farms and businesses owned, stocks, bonds, cash accounts, and other assets.

Table 3: SECOND MOMENTS OF SIMULATED AND U.S. DATA

	Model 3000 obs.	U.S. Data 1964-2006
$\sigma(\ln Y)$	.033	.041
$\sigma(\ln C)$	.020	.021
$\sigma(\ln H)$	.013	.042
$\sigma((\ln H)_{HP})$	.007	.018
$\text{corr}(\ln Y, \ln C)$	0.84	0.83
$\text{corr}(\ln Y, \ln H)$	0.80	0.56
$\text{corr}(\ln C, \ln H)$	0.37	0.51

Notes: $\sigma(\cdot)$ is sample standard deviation, $\text{corr}(\cdot)$ is sample correlation, and $(\ln H)_{HP}$ denotes HP-filtered (smoothing parameter 1,600) log hours. Unless noted otherwise, we extract a linear trend from the U.S. data before computing the sample moments.

Table 4: Prior Distributions for DSGE Model Estimation

Name	Domain	Density	Mean	S.D.
r_A	$\mathbb{R}^+$	Gamma	4.00	2.00
ν	$\mathbb{R}^+$	Gamma	1.00	0.50
$\ln \bar{A}$	$\mathbb{R}$	Normal	0.00	10.0
$\ln \bar{B}$	$\mathbb{R}$	Normal	0.00	10.0
ρ_A	$[0, 1)$	Beta	0.50	0.25
ρ_B	$[0, 1)$	Beta	0.50	0.25
σ_A	$\mathbb{R}^+$	Inv. Gamma	.012	.007
σ_B	$\mathbb{R}^+$	Inv. Gamma	.012	.007
σ_Z	$\mathbb{R}^+$	Inv. Gamma	.012	.007

Notes: The means and standard deviations of priors. The following parameters are fixed: $\alpha = 0.64$, $\delta = 0.025$, $\rho_Z = 0.99$. Moreover, we fix the policy parameters τ_H, τ_K, and χ at their "true" values. r_A is the annualized discount rate $r_A = 400 \times (1/\beta - 1)$

Table 5: PARAMETER ESTIMATES

	Simulated Data / Benchmark				U.S. Data	
	$T = 200$		$T = 2,500$		$T = 168$	
	Mean	90% Intv.	Mean	90% Intv	Mean	90% Intv
r_A	2.834	[2.682, 2.978]	2.774	[2.714, 2.833]	3.700	[3.253, 4.219]
ν	1.723	[1.573, 1.856]	2.143	[2.008, 2.257]	0.343	[0.104, 0.601]
$\ln \bar{A}$	-0.259	[-0.263, -0.256]	-0.257	[-0.259, -0.256]	-0.250	[-0.275, -0.225]
$\ln \bar{B}$	-0.329	[-0.335, -0.324]	-0.316	[-0.319, -0.312]	-0.439	[-0.522, -0.367]
ρ_A	0.898	[0.889, 0.908]	0.914	[0.913, 0.917]	0.975	[0.961, 0.989]
ρ_B	0.762	[0.601, 0.925]	0.922	[0.915, 0.929]	0.983	[0.972, 0.998]
σ_A	0.005	[0.005, 0.006]	0.005	[0.005, 0.006]	0.006	[0.006, 0.007]
σ_B	0.003	[0.002, 0.003]	0.003	[0.003, 0.003]	0.007	[0.007, 0.008]
σ_Z	0.003	[0.002, 0.003]	0.003	[0.003, 0.003]	0.012	[0.010, 0.013]

Notes: The following parameters are fixed during the estimation: τ_H, τ_K, χ as in Table 1, $\delta = 0.025$, $\rho_Z = 0.99$. r_A is the annualized discount rate $r_A = 400 \times (1/\beta - 1)$.

Table 6: RELATIVE IMPORTANCE OF PREFERENCE SHOCKS

	B		Z	
	Mean	90% Intv.	Mean	90% Intv.
Benchmark Economy, $T = 200$				
Output	5	[2, 8]	5	[4, 6]
Consumption	3	[0, 7]	6	[4, 7]
Hours	33	[18, 45]	5	[3, 7]
Benchmark Economy, $T = 2,500$				
Output	9	[8, 10]	5	[4, 5]
Consumption	9	[8, 10]	4	[4, 5]
Hours	43	[41, 46]	4	[4, 4]
U.S. Data				
Output	45	[21, 68]	5	[2, 9]
Consumption	47	[21, 75]	6	[1, 10]
Hours	98	[97, 99]	1	[0, 1]

Notes: The entries correspond to percentages.

Table 7: Steady States and Estimates under Alternative Policies

	Bench- mark	Labor Tax Cut	Capital Tax Raise	More Transfers	1960 Policy	2004 Policy
			Parameter Estimates, $T = 2,500$			
r_A	2.77	2.56	2.74	2.84	2.53	2.75
	[2.71, 2.83]	[2.49, 2.62]	[2.66, 2.82]	[2.79, 2.91]	[2.46, 2.58]	[2.69, 2.81]
ν	2.14	1.44	2.23	3.58	1.22	2.10
	[2.01, 2.26]	[1.38, 1.51]	[2.11, 2.36]	[3.28, 3.87]	[1.17, 1.26]	[2.00, 2.20]
$\ln \bar{A}$	-0.26	-0.28	-0.25	-0.23	-0.30	-0.26
	[-0.26, -0.26]	[-0.29, -0.28]	[-0.26, -0.25]	[-0.23, -0.23]	[-0.30, -0.30]	[-0.26, -0.26]
$\ln \bar{B}$	-0.32	-0.32	-0.31	-0.30	-0.31	-0.32
	[-0.32, -0.31]	[-0.32, -0.31]	[-0.32, -0.31]	[-0.31, -0.30]	[-0.31, -0.31]	[-0.32, -0.32]
			Steady States in Heterogeneous-Agents Economy			
E	0.60	0.64	0.60	0.57	0.66	0.60
K	15.2	16.1	14.0	14.8	15.4	15.5
Y	1.48	1.53	1.43	1.44	1.51	1.49
Y/E	2.46	2.39	2.39	2.54	2.29	2.48
R_A	4.00	3.68	4.76	4.04	4.16	3.80

Notes: The following parameters are fixed during the estimation of the representative-agent model: τ_H, τ_K, χ as in Table 1, $\delta = 0.025$, $\rho_Z = 0.99$. r_A is the annualized discount rate $r_A = 400 \times (1/\beta - 1)$. As parameter estimates we report posterior means and 90% credible intervals (in brackets).

Table 8: Predictions of Policy Effects, $T = 200$

		Labor Tax Cut	Capital Tax Raise	More Transfers	1960 Policy	2004 Policy
H	"True"	6.06	-0.23	-5.45	9.44	-0.21
	90% Intv.	[2.96, 3.15]	[-0.31, -0.28]	[-3.22, -3.04]	[5.18, 5.51]	[-0.21, -0.20]
	p-Value	2.2E-308	1.5E-013	2.2E-308	2.2E-308	3.3E-002
C	"True"	7.33	-2.73	3.04	1.73	3.86
	90% Intv.	[7.84, 8.03]	[-3.63, -3.37]	[1.79, 1.98]	[2.25, 2.65]	[3.66, 3.71]
	p-Value	3.2E-025	2.5E-021	9.7E-089	8.1E-009	3.5E-035
Y	"True"	3.44	-2.89	-2.19	2.57	0.81
	90% Intv.	[2.96, 3.15]	[-4.07, -3.84]	[-3.22, -3.04]	[2.28, 2.63]	[0.36, 0.41]
	p-Value	1.3E-011	5.9E-052	9.0E-066	1.7E-001	1.2E-178
$\mathcal{W}$	"True"	4.51	-2.61	5.80	-3.09	4.07
	90% Intv.	[6.60, 6.68]	[-3.52, -3.25]	[3.10, 3.18]	[0.16, 0.44]	[3.75, 3.79]
	p-Value	2.2E-308	3.8E-020	2.2E-308	2.2E-308	1.2E-120

Notes: The benchmark policy is $\tau_H = 0.29$, $\tau_K = 0.35$, $\chi = 0.36$. The entries in the table refer to percentage changes relative to the benchmark policy. The last two rows ($\mathcal{W}$) contain welfare gains (if positive) or costs (if negative) in percentage terms, measured by (22), due to the policy change. "True" effects are computed from the means of the ergodic distributions of the heterogeneous-agents economy. 90% Intv. are predictive intervals computed from the posterior of the representative-agent model based on observations under the benchmark policy.

Table 9: Predictions of Policy Effects – More Transfers, $T = 200$

		Labor Tax Cut		More Transfers	
		Estimation Uncertainty	Invariance Uncertainty	Estimation Uncertainty	Invariance Uncertainty
H	"True"	6.06	6.06	-5.45	-5.45
	90% Intv.	[2.96, 3.15]	[2.26, 3.78]	[-3.22, -3.04]	[-4.12, -2.32]
	p-Value	2.2E-308	5.3E-011	2.2E-308	1.8E-005
C	"True"	7.33	7.33	3.04	3.04
	90% Intv.	[7.84, 8.03]	[6.53, 8.78]	[1.79, 1.98]	[0.67, 3.10]
	p-Value	3.2E-025	3.2E-001	9.7E-089	5.5E-002
Y	"True"	3.44	3.44	-2.19	-2.19
	90% Intv.	[2.96, 3.15]	[1.51, 4.47]	[-3.22, -3.04]	[-4.70, -1.56]
	p-Value	1.3E-011	3.2E-001	9.0E-066	1.5E-001
$\mathcal{W}$	"True"	4.51	4.51	5.80	5.80
	90% Intv.	[6.60, 6.68]	[5.70, 7.58]	[3.10, 3.18]	[2.18, 4.11]
	p-Value	2.2E-308	9.7E-005	2.2E-308	3.4E-006

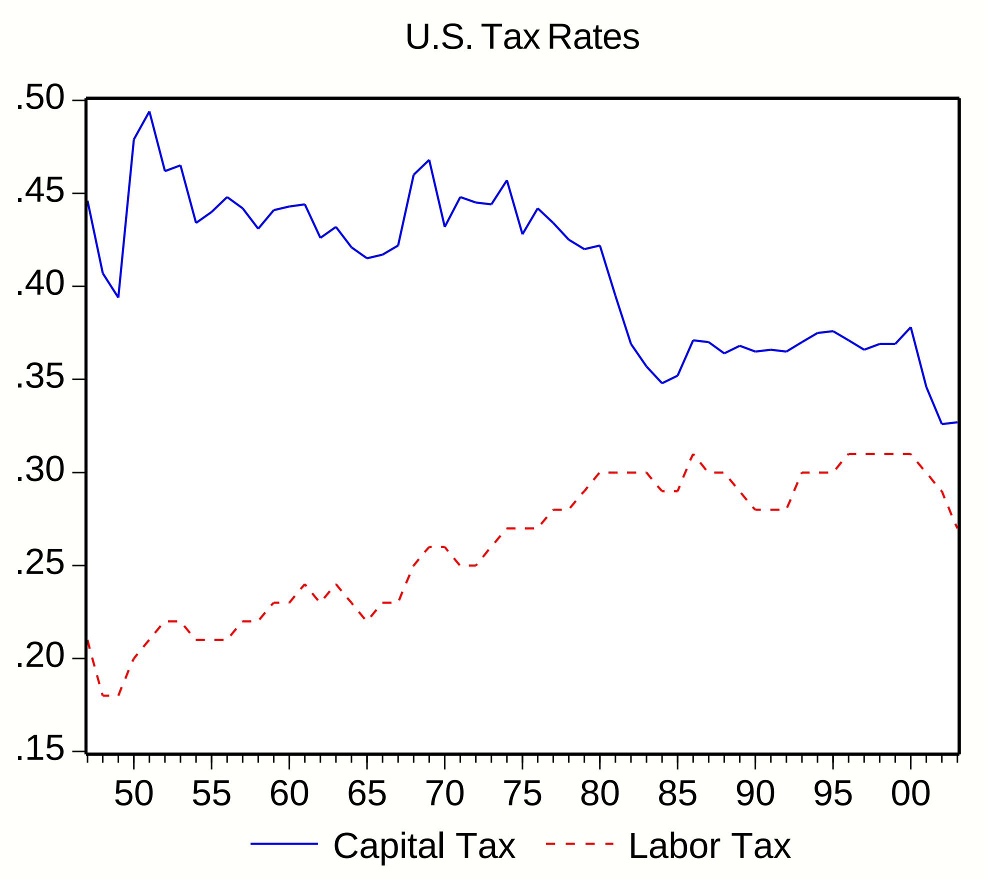
Notes: The entries in the table refer to percentage changes relative to the benchmark policy.

Table 10: Parameter Estimates for Alternative Model Economies and Measurements, $T = 2,500$

	Divisible Labor			Complete Markets			Efficiency Hours		
	Bench- mark	Labor Tax Cut	More Transfers	Bench- mark	Labor Tax Cut	More Transfers	Bench- mark	Labor Tax Cut	More Transfers
r_A	2.72	2.45	2.81	2.50	2.50	2.48	2.75	2.54	2.82
	[2.68, 2.77]	[2.40, 2.50]	[2.76, 2.86]	[2.47, 2.54]	[2.46, 2.54]	[2.45, 2.52]	[2.70, 2.80]	[2.48, 2.60]	[2.74, 2.88]
ν	0.37	0.37	0.38	1.42	1.33	1.53	0.64	0.54	0.80
	[0.37, 0.38]	[0.36, 0.37]	[0.37, 0.38]	[1.42, 1.43]	[1.33, 1.34]	[1.52, 1.53]	[0.62, 0.67]	[0.52, 0.56]	[0.77, 0.84]
$\ln \bar{A}$	0.34	0.34	0.34	-0.24	-0.25	-0.23	0.01	0.00	0.01
	[0.34, 0.34]	[0.33, 0.34]	[0.34, 0.35]	[-0.24, -0.24]	[-0.26, -0.25]	[-0.23, -0.23]	[0.00, 0.01]	[0.00, 0.01]	[0.00, 0.01]
$\ln \bar{B}$	-1.53	-1.53	-1.54	-0.35	-0.35	-0.34	-0.81	-0.82	-0.79
	[-1.53, -1.53]	[-1.53, -1.53]	[-1.54, -1.53]	[-0.35, -0.35]	[-0.35, -0.35]	[-0.34, -0.34]	[-0.81, -0.81]	[-0.82, -0.82]	[-0.79, -0.79]

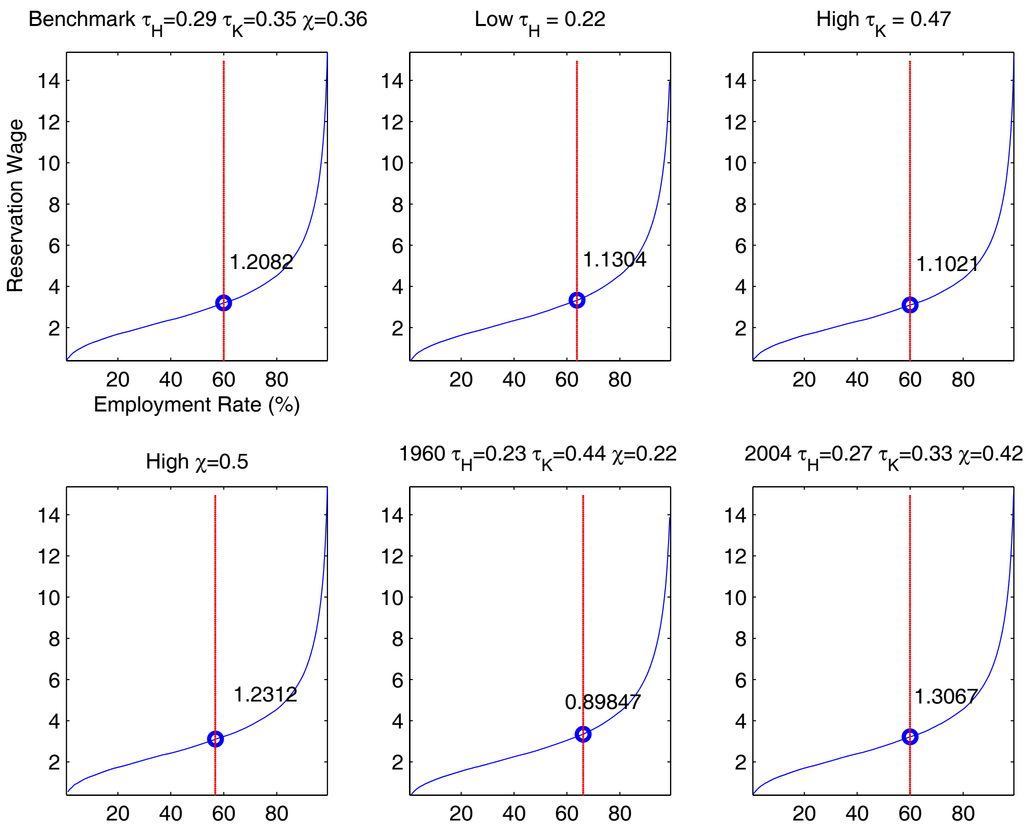
Notes: The following parameters are fixed during the estimation τ_H, τ_K, χ as tabulated, $\delta = 0.025$, $\rho_Z = 0.99$. r_A reflects the annualized discount rate $r_A = 400 \times (1/\beta - 1)$. As parameter estimates we report posterior means and 90% credible intervals (in brackets).

Figure 1: U.S. Capital and Labor Tax Rates



Notes: The data are taken from Chen, Imrohoroglu, and Imrohoroglu (2007).

Figure 2: Employment Rate Based on the Reservation Wage Distribution



Notes: Each curve represents the employment rate (on the x-axis) at a given wage rate (y-axis). The vertical line denotes the steady-state level of employment under the benchmark and the no-transfer policy regimes. The numbers in the plots indicate the elasticity of employment with respect to wages around the steady-state employment rate.

A Derivations for the Representative-Agent Model

In this appendix, we collect the first-order conditions (and their log-linear approximation around the steady state) of the representative-agent model we use to fit the time series generated from the heterogeneous-agents economy.

First-Order Conditions: The first-order conditions (FOCs) associated with the Household Problem are:

$$\begin{aligned} \lambda_t &= \frac{Z_t}{C_t} \\ \lambda_t &= \beta \mathbb{E}_t[\lambda_{t+1}(1+(1-\tau_K)R_{t+1})] \\ H_t^{1/\nu} &= (1-\tau_H)\frac{\lambda_t}{Z_t} W_t B_t^{1+1/\nu} \end{aligned}$$

Notice that the preference shock Z_t drops out of the labor supply function:

$$H_t^{1/\nu} = (1-\tau_H)\frac{1}{C_t} W_t B_t^{1+1/\nu}.$$

The FOCs of the firms problem are provided in (4).

Steady States: We subsequently denote the deterministic steady-state values by

$$\bar{H}, \bar{K}, \bar{\lambda}, \bar{C}, \bar{Y}, \bar{A}, \bar{B}, \bar{W}, \bar{G}, \bar{R}.$$

The steady state value of Z_t is equal to one. It is convenient to express the model in terms of ratios relative to steady-state hours worked. The first-order conditions in the steady state become

$$\begin{aligned} \bar{R} &= \frac{1/\beta - 1}{1-\tau_K}, \quad \left(\frac{\bar{H}}{\bar{B}}\right)^{\frac{1}{\nu}} = (1-\tau_H)\frac{\bar{B}}{\bar{C}}\bar{W}, \\ \frac{\bar{K}}{\bar{H}} &= \left(\frac{\bar{A}(1-\alpha)}{\bar{R}+\delta}\right)^{\frac{1}{\alpha}}, \quad \bar{W} = \alpha \bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha}. \end{aligned}$$

Hence,

$$\frac{\bar{H}}{\bar{B}} = \left(\frac{(1-\tau_H)\bar{W}}{\bar{C}/\bar{H}}\right)^{\frac{\nu}{1+\nu}}.$$

Moreover, the production function can be expressed as

$$\frac{\bar{Y}}{\bar{H}} = \bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha}.$$

The government budget constraint leads to

$$\frac{\bar{T}}{\bar{H}} = \chi\left(\tau_H \bar{W} + \tau_K \bar{R}\frac{\bar{K}}{\bar{H}}\right), \quad \frac{\bar{G}}{\bar{H}} = (1-\chi)\left(\tau_H \bar{W} + \tau_K \bar{R}\frac{\bar{K}}{\bar{H}}\right)$$

and the market clearing condition can be written as

$$\frac{\bar{Y}}{\bar{H}} = \frac{\bar{C}}{\bar{H}} + \delta\frac{\bar{K}}{\bar{H}} + \frac{\bar{G}}{\bar{H}}.$$

We can now write the consumption-hours ratio as

$$\begin{aligned}
\frac{\bar{C}}{\bar{H}} &= \bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha} - \delta\frac{\bar{K}}{\bar{H}} - (1-\chi)\left(\tau_H\bar{W} + \tau_K\bar{R}\frac{\bar{K}}{\bar{H}}\right) \\
&= \bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha} - (\delta + (1-\chi)\tau_K\bar{R})\frac{\bar{K}}{\bar{H}} - (1-\chi)\tau_H\alpha\bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha} \\
&= [1-(1-\chi)\tau_H\alpha]\bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha} - (\delta + (1-\chi)\tau_K\bar{R})\frac{\bar{K}}{\bar{H}}.
\end{aligned}$$

Hence, the steady state of hours worked is given by

$$\begin{aligned}
\bar{H} &= \bar{B}\left(\frac{(1-\tau_H)\alpha\bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha}}{[1-(1-\chi)\tau_H\alpha]\bar{A}\left(\frac{\bar{K}}{\bar{H}}\right)^{1-\alpha} - (\delta + (1-\chi)\tau_K\bar{R})\frac{\bar{K}}{\bar{H}}}\right)^{\frac{\nu}{1+\nu}} \\
&= \bar{B}\left(\frac{(1-\tau_H)\alpha}{[1-(1-\chi)\tau_H\alpha] - (\delta + (1-\chi)\tau_K\bar{R})\bar{A}^{-1}\left(\frac{\bar{K}}{\bar{H}}\right)^{\alpha}}\right)^{\frac{\nu}{1+\nu}} \\
&= \bar{B}\left(\frac{(1-\tau_H)\alpha}{[1-(1-\chi)\tau_H\alpha] - [\delta/(\bar{R}+\delta) + (1-\chi)\tau_K(\bar{R}/(\bar{R}+\delta))](1-\alpha)}\right)^{\frac{\nu}{1+\nu}}
\end{aligned}$$

Log-Linear Approximation: Denote the percentage gap from the steady-state value of each variable by

$$\widehat{H}_t, \widehat{K}_{t+1}, \widehat{\lambda}_t, \widehat{C}_t, \widehat{Y}_t, \widehat{A}_t, \widehat{B}_t, \widehat{W_t}, \widehat{G}_t, \widehat{Z}_t, \widehat{R_t}.$$

We obtain the following equations:

$$\begin{aligned}
[\bar{R}/(\bar{R}+\delta)]\widehat{R}_t &= \widehat{A}_t + \alpha\widehat{H}_t - \alpha\widehat{K}_t \\
\widehat{W}_t &= \widehat{A}_t + (\alpha-1)\widehat{H}_t + (1-\alpha)\widehat{K}_t \\
\widehat{\lambda}_t &= -\widehat{C}_t + \widehat{Z}_t \\
\widehat{\lambda}_t &= \mathbb{E}_t[\widehat{\lambda}_{t+1} + (1-\beta)\widehat{R}_{t+1}] \\
\nu^{-1}\widehat{H}_t &= -\widehat{C}_t + \widehat{W}_t + (1+\nu^{-1})\widehat{B}_t \\
\bar{Y}\widehat{Y}_t &= \bar{C}\widehat{C}_t + \bar{K}\widehat{K}_{t+1} - (1-\delta)\bar{K}\widehat{K}_t + \bar{G}\widehat{G}_t \\
(1-\chi)\widehat{G}_t &= \frac{\tau_H\alpha[\widehat{W}_t + \widehat{H}_t] + \tau_K(1-\alpha)[\bar{R}/(\bar{R}+\delta)]\widehat{Y}_t}{\tau_H\alpha + \tau_K(1-\alpha)[\bar{R}/(\bar{R}+\delta)]} \\
\widehat{Y}_t &= \widehat{A}_t + \alpha\widehat{H}_t + (1-\alpha)\widehat{K}_t \\
\widehat{A}_t &= \rho_A\widehat{A}_{t-1} + \sigma_A\epsilon_{A,t} \\
\widehat{B}_t &= \rho_B\widehat{B}_{t-1} + \sigma_B\epsilon_{B,t} \\
\widehat{Z}_t &= \rho_Z\widehat{Z}_{t-1} + \sigma_Z\epsilon_{Z,t}.
\end{aligned}$$

If $\chi = 0$ then $\bar{G} = 0$ and we compute the level of government spending rather than percentage deviations from a steady state that is zero.

B Aggregate Data Sources

Aggregate capital and labor tax rates are obtained from Chen, Imrohoroglu, and Imrohoroglu (2007). As a measure of hours we use the Aggregate Hours Index (PRS85006033) published by the Bureau of Labor Statistics. The remaining data series are obtained from the FRED2 database maintained by the Federal Reserve Bank of St. Louis. Consumption is defined as real personal consumption expenditures on non-durables (PCNDGC96) and services (PCESVC96). Output is defined as the sum of consumption, consumption expenditures on durables (PCDGCC96), gross private domestic investment (GPDIC), and federal consumption expenditures and gross investment (FGCEC96). Output, consumption, and hours are converted into per capita terms by dividing by the civilian non-institutionalized population (CNP16OV). The population series is provided at a monthly frequency and converted to quarterly frequency by simple averaging. Finally we take the natural logarithm of output, consumption, and hours. We restrict the sample to the period from 1965:I to 2006:IV, using observations from 1964 to initialize lags. We remove linear trends from the log output and consumption series and demean the log hours series. To make the log levels of the U.S. data comparable to the log levels of the data simulated from the heterogeneous-agents economy, we adjust (i) detrended log output by the steady-state output level in the heterogeneous-agents economy under the benchmark tax policy, (ii) detrended log consumption by the steady state output level in the heterogenous agent economy plus the log of the average consumption-output ratio in the U.S. data, and (iii) demeaned hours by the steady state of log employment.