6. NONLINEAR CONSIDERATIONS ON ECONOMIC SYSTEMS' BEHAVIOUR

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Abstract

Recent work done within the CNCSIS IDEI ID_1046 financed project has resulted in some ideas related to complex system behaviour and to certain ways in which this behaviour may be described using nonlinear models especially in relation to the evolution of the PIB and its components. We are presenting here some of these results and the way an oscillatory response in industrial production may be used to determine the associated differential equation of evolution.

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Perception of complex systems

Along with Purica (2010) we may consider that the development of human infrastructure (energy, transport, communications, etc.) at a planetary level and the capability to work with large amounts of data has revealed the existence of limits with regard to both socio-technological and environmental developments. While the limits of technological development have proven to be of the saturation type, i.e., new technologies penetrate to replace the old, saturated ones, environmental evolution shows very clearly that this planet is all we've got. In our striving to master energies at the planetary level, which is not expected to happen in the foreseeable future, we remain with the hope that the environment will be resilient enough to absorb our errors due to our lack of knowledge and inability to accept our limits. Several decision reactions are possible:

'Whatever you do will change nothing' 'Anything you do will change everything'

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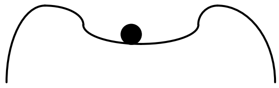


'Certain things you do will push the system beyond stable equilibrium, others will not'

The perception of change in complex systems and, accordingly, the reaction, show bifurcation-like behaviour especially when one acquires an awareness of the limits (environmental, technological, social, etc.). Linear mentalities like 'whatever you do will change nothing' or 'anything you do will change everything' must be changed in more subtle ways of acting that take into account second-order effects that characterize the behaviour of complex systems. For instance, when building dams to protect against sea-level increase, we should consider the fact that the production of cement for dams represents a source of CO_2, which contributes to the rise in sea level.

During the Middle Ages, the indicator of welfare was the quantity of gold one possessed. Accordingly, the 'research programmes' of those days were aimed at changing everything to gold. Since the emergence of energy-availability limitations, the indicators have changed. Also, the increased complexity of interactions among the various systems (energy, population, economy, environment, etc.) has lead to the introduction of aggregated indicators. The planetary view we have today requires the consideration of the meteo-geographical conditions of each region and the normalization of the specific indicator values in order to make better comparisons. This suggests a personalization of new energy-supply technologies being implemented in various regions, taking into account not only the geographical conditions, but also the social ones, in order to achieve maximum efficiency.

Taking decisions for development has always been based on some type of representation of the process. Various models have served as tools to devise or justify decisions. The mathematics behind these models is usually linear. Since the behaviour of the processes involved is highly non-linear, the approximations made were valid for restricted areas and time intervals. These models were not able to predict the limits beyond which a discontinuous behaviour would occur in systems evolution. Decisions of the type 'quit financing a technology and enhance others' are common in the economy. Only in recent years, non-linear models based on non-linear mathematical tools have made possible the prediction of discontinuous decisions which occur when certain system parameters cross some limit. Although the mathematics involved is more complicated with respect to the linear one, the representation of system evolution among limits is more straightforward.

Even if the limits are not accepted, they may sometimes be avoided or, in rare cases, crossed with the associated shocks. The capability to absorb shocks and still perform normally (resilience) measures the impact of our decisions for development on the environment, the economy, etc. Alternatively, accepting the limits opens the way to understanding the mutual interactions among the various systems, thus making it possible to change those limits in a sustainable symbiotic evolution.

Negotiating between energy and environmental concerns in development involves information which is not always available, and time constants that may be longer than what we have dealt with. The costs and financial measures implied may lead, for example, to capital accumulations which we are not prepared to control yet, lacking appropriate administrative structures, or may lead to unusually long payback times and the prospect of irreversibly damaging the environment. The present changes in energy generation, transmission, distribution and end-use systems, leading to more players in the market, have raised questions about the role of a regulator which would prevent chaos in the process and thus prevent shocks to the economy. Correlating global change with energy is one of the first projects to consider the interactions among various systems at a planetary scale, opening the way for closer international co-operation.

Geographical research on fuels along with scientific research on conversion technologies were one of the main reasons that led to the development of infrastructure, such as transport, telecommunications, etc., which in turn gave us the consciousness of, and the possibility to monitor, the influence of our activities on the environment.

The ability to perceive changes in the complex systems we interact with has influenced our ways of understanding and consequently modelling more complex behaviours. One should not forget that one of the first classes of models which show 'chaotic' behaviour was aimed at describing meteorological behaviour. (Lorenz, 1963)

Is there only one optimum?

In a very general context, the concept of optimum is linked to the existence of well-defined elements, summarized in the list below:

- existence of a space described by specific parameters of the considered system;
- the possibility to define the general magnitude of the space above, which depends on those parameters;
- criteria defining the interaction of the human observer with the parameters of that space, which are related to maxima or minima of the general magnitude of that space.

As a simple example, in physics, field theory characterized by a potential is well-known, especially in association with the extremes of the possible benefits or losses for the processes taking place in the space thus defined. In thermodynamics, energy plays such a role and the space in which the behavior of the system is observed is called "phase space". However, optimality is defined on the basis of useful energy and consumed energy, which is related to the interaction between the thermodynamics and the human observer. The idea that the world can be described by the general behavior of a unitary field influences physics nowadays and is one of the concepts with a strong impact on the generation of knowledge.

Changing parameters can lead the system from one extreme point of the general magnitude to several others, so that optimum can change into a new position in the considered space. Thus, changing the consideration criteria may result in the relocation of the optimum in another extreme point of the general magnitude. We can therefore define local and global optimums.

To make a link with the economy, we start with some statements of Nicholas Georgescu-Roegen[2]. He thinks that the product of the economic process, in addition to value, is "an immaterial flux, the pleasure of living." In this context, he shows that not only the production time is valuable, but also the rest cycle, which puts a whole new light on how production and consumption are taken into account. It is important to note, in light of the optimum, that Roegen seeks to define the overall magnitude, which is then associated with the dynamic processes of interaction intra-economy, as well as of the economy with the environment.

If we confine ourselves to identifying a general magnitude only, whose variation in the space of economic parameters may show extreme point, we remain confined in the economic theories of general equilibrium[3], where, according to the three requirements above for the existence of the optimum, the efficiency criteria are not introduced in relation to human observers. To go another step towards defining the optimums, Cournot explores the equilibrium positions in various markets and stresses the importance of interrelated analysis of these markets in a general economic system: "We have considered prices of other goods and income from other manufacturers as being given and unvarying, but in fact, the economic system is a whole in which all parts are interrelated and react with each other". Leontieff in the 1920's and 1930's used very successfully matrix algebra[4] to address the problem of the unitary behavior of economic sectors. Leontieff's considers linear systems of equations that could lead to determining an extreme position for the entire system.

Against this concept, Pareto introduces the human observer interacting with the economic system, thus opening the premises for defining optimums and efficiency. For example, he specified that the utility contains inherent contradictions, in that, if one considers consumption associated with utility (in the best sense of the word), then one can encounter cases such as alcohol consumption, which have a destructive component for the consumers. Pareto proposes the concept of *Ofelimity*[5], which has not actually spread in the economic language. The usage of Edgeworth's notion of indifference curves was more influential, and it refers to the combination of goods equally acceptable to consumers. Pareto developed the concept of maximum *Ofelimity* point where no small change can vary any individual's *Ofelimity*. The conclusion is that the change favors some over others and we cannot speak of a benefit for the whole community. It is stressed that there are an infinity of positions of

[2] Legea entropiei şi procesul economic, *Editura Politică, Bucureşti, 1979. Roegen describes the economic process through an equation (quasiequation as he states it) of the value associated with the process of entropy conversion from the environment by the human structures.*

[3] *Walras, as well as Jevons and Menger have published independently, in the years 1870, their theories of the value based on the concept of marginal utility.*

[4] *We remark that in the same period Heisenberg was developing quantum mechanics based on matrix algebra.*

[5] *Derived from Greek language as the power to satisfy whishes.*

maximum *Ofelimity*, each reflecting a different income distribution[6]. If we start from the idea that income distributions describe the state of an economic system, an entropy function may be introduced according to a standard definition by which to measure the state of pleasure derived from satisfying the needs of people, thus returning to Georgescu-Roegen's observations from which we started in the first place.

Blind use of mathematical concepts

Dimensional analysis in economics

The possibility to put certain processes into formulae and to make further calculations based on these formulae makes us sometime forget that the meaning of the mathematical symbols is valid provided the dimensions of the magnitudes involved are conserved as required by the theory of dimensional analysis.

A revealing example is given by Mayumi who showed that writing exponential functions with exponents having the 'money' dimension leads, e.g. in the Taylor series development of the exponential, to 'money' at the power of 1 (that has an economic meaning) but also to money at the power of two, three, and so on. Obviously the notion of square money, or cubic money, does not have an economic meaning. By contrast non-dimensional ratios may be safely used as exponents, as well as variables without dimensions, or groups of such variables that overall have no dimension.

Moreover, when we use blindly mathematical concepts, we are tempted to use the easier models (such as linear ones). Hence the lacks of these models must be avoided by using more complex models, as non-linear ones.

In this context the penetration of various models from other sciences into economic description is bringing the tendency to import some terminology from those sciences that is not correlated with the economic meaning of the variables and parameters of the process described. One example is 'gravitational' economics that uses a physical notion probably more with the intention to 'sell' a concept, applied to economics (that still has a certain scientific value) than the intention to maintain scientific consistency.

On this line one should praise Georgescu-Roegen for the fact that, although he uses notions such as energy, entropy, etc. he never mismatches their true meaning and extends the economic science in a consistent blending with the environment.

Dynamics in economics

Entrenched economic thinking starts with the idea that 'the end point of a dynamic process is the state of static equilibrium', which may even be the case in some limited linear systems and for a short time interval. The concept of long run cannot be generally applied. As Keynes (1923) says: "But this long run is a misleading guide to current affairs. In the long run we are all dead. Economists set themselves too easy, too useless a task if in tempestuous seasons they can only tell us that when the storm is long past the ocean is flat again".

[6] *From this point of view the Pareto oltimum is not necessarily a global optimum but a local one.*

As Fisher later observed (Keen, 2004), equilibrium conditions in the absence of disturbances are irrelevant, because disturbances will always occur. Whether equilibrium is stable or not, disequilibrium will be the state in which we live.

Nowadays, the science of complexity is bringing in mathematical entities that allow us to start grasping the real behavior of economic systems where dynamics and nonlinearity are the rule while equilibrium is the exception.

An oscillatory example for industrial production

In Purica, 2010 and Purica and Caraiani, 2009, we have shown that the shocks in the economy are triggering a typical exponentially amortized oscillatory response, e.g. in industrial activity, that allowed the determination of the specific coefficients of the second order differential equation describing the process.

The basic formulae for the above are given below:

A	Φ	T_d	1/a	
0.18	1.52	45.3	130	- parameters from fitted data above

ω_d= 0.138701662 T_d= 45.3 months (years d = 3.775)

Φ = 1.515393705

EXP(-ax)*A*SIN(2* π /T_d*x+ Φ) - characteristic function

where:

ω_n=A*ω_d undumped natural frequency

Φ =atan(ω_d/a) phase

ω_d=2*π/T_d dumped natural frequency

ζ_i=a/ω_n dumping ratio

Deduction of the values of interest is given below:

a=ζ_i*ω_n

ω_d=ω_n*SQRT(1-ζ_i^2)

a=ω_d(ζ_i/(1-ξ_i^2))

ζ_i=SQRT(1/(1+(ω_d/a)^2))

ζ_i= 0.055374283

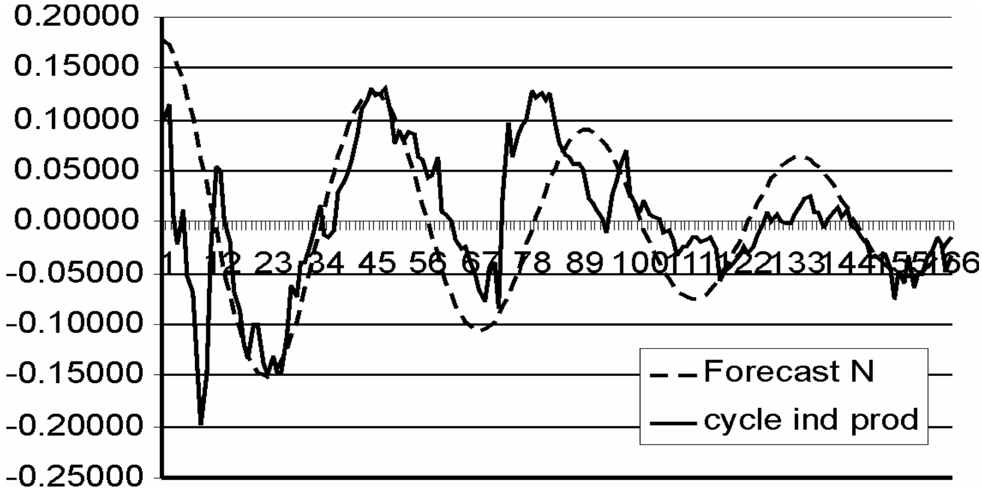
ω_n=a/ξ_i = 0.138914804 T_n= 45.23049473 months (years n = 3.769207894)

And the differential equation results as:

$$\frac{d^2y}{dt^2} - 0.015\frac{dy}{dt} + 0.019y = 0.019u$$

Figure 1

Industrial production response after the 1990 shock



Source: Purica, 2010.

In a separate paper we will analyze the implications of using such an approach to determine the later response to other type of discontinuous inputs such as the crisis at the end of 2008 in Romania that suggests time constants to recovery of the order of 2 years.

Conclusions

Looking at the above results, stemming from recent work done within the CNCSIS IDEI ID_1046 financed project, the basic ideas related to complex system behaviour have been put in light of the new patterns of nonlinear modelling and certain ways in which this behaviour may be described especially in relation to the evolution of the PIB and its components. We are presenting here some of these results and an example of the way an oscillatory response in industrial production may be used to determine the associated differential equation of evolution. Also, some considerations are made on important misuses of models in economical systems that could lead to strange or even wrong conclusions. It is important to note that a simple application of oscillatory behaviour may have a good power of prediction especially in crisis periods.

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