Strategic Ambiguity in Electoral Competition*

Enriqueta Aragonès+ and Zvika Neeman++

First version: January, 1994

This version: Abril, 1996

ABSTRACT

Many have observed that political candidates running for election are often purposefully expressing themselves in vague and ambiguous terms. In this paper we provide a simple formal model of this phenomenon. We model the electoral competition between two candidates as a two-stage game. In the first stage of the game two candidates simultaneously choose their ideologies, and in the second stage they simultaneously choose their level of ambiguity. Our results show that ambiguity, although disliked by voters, may be sustained in equilibrium. The introduction of ambiguity as a strategic choice variable for the candidates can also serve to explain why candidates with the same electoral objectives end up "separating," that is, assuming different ideological positions.

---

* We thank Salvador Barbera, Jim Fearon, Tim Feddersen, Michael Jones, Jane Marrinan, Andreu Mas-Colell, Roger Myerson, Don Saari and especially Itzhak Gilboa for their many comments and suggestions that have improved the paper considerably.

+ Departament d'Economia, Universitat Pompeu Fabra. Carrer Balmes 132, 08008 Barcelona (Spain).

++ Department of Economics, Boston University. Boston, MA 02215.

1. Introduction

Many have observed that political candidates running for election are often purposefully expressing themselves in vague and ambiguous terms. Moreover, the candidates' ambiguity typically involves precisely those issues which stand in the center of public debate. As Downs (1957) has observed, it is on the " critical issues" that candidates perceive incentives to equivocate, or to " becloud their policies in a fog of ambiguity." Downs argues that candidates have very good reasons to be ambiguous: a candidate who advocates an ambiguous platform during the campaign enjoys greater freedom in implementing his policies once he wins the election without having to sacrifice his credibility. Shepsle (1972) adds that "...we can accept with Downs the assumption that politicians do not lie – that false information does not enter the communications system – while still acknowledging the politician's advantage in speaking " half-truths" and in varying his appeals with variations in audience and political climate."

From the candidates' perspective the level of ambiguity presented with their platforms is the result of a conscious decision. A candidate's decision regarding his level of ambiguity can be interpreted as a choice of the candidate's level of commitment to his ideology A candidate who advocates an explicit and unambiguous platform is actually committing himself to implement more specific policies. On the other hand, a candidate who presents an ambiguous platform is less committed, avoiding promises which can be attributed to him later. Since we accept the assumption that candidates do not lie, it follows that the candidates' level of ambiguity actually determines their level of commitment to their ideology.

If we assume that committing to an ideology is costly for a candidate, then we will have that candidates would prefer ambiguous platforms to more specific ones. They will not want to release information about their future policies to the voters in order to have greater freedom of choice once in office.

A theory that supports this assumption is what Kreps (1979) names "Preference for Flexibility." This theory analyzes individual decisions that constrain the choices that will be feasible in a later stage. In an earlier stage the individual has to choose the set of alternatives that will be available for his choice in the future. If the individual is unsure about his subsequent preferences, he will show in the early stage a desire for flexibility, which translates into the following axiom: the union of two sets may be strictly preferred to each one taken separately[1]. If candidates care about their reputation, the choice of a platform during a campaign constrains their choices of policies in case they win the election. If they are unsure about what their preferences on policies will be in case they win the election, candidates will have a desire for flexibility that translates into a preference for ambiguity.

The analysis of the concept of freedom of choice made by Sen (1988) also supports our assumption of preference for ambiguity. Sen distinguishes between the *instrumental* relevance of freedom (as the value for things as means to other ends) and the *intrinsic* importance (their value as ends in their own right). Thus, the availability of a larger number of alternatives for the winner of the election improves his well-being.

Another reason to prefer ambiguity may be the following: a vague candidate enjoys greater freedom in choosing his policy and can therefore "sell" his policy plan to lobbyist groups, thereby increasing his post-election base of support and possibly his party's budget (see Morton and Myerson (1992)).

On the other hand, voters, and especially risk averse ones, may prefer less ambiguous candidates. Less ambiguous candidates advocate more specific campaign messages and therefore the uncertainty associated with their implemented policy is smaller. From the voters' perspective, the choice of the ambiguity level is positively associated with the variance of the candidate's future policies. Thus, risk averse voters will prefer candidates to be unambiguous.

Thus, we have distinguished several conflicting forces that may affect candidates' ambiguity levels. Candidates would like to win elections with ambiguous platforms, while voters prefer unambiguous candidates. Since candidates have to compete for the votes, they will face a trade-off. The purpose of this paper, then, is to understand the way in which these forces interact. We present a game theoretic model that allows us to determine the equilibrium levels of candidates' ambiguity. Maybe more interestingly, our approach enables us to identify a way in which candidates' ambiguity levels and candidates' ideologies interact with each other.

[1] For example, if a decision maker prefers alternative a to alternative b, then the set $\{a\}$ will be as good as the set $\{a,b\}$. If, however, the decision maker is unsure about his future preferences, then the set $\{a,b\}$ may be strictly preferred to the set $\{a\}$.

In our model candidates' platforms are represented by *sets of policies.* Thus, whereas in the standard spatial model candidates are constrained to choose a single policy which they promise to implement once they win the election, in this paper candidates have the option to remain somewhat vague regarding the policy that they will implement in case they win the election, that is, to choose the size of the set of policies. The center of the candidate's "policy-set" is determined by the candidate's choice of ideology and the size of the set is determined by the candidate's level of ambiguity. A more ambiguous candidate chooses a larger policy-set, and therefore, in case he wins the election, he can choose to adopt a policy from a larger number of them. In other words, he is less constrained by his campaign promises, thus he can implement more expedient policies as the need arises. For example, while an unambiguous leftist candidate has to implement a leftist policy if he wins the election, an ambiguous leftist candidate might also implement a centrist policy if it proves more expedient.

In contrast to the standard spatial voting model, our more general treatment allows us to distinguish between candidates' policies and candidates' ideologies. While there is an obvious relationship between the two, they need not coincide, and indeed may belong to different spaces (see, Hinich and Munger (1992)). While a policy refers to a specific plan of action, an ideology refers to a broad and not necessarily precise description of one's convictions and positions concerning various issues that stand at the center of public debate. The act of joining a party, for instance, can serve as an example of ideological identification. Another example is the New-Hampshire primaries, in which a candidate already has to associate himself with a certain ideology. The mere fact that the candidate competes in the primaries of the Democratic or the Republican party is sufficient to distinguish him ideologically. Furthermore, each candidate has to express *some* opinions to draw voters' attention. Still, at this stage of the campaign the candidates' positions on the issues that are at stake may still be rather vague. As the campaign unfolds, the candidates have many opportunities to make themselves more explicit, say, in interviews, debates, talk shows, and so forth.

We model the election process as a two stage game. In the first stage, two candidates simultaneously announce their ideological positions. In the second stage the candidates simultaneously decide how ambiguous they want to be. Since in the second stage of the game the ideologies of the candidates are publicly known, the candidates can choose their levels of ambiguity conditional on the ideology choices (of both of them) in the first stage. After the two candidates voiced their ideological credo and chose their ambiguity levels or degree of commitment to their ideologies, election takes place and the winning candidate is determined by majority rule.

Regarding candidates' preferences, we assume that candidates do not have any *a-priori* preference for any ideological position. Rather, they wish to win the election while being as ambiguous as possible. We assume that winning the election results in some "utility" for a candidate which depends on his level of ambiguity, while losing the election gives them a zero payoff. We assume that candidates do not know the exact distribution of voters' preferences, instead, they share common beliefs over them. We also make the standard assumption that the candidates are expected utility maximizers; that is, they maximize the product of the probability of winning the election and the utility of assuming office. Thus the choice of ideology affects their payoffs only through affecting their *chances* of winning the election. By contrast, the choice of ambiguity level may affect both the probability of winning the election and the utility of governing. Thus, the second strategic choice variable, namely, the level of ambiguity may confront a candidate with a trade-off: it will often be the case that a higher level of ambiguity (i.e., low level of commitment) decreases the probability of winning the election, but increases its desirability.

We assume that voters have single peaked preferences on the policy space. Before the election voters learn the platforms of both candidates and use this information to estimate the policy that each candidate will implement in case he wins the election. Thus, voters' preferences on policies translate into a decision rule that is defined on the parameters of the platforms: ideologies and ambiguity. In order to make their decision, voters take into account the distance of the candidates' ideology from their ideal point, and the level of ambiguity chosen, and think of them as the mean and the variance of a random variable whose realization represents the policy that will be actually implemented after the election. We assume that voters maximize their expected utility and vote sincerely. In case of indifference, we assume that voters vote randomly for either candidate.

We show the solution to this game in two different scenarios: in the first model the choices of ideology and ambiguity levels are discrete, while in the second one they are continuous. In both models a subgame perfect equilibrium always exists. Two kinds of equilibria emerge in these models. In the first equilibrium both candidates choose the median voter's preferred ideology and choose a minimal level of ambiguity. In the second equilibrium the candidates differentiate themselves ideologically and choose identical, positive levels of ambiguity. What determines which equilibrium prevails is the ratio between the utility of assuming office and the (dis)utility of being less ambiguous. Not surprisingly, when candidates value the fact of winning the election and ambiguity is not very rewarding (relative to winning the election) the first equilibrium prevails, and when the candidates care less about winning the election per-se and more about their freedom once they win the election, the second equilibrium prevails. In intermediate parameter values both equilibria may coexist simultaneously.

The intuition underlying the first equilibrium is identical to that of the standard spatial voting model, namely, both candidates compete by choosing what they believe to be the median voter's preferred ideology in the first stage of the game, knowing that by doing so they will be constrained to choose a very specific platform (low ambiguity). These strategies constitute an equilibrium when ambiguity is not very rewarding for candidates. In this case, the main interest of the candidates is to maximize the probability of winning the election.

The intuition which underlies the second equilibrium is more interesting. Candidates differentiate themselves ideologically in order to soften the second stage competition in ambiguity. In this equilibrium candidates face a trade-off between the probability of winning the election and their level of utility in case they win the election (determined by their level of ambiguity). Each candidate can guarantee a 50% probability of winning the election by adopting his opponent's ideology, but when ambiguity is valuable, one of the candidates may increase his expected payoff by sacrificing some probability of winning and allowing himself an ambiguous platform in case of winning. That is, at an equilibrium with differentiated ideologies, one candidate may have a lower probability of winning the election. However, he realizes that should he move closer to the other candidate's ideological position, the other candidate would retaliate by choosing to be less ambiguous in the second stage of the campaign, thereby forcing the first candidate to respond by lowering his own ambiguity level.

The rest of the paper is organized as follows: in section 2 we present the main assumptions of the model that allows us to illustrate our arguments. In section 3 we solve the model for the case of finite sets of alternatives. In section 4 we present a more general (continuous) model and show that the results extend naturally to this case as well. In section 5 we discuss the relationship of this paper to the existing political and economic literature. Section 6 concludes. A detailed formal derivation of the results is relegated to the appendix.

2. The main assumptions.

As noted in the introduction, we model the electoral competition between two candidates as a two stage game and analyze the subgame perfect equilibria of this game. We now describe the electoral game, starting with the preferences of the candidates and the voters.

We denote the two candidates by 1 and 2. Let denote the set of ideologies available to the candidates, and let À denote the set of levels of ambiguity. In the first stage of the game, the candidates simultaneously choose their ideologies (I_1, I_2) where I_i for i $\{1,2\}$. In the second stage of the game, the candidates simultaneously announce their ambiguity level (a_1, a_2), or the degree of their commitment to their ideologies (I_1, I_2), which at this stage are publicly known. Formally, a candidate's (pure) strategy can be described by a vector (I, f). I denotes the ideology chosen by the candidate and f denotes the candidate's choice of an ambiguity level as a function of the ideology choices of the first stage. Since in the second stage of the electoral game the ideologies of the candidates are publicly known, the candidates can choose their levels of ambiguity conditional on them. Formally, the level of ambiguity f is a function that maps the ideologies that were chosen in the first stage into À, or f: × → À. The strategy of candidate i $\{1,2\}$ is denoted by (I_i, f_i).

We assume that the candidates have identical utility functions which are increasing in the probability of winning the election and with the level of ambiguity. Formally, the utility function of candidate i is $U_i(I_1, f_1; I_2, f_2) = P(I_1, a_1; I_2, a_2)u(a_i)$ where $a_i = f_i(I_1, I_2)$ is candidate i's level of ambiguity; $P_i(I_1, a_1; I_2, a_2)$ denotes the probability that candidate i wins the election given the candidates' choices of ideologies and levels of ambiguity; and $u(a)$ is an increasing function that represents the utility of winning the election given the level of ambiguity chosen by the candidate in his platform.

During the campaign voters learn the platforms presented by the candidates. Therefore, they learn the candidates' ideologies and their levels of ambiguity. The decision rule of the voters is defined on these two variables as a result of the maximization of their expected utility. Given two candidates with the same level of ambiguity, a voter prefers the one whose ideology is closer to her ideal point. We also assume that voters dislike ambiguous candidates. In general, we will assume that given two candidates with the same ideology, all voters prefer the one with the lowest level of ambiguity. We will only relax this assumption in a special case that gives the same qualitative results, in which the structure of the ideology set calls for a more natural description of the voters' preferences.

Voters vote sincerely. That is, they vote for the candidate who is ranked higher according to their decision rule. When a voter is indifferent between the two candidates, she votes randomly for either candidate.

We assume that candidates do not know the distribution of the voters' ideal points, instead they have a common belief about this distribution. Alternatively, we could assume that candidates know the exact distribution of the voters' ideal points, but voter turnout is random. If the candidates knew the preferences of the median voter, the choice of ideologies would be trivial: both candidates would choose the most preferred ideology by the median voter and the competition of the second stage would lead both candidates to choose unambiguous platforms. Thus, it is essential that candidates are uncertain about the preferences of the median voter in order to derive more interesting results.

The temporal aspect of the choice of the platforms is also a necessary condition to have a result other than Downs' *median voter*. If the choice of ideology and ambiguity was simultaneous, candidates would never choose different ideologies in equilibrium, since approaching the position of the other candidate would always be a profitable deviation. The choice of platforms in two stages allows the candidates to commit, with their choices of ideologies at the beginning of the campaign, to a softer competition in ambiguity in the second stage.

We now proceed to solve two different specifications of this model. First we solve a very simple case, in which the number of ideologies and levels of ambiguity available to the candidates are minimal, but enough to show the choice in equilibrium of different ideologies and ambiguous platforms. Then we analyze a general case, with a continuum of ideologies, and a continuum of levels of ambiguity, that reproduce the same qualitative results.

3. A Discrete Model of Strategic Ambiguity.

In this model, we assume that, in the first stage of the game, a candidate can choose to be either "*Leftist*", "*Centrist*", or "*Rightist*". Thus, we have that the set of ideologies is $=\{L,C,R\}$. In the second stage of the game, the level of ambiguity can take only two values, $a \quad \{a_l, a_h\}$ where $0 \leq a_l < a_h$. a_l stands for a choice of a low level of ambiguity, and a_h stands for a choice of a high level of ambiguity.

We assume that the utility that candidates derive from being in office is represented by $u(a) = k + a$, where a is the level of ambiguity of the candidate's platform, and k is a positive constant to be interpreted as the utility that the candidate derives from winning the election *per se*. Note that as k increases, the significance of ambiguity in the candidate's payoff decreases and the model "converges" to the usual Downsian model.

The voters in this model are assumed to belong to three main blocs: *Leftist*, *Centrist*, and *Rightist*. The preferences of the voters depend only on their ideological identification, that is, on their blocs. We present the voters' preferences on platforms in the following table: (the alternatives are ranked in decreasing order from top to bottom),

Leftist	*Centrist*	*Rightist*
(L,a_l)	(C,a_l)	(R,a_l)
(L,a_h)	(C,a_h)	(R,a_h)
$(C,a_l)\,(C,a_h)$	$(L,a_h)\,(R,a_h)$	$(C,a_l)\,(C,a_h)$
(R,a_h)	$(L,a_l)\,(R,a_l)$	(L,a_h)
(R,a_l)		(L,a_l)

Voters' preferences are based on the premise that candidates are likely to adopt policies that agree with their ideology and less ambiguous candidates are even more likely to do so. If an ambiguous candidate implements a policy that is not his stated ideology, voters assume that centrist candidates are equally likely to drift to either side of the policy space, while extremist candidates are more likely to drift to the center. Thus, the voters' decision rule is lexicographic: when comparing two candidates, a voter always prefers the one who is ideologically closer to her. Only if the ideologies of the candidates are identical, does the voter consider their ambiguity levels. The preference for ambiguity depends on the ideology of the voter, and, specifically, on whether she would like the candidate to "drift" from his stated ideology. A leftist voter, for example, has the following preferences: she prefers a candidate which stands for a leftist ideology to a centrist candidate, and a centrist candidate is obviously preferred to a rightist candidate. As for the ambiguity of the candidates - an unambiguous leftist candidate is the best, a more ambiguous leftist candidate is not as good, but is still better than anyone else. The voter is indifferent between the levels of ambiguity of centrist candidates. Lastly, an ambiguous rightist candidate is preferred to an unambiguous one - which is worse yet. The preferences of a rightist voter are symmetric: first comes an unambiguous rightist candidate, then a more ambiguous rightist candidate, and so on. A centrist voter prefers an unambiguous centrist candidate the most, then she prefers a more ambiguous centrist candidate. She is indifferent between ambiguous or lowly committed leftist and rightist candidates, and is most averse to unambiguous or highly committed extreme candidates. It is important to note that these voters' preferences can be justified on grounds of stochastic dominance. Namely, a leftist voter prefers an unambiguous or a highly committed leftist candidate to a more ambiguous or less committed leftist candidate because the probability that the latter will implement a centrist or rightist policy is higher. Similarly, an ambiguous leftist candidate is preferred to a centrist candidate and so on.[2]

Voters vote sincerely. That is, they vote for the candidate who is ranked higher in their preference profile. When a voter is indifferent between the two candidates, she votes randomly for either candidate.

Without loss of generality, we normalize the size of the population to be 1. We denote the size of the "*Leftist*" bloc by n_L, the size of the "*Centrist*" bloc by n_C, and the size of the "*Rightist*" bloc by n_R. Each bloc has a non-negative size and

[2] Similar results obtain when voters' preferences are such that between two candidates who chose the same ideology, voters always prefer the candidate with a lower level of ambiguity. However, we believe that the preferences above are more natural.

$n_L + n_C + n_R = 1$. Each voter knows the bloc to which she belongs, or alternatively, she knows her preferences. This information, however, is unobservable to the candidates. The candidates do not know the exact sizes of the voters' blocs, but they have beliefs about them. Specifically, we assume that the candidates have an identical prior distribution defined over n_L, n_C, and n_R. In general, the beliefs of the candidates can be described by a probability distribution over the two dimensional simplex as in figure 1.

Each point in the figure corresponds to a different distribution of bloc's sizes. The respective sizes of the leftist and rightist blocs are depicted by the axes, and the size of the centrist bloc corresponds to the distance of the point from the diagonal line connecting the points (0,1) and (1,0). Thus, for example, the probability that the leftist bloc forms a majority corresponds to the integral of the distribution function over the area denoted by δ, the probability that the rightist bloc forms a majority corresponds to the integral of the distribution function over the area denoted by α, and the probability that the number of leftist voters exceeds that of the rightist voters corresponds to the integral of the distribution function over the area denoted by $\gamma + \delta$.

As we demonstrate in the sequel, the exact distribution of the sizes of voters' blocs is immaterial. For our results, the information contained in the distribution can be summarized by the following two probabilities: the probability that the leftist bloc forms a majority, or $P\left(n_L > 1/2\right)$; and the probability that the rightist block forms a majority, or, $P\left(n_R > 1/2\right)$. We focus our attention on the case where the median voter, as perceived by the candidates, belongs to the centrist bloc of voters. That is, we assume that $0 \le P\left(n_L > 1/2\right), P\left(n_R > 1/2\right) \le 1/2$.[3]

In a subgame perfect equilibrium, the candidates choose their ideologies in the first stage while taking into account the implications of their choices to the second stage game. In the second stage, they continue to play their equilibrium strategies, as foreseen in the first stage of the game. Our results depend on the relative significance of ambiguity as expressed by the ratio $\frac{k + a_l}{k + a_h}$, denoted ρ. We summarize the results in the following theorem.

[3] Relaxing this assumption yields "less interesting" equilibria where both candidates choose a leftist or rightist ideology according to the location of the median voter.

THEOREM 1 *If* $\frac{1}{2} \leq \rho = \frac{k + a_l}{k + a_h} < 1$, *in the electoral game described above, generically, there exists a unique subgame perfect equilibrium whose outcomes are as follows:*

(*a*) *When* $P(n_L > \frac{1}{2}) < \frac{\rho}{2}$ *and* $P(n_R > \frac{1}{2}) < \frac{\rho}{2}$, *both candidates choose a centrist ideology and a low level of ambiguity.*

(*b*) *When* $\frac{\rho}{2} < P(n_L > \frac{1}{2})$ *and* $P(n_L > \frac{1}{2}) > P(n_R > \frac{1}{2})$ *one candidate chooses a leftist ideology, the other a centrist ideology, and both choose a high level of ambiguity.*

(*c*) *When* $\frac{\rho}{2} < P(n_R > \frac{1}{2})$ *and* $P(n_R > \frac{1}{2}) > P(n_L > \frac{1}{2})$ *one candidate chooses a rightist ideology, the other a centrist ideology, and both choose a high level of ambiguity.*

(The proof of the theorem is relegated to the appendix.)

Figure 2 depicts the ideologies chosen as equilibrium outcomes. On the borders between the different areas of the figure (that is, on the lines), the possible equilibrium outcomes are those of the bordering areas.

When ρ is small, the significance of ambiguity in the payoff of the candidates is very high, and it is not surprising that in equilibrium both candidates would choose ambiguous platforms[4]. This theorem focuses on the more interesting case in which ρ is not too small, and it shows that we can only find two kind of equilibria (up to renaming the candidates). In one of the equilibrium, the two candidates choose the same ideology in the first stage of the game, and a low level of ambiguity in the second stage. We find this equilibrium when the candidates believe that it is very unlikely that the median voter belongs to any of the extreme blocs. Thus, a candidate that deviates from the Centrist ideology would sacrifice an important part of his payoff due to the decrease in the probability of winning the election.

[4] If $\rho < \frac{1}{2}$ our analysis shows that equilibria where both candidates choose the same ideology and a high level of ambiguity exist. We find these equilibria – as well as the assumption that the level of ambiguity is very important to the candidates (namely, $\frac{\rho}{2} < \frac{1}{2}$) – somewhat less interesting.

When candidates believe that there is a chance that the median voter belongs to one of the extreme blocs, it is profitable for one of the candidates to choose the ideology of this bloc, even if it means sacrificing a little probability of winning, because he can compensate this loss with the gains derived of an ambiguous platform. Therefore, in the second kind of equilibrium, candidates choose different ideologies and a low high level of ambiguity in the second stage.

Notice that the way in which the candidates measure whether it is likely or not that the median voter belongs to a certain ideological bloc is relative to the ratio of the payoffs from ambiguity. As the value of this ratio increases (either k is very high, or the values of a_h and a_l are very similar), the incentives of the candidates to choose different ideologies disappear. Thus, the results can be summarized as follows: when the value of ambiguity is high relative to the value of winning the election *per se*, in equilibrium candidates will choose different ideologies and ambiguous platforms, otherwise, they will choose the same ideology and low levels of ambiguity.

4. A Continuous Model of Strategic Ambiguity.

In this section we present a continuous version of the same electoral game. In the first stage of the game the candidates choose their ideologies I_i . Notice that the space of ideologies in this case is the real line. We denote the ambiguity level of candidate *i* by $a_i \quad {}_+$, and it can be any non negative number. As before, in the second stage of the game, candidates determine their ambiguity levels $f_i(I_1, I_2) \quad {}_+$ conditional on their ideology choices of the first stage. The utility that a candidate derives from winning the election is represented by $u(a) = k + a^2$.[5]

A voter with an ideal point v derives a utility $u_v(p) = -(p - v)^2$ when policy p is implemented. Voters interpret a candidate's choice of an ideology I and an ambiguity level a as inducing a distribution $\pi(I, a)$ over his implemented policy once in office which is uniform over the interval $[I - a, I + a]$. Voters vote for the candidate that maximizes their expected utility. That is, a voter with an ideal point v votes for the candidate that maximizes her expected utility $U_v(I,a) = E_{\pi(I,a)}[u_v(p)] = -(I - v)^2 - \dfrac{a^2}{3}$. In case of indifference, she votes randomly.

[5] Notice that unlike in the previous section, candidates' utilities depend on their level of ambiguity squared. We make this technical assumption to simplify the analysis.

Candidates are uncertain about the distribution of voters' ideal points. More specifically, we assume that the candidates believe that the ideal point of the median voter is uniformly distributed over the interval $[0,1]$.

We summarize the results in the following theorem.

THEOREM 2 *In the electoral game described above,*

- *when* $\frac{33}{16} < k$, $I_1 = I_2 = 1/2$, *and* $a_1 = a_2 = 0$ *is the unique (up to renaming the names of the parties) subgame perfect equilibrium outcome.*
- *when* $0 \le k < 4/3$, $I_1 = -1/4$, $I_2 = 5/4$, *and* $a_1 = a_2 = 9/2 - k$ *is the unique subgame perfect equilibrium outcome.*
- *when* $4/3 \le k \le \frac{33}{16}$, *the two previous equilibrium outcomes are the only subgame perfect equilibria outcomes.*

(The proof of the theorem is relegated to the appendix.)

As in the previous case, the results depend on the importance of the level of ambiguity to the candidates. In this model the value of ambiguity for the candidates can be measured by the size of k. When ambiguity does not play a major role in the candidates preferences, that is when k is large, both candidates choose the median voter's ideology in the first stage of the game and strongly commit to it. When, on the other hand, the candidates value the flexibility in choosing their subsequent policy more, that is when k is small, the candidates choose different ideological positions in the first stage of the game in order to relax the ambiguity competition in the second stage.

Thus, the results of this model replicate the ones found in the discrete one. Notice that, unlike in the discrete case where the subgame perfect equilibrium was unique, in the continuous case, the two kinds of equilibria coexist for a certain range of the values of k.

5. Related Literature

5.1 Political Science Literature

As mentioned in the introduction, the notion of strategic ambiguity has been extensively dealt with in political science literature. (For a survey of this literature, see Shepsle (1972)) This literature has lead to several attempts of formal modelling of strategic ambiguity. Generally, these formal models have employed the assumptions of the standard spatial model. Ambiguous strategies were represented as probability distributions (lotteries) over the policy space. Zeckhauser (1969) is probably the earliest formal discussion of ambiguous policy formation. He shows that under certain conditions, a lottery over some subset of the alternatives can defeat the median position, and that a component of this lottery can defeat the lottery itself. Thus, an alternative that wins a majority of the vote may not exist. However, he shows that if an equilibrium of the m-dimensional election game exists, it must be in unambiguous strategies. Shepsle (1972) shows that if only uniform lotteries are permitted and the incumbent is restricted to select a less ambiguous lottery than the challenger, there exist voter preferences such that the challenger's choice will command more votes than any policy available to the incumbent. Mckelvey (1980) studies the effect of the introduction of a fixed amount of ambiguity (or variance). He shows that it has no effect on the location or existence of equilibria in unidimensional models. For higher dimensions, assuming that voters' utility functions are multivariate normal density functions, the introduction of ambiguity does not disrupt equilibria when they exist.

In contrast to the results of this paper, most of the former literature on strategic ambiguity did not differ qualitatively from the standard spatial model literature. Ambiguous policies where chosen by candidates only in special cases of models with asymmetric assumptions on the behavior of candidates. One exception is found in Alesina and Cukierman (1990). In their model candidates have ideal points in the policy space, and the incumbent faces the trade-off between implementing his ideal point and implementing the policy that maximizes his chances of reelection. In their model, voters are not perfectly informed about the preferences of the candidates, and the level of ambiguity is defined as the variance of the noise between the policy outcome observed by the voters and the policy instrument chosen by the candidates. Thus, as in our case, ambiguity allows candidates to exploit this trade-off.

5.2 Related Economic Literature

The model presented here is reminiscent of a variation of Hotelling's (1929) model, due to D'Aspremont, Gabszewicz, and Thisse (1979). In Hotelling's model, two sellers choose locations on a line of finite length, to be thought of as "main-street", and then compete in prices. Consumers are evenly distributed along the line and each one of them consumes exactly one unit of the product, irrespective of its price. Each consumer buys from the seller who quotes the least delivered price, that is, the mill price plus the transportation costs which where assumed to be linear with respect to the distance between the consumer and the seller. In this model, Hotelling derived the Principle of Minimal Differentiation: both sellers will tend to position themselves at the center of the market.

D'Aspremont, Gabszewicz, and Thisse considered a slightly modified version of Hotelling's model where consumers have quadratic transportation costs as a function of the distance. They show that, in the first stage of the game, the sellers locate as far from one another as possible - the first seller locates at the leftmost end of the line segment, and the second seller locates at the rightmost end of the line segment. The intuition behind this result is that the sellers can soften the price competition of the second stage by locating far away from each other in the first stage of the game. The sellers do not have an incentive to move closer to the median consumer because if they do so the other seller will retaliate in the second stage by cutting his prices and escalating the price war. Locating as far as possible from the other seller allows the sellers to charge higher prices without losing their consumers.

The intuition of the two-stage competition in our model is similar to theirs. Voters dislike ambiguous candidates, in the same fashion than consumers, in Hotelling's model, dislike high prices. While candidates like ambiguity, as sellers benefit from high prices. The main difference between the two models is that in our case candidates are concerned with the probability of winning the election, rather than the maximization of their share of the vote (the share of the market in Hotelling's model). This discontinuity of the candidates' utility (with respect to the share of the voters at $\frac{1}{2}$) makes our model and results rather different. In particular the principle of *maximal* differentiation does not hold in our model. But the intuition behind the equilibrium in which candidates choose different ideologies and ambiguous platforms is what drives Hotelling result: candidates differentiate in the first stage of the game in order to soften the competition in the second stage.

6. Conclusion

In this paper we try to find a reason why political candidates often express electoral promises in vague and ambiguous terms. One of the results of this analysis offers an answer to why political candidates with the same electoral objectives end up making different electoral promises. These two questions represent important puzzles in the political science and, as such, have been widely discussed. Nevertheless, we think that the answer that we offer is very innovative, with respect to the existing literature.

The first important feature is the link we present between these two puzzles. Offering candidates the opportunity to present ambiguous platforms, gives them incentives to choose different ideologies. As a result we obtain an explanation of why Downs' result does not necessarily hold.

The existing literature offers three possible explanations to policy and ideological differentiation. Probabilistic voting (see e.g., Hinich, 1977), parties with different policy preferences (see e.g., Wittman, 1983), and sequential entry (Palfrey, 1984). By contrast, our model suggests that the strategic role of ambiguity can account for this phenomena.

Incorporating the choice of the level of ambiguity adds a new strategic dimension to the standard model of electoral competition. The candidates may have an incentive to differentiate themselves in the ideology space so that they can soften the competition in the ambiguity space. Hence, this model generalizes the result of Downs (1957) by showing that the median voter result, where both candidates choose the same ideological position, holds only as a special case. Yet, the spirit of the median voter result is retained. From the voters' perspective, ambiguity (or low commitment) blurs the ideological differences between the candidates. Less committed candidates that have chosen different ideologies during the campaign might end up choosing similar policies in case they win the election because by choosing ambiguous platforms, the policies available to both candidates may coincide.

Generally, a candidate can choose to become very explicit with respect to certain issues, and extremely ambiguous or vague with respect to other issues. For clarity of exposition, we introduce a simple model with only one ideology dimension into the formal models and assume that a candidate simply has to choose whether he is ambiguous or not. Thus, these models make the simplest assumptions possible while still capturing the fact that the candidates are free to decide how much information they release regarding their future policies.

Finally, we emphasize that any model that shares the underlying features of our models, namely, a two-stage game where the candidates and voters have opposed preferences regarding the outcome of the second stage of the game, and uncertainty about the median voter's preferences, will yield similar results: candidates may choose to differentiate themselves in the first stage of the game in order to relax the competition in the second stage of the game. The interpretation of the formal model presented here, of strategic ambiguity, is not the only possible one. For example, another interpretation might be the strategic choice of the level of corruption. As in our model, both candidates can be thought of as sharing a common interest for higher personal corruption that may impair their chances of winning the election. Thus, when the benefit from corruption is sufficiently high, the candidates will differentiate themselves ideologically in the first stage of the game so that they will be able to relax the competition in the second stage of the game and be more corrupt. (Myerson (1993) offers related analysis.)

References

Alesina, A. and A. Cukierman (1990) The politics of ambiguity. *Quarterly Journal of Economics* 105, 829-850.

D'Aspremont, C., Gabszewicz, J. J. and Thisse, J. F. (1979) On Hotelling's stability in competition. *Econometrica* 47, 1045-1070.

Downs, A. (1957) *An Economic Theory of Democracy*. New York: Harper and Row.

Hinich, M. (1977) Equilibrium in spatial voting: the median voter result is an artifact. *Journal of Economic Theory* 16, 208-219.

Hinich, M. and Munger, M. C. (1992) A spatial theory of ideology. *Jouurnal of Theoretical Politics* 4, 5-30.

Hotelling, H. (1929) Stability in Competition. *Economic Journal* 39, 41-57.

Kreps, D. M. (1979) A representation theorem for "preference for flexibility." *Econometrica* 47, 565-577.

McKelvey, R. D. (1980) Ambiguity in spatial models of policy formation. *Public Choice* 35, 385-402.

Morton, R. B. and Myerson, R. B. (1992) Campaign spending with impressionable voters. Mimeo, Northwestern University.

Myerson, R. B. (1993) Effectiveness of electoral systems for reducing government corruption: a game theoretic analysis. *Games and Economic Behavior* 5, 118-132.

Palfrey, T. (1984) Spatial equilibrium with entry. *Review of Economic Studies* 51, 139-156.

Sen, A. (1988) Freedom of Choice. *European Economic Review* 32, 269-294.

Shepsle, K. A. (1972) The strategy of ambiguity: uncertainty and electoral competition. *American Political Science Review* 66, 555-568.

Wittman, D. A. (1983) Candidate motivation: a synthesis of alternatives. *American Political Science Review* 77, 142-157.

Zeckhauser, R. (1969) Majority rule with lotteries on alternatives. *Quarterly Journal of Economics* 83, 696-703.

Appendix: Proofs

PROOF OF THEOREM 1 We start by computing the equilibria of the second stage games. Symmetry considerations imply that in order to analyze the second stage game, we need to study only four different classes of second stage games:

(1) Where both candidates have chosen a centrist ideology in the first stage, or $G(C,C)$.

(2) Where both candidates have chosen an identical ideological position in the first stage, but not the centrist one, $G(L,L)$ or $G(R,R)$.

(3) Where the candidates have chosen adjacent ideological positions in the first stage, $G(L,C)$, $G(C,L)$, $G(C,R)$, or $G(R,C)$.

(4) Where the candidates have chosen extreme ideological positions in the first stage of the game, $G(L,R)$ or $G(R,L)$.

First, consider the game $G(C,C)$**.** Notice that since leftist and rightist voters are indifferent to the results of the elections, the vote of the centrist voters determines the winner.

$G(C,C)$	a_l	a_h
a_l	$\tfrac{1}{2}(k+a_l)$, $\tfrac{1}{2}(k+a_l)$	$k+a_l$, 0
a_h	0, $k+a_l$	$\tfrac{1}{2}(k+a_h)$, $\tfrac{1}{2}(k+a_h)$

It is straightforward to verify that when $\dfrac{k+a_l}{k+a_h} > \tfrac{1}{2}$, (a_l,a_l) is the only equilibrium of the game $G(C,C)$.

In the game $G(L,C)$ leftist voters vote for the leftist party and centrist and rightist voters vote for the centrist party.

$G(L,C)$	a_l	a_h
a_l	$\left(1-P\left(n_L > \frac{1}{2}\right)\right)(k+a_l)$ $P\left(n_L > \frac{1}{2}\right)(k+a_l)$	$\left(1-P\left(n_L > \frac{1}{2}\right)\right)(k+a_h)$ $P\left(n_L > \frac{1}{2}\right)(k+a_l)$
a_h	$\left(1-P\left(n_L > \frac{1}{2}\right)\right)(k+a_l)$ $P\left(n_L > \frac{1}{2}\right)(k+a_h)$	$\left(1-P\left(n_L > \frac{1}{2}\right)\right)(k+a_h)$ $P\left(n_L > \frac{1}{2}\right)(k+a_h)$

Strict dominance considerations imply that (a_h, a_h) is the unique equilibrium of this game as well as of the games $G(C,L)$, $G(C,R)$ and $G(R,C)$. In the game $G(L,L)$ the less ambiguous candidate gets the vote of the leftist bloc and the more ambiguous candidate gets the vote of the centrist and rightist voters. Therefore, again, strict dominance considerations imply that (a_h, a_h) is the unique equilibrium. Similarly, (a_h, a_h) is the unique equilibrium of the game $G(R,R)$ as well.

In the game $G(L,R)$ leftist voters vote for the leftist candidate, rightist voters vote for the rightist candidate, and centrist voters vote for the more ambiguous of the two candidates.

$G(L,R)$	a_l	a_h
a_l	$P(n_R > n_L)(k+a_l)$ $P(n_L > n_R)(k+a_l)$	$\left(1-P\left(n_L > \frac{1}{2}\right)\right)(k+a_h)$ $P\left(n_L > \frac{1}{2}\right)(k+a_l)$
a_h	$P\left(n_R > \frac{1}{2}\right)(k+a_l)$ $\left(1-P\left(n_R > \frac{1}{2}\right)\right)(k+a_h)$	$P(n_R > n_L)(k+a_h)$ $P(n_L > n_R)(k+a_h)$

It is straightforward to verify that (a_h, a_h) is the only equilibrium of $G(L,R)$. Analogously, it is also the only equilibrium of $G(R,L)$.

LEMMA *In a subgame perfect equilibrium, the candidates do not choose* (L,R), (R,L), (L,L), *or* (R,R) *in the first stage of the game.*

Proof We show that the candidates do not choose (L,R) in the first stage of the game. Since (a_h, a_h) is the unique equilibrium played after the candidates choose (L,C) in the first stage of the game, by deviating and choosing C, the second candidate gets the vote of the centrist voters and so increases his probability of winning the elections from $P(n_R > n_L) = 1 - P(n_L > n_R)$ to $1 - P(n_L > 1/2)$ without changing his level of ambiguity. A similar argument shows that, in a subgame perfect equilibrium, the candidates do not choose (R,L) either. In much the same way, candidates do not choose (L,L) or (R,R) in a subgame perfect equilibrium. In a subgame perfect equilibrium, the fact that $0 \le P(n_L > 1/2), P(n_R > 1/2) \le 1/2$ implies that by deviating to the center a candidate increases his probability of winning without decreasing his level of ambiguity. ■

Thus, up to renaming the candidates, only (C,C), (L,C) and (C,R) can be chosen in the first stage of the electoral game in a subgame perfect equilibrium. To complete the proof of the theorem notice that when $\rho/2 > \max\{P(n_L > 1/2), P(n_R > 1/2)\}$, $(C,a_l;C,a_l)$, is the unique subgame perfect equilibrium of the electoral game, when $P(n_L > 1/2) > \rho/2$ and $P(n_L > 1/2) > P(n_R > 1/2)$, $(L,a_h;C,a_h)$ is the unique subgame perfect equilibrium of the electoral game, and when $(n_R > 1/2) > \rho/2$ and $P(n_L > 1/2) < P(n_R > 1/2)$, $(C,a_h;R,a_h)$ is the unique subgame perfect equilibrium of the electoral game. In case of equalities, all the equilibria of the neighboring regions are possible. ■

PROOF OF THEOREM 2 As in the proof of theorem 1, we compute the subgame perfect equilibria through backward induction. First, we compute the equilibrium levels of ambiguity as a function of the ideology choices of the first stage of the electoral game and then we compute the equilibrium's ideologies. In a subgame perfect equilibrium (SPE), if the candidates choose the same ideologies in the first stage of the game, all voters vote for the candidate that chooses a lower level of ambiguity in the second. Therefore, the only second stage SPE involves both candidates choosing zero ambiguity. Suppose then that $I_1 < I_2$. For $i \in \{1,2\}$, denote $\alpha_i = a_i^2$. Thus, when the parties choose I_1, a_1 and I_2, a_2 the ideal point of the indifferent voter is

$$v^* = \frac{I_1 + I_2}{2} + \frac{\alpha_2 - \alpha_1}{6(I_2 - I_1)}.$$

The probabilities with which the candidates win the election are,

$$P_1(I_1, a_1; I_2, a_2) = \begin{cases} 0 & v^* \le 0 \\ v^* & 0 \le v^* \le 1 \\ 1 & 1 \le v^* \end{cases} \qquad P_2(I_1, a_1; I_2, a_2) = \begin{cases} 1 & v^* \le 0 \\ 1 - v^* & 0 \le v^* \le 1 \\ 0 & 1 \le v^* \end{cases}.$$

Therefore, the candidates' respective utilities are,

$$U_1(I_1, a_1; I_2, a_2) = \begin{cases} 0 & v^* \le 0 \\ v^*(k + \alpha_1) & 0 \le v^* \le 1 \\ k + \alpha_1 & 1 \le v^* \end{cases} \quad U_2(I_1, a_1; I_2, a_2) = \begin{cases} k + \alpha_2 & v^* \le 0 \\ (1 - v^*)(k + \alpha_2) & 0 \le v^* \le 1 \\ 0 & 1 \le v^* \end{cases}$$

Notice that the utilities are either linear or quadratic in α. Thus, in the second stage of the game, when I_1, I_2 and α_2 are fixed, the optimal α_1 is,

$$\alpha_1^*(I_1, I_2, \alpha_2) = \max\left\{0, \frac{1}{2}\left(3\left(I_2^2 - I_1^2\right) - k + \alpha_2\right), 3\left(I_2^2 - I_1^2\right) - 6(I_2 - I_1) + \alpha_2\right\}.$$

When I_1, I_2 and α_1 are fixed, the optimal α_2 is,

$$\alpha_2^*(I_1, I_2, \alpha_1) = \max\left\{0, \frac{1}{2}\left(-3\left(I_2^2 - I_1^2\right) + 6(I_2 - I_1) - k + \alpha_1\right), -3\left(I_2^2 - I_1^2\right) + \alpha_1\right\}.$$

We represent the α_i^*'s as reaction functions. We distinguish between two cases (1) where $k \ge 6(I_2 - I_1)$, and (2) where $k \le 6(I_2 - I_1)$. When $k \ge 6(I_2 - I_1)$,

$$\alpha_1^*(I_1, I_2, \alpha_2) = \begin{cases} 0 & 0 \le \alpha_2 \le 6(I_2 - I_1) - 3\left(I_2^2 - I_1^2\right) \\ 3\left(I_2^2 - I_1^2\right) - 6(I_2 - I_1) + \alpha_2 & 6(I_2 - I_1) - 3\left(I_2^2 - I_1^2\right) \le \alpha_2 \end{cases}$$

$$\alpha_2^*(I_1, I_2, \alpha_1) = \begin{cases} 0 & 0 \le \alpha_1 \le 3\left(I_2^2 - I_1^2\right) \\ \alpha_1 - 3\left(I_2^2 - I_1^2\right) & 3\left(I_2^2 - I_1^2\right) \le \alpha_1 \end{cases}$$

and when $k \le 6(I_2 - I_1)$,

$$\alpha_1*(I_1,I_2,\alpha_2)=\begin{cases}0 & 0\le\alpha_2\le k-3\left(I_2^2-I_1^2\right)\\ \frac{1}{2}\left(3\left(I_2^2-I_1^2\right)-k+\alpha_2\right) & k-3\left(I_2^2-I_1^2\right)\le\alpha_2\le 12\left(I_2-I_1\right)-3\left(I_2^2-I_1^2\right)-k\\ 3\left(I_2^2-I_1^2\right)-6\left(I_2-I_1\right)+\alpha_2 & 12\left(I_2-I_1\right)-3\left(I_2^2-I_1^2\right)-k\le\alpha_2\end{cases}$$

$$\alpha_2*(I_1,I_2,\alpha_1)=\begin{cases}0 & 0\le\alpha_1\le 3\left(I_2^2-I_1^2\right)-6\left(I_2-I_1\right)+k\\ \frac{1}{2}\left(6\left(I_2-I_1\right)-3\left(I_2^2-I_1^2\right)-k+\alpha_1\right) & 3\left(I_2^2-I_1^2\right)-6\left(I_2-I_1\right)+k\le\alpha_1\le 3\left(I_2^2-I_1^2\right)+6\left(I_2-I_1\right)-k\\ \alpha_1-3\left(I_2^2-I_1^2\right) & 3\left(I_2^2-I_1^2\right)+6\left(I_2-I_1\right)-k\le\alpha_1\end{cases}$$

We continue by incorporating the second stage equilibrium levels of ambiguity into the candidates' utilities, and compute the candidates' utilities as functions of the ideologies alone. Notice that the α_i*'s are continuous in I_1 and I_2 and therefore the candidates' utilities are continuous in I_1 and I_2.

We start by analyzing the simpler case, where $k\ge 6\left(I_2-I_1\right)$. We distinguish three subcases. ***(i)*** Suppose that $0<I_2+I_1<2$. This implies that $0<v*<1$, and therefore $\alpha_1*=\alpha_2*=0$, $U_1=\frac{I_2+I_1}{2}k$ and $U_2=\left(1-\frac{I_2+I_1}{2}\right)k$. ***(ii)*** Suppose that $I_2+I_1\le 0$. It follows that $v*=0$, $\alpha_1*=0$ and $\alpha_2*=-3\left(I_2^2-I_1^2\right)$. Consequently $U_1=0$ and $U_2=k-3\left(I_2^2-I_1^2\right)$. Lastly, in case that ***(iii)*** $2\le I_2+I_1$. It follows that $v*=1$, $\alpha_1*=3\left(I_2^2-I_1^2\right)-6\left(I_2-I_1\right)$ and $\alpha_2*=0$. Thus, $U_1=k+3\left(I_2^2-I_1^2\right)-6\left(I_2-I_1\right)$ and $U_2=0$. In all the above cases, at least one of the candidates can always benefit by locating closer to the other candidate in the first stage of the game. Therefore, we conclude that no equilibrium exists in this range of ideology choices.

We now analyze the more complicated case where $k\le 6\left(I_2-I_1\right)$. We distinguish six cases. The six cases correspond to the six possibilities of matching the slopes of α_1* and α_2* which are 0, $\frac{1}{2}$,or 1. (Three of the nine possibilities of matching the slopes are impossible.) We number these cases **(1.1)**, **(1.2)**, **(2.1)**, **(1.3)**, **(3.1)**, and **(2.2)**. (**(1.3)**, for example, represents the region where α_1* has slope 0 and α_2* slope 1.) We represent the regions that correspond to these cases in figure 3. As we show in the sequel, candidates' utilities in regions (*i*), (*ii*) and (*iii*) coincide with those of regions (1.1), (1.3) and (3.1) respectively.

We now show that except for region (2.2), all regions of ideology choices do not admit the existence of an equilibrium.

(1.1) When $\left\{\begin{array}{l} 3\left(I_2^2 - I_1^2\right) - 6\left(I_2 - I_1\right) + k \ge 0 \\ k - 3\left(I_2^2 - I_1^2\right) \ge 0 \end{array}\right\}$, $\alpha_1 * = \alpha_2 * = 0$. It follows that $v* = \frac{I_2 + I_1}{2}$. Since $k \le 6\left(I_2 - I_1\right)$ it follows that $0 \le v* \le 1$. Therefore, in this region $U_1 = \frac{I_2 + I_1}{2}k$ and $U_2 = \left(1 - \frac{I_2 + I_1}{2}\right)k$. Since both candidates can increase their utility by moving closer to the other candidate in the first stage of the game, no equilibrium exists for this range of ideology choices.

(1.3) When $\left\{3\left(I_2^2 - I_1^2\right) + 6\left(I_2 - I_1\right) - k \le 0\right\}$, $\alpha_1 * = 0$ and $\alpha_2 * = -3\left(I_2^2 - I_1^2\right)$. It follows that $v* = 0$. Therefore, in this region $U_1 = 0$ and $U_2 = k - 3\left(I_2^2 - I_1^2\right)$. Since candidate 1 can guarantee himself a positive utility by choosing candidate's 2 ideology in the first stage of the game, no equilibrium exists for this range of ideology choices.

(3.1) When $\left\{-3\left(I_2^2 - I_1^2\right) + 12\left(I_2 - I_1\right) - k \le 0\right\}$, $\alpha_1 * = 3\left(I_2^2 - I_1^2\right) - 6\left(I_2 - I_1\right)$ and $\alpha_2 * = 0$. It follows that $v* = 1$. Therefore, in this region $U_1 = 3\left(I_2^2 - I_1^2\right) - 6\left(I_2 - I_1\right) + k$ and $U_2 = 0$. Since candidate 2 can guarantee himself a positive utility by choosing candidate's 1 ideology in the first stage of the game, no equilibrium exists for this range of ideology choices.

(2.2) When $\left\{\begin{array}{l} k - \left(I_2^2 - I_1^2\right) - 2\left(I_2 - I_1\right) \le 0 \\ \left(I_2^2 - I_1^2\right) - 4\left(I_2 - I_1\right) + k \le 0 \end{array}\right\}$, $\alpha_1 * = \left(I_2^2 - I_1^2\right) + 2\left(I_2 - I_1\right) - k$ and $\alpha_2 * = -\left(I_2^2 - I_1^2\right) + 4\left(I_2 - I_1\right) - k$. It follows that $v* = \frac{1}{3} + \frac{I_2 + I_1}{6}$. We claim that $0 \le v* \le 1$. Notice that $0 \le v* \Leftrightarrow I_2 + I_1 \ge -2$ and that $v* \le 1 \Leftrightarrow I_2 + I_1 \le 4$ and that both inequalities hold. Therefore, in this region $U_1 = \left(\frac{1}{3} + \frac{I_2 + I_1}{6}\right)\left(2\left(I_2 - I_1\right) + \left(I_2^2 - I_1^2\right)\right)$ and $U_2 = \left(\frac{2}{3} - \frac{I_2 + I_1}{6}\right)\left(4\left(I_2 - I_1\right) - \left(I_2^2 - I_1^2\right)\right)$. The only possible equilibrium in this region is $I_1 * = -\frac{1}{4}$, $I_2 * = \frac{5}{4}$, and $\alpha_1 * = \alpha_2 * = \frac{9}{2} - k$ when $k \le \frac{9}{2}$.

(2.1) When $\left\{\begin{array}{l} k - 3\left(I_2^2 - I_1^2\right) \le 0 \le 12\left(I_2 - I_1\right) - 3\left(I_2^2 - I_1^2\right) - k \\ 0 \le \left(I_2^2 - I_1^2\right) - 4\left(I_2 - I_1\right) + k \end{array}\right\}$, $\alpha_1 * = \frac{1}{2}\left(3\left(I_2^2 - I_1^2\right) - k\right)$ and $\alpha_2 * = 0$. It follows that $v* = \frac{I_2 + I_1}{4} + \frac{k}{12\left(I_2 - I_1\right)}$. We claim that $0 \le v* \le 1$. Notice that $v* \ge 0 \Leftrightarrow 3\left(I_2^2 - I_1^2\right) + k \ge 0$ and $v* \le 1 \Leftrightarrow 12\left(I_2 - I_1\right) - 3\left(I_2^2 - I_1^2\right) + k \ge 0$. Therefore, in this region $U_1 = \left(\frac{I_2 + I_1}{4} + \frac{k}{12(I_2 - I_1)}\right)\frac{1}{2}\left(3\left(I_2^2 - I_1^2\right) + k\right)$ and $U_2 = \left(1 - \frac{I_2 + I_1}{4} - \frac{k}{12(I_2 - I_1)}\right)k$. We claim that no equilibrium exists for this range of ideology choices. In this region, $\frac{dU_2}{dI_2} = -\frac{k}{4} + \frac{k^2}{12(I_2 - I_1)^2}$ and therefore 2's best response to 1 is to set $I_2 = I_1 + \sqrt{\frac{k}{3}}$ whenever it is possible in this region and to set I_2 on the boundaries of the region when it is not possible. Since the utilities of the candidates are continuous in the ideologies, the analysis of the other regions shows that there can be no equilibrium on the boundaries of 2.1. Specifically, 2.1 borders with regions 1.1, 3.1, and 2.2. We already proved that no equilibrium exists in regions 1.1 and 3.1 including their boundaries and the only equilibrium that may exist in region 2.2 does not lie on its boundary with region 2.1. We now show that there can be no interior equilibrium in 2.1 either. In any interior equilibrium, 2 sets $I_2 = I_1 + \sqrt{\frac{k}{3}}$. 1's utility in this case is $\frac{1}{2}\left(\sqrt{3k}I_1^2 2kI_1 + \frac{k^2}{\sqrt{3k}}\right)$. This is a convex function and therefore cannot be maximized in the interior of 2.1.

(1.2) When $\left\{\begin{array}{c} \left(I_2^2 - I_1^2\right) + 2\left(I_2 - I_1\right) - k \le 0 \\ 3\left(I_2^2 - I_1^2\right) - 6\left(I_2 - I_1\right) + k \le 0 \le 3\left(I_2^2 - I_1^2\right) - 6\left(I_2 - I_1\right) - k \end{array}\right\}$, $\alpha_1{*} = 0$ and $\alpha_2{*} = \frac{1}{2}\left(6\left(I_2 - I_1\right) - 3\left(I_2^2 - I_1^2\right) - k\right)$. It follows that $v{*} = \frac{1}{2} + \frac{I_2 + I_1}{4} - \frac{k}{12(I_2 - I_1)}$. We claim that $0 \le v{*} \le 1$. Notice that $0 \le v{*} \Leftrightarrow 6\left(I_2 - I_1\right) + 3\left(I_2^2 - I_1^2\right) - k \ge 0$ and that $v{*} \le 1 \Leftrightarrow -3\left(I_2^2 - I_1^2\right) + 6\left(I_2 - I_1\right) + k \ge 0$. Therefore, in this region

$$U_1 = \left(\frac{1}{2} + \frac{I_2 + I_1}{4} - \frac{k}{12(I_2 - I_1)}\right)k$$

and

$$U_2 = \left(\frac{1}{2} - \frac{I_2 + I_1}{4} - \frac{k}{12(I_2 - I_1)}\right)\frac{1}{2}\left(-3\left(I_2^2 - I_1^2\right) + 6\left(I_2 - I_1\right) + k\right).$$

Symmetry arguments (to region 2.1) imply that no equilibrium exists in this range of ideology choices.

Thus, we have identified two candidate equilibria

(i) $I_1{*} = I_2{*} = \frac{1}{2}$ and $\alpha_1{*} = \alpha_2{*} = 0$. And,

(ii) $I_1{*} = -\frac{1}{4}$, $I_2{*} = \frac{5}{4}$, and $\alpha_1{*} = \alpha_2{*} = \frac{9}{2} - k$.

We claim that (i) holds as SPE for any $k \geq \frac{4}{3}$ and that (ii) holds as SPE for any $k \leq \frac{33}{16}$. To verify the first claim notice that when $I_1{*} = I_2{*} = \frac{1}{2}$ and $\alpha_1{*} = \alpha_2{*} = 0$ the candidates will benefit mostly from deviating into region 2.2. Symmetry considerations imply that it is sufficient to check the equilibrium against a deviation of one candidate only. Candidate 1 will benefit mostly by deviating into region 2.2 and choosing $I_1 = -\frac{1}{2}$ which will give him a utility of $\frac{2}{3}$, hence the bound $k \geq \frac{4}{3}$. (It is straightforward to verify that 1 will not deviate into region 1.1 where he would rather locate as close as possible to candidate 2, to region 1.3 where he gets a utility of 0, nor to region 1.2 where his maximal utility is $\left(\frac{1}{2} - \frac{k}{12}\right)k \leq \frac{k}{2}$. Similarly, 2 will not deviate to regions 1.1, 3.1 and 2.1, and therefore by applying symmetry again we conclude that 1 will not deviate and choose $I_1 > I_2$ either.) To verify the second claim notice that when $I_1{*} = -\frac{1}{4}$, $I_2{*} = \frac{5}{4}$, and $\alpha_1{*} = \alpha_2{*} = \frac{9}{2} - k$, candidate 1 will benefit mostly by deviating into region 3.1 where his utility is $U_1 = k + 3\left(I_2^2 - I_1^2\right) - 6\left(I_2 - I_1\right)$. Analogously, candidate 2 will benefit mostly by deviating into region 1.3 where his utility is $U_2 = k - 3\left(I_2^2 - I_1^2\right)$. As before, it is sufficient to verify that the equilibrium is immuned against a deviation of one candidate. The highest utility that candidate 1 can achieve in region 3.1 is obtained when $I_1 = 1$ and equals $k + \frac{3}{16}$ while at equilibrium 1's utility equals $\frac{9}{4}$. Hence the bound $k \leq \frac{33}{16}$. (It is straightforward to verify that 1 will not deviate into region 1.2 where he is forced to choose a zero level of ambiguity, nor to region 1.3 where he gets a utility of 0. Similarly, it is easy to see that 1 will not deviate and choose $I_1 > I_2$. It is more difficult to see that deviating into region 3.1 is preferable to deviating into region 2.1, but intuitively, 1 is better off in 3.1 than in 2.1.) This completes the proof of the theorem.