Consumer Choice in Competitive Location Models: Formulations and Heuristics

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Abstract

A new direction of research in Competitive Location theory incorporates theories of Consumer Choice Behavior in its models. Following this direction, this paper studies the importance of consumer behavior with respect to distance or transportation costs in the optimality of locations obtained by traditional Competitive Location models. To do this, it considers different ways of defining a key parameter in the basic Maximum Capture model (MAXCAP). This parameter will reflect various ways of taking into account distance based on several Consumer Choice Behavior theories. The optimal locations and the deviation in demand captured when the optimal locations of the other models are used instead of the true ones, are computed for each model. A metaheuristic based on GRASP and Tabu search procedure is presented to solve all the models. Computational experience and an application to 55-node network are also presented.
Keywords: distance, competitive location models, consumer choice behavior, GRASP, tabu.

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1. INTRODUCTION

Traditional Competitive Location Theory assumes that consumers patronize the closest store, but does this judgment reflect the real behavior of consumer's choice?
It seems more realistic to consider that the consumer does not consider distance alone.

Consumer Choice Behavior literature studies the key variables that a customer takes into account to patronize one or other facility. This literature usually assumes that the customer not only cares about patronizing the closest facility but also considers others variables to make the decision.
The development of this new branch of theory has been very extensive. Basically, all these models are based on Newton's Law of Gravitation[1] (1686). Reilly (1929) and Converse (1949) adapted it to the economic case, assuming that "the probability that a customer patronizes a facility is proportional to its attractiveness and inversely proportional to a power of distance to it".

In the next step, the Luce axiom of discrete choice[2] was introduced in these models by Huff (1964). The Huff probability formulation uses distance (or travel time) from retail centers and the size of retail centers as inputs to find the probability of consumers shopping at a given retail outlet.

Finally, Nakanishi and Cooper (1974) extended Huff's model by including a set of attractiveness attributes to the store (not only one attribute as in Huff's model). This more general statement was known as the Multiplicative Competitive Interaction (MCI) Model. Its formulation is as follows:

$$P_{ij} = \frac{\prod_{k=1}^{q} X_{kij}^{\beta_k}}{\sum_{j=1}^{n} \prod_{k=1}^{q} X_{kij}^{\beta_k}}$$

where, P_{ij} is the probability of customers living at site *i* shopping at store *j*, X_{kij} is the *k*-th attribute describing store *j* attracting customers from site *i*, and β_k is the estimated parameter reflecting sensitivity of customers with respect to attribute *k*. The MCI models were based on consumer surveys and customer origin studies to determine the most significant attribute in each market (attributes such as consumer evaluation of store image, store appearance, service levels, and also objective measures as the numbers of checkout counters and availability of credit card services).

[1] Newton's Law of Gravitation studies the force between planets and stars in the universe. This law states that the force between two bodies is proportional to the product of the masses of the bodies and inversely proportional to the square of the distance between them.
[2] Luce axiom applied to this case assumes that customer choose the optimal location option as a function of the utility of this option with respect to the level of utility of the other options.

Another line of study within the retail store area is the **Competitive Location Literature.** This one addresses the issue of optimally locating firms that compete for clients in space. The first study of this line is due to Hotelling work (1929), where consumers were assumed to patronize the closest facility. Different models based on this assumption of consumer behavior have been developed. The most relevant ones are based on Voronoi Diagrams and Location-Allocation models that jointly determine the optimal location of service facilities and the allocation of service areas to them (Hodgson (1978)).

Several lines of work have been developed in this field. The key one for this paper was developed by ReVelle (1986). ReVelle and his followers have constructed a group of models that examined competition among retail stores in a spatial market. The basic model was the Maximum Capture Problem (MAXCAP, ReVelle (1986)). This model analyzes the location of servers by an entering firm that maximizes its market share captured in a market where competitor servers are already in position. This model has been adapted to different situations. The first modification introduced facilities that are hierarchical in nature and where there is competition at each level of the hierarchy (Serra, et. al. (1992)). A second extension took into account the possible reaction from competitors to the entering firm (Serra and ReVelle (1994)). Finally, another modification of the MAXCAP problem introduced scenarios with different demands and / or competitor locations (Serra and ReVelle (1996)). A good review of these models can be found in Serra and ReVelle (1996).

All these Competitive Location theories find optimal locations assuming that customers patronize the closest shop. But as we have seen previously, theories of Consumer Choice Behavior conclude that consumers take into account other attributes of the stores apart from distance when choosing which outlet to patronize.

This last statement sheds light on the next direction of research in Location theory, trying to include theories of Consumer Choice Behavior. Three papers are particularly worth noting from the brief review of the literature on this new approach.

Karkazis (1989) considered two criteria for which customers would decide which shop to patronize. On the one hand, a Level Criterion based on the preference of a customer on the site of the facility and on the other hand a Distance Criterion based on the closeness to the store. The problem was solved in a dynamic fashion, as there was a trade-off between both criteria.

Another important paper in this direction is the one by Eiselt and Laporte (1989). They analyze a conditional location problem on a weighted network. To do this, they generalized ReVelle's finding of the MAXCAP formulation in order to include parameters based on Gravity models and Voronoi diagrams. These parameters or attraction functions were defined in terms of consumer facility distances and facility weights. The purpose of the paper is to locate an additional facility and determine simultaneously the optimal weight of that new facility. The model was solved using a simple procedure, which considers in turn all candidates' locations.

Finally, a more recent paper by Santos-Peñate, et.al. (1996) analyzes the choice of the location and the optimal level of a service center's attractiveness for a firm that wants to enter in a market where the competitor's firm is already operating. The maximizing profit model was a modification of the traditional competitive location model using Huff's model and the Multiplicative Competitive model for consumer choice behavior. The model was simply solved using a Greedy Adding procedure together with a Teitz and Bart algorithm.

This paper also follows this new direction of research. In order to incorporate these theories of Consumer Choice Behavior in Competitive Location models, two issues have to be analyzed: (1) Which is the best way to include distance in Competitive Location models? and (2) Which key attributes of stores (apart from distance) have to be included in Competitive Location models?

Given to the complexity of these questions, this paper tries to answer only the first question. Do we have to take into account the various theories of consumer choice behavior to introduce distance in one or another way in the location models? How should we include distance in our location models?
To do this, we consider different ways of defining a key parameter of the basic model. This parameter will reflect the various ways of taking distance into account based on different Consumer Choice Behavior theories. The basic MAXCAP model (Maximum Capture model, ReVelle (1986)) considered the traditional assumption where consumers patronize the closest facility, only comparing its *distance to the closest facility for different chains*. In this model, the closest facility captures all the demand (basic idea: *all or nothing capture*). The Multiplicative Competitive Interaction model (MCI, Nakanishi and Cooper (1974)) and the Proportional Customer Preference model (Serra, et.al. (1997)) are based on the same idea (basic idea in Hakimi (1990)). This idea states that customers do not choose the chain, instead they select probabilities that are *functions of their distance to all outlets*, then the demand captured in each node by each outlet is *proportional* to the distance from node *i* to all the outlets, regardless of ownership. The difference between both models is the incorporation consumers' sensitivity to the distance involved. MCI introduced this parameter, while the Proportional Customer Preference model assumes that this sensibility is equal to 1. Finally, the Partially Binary Preference model (Serra, et.al. (1997)) assumes that consumers patronize *the closest facility of the chosen chain*. In this case, the capture obtained in demand node *i* by each firm is *proportional* to the distance from node *i* to the closest facility.

To solve this model, a new metaheuristic has been developed based on two well-known metaheuristics, TABU search and Greedy Randomized Adaptive search procedure (GRASP) method.

After the application of the heuristic to several numerical cases, we will be able to analyze whether the optimality of the locations substantially differ or not depending on the Consumer Behavior theory we take into account.

The paper is organized as follows. In section 2, the decision models are presented. In section 3, a metaheuristic based on Tabu search and Greedy Randomized Adaptive search is developed. Section 4 presents some computational experience in different sized networks. In section 5, a 55-node network example is presented. Finally, section 6 includes some concluding remarks.

2. THE MODELS

In all the models, the basic problem states that a new firm (from now on Firm A) wants to enter with p servers in a market in order to obtain the maximum capture, given that it has to compete with q existing outlets. These competitors can belong to one or more firms, but without loss of generality it is assumed that there is only one competing firm (Firm B) operating in the market; as was assumed by ReVelle (1986).

These models study the location of retail facilities in discrete space. The models make the following assumptions:

(1) The customer wants to buy a unit of a specific product; i.e. we do not take into account multipurpose shopping behavior.
(2) The product sold is homogeneous, in the sense that the customer goes to buy the same product at all the outlets.
(3) Unit costs are the same in all stores regardless of ownership.
(4) Both firms are profit maximizing.
(5) The spatial market is defined by a connected graph. At each vertex of the graph, there is a local market with a given number of consumers that generates a demand for the product.
Potential locations for the services are also pre-specified (note that all outlets are allowed to locate only at the vertices of the graph).
(6) Under equal conditions (in terms of distance) the existing firm captures the demand [3].

[3] Note that here, we use the Hakimi assumption (1986) that states that in case of equal distance to the outlets from a node, demand is fully allocated to the existing firm. But this is not a key issue because it is easy to modify the models if we change this assumption (for example, the entrant firm will keep all the demand under equal conditions, or split the demand between both models).

Defining a key parameter of the models ($\rho(x_{ij})$) in a different way, we will be able to reflect the various ways of taking distance into account based on several Consumer Choice Behavior theories.
Solving these models will provide the optimal locations for the entering firm in each case. But, how can we analyze whether the optimal solution changes dramatically when applying these different models? To do so, we will compute the deviation in demand captured by each model and the optimal locations it provides and compare these with the optimal locations provided by other models.

Deviation–*ij*: $$\left[Z_i^*\left(Loc_i\right) - Z_i\left(Loc_j\right)\right] \Big/ Z_i^*\left(Loc_i\right) \qquad " \ i \ ," \ j$$

Where, $Z^i(Loc_i)$ is the demand captured by model *i* when the optimal location found by model *i* is used and $Z^i(Loc_j)$ is the demand captured by model *i* when the optimal location found by model *j* is used.

Analysis of these deviations will give an idea of the importance of introducing distances in different ways. Basically,

- *If these deviations are not significant* .The conclusion will be that does not matter how distance is included because the demand captured by the optimal locations will be similar in all the models.
- *If these deviations are significant.* The conclusion will be that before applying a location model (in order to find the optimal locations), we have to analyze which consumer behavior better represents the one analyzed. This prior analysis will tell us how to introduce distance in Competitive Location models.

Model 1: Maximum Capture model (MAXCAP).

We will use the Maximum Capture model (MAXCAP) of ReVelle (1986) in the P-median-like formulation as follows:

$$\text{MAX} \quad Z_1 = \sum_{i \in I} \sum_{j \in J} a_i \ \rho(x_{ij}) \ x_{ij} \qquad (1)$$

Subject to

$$\sum_{j \in J} x_{ij} = 1 \qquad \forall i \in I \qquad (2)$$

$$x_{ij} \leq x_{jj} \qquad \forall i \in I, \ \forall j \in J \qquad (3)$$

$$\sum_{j \in J} x_{jj} = p \qquad (4)$$

$$x_{ij} = (0,1) \quad x_{jj} = (0,1) \quad \forall i \in I, \ \forall j \in J$$

Where the parameters are:

$i, I =$ Index and set of local markets that are located at the vertex of the graph.

$j, J =$ Index and set of potential locations for firm A's outlets.

$J^B (\in J) =$ The set of actual locations of the *q* firm B's outlet.

$d_{ij} =$ The network distances between local market *i* and an outlet in *j*.

$a_i =$ Demand at node *i*.

And the variables are defined as follows:

$x_{ij} =$ 1, if demand node *i* is assigned to node *j*; 0, otherwise.

$x_{jj} =$ 1, if an outlet of firm's A is opened at node *j*; 0, otherwise.

The constraint set basically states that: constraint set (2) forces each demand node *i* to assign to only one facility. But for a demand node *i* to be assigned to a facility at *j*, there has to be a facility open at *j*; this is achieved by constraint set (3). Finally, constraint (4) sets the number of outlets to be opened by firm A. The objective function defines the total capture that firm A can achieve with the siting of its *p* servers. The definition of $\rho(x_{ij})$ is the key parameter for our work. Basically, it will reflect the proportion of demand captured by an outlet at *j* from a demand node *i*. The definition of this parameter will depend on the Choice Consumer Behavior model we consider.

In this case, the MAXCAP model uses the traditional view of **all or nothing capture** due to the distance criterion. This is the general assumption where consumers patronize the closest shop. In other words, for each demand node, **consumers compare the distance between the closest firm A server and the closest firm B server.** Applied to this problem, an outlet of firm A located at *j* will capture all the demand in *i* if its distance to *i* is less than the distance between local market *i* and the closest B server. Thus, under this assumption the definition of $\rho(x_{ij})$ is as follows:

$$\rho(x_{ij}) = 1, \text{ if } d_{ij} < d_{ib_i}; \ 0, \text{ otherwise.}$$

Where, d_{ib_i} is the distance from node *i* to the closest B server.

This criterion is a usual assumption in the most important Competitive Location models as ReVelle (1986), Serra et. al. (1992,1994,1996),

Models 2 and 3.

The next two models are based on the idea that the probability that a customer at location *i* will shop at retail facility *j* is a **relative function of all its distance to the outlets.**

The basic idea is that demand captured at each node by each outlet is proportional to the distance from node *i* to all the outlets, regardless of ownership.

For these two cases, the formulation of the problem is as follows:

$$\text{MAX} \quad Z_{2,3} = \sum_{i \in I} \sum_{j \in J} a_i \, \rho\left(x_{ij}\right) \; x_{i,j} \qquad (1)$$

Subject to

$$x_{ij} \le x_{jj} \qquad \forall i \in I, \; \forall j \in J \quad (3)$$

$$\sum_{j \in J} x_{jj} = p \qquad (4)$$

$$x_{ij} = (0,1) \qquad \forall i \in I, \; \forall j \in J$$

This formulation is similar to the one in the P-median problem, except in that we do not include the constraint set (2). The set of constraints (2): $\sum_{j \in J} x_{ij} = 1 \quad \forall i \in I$, forces each demand node *i* to assign to only one facility. In these models we do not need this set of constraints as we control this effect including $\rho\left(x_{ij}\right)$ (jointly with a_i) in the objective function. $a_i \rho\left(x_{ij}\right)$ is the part of the demand in *i* (in absolute value) that a facility located at *j* captures from the total demand on *i* (a_i).Then, in the model we avoid the multiple incorporation of a_i in absolute values by different location assignments but we allow the assignment of different parts of a_i to different facilities.

Model 2: Multiplicative Competitive Interaction (MCI) model

The Consumer Choice Behavior Literature assumes that customers consider other variables apart from the distance to choose the facility they will patronize. The best example of this literature is the MCI model developed by Nakanishi and Cooper (1974).

In this paper, we use the version of the MCI model made by Jain and Mahajan (1979). They took into account that the characteristics of a retail facility could come from two sets. The first one includes the characteristics which are *independent* of the consumer's point of origin (e.g.: quality of product and services, in-store convenience level, price of the product, sales area in the store,). The other set includes the characteristics which are *dependent* on the consumer's point of origin (e.g.: distance or travel time).

Thus, the definition of ρ_{ij} based on this simple MCI modification is as follows:

$$\rho_{ij} = \left(\prod_{k=1}^{s} A_{kj}^{\beta_k}\right)\left(\prod_{e=1}^{r} B_{eij}^{\beta_e}\right) \Big/ \sum_{j=1}^{m}\left[\left(\prod_{k=1}^{s} A_{kj}^{\beta_k}\right)\left(\prod_{e=1}^{r} B_{eij}^{\beta_e}\right)\right]$$

Where,

ρ_{ij} = The probability that a customer at location *i* will shop at retail facility *j*. (The proportion of capture that an outlet in *j* will achieve by demand node *i*)

A_{kj} = The *k*-th attribute of the retail facility *j* which is independent of the consumer's point of origin; *k* = 1,*s*.

B_{eij} = The *e*-th attribute of the retail facility *j* which is dependent on the consumer's point of origin, *e* = 1, *r*.

m = *p* + *q*, total number of outlets in the market.

β_k, β_e = Empirically determined parameters, which reflect the sensitivity of the retail outlet characteristics on the probability to shop at a particular store.

As the objective of the paper is the analysis of the way of introducing distance in Competitive Location models, we can assume that all outlets are similar (for example; pharmacies,). Then, the attributes of the outlets that are independent of the customer's point of origin are equal for all the outlets and equal to 1.

Assumption 1: $A_{kj} = 1 \Rightarrow \prod_{k=1}^{s} 1^{\beta_k} = 1$

Also we can assume that the only relevant attribute dependent on the consumer's point of origin is the distance.

Assumption 2: $B_{eij} = d_{ij}$ *as* $e = 1$.

As the distance is a disutility for the consumer, we will advance that b_d will be negative. To make comparisons easier, we will put this parameter in absolute value and remove the distance to the denominator.

At the same time, the summation of the denominator can be decomposed, using the definition $m = p + q$, as the summation of the distance to firm A 's located outlets (*p*) and the firm B's located outlets (*q*). Then, using the notation of the MAXCAP model and applying all these assumptions and simplifications, we can rewrite the definition of $\rho(x_{ij})$.

$$\rho(x_{ij}) = \frac{1 / d_{ij}^{\,b_d}}{\sum_{j \in J} \left(1 / d_{ij}^{\,b_d} \right) x_{jj} + \sum_{j \in J^B} \left(1 / d_{ij}^{\,b_d} \right)}$$

$\rho(x_{ij})$ is the proportion of capture that an outlet in *j* will achieve at demand node *i*. The basic idea here is that customers select stores (regardless of the ownership) with probabilities that are inversely proportional to a function of their distances, taking into account of consumers' sensitivity to distance.

Model 3: Proportional Customer Preference model

A new line of Consumer Behavior theory developed by Serra et.al. (1997) considers the existence of interaction among outlets that affect consumer decision. To reflect this behavior pattern, they develop a model assuming that all outlets compete for customers. A customer does not choose the chain; instead he selects stores with probabilities that are inversely proportional to functions of his distance to all the stores. Thus, the basic idea is that the demand captured in each node by each outlet is **proportional to the distance from node i to all the outlets, regardless of ownership**.

In this case, the definition of $\rho(x_{ij})$ is as follows[4]:

[4] Note that this model is non-linear since there are x_{jj} variables in the denominator.

$$\rho(x_{ij}) = \frac{1/d_{ij}}{\sum_{j \in J} \left(1/d_{ij}\right) x_{jj} + \sum_{j \in J^B} \left(1/d_{ij}\right)}$$

Where, $\rho(x_{ij})$ is again the proportion of capture that an outlet in *j* will achieve by demand node *i*.

We should observe that the only difference in the definition of $\rho(x_{ij})$ between this model and the MCI model is the parameter β_d (i.e. sensitivity of consumers to the distance attribute in the choice among outlets). The sensibility of Location Models to distance can therefore only be found by comparing the deviation of optimality of the locations founds in both models.

Model 4: Partial Binary Preferences

In Serra et.al. (1997), a second model is defined which reflects partial binary customer preferences. This states that consumers patronize the **closest outlet of the chosen chain.** Thus, the proportion of times that he picks one outlet is inversely proportional to some function of the distance.

In this case, the capture obtained in demand node *i* by each firm is **proportional** to the distance from node *i* to the closest facility. Therefore, the definition of $\rho(x_{ij})$ in this case is as follows[5]:

$$\rho(x_{ij}) = \frac{d_{i\beta_i}}{d_{i\alpha_i} + d_{i\beta_i}}$$

Where,

$d_{i\alpha_i}$ = Distance from demand node *i* to the closest A server.

$d_{i\beta_i}$ = Distance from demand node *i* to the closest B server (the competing firm)

$\rho(x_{ij})$ = The proportion of capture that an outlet in *j* will achieve by demand node *i*.

Finally, it should be noted that the formulation in this case is equal to the MAXCAP formulation.

3. A META-HEURISTIC TO SOLVE THE MODEL

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[5] Note that since α_i is in the faction's denominator of the parameter (i.e. in the objective), the model will always assign node *i* to the closest firm A's facility.

The models presented in the previous section are combinatorial optimization problems[6]. Many combinatorial problems are intractable and belong to the class of *NP*-Hard (non-deterministic polynomial-time complete) problems. In our case, the p-Median problem is *NP*-Hard on a general graph (Kariv and Hakimi; 1979).

The common belief in this field is that no efficient algorithm could ever be found to solve these inherently hard problems. Heuristics (or approximate algorithms) are considered one of the practical tools for solving hard combination optimization problems.

Several heuristics have been studied to solve the p-Median problem. Those heuristics can be grouped in two classes (Golden, et.al (1980)): construction algorithms and improvement algorithms.
The former type tries to build a good solution from the beginning. In this group we can find the well-known greedy adding and greedy subtracting algorithm.
The second class of algorithms use a known starting solution and try to improve on it. The best representative of this group is the well-known Teitz and Bart (1968) one-opt heuristic.
This method has been successful applied in Serra and Marianov (1996). But this heuristic has some problems. The first well-known problem is the possibility to find local optima and the second one is a more recent one found by Rosing (1997). He has demonstrated that the solution provided by an interchange heuristic (p.e. the case of the Teitz and Bart heuristic applied to the p-Median problem) deteriorates, when either the number of demand nodes and / or the number of facilities to be located increases. This deterioration can be reflected both in the probability of finding the optimal solution and in the closeness of a typical solution to the optimal one.

The most recent development in approximate search methods for solving complex optimization problems is known as ***Metaheuristics***.

A metaheuristic is a process which applies a subordinate heuristic at each step which has to be designed for each particular problem. Although there is no guarantee of optimality of these methodologies; Metaheuristics have proved highly successful in obtaining high quality solutions to many real world complex problems.

The basic families of metaheuristics are: genetic algorithms, greedy random adaptive search procedures, problem-space search, simulated annealing, tabu search, threshold algorithms and heuristic concentration (good review in Osman (1995)).

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[6] Combinatorial Optimization problems are normally easy to describe but difficult to solve (Osman (1995))

The more recent Metaheuristic methods applied to Competitive Location theories are: **Tabu Search, Heuristic Concentration and GRASP.**

In essence, **Tabu Search** (Glover, 1977,1989,1990) explores a part of the solution space by repeatedly examining all neighborhoods of the current solution, and moving to the best neighborhood even if this deteriorates the objective function. This approach tries to avoid being trapped in a local optimum. In order to avoid the cycling solution that has recently been examined, these are inserted in a tabu list that is constantly updated. Additionally, several criterias of flexibility can be included in the tabu search as aspiration, intensification, diversification and stopping criteria.

This method has been successfully applied to a wide variety of location problems: p-hub location problems (Klincewicz (1992) and Marianov, et.al. (1997)), (r | *Xp)*- Medianoid and (r | *p*)- Centroid Problems (Benati and Laporte (1994)), the Vehicle Routing Problem (Gendreau, et.al. (1994)) and p-Median problem (Rolland, et.al. (1996)).

Another important metaheuristic is the **Greedy Randomized Adaptive Search (GRASP)** (developed by Feo and Resende (1989)). GRASP is a type of a more general class of metaheuristic called Problem-space methods. The Problem-space methods were a class of heuristics superimposed on fast problem-specific constructive procedures. The aim was to generate many different starting solutions that can be improved by local search methods.

GRASP is an iterative process; each GRASP iteration consists of two phases: a construction phase and a local search phase. The best overall solution is kept as the result.

In the first phase, a feasible solution is iteratively constructed, one element at a time. At each construction iteration, the choice of the next element to be added is determined by ordering all elements in a candidate list with respect to a greedy function. The heuristic is adaptive because the benefits associated with every element are updated at each iteration of the construction phase.

In order to find a Restricted Candidate List (RCL), the Cardinality and Value restrictions were applied to the ordered candidates. The Cardinality restriction restricts the initial length of the RCL, while the value restriction restrict the candidates by the value of its greedy function. The probabilistic component appears in this phase by randomly choosing one of the best candidates in the list, but usually not the top candidate. This choice technique of the GRASP construction phase allows different solutions to be obtained at each GRASP iteration, but (as in other many deterministic methods) this solution is not guaranteed to be locally optimal with respect to simple neighborhood definitions. Hence, the second GRASP phase tries to improve each constructed solution. Usually, a local optimization procedure such as a two-exchange is used in this second part.

GRASP has been applied successfully to several combinatorial problems such as: p-hub location problems (Klincewicz (1992)), Quadratic Assignment Problems (Li, et.al. (1994)), Maximum

Independent Set Problem (Feo, et.al. (1994)), Satisfiability Problem (Resende and Feo (1996)) and Dense Quadratic Assignment Problems (Resende, et.al. (1996)).

Finally, the more recently metaheuristic is the **Heuristic Concentration (HC)** (Rosing and ReVelle, 1996, 1997; Rosing, 1997). The development of this new metaheuristic was the result of the observation that the author made of the fact that different random trials of an interchange heuristic generally give solutions that are highly similar to the specific demand nodes selected to be facilities. HC has two stages. In stage one, Concentration Set (CS) is constructed by multiple random-start runs of an interchange heuristic. Then, in stage two, the best solution extracted from the CS is found by using an exact procedure (p.e. integer linear program) or a good solution (possible optimal) by a heuristic.

In this paper, we develop a metaheuristic that will be applied to all the models described in the previous section. Basically this new heuristic is based on Tabu Search and GRASP heuristics previously described. In essence, the metaheuristic has two phases. In the first one, we construct a good initial solution using the GRASP procedure. In the second phase, we improve the previous solution found applying the well-known Tabu Search heuristic including the aspiration and diversification criterion.

<u>Formal description of the metaheuristic procedure</u>

PHASE 1: GRASP

1. K=1.

Construction Phase (Construct a Greedy Randomized Solution)

2. Let $i \longrightarrow i + 1$ (LOCPi)
3. Compute Z^m for all nodes j not in $LOCP_i$ (or in $LOCQ_i$). Relabel the solution $LOCP_j$ in decreasing order of Z^m ($LOCP_j$). Relabel all vertices accordingly. Apply the Cardinality (BETA) and Value (ALPHA) Restrictions to construct the Restricted Candidate List.
4. Choose Randomly from among the elements of the Restricted Candidate List with each element having equal probability. Add this random choice to set $LOCP_i$.
5. If $i \leq P$, go to step 2.

Local Search phase: Teitz and Bart.

6. Start with the initial solution set $LOCP_i$ found in the construction phase. Compute $Z^m{}_c(LOCP_i)$.
7. Let $i \longrightarrow i + 1$ (LOCPi)
8. Let $e \longrightarrow e + 1$; $e \in$ (empty nodes). Relocate $LOCP_i$ to an empty node. Compute $Z^m{}_{en}(LOCP_i)$. If $Z^m{}_{en}(LOCP_i) > Z^m{}_{en}(LOCP_i)$, keep the relocation and relabel $Z^m{}_{en}(LOCP_i)$ as $Z^m{}_{STEP8}(LOCP_i)$. Restart step 8 until all the empty nodes are checked.

   If $i \leq P$, go to step 7.

9. If $Z^m_{STEP8}(LOCP) > Z^m_c(LOCP)$, then go to step 7.

10. If $K < MAXITER$, $K \rightarrow K+1$ and go to step 1. Updating the Best Solution found $Z^m_{BEST}(LOCP)$ in each GRASP iteration.

PHASE 2: TABU SEARCH (following Benati and Laporte (1994))

1. Let $t = 0$ (number of iterations of the TABU procedure).
2. Set $Z^m_0(LOCP) = Z^m_{BEST}(LOCP)$, the best solution found in GRASP. Set $LOCP_i^0 = LOCP_i^*$, for i=1,..,P the optimal locations found in GRASP.
3. $LOCP_i^0 \rightarrow LOCP_{i+1}^0$ (for each located node). Consider all solutions of the neighborhood of $LOCP_i^0$, obtained by exchanging a facility from node $LOCP_i^0 \in LOCP$ to a neighborhood node $LOCPngh_i \notin LOCP$. Relabel the solution LOCP in decreasing order of $Z^m(LOCPngh_i)$. Relabel all vertices accordingly.
4. If ($Z^m(LOCPngh_i) > Z^m_{BEST}(LOCP_i^0)$) or $LOCPngh_i$ is not tabu, then set $Z^m_{BEST}(LOCP_i^0) = Z^m(LOCPngh_i)$, the outlet is located in $LOCPngh_i$ and $LOCP_i^0$ is declared tabu until $t + \theta$, where θ is a pre-fixed value, and go to step 3. Otherwise, set $i = i + 1$.

   If all nodes visited are tabu and none improves the objective, then the model chooses the node with the lowest tabu tag ($t + \theta$) and lift the tabu status of $LOCPngh_i$. Go to step 3.
5. If t is less than a pre-fixed upper bound T; let $t \rightarrow t + 1$ and go to step 1.
6. If the iteration for the given starting solution is over, the model starts a new procedure with an initial solution equal to the NP least visited nodes. Go to step 1, the first time, otherwise, stop.

Comments about the meta-heuristic:

- Step 4 of Tabu Search, tabu status can be canceled if this permits an improvement in the objective. This rule is called *the aspiration criterion*

- Step 6 of Tabu Search, states *the diversification criterion,* that allows a broader exploration of the solution space by starting from locations that have been less well explored.

4. COMPUTATIONAL EXPERIENCE

The algorithm has been applied to several randomly generated networks. These networks have number of nodes equal to 20,30 and 50. For each *n*, three different number of outlets are located so that *p*=2,3,4;

while the number of the established firm are pre-fixed $q = 5$[7]. Finally, for each n and each p, ten networks were randomly generated. Therefore, a total of 90 networks were generated.

For each of these networks, nodes $n \in (0,1000)^2$ were generated following a uniform distribution. The Euclidean distances between nodes were computed. The neighborhood for each node is defined as the randomly (2-6) closest nodes using the Euclidean distance criterion. The demand in each node was randomly generated within the (800,1000) interval again following a uniform distribution.

For each 90 networks and for each model, the metaheuristic solutions were compared to the optimal ones. Optimal solutions were obtained by enumeration.

In phase 1 (GRASP) of the metaheuristic, K was equal to 20. In step 3 of this phase 1, we set the cardinality restriction as thirty- percent of the network size and the value restriction as fifty- percent of the best candidate objective. In phase 2 (Tabu Search) of the algorithm, the stopping criterion imposed was T equal to 40. In step 3 of this phase 2, the θ was set equal to 5. In step 6 of this phase 2, the diversification criterion is applied starting four times with the less visited nodes.

Finally, in the MCI model, the sensitivity of consumers to quadratic distance is used in all the cases (b_d).

The heuristic was programmed in FORTRAN77 and executed in a Pentium PC 133 with 16mb of RAM.

<u>Basic results regarding the behavior of the metaheuristic</u>

The results of the behavior of the metaheuristic are shown in Tables 1 (Tables 1a, 1b and 1c correspond to the *20-*, *30-*, *50-*nodes network respectively) and Tables 2 (Tables 2a, 2b and 2c correspond to the *20-*, *30-*, *50-*nodes network respectively).

Column 3 of Table 1 shows the percentage of times that the optimal solution was found by phase 1 of the algorithm, making it unnecessary to execute phase 2. For example, for *n* = 20, *p* =3 and *model* = 2; 80 % of the solutions found with this phase were optimal. Column 4 of table 1 shows the percentage of times that the optimal solution was found by phase 2 in the cases where the optimal solution was not found in the previous phase. For example, in all the runs of 20-network size, the TABU procedure (phase 2 of algorithm) found the optimal solution. Finally, the last columns indicate the average deviation from optimality where the algorithm failed to find the optimal solution. Only, 6 solutions out of the 360 runs

[7] For each network size, the outlets of firm B were located:

Outlets of firm B	**20-network**	**30-network**	**50-network**
Nodes	4,7,11,17,19	4,7,17,22,27	4,21,22,36,38

were non-optimal based on our comparison with complete enumeration. In general, the average deviation from optimality did not exceed 1%, except when n = 30, p =4 and *model* = 1, where the deviation from optimality was equal to 7.6 %.

Tables 2 shows the average execution time in seconds spent per phases by global metaheuristic and per enumeration procedure. Notice that the algorithm becomes very useful when the network size is greater than 30 nodes and we have to locate 3 or more entering outlets. In these cases, the time spent by the algorithm is less than the one for the enumeration procedure. For example, in n = 50, p = 4, *model* = 2, the time spent by the algorithm is 21.733 seconds while the enumeration procedure spent 658.374 seconds to find the same solution.

Although the average computing time of the metaheuristic increases with the number of nodes and the number of outlets, it is worth using in these cases, because it saves a lot of time compared enumeration procedure.

Basic results for the comparison of the models

In Tables 3, 4 and 5 the average deviation and the maximum deviation (in brackets) in demand captured (as is defined in page 6) of the *20*-nodes, *30*-nodes, *50*-nodes runs are shown. From these tables, we can extract the following basic conclusions:

- The greatest deviation in demand captured is the one found when we use the optimal locations of model 2,3,4 while the true model is the first one (the traditional MAXCAP model). This behavior is constant for all the network size run.

  For example, in n = 20, p =3, the deviation in capture by the use of optimal locations of model 3 (column 4) in relation to the use of the optimal location of model 1 (the true one's) is around 34.82 % on average.

- The smallest deviation in demand captures is achieved in two cases. When we use the optimal location of model 2 when the true model is the third one and reversibly, when we use the optimal locations of model 3 while the true model is the second one. For example, in n = 30, p = 3, the deviation in demand captured by the use of optimal location of model 3 when the true one is the second model is 1.78 % on average. And reversibly, the deviation of using the optimal location of model 2 when the true model is the third one is 0.97 % on average.

  As the only difference between model 2 (MCI model) and model 3 (Proportional Customer Preference's model) is the introduction of the sensitivity of consumers to the quadratic distance in the choice among outlets, we can conclude that the introduction of this sensitivity in the Competitive Location models is not important in terms of optimality.

- Finally, it seems that the use of the optimal locations of the traditional MAXCAP model produces the smallest deviation in demand captured. On average, this deviation is less than 10 % in all cases, but looking at the respective maximum deviations there seems to have a significant variation.

For example, in n = 50, p =3, when we use the optimal location of model 1, the average deviation in the second model was 6.85 % (the maximum deviation was 10.52 %), the average deviation in the third model was 6.27 % (the maximum deviation was 11.9 %) and the average deviation in the fourth model is 4.9 % (the maximum deviation was 8.61 %).

We can conclude that these deviations are significant. So, before applying one of the defined location models, we have to analyze which kind of consumer behavior we are dealing with. This analysis will tell us how to introduce distance in the location model. But if this analysis cannot be made or it is too costly, the location model that we will have to use is the traditional MAXCAP model (model 1) as this will give the smallest deviation in demand captured whatever the true model is.

5. AN EXAMPLE

In this case, the four models have been applied to a 55-node network (Swain 1974, Figure 4), where the total demand to capture is 3575. The demand at each node is indicated in Table 6.
As in the previous section, Firm B is already operating five outlets in the market. They are located at nodes 4, 21, 22, 36, 38. Three different scenarios are examined with regard to the number of outlets to be located by Firm A (p = 2, 3 and 4).

The market captured by the outlets located by Firm A in each scenario and for each model is presented in Table 7. The final locations of these new outlets of Firm A in each scenario and for each model are shown in Table 8.

Basic results of the behavior of the metaheuristic

Table 7. Values of the objective per phases, global metaheuristic and enumeration in 55-nodes network

p	Model	GRASP	TABU	Enum.	Deviation
2	1	1462	1462	1462	0 %
	2	1468.587	1468.587	1468.587	0 %
	3	1392.797	1392.797	1392.797	0 %
	4	1402.409	1402.409	1402.409	0 %
3	1	1764	1764	1764	0 %
	2	1742.691	1767.926	1767.926	0 %
	3	1711.473	1711.473	1711.473	0 %
	4	1474.34	1474.34	1474.34	0 %
4	1	2000	2000	2000	0 %
	2	1970.905	1970.905	1979.321	0.42 %
	3	1952.43	1952.43	1952.43	0 %
	4	1524.85	1542.85	1542.85	0 %

Table 8. Locations per phases of metaheuristic and enumeration in 55-nodes network

p	Model	GRASP	TABU	Enum.
2	1	13,42	13,42	5,42
	2	2,4	2,4	2,4
	3	2,4	2,4	2,4
	4	5,3	5,3	5,3
3	1	13,42,17	13,42,17	13,42,17
	2	4,3,1	4,3,5	4,3,5
	3	4,2,3	4,2,3	4,2,3
	4	5,31,33	5,31,33	5,31,33
4	1	30,13,42,17	30,13,42,17	5,17,30,42
	2	4,3,2,5	4,3,2,5	3,4,5,7
	3	2,3,4,5	2,3,4,5	2,3,4,5
	4	41,31,33,5	41,31,33,5	5,31,33,41

From the previous tables, we can point out that the optimal locations found by the Multiplicative Competitive Interaction model (model 2) and by the Proportional Customer Preference's model (model 3) are nearly the same ones. This fact explains why the deviations produced when using the locations of model 2 to evaluate the demand captured by model 3 and reversibly are the smallest ones. For example, in p = 2, there are no deviations in these cases (as is shown in Table 9).

Basic results for the comparison of the models

Tables 9, 10, 11 show the deviation in demand captured (as is defined in page 6) for the three scenarios. From these tables, we can extract the same conclusions found in computational experience. On the one hand, the greater deviation in demand captured is the one found in Table 9 using the optimal location of model 2 and 3 while the true model is the MAXCAP (model 1). These deviations are 63.54%. On the other hand, the use of the optimal locations of the traditional MAXCAP model is the one that produces the smallest deviation in demand captured. This deviation is less than 8.1% in all cases.

Table 9. Deviation for 55-nodes network. Case p = 2.

MODEL	Loc1	Loc2	Loc3	Loc4
1	0 %	63.54 %	63.54 %	16.76 %
2	6.97 %	0 %	0 %	3.59 %
3	5.42 %	0 %	0 %	3.03 %
4	4.64 %	31.15 %	31.15 %	0 %

Table 10. Deviation for 55-nodes network. Case p = 3.

MODEL	Loc1	Loc2	Loc3	Loc4
1	0 %	31 %	49.26 %	27.66 %
2	7.4 %	0 %	0.31 %	6.6 %
3	8.03 %	0.38 %	0 %	7.69 %
4	2.8 %	14.95 %	28 %	0 %

Table 11. Deviation for 55-nodes network. Case p = 4.

MODEL	Loc1	Loc2	Loc3	Loc4
1	0 %	31.05 %	33.45 %	16.8 %
2	6.26 %	0 %	0.42 %	5.8 %
3	7.35 %	0.78 %	0 %	8 %
4	3.6 %	20 %	17.77 %	0 %

6. CONCLUSIONS

This paper had tried to follow the new direction of research in Competitive Location Theory, which incorporates the theories of Consumer Choice Behavior in its models. This study tries to establish just how important the precise method of including distance in Competitive Location Models is in determining optimal locations. To do this, we have considered different ways of defining a key parameter in the basic MAXCAP model that would reflect various ways of taking distance into account based on several Consumer Choice Behavior theories.

The basic Maximum Capture model (MAXCAP) uses the traditional view of all or nothing capture by outlets, where consumers compare distance to the closest facility of the other chain. The Multiplicative Competitive Interaction (MCI) model and the Proportional Customer Preference model are based on the same idea: proportional capture where consumers select with probabilities that are functions of their distance to all the outlets. The difference between both models is the introduction in the MCI model of the sensitivity of consumers to quadratic distance. Finally, the Partial Binary Preference model assumes that consumers patronize the closest facility of the chosen chain and the capture is proportional.

In order to analyze if the optimality of locations changes dramatically when applying these different models, we have computed the deviation in demand captured by the use of the optimal location of the true model in relation to the use of the optimal locations of the other models.

We have developed a metaheuristic to solve all the models. This is based on two well-known metaheuristics; GRASP and tabu search procedure. Metaheuristic behaved very well in finding the optimal locations, as only 6 of the 360 runs were non-optimal and the average deviation from optimality did not exceed 1%, except in one case where the deviation was equal to 7.6%.

One can conclude from the computational experience that:

- The greatest deviation in demand captured is the one found when the MAXCAP is the true model, but uses the optimal locations found by the other models.

- The introduction of consumers sensitivity to the quadratic distance is unimportant in optimality terms since the smallest deviations in demand captured are the ones between MCI model and Proportional Customer Preference model.

- The deviations are generally significant. This suggests that prior analysis of consumer choice behavior is needed so that one can decide how best to include distance. But if this analysis cannot be made or it is too costly, we will have to use the traditional MAXCAP model as it is the one which gives the smallest deviation in demand captured (on average, less than 10%) whatever the true model is.

The application of these models to the 55-nodes network case have confirmed these results.

Future research will focus on which key attributes of the store, based on the Consumer Choice Behavior theories, have to be included in the Competitive Location models.

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ANNEX

Table 1a : Behavior of the metaheuristic in 20-nodes networks.

p	Model	GRASP	TABU	Deviation vs. enum.
2	1	100 %		
	2	100 %		
	3	80 %	100 %	
	4	100 %		
3	1	100 %		
	2	80 %	100 %	
	3	70 %	100 %	
	4	70 %	100 %	
4	1	70 %	100 %	
	2	30 %	100 %	
	3	10 %	100 %	
	4	10 %	100 %	

Table 1b : Behavior of the metaheuristic in 30-nodes networks.

p	Model	GRASP	TABU	Deviation vs. enum.
2	1	90 %	100 %	
	2	100 %		
	3	40 %	100 %	
	4	100 %		
3	1	100 %		
	2	100 %		
	3	80 %	100 %	
	4	80 %	100 %	
4	1	50 %	80 %	7.6 %
	2	50 %	80 %	0.78 %
	3	60 %	75 %	0.06 %
	4	90 %	100 %	

Table 1c : Behavior of the metaheuristic in 50-nodes networks.

p	Model	GRASP	TABU	Deviation vs. enum.
2	1	100 %		
	2	80 %	100 %	
	3	70 %	100 %	
	4	100 %		
3	1	100 %		
	2	80 %	100 %	
	3	70 %	100 %	
	4	100 %		
4	1	90 %	100 %	
	2	60 %	75 %	0.4 %
	3	30 %	71.43 %	0.202 %
	4	100 %		

Table 2a : Time spent by metaheuristic and enumeration in 20-nodes network

p	Model	GRASP	TABU	TOTAL[8]	ENUM
2	1	0.132	0.12	0.252	0.01
	2	1.259	1.336	2.595	0.141
	3	0.169	0.209	0.378	0.011
	4	0.18	0.116	0.296	0.012
3	1	0.213	0.171	0.384	0.1
	2	1.535	2.295	3.83	1.088
	3	0.215	0.349	0.564	0.166
	4	0.203	0.197	0.4	0.01
4	1	0.182	0.257	0.439	0.388
	2	1.403	3.389	4.792	5.376
	3	0.247	0.52	0.767	0.835
	4	0.162	0.269	0.431	0.406

[8] Total = Grasp + Tabu . The reading / creating data time is not included because what we want to compare is the time spent by the metaheuristic with respect to the time spent by the enumeration procedure.

Table 2b : Time spent by metaheuristic and enumeration in 30-nodes network

p	Model	GRASP	TABU	TOTAL	ENUM
2	1	0.287	0.175	0.462	0.056
	2	2.368	2.004	4.372	0.513
	3	0.34	0.319	0.659	0.073
	4	0.29	0.186	0.476	0.05
3	1	0.45	0.298	0.748	0.45
	2	3.882	3.702	7.584	5.811
	3	0.66	0.555	1.215	0.901
	4	0.38	0.323	0.703	0.469
4	1	0.418	0.416	0.834	3.275
	2	3.943	5.563	9.506	46.068
	3	0.682	0.833	1.515	7.092
	4	0.506	0.433	0.939	3.426

Table 2c : Time spent by metaheuristic and enumeration in 50-nodes network

p	Model	GRASP	TABU	TOTAL	ENUM
2	1	0.757	0.307	1.064	0.185
	2	7.898	3.442	11.34	2.426
	3	0.99	0.509	1.499	0.366
	4	0.923	0.342	1.265	0.227
3	1	1.322	0.505	1.827	3.592
	2	13.529	6.369	19.898	47.37
	3	1.666	0.95	2.616	7.173
	4	1.623	0.554	2.177	3.871
4	1	1.69	0.705	2.395	45.175
	2	11.896	9.837	21.733	658.374
	3	1.718	1.469	3.187	99.602
	4	2.231	0.763	2.994	48.73

Table 3a. Average (maximum) deviation for 20-nodes network.Case p = 2.

MODEL	Loc1	Loc2	Loc3	Loc4
1	0 %	27.24 % (53.21 %)	47.05 % (77.98 %)	13.19 % (35.15 %)
2	8.51 % (20.42 %)	0 %	2 % (8.2 %)	4 % (11.4 %)
3	8.94 % (18.02 %)	1.47 % (4.83 %)	0 %	5.7 % (12.2 %)
4	4.26 % (9.24 %)	7.3 % (21.99 %)	17.66 % (30.15 %)	0 %

Table 3b. Average (maximum) deviation for 20-nodes network.Case p = 3.

MODEL	Loc1	Loc2	Loc3	Loc4
1	0 %	34.81 % (54.75 %)	34.82 % (54.75 %)	14.89 % (34.78 %)
2	7.28 % (12.41 %)	0 %	0.36 % (1.04 %)	2.98 % (7.63 %)
3	7.07 % (11.77 %)	0.32 % (1.83 %)	0 %	3.8 % (7.85 %)
4	4.24 % (9.27 %)	16.5 % (31.84 %)	13.27 % (22.75 %)	0 %

Table 3c. Average (maximum) deviation for 20-nodes network.Case p = 4.

MODEL	Loc1	Loc2	Loc3	Loc4
1	0 %	34.6 % (76.09 %)	41.42 % (64.87 %)	17.1 % (30.57 %)
2	9.79 % (12.43 %)	0 %	1.96 % (3.87 %)	6.15 % (12.2 %)
3	8.63 % (13.65 %)	1.18 % (2.51 %)	0 %	5.71 % (11.06 %)
4	8.47 % (11.35 %)	17.84 % (51.61 %)	20.73 % (37.78 %)	0 %

Table 4a. Average (maximum) deviation for 30-nodes network.Case p = 2.

MODEL	Loc1	Loc2	Loc3	Loc4

1	0 %	15.65 % (34.96 %)	27.32 % (68.34 %)	10.93 % (19.43 %)
2	5.75 % (12.01 %)	0 %	0.59 % (2.57 %)	0.94 % (3.71 %)
3	6.27 % (11.39 %)	1.4 % (6.26 %)	0 %	2.64 % (7.49 %)
4	3.01 % (9.39 %)	2.97 % (13.78 %)	6.85 % (23.86 %)	0 %

Table 4b. Average (maximum) deviation for 30-nodes network.Case p = 3.

MODEL	Loc1	Loc2	Loc3	Loc4
1	0 %	19.46 % (46.77 %)	35.17 % (72.08 %)	15.42 % (33.89 %)
2	9.83 % (17.97 %)	0 %	1.78 % (5.68 %)	1.47 % (5.05 %)
3	7.17 % (11.43 %)	0.97 % (3.25 %)	0 %	2.29 % (6.49 %)
4	6.01 % (11.52 %)	5.17 % (17.43 %)	13.81 % (34.26 %)	0 %

Table 4c. Average (maximum) deviation for 30-nodes network.Case p = 4.

MODEL	Loc1	Loc2	Loc3	Loc4
1	0 %	26.25 % (52.78 %)	35.6 % (46.82 %)	11.75 % (19.02 %)
2	8.75 % (19.12 %)	0 %	1.75 % (3.12 %)	2.75 % (5.87 %)
3	7.14 % (13.87 %)	1.18 % (2.93 %)	0 %	3.58 % (7.92 %)
4	6.76 % (14.78 %)	13.09 % (26.19 %)	18.96 % (34.25 %)	0 %

Table 5a. Average (maximum) deviation for 50-nodes network.Case p = 2.

MODEL	Loc1	Loc2	Loc3	Loc4
1	0 %	13.02 % (27.21 %)	24.89 % (71.96 %)	14.11 % (32.92 %)
2	7.36 % (12.36 %)	0 %	1.06 % (6.05 %)	0.82 % (2.42 %)

3	5.71 % (10.78 %)	0.88 % (5.18 %)	0 %	1.22 % (3.24 %)
4	3.76 % (8.67 %)	0.4 % (1.31 %)	6.81 % (45.3 %)	0 %

Table 5b. Average (maximum) deviation for 50-nodes network.Case p = 3.

MODEL	Loc1	Loc2	Loc3	Loc4
1	0 %	17.52 % (40.35 %)	31.27 % (51.95 %)	12.17 % (25.8 %)
2	6.85 % (10.52 %)	0 %	1.59 % (3.84 %)	0.76 % (3.48 %)
3	6.27 % (11.9 %)	1.35 % (3.79 %)	0 %	2.62 % (5.26 %)
4	4.9 % (8.61 %)	2.36 % (9.18 %)	9.89 % (15.58 %)	0 %

Table 5c. Average (maximum) deviation for 50-nodes network.Case p = 4.

MODEL	Loc1	Loc2	Loc3	Loc4
1	0 %	22.88 % (44.34 %)	40.67 % (57.54 %)	12.55 % (24.72 %)
2	5.75 % (11.99 %)	0 %	2.53 % (5.13 %)	2.29 % (4.12 %)
3	5.24 % (7.79 %)	1.03 % (2.96 %)	0 %	3.58 % (8.19 %)
4	4.46 % (7.93 %)	8.63 % (33.34 %)	14.87 % (28.63 %)	0 %

Table 6. 55-nodes network demand.

Node	Demand	Node	Demand	Node	Demand
1	120	20	77	39	47
2	114	21	76	40	44
3	110	22	74	41	43
4	108	23	72	42	42
5	105	24	70	43	41
6	103	25	69	44	40
7	100	26	69	45	39

8	94	27	64	46	37
9	91	28	63	47	35
10	90	29	62	48	34
11	88	30	61	49	33
12	87	31	60	50	33
13	87	32	58	51	32
14	85	33	57	52	26
15	83	34	55	53	25
16	82	35	54	54	24
17	80	36	53	55	21
18	79	37	51		
19	79	38	49		