A Dynamic Choice Model of Hybrid Behavior in the Attribute-Space.*

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November 2001.

Abstract: This paper presents a dynamic choice model in the attribute space considering rational consumers that discount the future. In light of the evidence of several state-dependence patterns, the model is further extended by considering a utility function that allows for the different types of behavior described in the literature: pure inertia, pure variety seeking and hybrid. The model presents a stationary consumption pattern that can be inertial, where the consumer only buys one product, or a variety-seeking one, where the consumer buys several products simultaneously. Under the inverted–U marginal utility assumption, the consumer behaves inertial among the existing brands for several periods, and eventually, once the stationary levels are approached, the consumer turns to a variety-seeking behavior. An empirical analysis is run using a scanner database for fabric softener and significant evidence of hybrid behavior for most attributes is found, which supports the functional form considered in the theory.

Keywords: consumer choice models, state-dependence models, consumption patterns, variety seeking, hybrid behavior.

JEL: C35, D91, M39

*This research was done during my visit to the Department of Economics of the London Business School. I thank Bill Putsis and Bruce Hardie, from the Marketing Department, for their helpful comments and suggestions. I gratefully acknowledge the financial support of the Generalitat de Catalunya under grant BBR-99-37. All errors remain responsibility of the author.

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1 Introduction:

State-dependence models of consumer choice have gained relevance in Marketing, supported by empirical evidence that an item's purchase probabilities vary over time depending on the previously purchased history. In some situations the purchase of an item by a consumer decreases the probability that it will be purchased on the next occasion. This pattern is known in the literature as a "variety-seeking" behavior and comes into play when consumers become satiated by the constituent attributes derived from consuming the implied item (McAlister 1982, Givon 1984, Lattin and McAlister 1985). As a result, a consumer is assumed to derive no utility from consuming the same item over several periods. On the contrary, when the choice of an item leads to an increase in the probability of selecting it on a subsequent choice occasion, there is a reinforcing e®ect on consumer habit and the pattern is said to be inertial. Several models considered in the literature account for this variety-avoiding behavior: "learning" (Kuehn 1962), "last purchase loyalty" (Morrison 1966), "inertia" (Jeuland 1979), "variety avoiding" (Givon 1984) and "loss aversion" (Tversky and Kahneman 1991).

Researchers have proposed ¯rst-order models of brand choice, ¯nding substantial empirical evidence in favor of both variety seeking (e.g. McAlister 1982, Lattin and McAlister 1985, Kahn, Kalwani and Morrison 1986) and inertia (e.g. Jeuland 1979, Guadagni and Little 1983, Hardie, Johnson and Fader 1993). In most of these studies the brand has been the only decision variable considered in the consumer choice model. However, later empirical researches have considered an extended multiattribute framework, allowing for di®erent levels of variety seeking or inertia for each characteristic (see Lattin 1987 and Fader and Hardie 1996).

None of the referred models allows for a behavior where both inertia and variety seeking may coexist within the individual for the same attribute. However, some empirical evidence consistent with a mixed behavior has been reported: Wierenga (1974) observed that consumers tend to °uctuate between repeat purchasing and brand-switching behavior for frequently purchased products; and Bawa (1990) ¯nds evidence of mixed pattern in a two-brand framework for some frequent-purchase categories.[1] From now on, we refer to this mixture of inertia and variety seeking as "hybrid" behavior. Some theoretical framework in the psychology literature accounts for a hybrid behavior: Berlyne (1963, 1970) proposes that the attractiveness

[1]Considering the brand as the only valuable attribute, Bawa ¯nds evidence of hybrid behavior in the facial tissue and paper towel categories using panel datasets at a household level. However, the model assumes that each time a brand switch occurs, the choice process "renews", so the state-dependence assumption is only valid for the last purchases after the last switch. This assumption seems to be too restrictive for categories where consumers constantly seek variety.

of a stimulus is an inverted-U shaped function of its level of familiarity. According to this theory, an inertial behavior comes into play when the individual is exposed to a relatively unfamiliar stimulus and there is a tendency to repeat it, increasing the level of familiarity. High familiarity, on the other hand, brings the variety-seeking pattern into play, leading to an increasing tendency to look for other stimuli. According to this theory, if repeat purchasing of an item leads to greater familiarity with it, the consumer will switch from an inertial to a variety-seeking pattern once a certain number of repeat purchases are made.

The deterministic variety-seeking models in the tradition of Jeuland (1978) and McAlister (1982) assume a decreasing marginal relationship between the attribute inventories and the resulting utility.[2] This assumption implies strictly convex indifference curves over all the attribute space and a variety-seeking behavior even when new attributes are launched. However, this result is not consistent with the empirical evidence for unfamiliar attributes (see Wierenga 1974 and Bawa 1990). As stated by Berline (1963, 1970), for unfamiliar items the level of attractiveness increases through consumption, reinforcing the tendency to repeat the purchase. In fact, the ability to enjoy new attributes is a process that usually requires successive consumption of the same item: It takes time and several trials for a consumer to get used to new musical styles or to develop the ability to appreciate wines. This gradual increasing valuation of a product through its successive consumption may be explained assuming an inverted-U partial utility function, as proposed by Bawa (1990).

State-dependence assumptions imply that present decisions will a®ect future utilities. This intertemporal interaction between present consumption and future preferences requires a dynamic framework to be modeled. Investment is an intertemporal concept and can be de¯ned as the employment of money or resources in the acquisition of anything from which a future pro¯t is expected. This concept has extensively been used in all sorts of economic models to study both ¯rm and household optimal decisions (e.g. research and development, advertising, household production, economic growth, etc.). Multiple-purchase decisions like buying a package of assorted yogurts for future consumption require considering rational agents that discount the future to be modeled. As long as future tastes are continuously modi¯ed by the stock of attributes accumulated by the past consumption history, a rational agent will plan the consumption pattern across several periods as an attribute-investment process.

[2] Some of the state-dependence choice models assume uncertainty as a way to account for a variety-seeking behavior. However, as pointed out by McAlister and Pessemier (1982), alternation among familiar brands involves no risk. For most of the categories, the expected utility of consuming a second unit after the ¯rst trial is fairly deterministic. The decision to try a new brand may be considered a risky one as far as the consumer does not know the levels of attributes derived from its consumption, but once tried, choosing the new brand within a consumption pattern is basically a deterministic decision.

In this article the McAlister (1982) model is extended by considering an economic consumer-choice model with rational consumers that discount the future. The McAlister model can be considered a particular case where consumers are myopic and do not discount future utilities when taking present decisions. The resulting dynamic problem is solved analytically and the optimal consumption paths are characterized. In light of the evidence of several state-dependence patterns, the McAlister model is further extended by considering a utility function in a manner that allows for the di®erent types of behavior described in the literature: pure inertia, pure variety seeking and hybrid. The optimal consumption plan since an agent tries a new category, departing from a zero stock level of attributes is characterized. Under the inverted-U marginal utility assumption (Bawa 1990) the consumer behaves inertial among the existing brands for several periods, and eventually, once the stationary stock levels are approached, the consumer may turn to a variety-seeking behavior.

An empirical analysis is run to estimate the marginal utility functions for several attributes using a scanner database for fabric softener previously used by Fader & Hardie (1996). An extended non-linear functional form for the marginal utility functions gives a statistically stronger result than the linear version. Empirical evidence of hybrid behavior is found for most attributes, so the theoretical hybrid functional form is empirically supported by the data.

The outline of the paper is as follows. In section 2 the model is introduced, along with a basic discussion of the long-run optimal stationary path, and the main analytical results for the characterization of both inertial and variety-seeking long-run patterns are presented. In section 3 the consumption patterns for new categories or new attributes are studied, assuming a utility functional form that allows for inertial, variety-seeking and hybrid behaviors. In section 4 the indivisible-good version of the model is developed, to account for more realistic choice and consumption processes. In section 5 the empirical analysis is described and the main results that support the theoretical framework are highlighted. The conclusions are presented in section 6 with some managerial implications of the results and proposals for further developments and extensions of the model.

2 The General Model

The general model developed in this section describes the optimal consumption path in a dynamic framework. The objective is to model the consumption pattern within a frequent-purchase category, in a manner that allows for the di®erent types of behavior described in the literature: pure inertia, pure variety seeking, and hybrid inertia and variety seeking.

Consistent with the characteristics models [see Lancaster (1971)] the approach followed relates the preference for a product to the preference contributions of the attributes derived from its consumption. In line with models like the ones proposed by McAlister (1982) and McAlister and Pessemier (1982), the utility in each consumption period is derived from the attribute inventories accumulated when an item is consumed. The attribute inventory is hypothesized to depreciate continuously, and experiences discrete increments each period an item containing this attribute is consumed. The attribute inventory for the characteristic j in period t, say $z_j(t)$, is determined by the following law of motion:

$$z_j(t) = i_j(t) + (1 - \lambda_j) z_j(t - 1) \quad (1)$$

where $i_j(t)$ is the amount of attribute j derived from consumption of an item in period t, and λ_j is the corresponding depreciation rate.

As proposed by Lancaster (1971), when an item is consumed, the contribution to each attribute is assumed to be determined by the following linear consumption technology:

$$i_j(t) = \sum_{i=1} b_{ji} q_i(t) \quad (2)$$

where $q_i(t)$ is the quantity of good i consumed at period t and b_{ji} is the quantity of the characteristic j contained in one unit of good i. If there were two homogeneous goods (identical proportions of the constituent characteristics) the agent would only consume the efficient one on the basis of cost and the demand for the other good would be zero. Then, without loss of generality, from now on it will be assumed that all the available goods are differentiated.

The consumer is a utility maximizer, responding to a local temporal budget constraint. In line with Thaler (1985), the budgeting process is assumed to occur on a periodical basis for each category. Given the time and category specific budget constraint, the consumer evaluates purchases as situations arise. Assuming a previously determined budget for every period, say m(t), and a vector of exogenous prices p(t), the consumer will face the following restriction:

$$\sum_{i=1} p_i(t) q_i(t) \leq m(t) \quad (3)$$

The one-period utility function is expressed in terms of the stock of the embodied characteristics. Assuming additive separability among the partial contributions to utility made by each constituting attribute, the utility in period t can be expressed as follows:[3]

$$u(t) = \sum_{j=1}^{J} u_j\,[z_j(t)]$$

In the described framework, the decisions on present consumption affect the future utilities through the depreciated stock of attributes. For every depreciation rate, $\lambda_j < 1$, the consumer will be deriving future utilities from the present attribute inventory accumulated when a good is consumed. Choosing the optimal consumption pattern becomes a dynamic problem. Assuming that the future utility is discounted at a rate δ; a rational consumer decides the optimal consumption path by maximizing the discounted flow of utilities derived from the attribute levels reached in every period. Given the initial stock of attributes, z_0, the optimization problem for the consumer will consist of choosing the amount of items to be consumed in each period to maximize the discounted flow of utilities subject to the laws of motion for every attribute (1), the consumption technology (2), and the budget constraint in every period (3).

As defined by Lancaster (1971), in a complex economy the number of available brands, I, exceeds the number of attributes, J. If this is the case, for every attribute vector there may be more than one goods vector, but the optimizing consumer will choose the most efficient combinations in the subset of goods that constitute the attribute frontier. This gives a one-to-one relationship between the efficient goods and the attributes with an implied zero demand for the remaining $I - J$ goods. On the other side, if the market is characterized by a wide range of characteristics the opposite case may be presented, with more attributes than available brands. Some products like automobiles are made up of a great amount of constituent attributes, in what Lancaster defines as a simple economy. The economy is simple in the sense that the number of available items do not allow for acquiring every combination of attributes, so the consumer is limited to the available subset of implied combinations. In this economy the consumer will be limited to choose in the subspace of dimension I generated by the available set of items. The optimization problem can be solved by

[3] In an additively separable specification it is assumed that there are no interactions among the attributes. This assumption is implicit in economic models with Cobb-Douglas and logarithmic preferences. In empirical choice models this additive functional form usually produces good predictions and explains a large part of the total variance in empirical research (Dawes and Corrigan 1974; Green and Srinivasan 1978). However, in the empirical analysis the functional form can be extended by including interaction terms among attributes. According to Johnson, Meyer and Ghose (1989) interactions among attributes are not always statistically significative. They show that adding interaction terms may have a positive effect in the Pearson's validation r when the attributes are highly correlated in the choice set, but in orthogonal settings decreases validation correlations and appears simply to result in "overmodeling."

considering the reduced system of any arbitrarily chosen subset of I attributes, with the remaining $J - I$ attributes being implicitly determined.

In both the simple and the complex economies there is a one-to-one relationship between the efficient goods and the implied attributes. The square matrix B of the reduced system can be inverted and without loss of generality the maximization problem can be expressed in the attribute space. The one-period budget restriction can then be expressed in terms of the level of attributes derived from consumption of the goods purchased:

$$\sum_{j=1}^{J} p'_j(t) i_j(t) \leq m(t) \qquad \forall t$$

where the price of acquiring one unit of attribute j at time t; denoted by $p'_j(t)$, is a linear function of the good-price vector $p(t)$ and the inverse of the technology matrix:

$$p'_j(t) = \sum_{i=1}^{I} b_{ij}^{-1} p_i(t)$$

The optimization problem is now defined in the attribute inventory space:

$$\max_{\{i_1(t);i_2(t);\dots;i_J(t)\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} (1-\delta)^t \sum_{j=1}^{J} u_j\left[z_j(t)\right] \qquad \text{(P)}$$

subject to:

$$z_j(t) = i_j(t) + (1-\lambda_j) z_j(t-1) \quad \forall j; \forall t$$

$$\sum_{j=1}^{J} p'_j(t) i_j(t) \leq m(t) \qquad \forall t \qquad (4)$$

$$\sum_{j=1}^{J} b_{ij}^{-1} i_j(t) \geq 0 \quad \forall i; \forall t \qquad (5)$$

$$z_j(0) = z_{j0} \qquad \forall j$$

where the optimal consumption pattern consists of choosing the investment of constituent attributes $i(t)$ derived from consuming the selected goods, at every period, stocking them through the respective equations (1), in order to maximize the discounted flow of utilities derived from their inventory levels $z_j(t)$. The equations (4) correspond to the budget restrictions at every period t, and the inequalities (5) correspond to the positivity restrictions for the quantities of goods consumed in every period, $q_i(t)$.

2.1 The Optimal Consumption Pattern

Standard dynamic optimization problems consider a continuous, strictly increasing, twice differentiable, and concave utility function, and a compact and convex set defined by the constraint equation system. Under these assumptions, the following first-order conditions characterize the interior solution when nonsatiation of the attributes is also assumed:

$$\sum_{j=1}^{J} p'_j(t) z_j(t) = m(t) + \sum_{j=1}^{J} p'_j(t)(1-\lambda_j) z_j(t-1) \qquad t = 1, 2, \ldots \tag{6}$$

$$\frac{p'_v(t)}{p'_w(t)} = \frac{\sum_{n=0}^{\infty} (1-\delta)^n (1-\lambda_v)^n u'[z_v(t+n)]}{\sum_{n=0}^{\infty} (1-\delta)^n (1-\lambda_w)^n u'[z_w(t+n)]}; \quad \forall v, w \in J \qquad t = 1, 2, \ldots \tag{7}$$

The first equation is the budget restriction expressed in terms of the attribute inventories $z(t)$ and defines the maximum stocks attainable given the price vector $p(t)$, the budget $m(t)$, and the depreciated stock-of-attribute vector from the previous period $z(t-1)$. This is a first-order difference equation, so at every period, the attribute frontier is a function of the previous-period optimal stocks of attributes. The second equation characterizes interior solutions where strictly positive quantities of all goods are consumed. For interior solutions the price ratio between every two attributes equals the ratio between the discounted flows of future marginal utilities derived from an additional unit of attribute. When interiority is presented the consumer behaves variety-seeking as all the durable goods are consumed simultaneously at every period. However, the model may also present non-interior solutions where only one good is consumed along the optimal pattern. When products are close substitutes the optimal consumption pattern may not be interior, implying an inertial behavior, even under strongly concave preferences.[4]

In a dynamic model rational agents create expectations on future prices and budget to choose the optimal consumption plan. Assuming that in a frequent-consumption category the agent expects the price vector and the budget to be constant for several periods, the consumption path approaches a stationary pattern. If the price vector $p(t)$ and the budget $m(t)$ remain constant, the restriction (6) will monotonically converge to a long-run budget restriction, where in every period the consumer faces the same attribute frontier. Once the stationary restriction is reached, the budget for every period is completely used to restore the depreciated levels of attributes. If this is the case, the consumption path has reached a stationary pattern

[4] A utility function is said to be strongly concave when the marginal utility approaches infinity as the related attribute goes to 0. As a result, the indifference curves never cross the axes, so a strictly positive level of every attribute is needed to derive utility.

where the purchased quantities and the attribute stocks remain constant period by period. In the long run the consumer splits the budget to restore the depreciated stocks of attributes, maintaining an optimal proportion among the different attributes.

It is unrealistic to think of a stationary consumption pattern in a market where prices vary frequently, and new attributes or new products are constantly being launched. In a dynamic market the economy will always be in a transitional path. However, rational consumers choose the optimal purchase patterns according to the expected future flow of utilities stated in problem (P). As in every standard concave dynamic problem, the steady state governs the transitional dynamics, so any optimal path out of the steady state from any initial conditions is driven by the convergence dynamics towards it. If there is a change in prices or any other variable assumed to be constant, the long-run frontier will shift and the problem will present a new steady state, but the consumption pattern will again be governed by the convergence dynamics. This is the main rationale: to study the steady state as the long-run levels at which any consumption pattern monotonically approaches even in a dynamic market where the marketing conditions vary constantly.

Definition 2.1: A stationary consumption path is defined as a consumption sequence, $\{q(t);\ t = 1, 2, \ldots\}$ that solves the optimization problem (P) for a certain initial condition, z_0, such that the resulting vector of attribute stocks along the path, $\{z(t);\ t = 1, 2, \ldots\}$ remains fixed over time.

From now on, without loss of generality, our analysis will be limited to the case where consumers derive utility from two different attributes and two items within a certain category. The following results and conclusions can be extended to a higher-dimension problem. Let's assume that the exogenous budget $m(t)$ and the price vector $p(t)$ remain constant, so a convergence process towards a stationary pattern will be presented. The optimal consumption path characterized by the first order conditions will eventually converge to a steady-state consumption path. Imposing stationarity in the budget restriction (6) and in the first-order condition (7), the following equation system is obtained:

$$m = \lambda_1 p'_1 z^*_1 + \lambda_2 p'_2 z^*_2 \qquad (8)$$

$$\frac{p'_1}{p'_2} = \frac{[1 - (1 - \delta)(1 - \lambda_2)]\, u'(z^*_1)}{[1 - (1 - \delta)(1 - \lambda_1)]\, u'(z^*_2)} \qquad (9)$$

where

$$p'_j(t) = p_1(t) b_{1j}^{-1} + p_2(t) b_{2j}^{-1} \qquad j = 1, 2 \quad \forall t$$

The second condition is only valid when the utility function is concave at the stationary value and the demands for both goods are strictly positive (interior solutions). If this is not the case, the consumption pattern would be inertial and the consumer would only purchase one good.

Definitions 2.2: A variety-seeking steady-state consumption path for the optimization problem (P) is a steady-state equilibrium with strictly positive consumption of both goods, q_1 and q_2: An inertial steady-state consumption path is a steady-state equilibrium in which only one of both goods is consumed.

Proposition 2.1: Consider the dynamic optimization problem (P), where the utility function $U(z_1(t); z_2(t))$ is strictly quasiconcave. When the budget $m(t)$ and the vector price $p(t)$ remain constant over time, the following three possible situations can be presented:

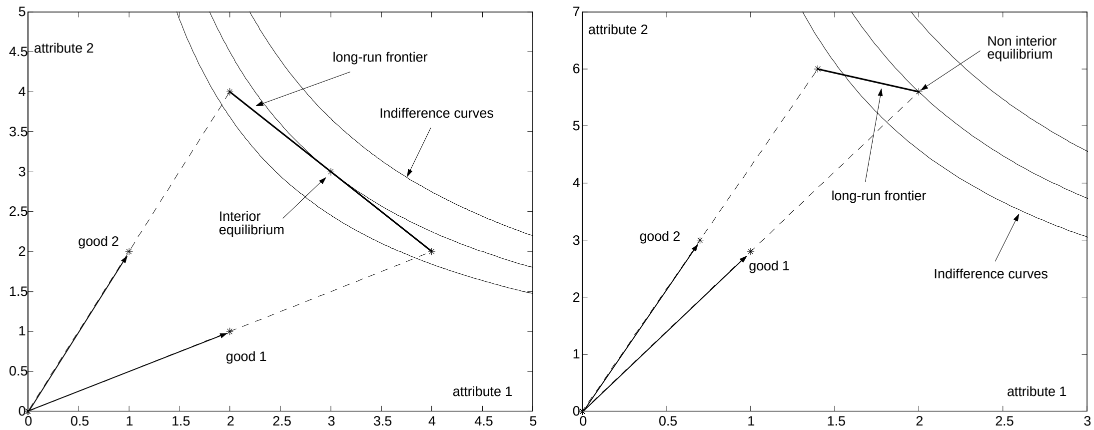
(a).- If $\frac{p_1^0}{p_2^0} \leq \frac{[1-(1-\delta)(1-\lambda_2)]u_1^0(\frac{mb_{11}}{p_1\lambda_1})}{[1-(1-\delta)(1-\lambda_1)]u_2^0(\frac{mb_{21}}{p_1\lambda_2})}$, the consumption pattern will converge to an inertial long-run path where only good q_1 is purchased.

(b).- If $\frac{p_1^0}{p_2^0} \geq \frac{[1-(1-\delta)(1-\lambda_2)]u_1^0(\frac{mb_{12}}{p_2\lambda_1})}{[1-(1-\delta)(1-\lambda_1)]u_2^0(\frac{mb_{22}}{p_2\lambda_2})}$, the consumption pattern will converge to an inertial long-run path where only good q_2 is purchased.

(c).- In all other cases the consumption pattern will converge to a variety-seeking long-run path where both goods q_1 and q_2 are simultaneously purchased in a fixed proportion.

The expression to the right side of condition (a) is the marginal rate of substitution MRS evaluated in the lower edge of the long-run frontier. If this value is higher than the price ratio, the consumer would be better off in the long run by purchasing only good q_1. The same intuition is valid for condition (b) in the upper edge of the long-run frontier. This edge corresponds to the long-run level of attributes if only q_2 is purchased. The quasiconcavity assumption imply convex indifference curves, so in all other cases the consumer will be better off by purchasing both goods in every period, and the consumption pattern will converge to a variety-seeking steady state.

The examples from figures 1a and 1b illustrate a variety-seeking and an inertial long-run equilibrium respectively. In the example from figure 1a the interiority condition (c) from proposition 2.1holds and the stationary stock levels of attributes approached in the long run correspond to an interior equilibrium where the consumer shares the budget between goods 1 and 2 in every period. However, in the example from figure 1b both goods are close substitutes in the attribute space, and the non-interior condition (a) from proposition 2.1 holds. When this is the case the agent maximizes his utility by consuming only the good 1. This inertial consumption 1 will eventually converge to the stationary stock level of attributes corresponding to the right side of the long-run frontier.



Figures 1a and 1b: Optimal consumption long-run patterns for a logarithmic utility function, $u(z) = \log(z)$. The parameter values are: $m = 1$; $p_1 = p_2 = 1$; $\lambda_1 = \lambda_2 = 0.5$; and $\delta = 2$. The consumption technology matrixes are $B = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0.7 \\ 2.8 & 3 \end{pmatrix}$ respectively.

3 The Hybrid Behavior

In this section the analysis is extended out of the steady state. The objective is to determine the optimal consumption patterns when the consumer tries a new category or a new attribute in an established category. In this case, the stock levels for one or some attributes are much lower that the long-run ones, so some simulations must be run out of the steady state to determine the optimal way in which the consumer accumulates the new attributes.

The utility function assumptions play a central role on the way the consumer behaves when non-familiar attributes are considered in the optimal consumption pattern. In line with the hybrid behavior approach presented by Bawa (1990), a utility function that accounts for an increasing marginal utility region followed by a decreasing marginal utility one is assumed. Consuming an unfamiliar item will reinforce its future acquisition and the consumer will behave inertial. There is no doubt that this is a dynamic process out of a stationary pattern, so in this section some simulations are run to determine the transitional patterns departing from zero levels of attributes.

A polynomial partial utility function that allows for the hybrid behavior is used for the analysis. The utility range is normalized to the (0,1) interval and each attribute presents a saturation point at level ℓ. Imposing these conditions, a cubic formulation for the utility function presents one degree of freedom, depending on the marginal utility at $z = 0$. The resulting formulation is:

$$u(z) = (c - 2)\left(\frac{z}{\ell}\right)^3 + (3 - 2c)\left(\frac{z}{\ell}\right)^2 + c\left(\frac{z}{\ell}\right) \tag{10}$$

where the parameter c is the marginal utility $u'(z)$ at $z = 0$. When $c = 0$ the utility function is convex for $z < \ell/2$. Higher values of c imply a smaller increasing-marginal-utility region. For $c \geq 1.5$ the marginal utility is downward slopping for all the domain $z > 0$. The implied marginal utilities and indifference curves in a two-attribute space are depicted in figures 2 and 3 respectively for several values of the parameter c.

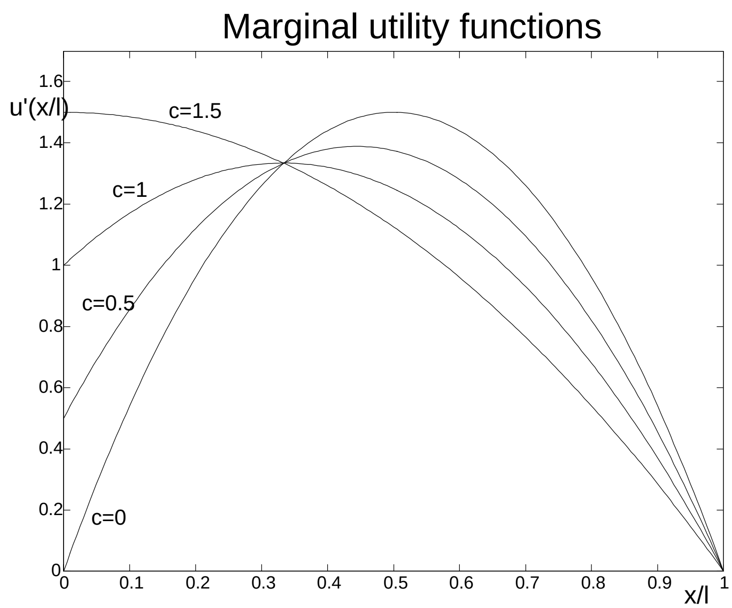
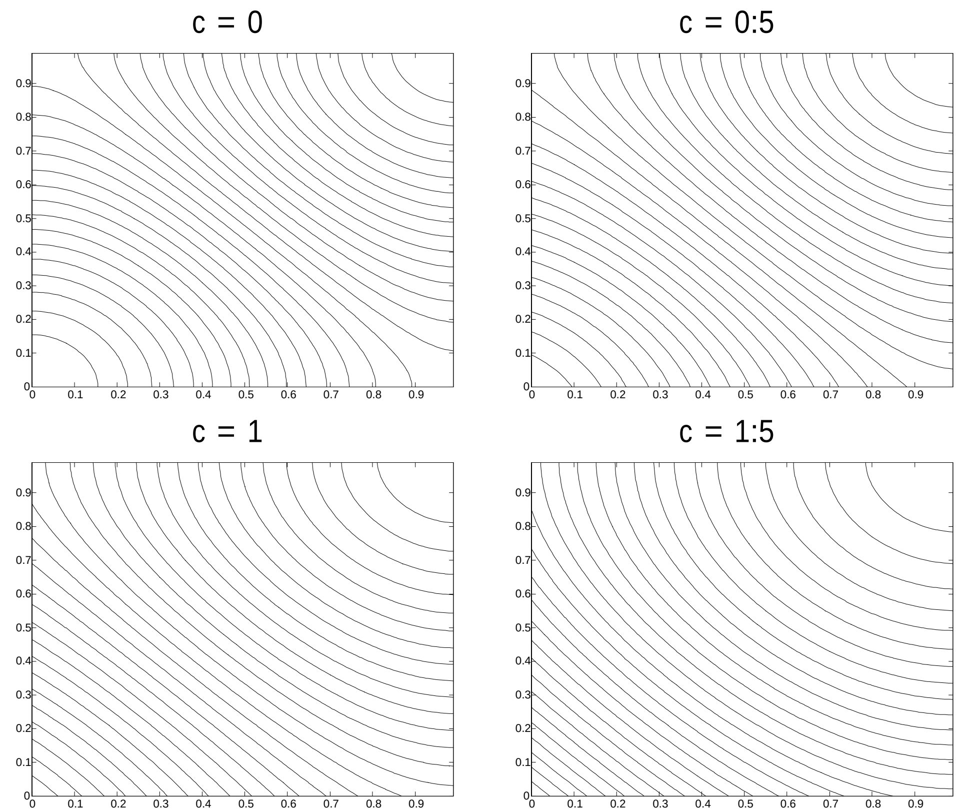


Figure 2: Marginal utility functions for several values of c.

The law of motion for the state variables $\{z_1(t); z_2(t)\}$ is derived by following a standard numerical technique. Departing from the steady-state neighborhood, we move backward through the first-order-condition dynamic system (6)-(7). It should be noted that for some periods far away from the steady state the positivity restrictions in the demands for both goods may be binding even for a quasiconcave region, implying a zero purchase level for one product. If this is the case, condition (7) does not hold.

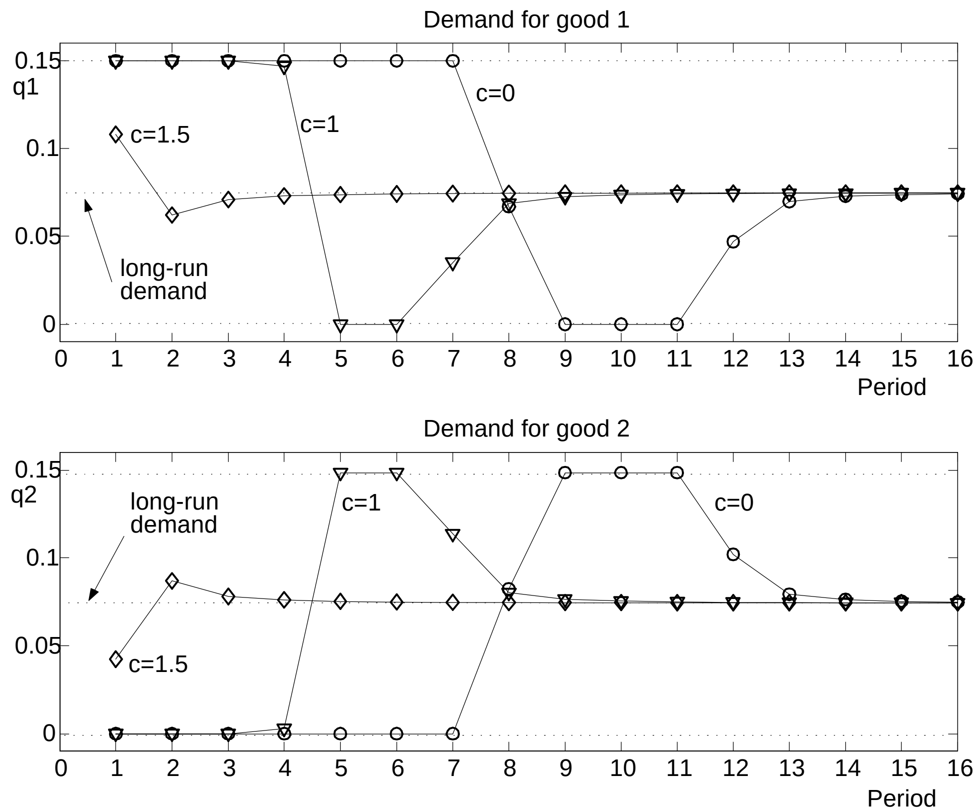
The shape of the partial utility function $u(z)$ conditions the variety-seeking or inertial pattern along the transition, depending on the convexity-concavity of the indifference curves. In all the following examples the long-run equilibrium is a variety-seeking one, as the interiority condition (c) from proposition 2.1 holds, so eventually the consumer will end up buying simultaneously both goods. However, the path departs from a zero level of attribute inventories, so the transitional dynamics illustrates the optimal consumption pattern while the consumer gets used to both attributes.



Figures 3a-3d: Indi®erence curves in a two-attribute space for several values of c

The demands for goods 1 and 2 are depicted in ¯gures 4a and 4b for several utility functions. When c = 1:5 the marginal utility function is decreasing in all the domain [0; `], so every additional unit consumed reduces the next-period marginal utility, discouraging its consumption. Along the transition path the consumer balances both attributes, so the proportion between them gradually approaches the long-run value. As a result, the consumer seeks variety not only in the long run, but also during the ¯rst consumptions.

When the utility function is hybrid the pattern is completely di®erent, as the agent maximizes the utility by consuming only one attribute since the ¯rst period. Its accumulation fosters its further acquisition over all the concave region of the indi®erence curves. As a result, the consumer behaves inertial by stocking only one attribute. Switching to the other product will not occur until the discounted marginal utility of acquiring an additional unit is lower than the marginal utility of consuming the ¯rst unit of the unknown product. From then on, the consumer will behave inertial with the second attribute, and so on, until the long-run variety-seeking equilibrium is approached. For expositional convenience and without loss of generality, the products considered in the previous analysis are not multiattribute, so the technology matrix considered in the examples is diagonal. However, when multiattribute products are considered, the consumer is more likely to choose inertial transitional patterns, as products become substitutes. The closer substitute the products are, the more inertial the consumption plan is.



Figures 4a and 4b: Demands for goods, departing from a zero level of attributes, and the following parameter values: $m = 0.15$; $p_1 = 1$, $p_2 = 1.01$, $\lambda_1 = \lambda_2 = 0.1$; $\delta = 0.1$; $B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

4 Indivisible Goods:

Divisibility is a standard assumption in most economic models, as the implied demand functions can be best understood if all goods and services are assumed to be divisible. However, only few goods like gasoline or electricity can be bought in any quantity desired. For the rest of the goods the consumer is restricted to purchase or consume among a limited set of available products and presentations. The indivisibility assumption brings reality to both the purchase and consumption processes and is of special interest for understanding the demand for most frequent-consumption categories.

This section considers the discrete version of the problem, where the agent is restricted to choose among a limited set of alternatives to be consumed. At every period the agent is able to determine the optimal consumption pattern, discounting the utility derived from future consumptions. There is empirical evidence that consumers purchase grocery products in weekly cycles (Dunn et al. 1983), but for most categories the consumption frequency is much higher and the multiple-purchase decisions consider the several consumptions over the interpurchase period. Unlike most of the deterministic state-dependence models described in the literature (see Jeuland 1979, McAlister 1982, and Tvresky and Kahneman 1991 among others), considering future utilities allows to determine the demanded set of products when the purchase is multiple.

Consistent with the choice models, at every period t the agent derives utility from consuming any of the n available indivisible goods, $x_1, x_2, ..., x_n \in X$; for a given category. Let's further assume that the consumer can afford any of the consumption alternatives, $p_{x_i}(t) \leq m(t)\ \forall t; \forall x_i \in X$. A consumption plan for a limited horizon of T periods is a sequence $\{x(1); x(2); ...; x(T)\};\ x(t) \in X$ where the agent selects one of the possible n products in each period. A rational consumer that maximizes the consumption flow considering T periods faces a discrete maximization problem of choosing among all the possible patterns. When the choice set is discrete, the standard dynamic programming techniques can not be used and the demand functions have to be determined by comparing the derived utility from choosing any of the n^T possible plans. Formally, given the initial stock of attributes, z_0, the consumer will choose the optimal consumption sequence for the T periods to maximize the discounted flow of utilities subject to the stocking processes for every attribute and the consumption technology:

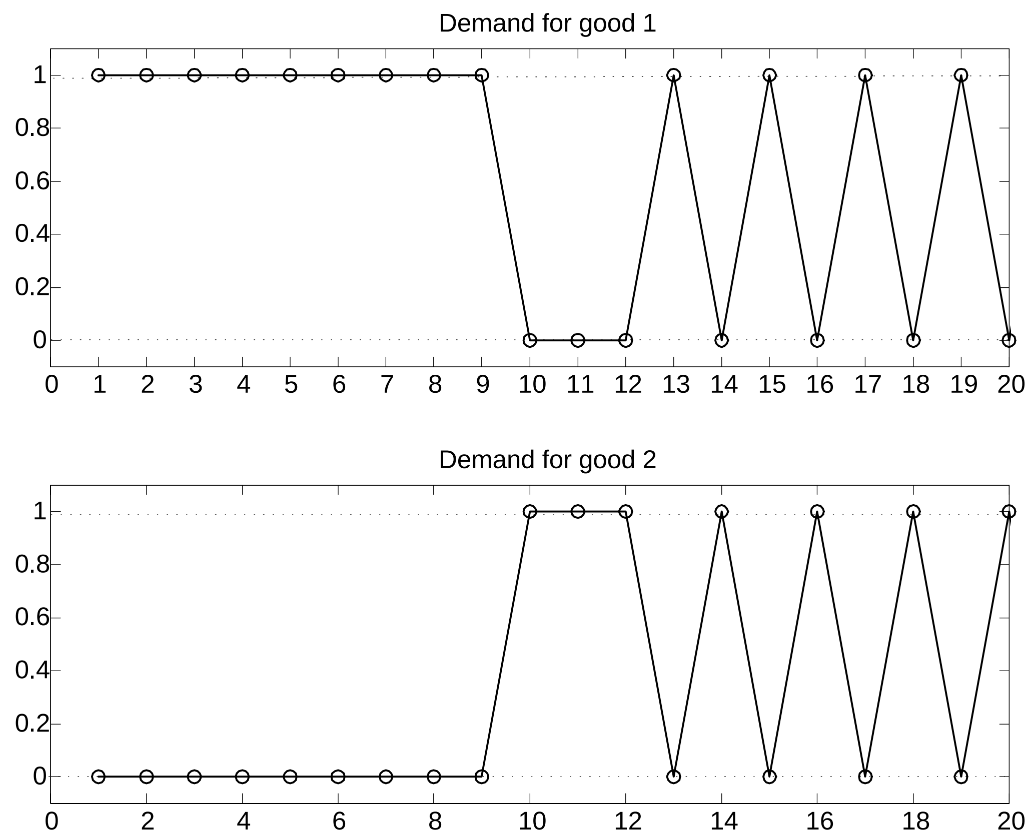
$$\max_{x_i(t) \in X;\ t=1,2,...,T} \sum_{t=1}^{T} (1-\delta)^t \sum_{j=1}^{J} u_j\left[z_j(t)\right] \tag{P}$$

subject to:

$$\begin{aligned} z_j(t) &= i_j(t) + (1-\lambda_j) z_j(t-1) \quad \forall j; \forall t \\ i_j(t) &= b_{ji} x_i(t) \qquad \forall j; \forall t \\ z_j(0) &= z_{j0} \qquad \forall j \end{aligned}$$

This version of the problem presents a similar qualitative behavior than the continuous-choice-set version studied in the previous sections. As expected, when a inverted-U marginal utility function is considered for both attributes the optimal consumption path follows the same hybrid pattern that the one presented in the divisible-good framework. The consumer behaves inertial during several periods until a high level of familiarity with the product containing the implied attributes is reached. At that level the decreasing marginal utility regions have been reached and the agent switches to consume a new unfamiliar product during several periods. Once the level of familiarity with the available products is high, the consumer shifts among them from period to period, seeking variety. As the finite horizon T increases, the consumption pattern converges to a stationary level of attributes in the long run.

The example from figure 5 illustrates the behavior described. Similar to the previous section, the consumer was assumed to derive utility from two different attributes, each one contained in one product. A time horizon of 20 periods and a U-inverted marginal utility function ($c = 0$) were also assumed. Some simulations were run to determine the consumption pattern in a new category departing from a zero level of attributes. The utilities derived from the possible n^T consumption patterns were compared to determine the optimal consumption plan. The implied demands for both goods are depicted.



Figures 5a and 5b: Demands for goods, departing from a zero level of attributes, and the following parameter values: $m = 2.25$; $p_1 = 1$, $p_2 = 1.01$, $\lambda_1 = \lambda_2 = 0.2$; $\delta = 0.9$; $c = 0$; $T = 20$; $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

In the previous section the goods are assumed to be divisible and the agent seeks variety by consuming simultaneously both products. However, simultaneous consumption of several goods is not possible in the indivisible-good version, and the resulting variety-seeking behavior implies switching among the existing items. Both results re°ect a similar pattern derived from the decreasing marginal utility assumption when the consumer reaches a high level of familiarity with the existing products.

5 Empirical Analysis

Early empirical work proposes ¯rst-order models of brand choice to account for variety-seeking (e.g. McAlister 1982, Lattin and McAlister 1985, Kahn, Kalwani and Morrison 1986) or inertial behavior (e.g. Jeuland 1979, Guadagni and Little 1983, Hardie, Johnson and Fader 1993). Later research allows for di®erent levels of salience for other characteristics than brand in a multiattribute framework (see Lattin 1987 and Fader and Hardie 1996). In this context individuals may simultaneously seek for variety in certain product attributes and exhibit loyalty towards others, but the partial utilities are assumed to be a linear function of the state-dependence variables, which does not allow for a simultaneous inertial and variety-seeking behavior depending on the stocked levels of the state variables.

In line with the theoretical framework, the objective of this empirical research is to model consumer choice on the basis of a set of product attributes for a certain category.[5] The evidence of both inertial and variety-seeking patterns simultaneously (hybrid pattern) is tested for some attributes within a certain category depending on their stock levels. We use a scanner database for fabric softener previously used by Fader and Hardie (1996; hereafter F&H) containing information on 9781 purchases over a $2\frac{1}{2}$-year period among 594 households. Following F&H, the accumulated stocks for the several categorical attributes are included as the state-dependence variables, in line with the pioneering methodology proposed by Guadagni & Little (1983; hereafter G&L). The dataset includes information on a total of 56 di®erent items available during 1991 and a total of 22 unique item attribute categories (10 brands, 4 sizes, 4 forms and 4 formulas). Additional information regarding prices and retail promotional activities for every item is also available at every purchase occasion. A detailed description of the constituent attributes for the 56 available items is provided by F&H (Table 1, pp 445).

The standard approach to model product choice with scanner data considers the multinomial logit (MNL) model (see McFadden 1974, G&L, Lattin 1987 and F&H

[5]As demonstrated by Fader & Hardie (1986), in categories were a relevant quantity of items are avaialable, the characteristics approach represents a powerful and parsimonious approach to modeling consumer choice.

among others). The MNL model computes the probability of choosing an alternative as a function of the marginal utilities of all the available alternatives. Given some standard assumptions it can be shown that the MNL model has the following functional form:

$$p_i = \frac{e^{\nu_i}}{\sum_J e^{\nu_j}}$$

where, ignoring household and time indices, p_i is the probability of choosing item i, and ν_i is the deterministic component of item i's utility.[6]

Our premise holds that the utility of an item is given by the sum of the utilities provided by the constituent attribute levels. Assuming no interactions, item i's utility, ν_i; can be expressed as an additively separable function of the stock level of the N constituent attributes and the values of the marketing variables for item i, X_i:[7] The heterogeneity among consumers is modeled by considering a state-dependence variable $STKAT$ that accounts for the accumulated stock of each attribute level.[8] The utility function for the item i is:

$$\nu_i = \sum_{n=1}^{N} g_n(STKAT_n) + \beta X_i \tag{11}$$

where the remaining βX_i term includes a standard set of marketing variables: regular price, price cut, and two dummy variables indicating promotional activities: displays and newspaper features.

Given the categorical nature of each of the N attributes, the functional form of the attribute-specific partial functions is as follows:

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[6]Following McFadden (1974), the conditional probability equations can be obtained by using a random utility framework and assuming that the stochastic components of utility are independent and identically distributed for each alternative as double exponential. This random utility assumption has been widely used in choice modelling.

[7]As in the case with most linear models, this additively separable specification can usually produce good predictions and explains a large part of the total variance (Dawes and Corrigan 1974; Green and Srinivasan (1978); Johnson, Meyer and Ghose 1989). However, interactions can easily be included in the model, but because of the large number of potential interaction effects, the process of adding interaction terms should be driven according to the knowledge of the product category.

[8]Guadagni & Little (1983) use the following exponentially smoothing formulation to weight the past purchase history on brand and size attributes for each household:

$LOY_{hj}(t+1) = \gamma LOY_{hj}(t) + (1-\gamma) i_{hjt}$

It is worth noting, by comparing the previous equation to equation (1), that the loyalty variable is a normalized measure of the stock level of implied attribute. When the investment from the previous equation is normalized to 1, the maximum stock level attainable after infinite periods of consecutive investment in the attribute is 1, and the previous-period stock of attribute depreciates at a rate (1-γ).

$$g_n(STKAT_n) = m_{in}\alpha_{0n} + \alpha_{1n} m_{in} STKAT + \alpha_{2n} m_{in} STKAT^2 \qquad (12)$$

where m_{in} is an elementary vector of length equal to the M_n categories of attribute n. The element of this vector corresponding to the category included in the item i is equal to 1. The vector α_{0n} includes the M_n attribute level-specific intercept terms for attribute n and the parameters α_{1n} and α_{2n} correspond to the quadratic functional form proposed for the stock level of the attribute. This formulation is a natural extension of the one-segment linear model presented by F&H, allowing for several consumption patterns for each attribute: pure inertia, pure variety seeking, and hybrid behavior.[9] In our theoretical model from previous sections the utility levels derived from consumption of the available items are determined by the increase in the stock of the constituent attributes. Given the depreciated stocks of attributes from the previous period, the utility derived from consuming the item i is a function of the marginal utilities of the implied attributes. The hybrid functional form from equation (10) allows for a quadratic marginal utility function, in line with the empirical model specification.

Up to this point, both the choice occasion and household indices have been suppressed. Let them be defined as t and h respectively, and let H be the number of households in the panel, T_h the number of choice occasions for household h, and δ_{it}^h be a purchase indicator equal to 1 if household h chooses the item i on purchase occasion t, and 0 otherwise. For the attribute-based model specification the model parameters are estimated by maximizing the Log-Likelihood function:

$$LL = \sum_{h=1}^{H} \ln \left[\prod_{t=1}^{T_h} \prod_{i} p_i^h(t)^{\delta_{it}^h} \right]$$

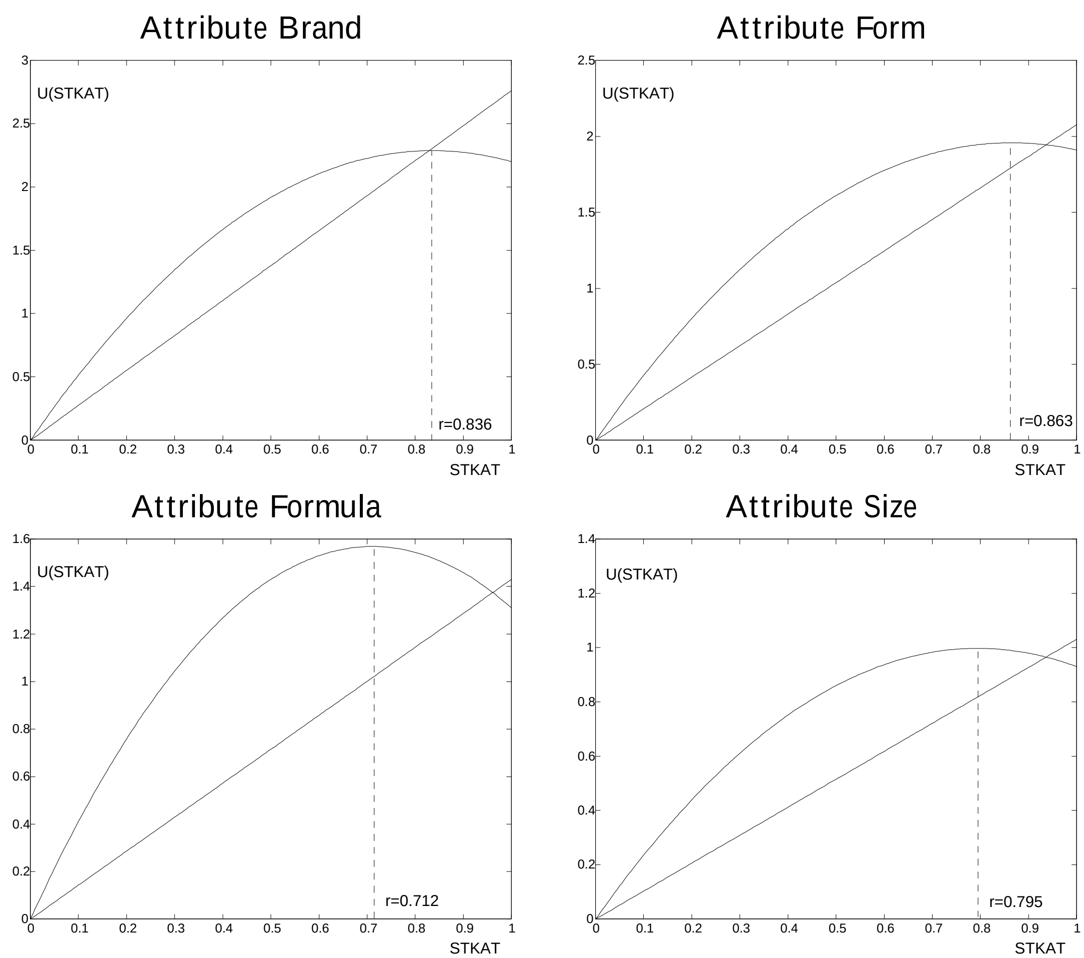
We begin by estimating the "attribute level-specific intercept-only" ALSIO model, which is the attribute-based version of the common item-specific intercept-only model used in several other studies (α_{1n}, α_{2n} and β are set to 0). Then the model is successively extended including several groups of variables. For every extension a Likelihood ratio test is conducted to determine the significance level of each group of explanatory variables. The second benchmark includes the four marketing-mix variables βX_i. The third is the F&H model and is equivalent to a G&L specification but with the 55 item-specific intercept terms replaced by 18 attribute level-specific intercept terms; α_{1n}. The last estimation corresponds to the extended version developed in this section that allows for a non-linear partial utility function. The likelihood values, the ratio test and the estimation results for the different extensions are shown in table 1.

[9] Fader and Hardie (1996) further extend the model allowing for multiple market segments and find that the two-segment model provides a better fit.

Table1: Estimation results:

	ALSIO model		MKT model		F&H model		Hybrid Model	
	Coef.	(t-stat)	Coef.	(t-stat)	Coef.	(t-stat)	Coef.	(t-stat)
Att. level intercepts:								
Brand:								
Bounce	-0.105	(-0.36)	0.257	(0.82)	0.162	(0.48)	0.007	(0.02)
Cling Free	0.372	(1.37)	0.215	(0.79)	0.382	(1.29)	0.238	(0.81)
Downy	0.137	(0.57)	0.599	(2.30)	0.420	(1.46)	0.202	(0.70)
Final Touch	0.131	(0.46)	0.252	(0.88)	0.226	(1.77)	0.382	(1.22)
Generic	1.119	(3.65)	-1.177	(-2.55)	-1.349	-2.58)	-1.241	(-2.42)
Private Label	0.061	(0.25)	-0.748	(-2.77)	-0.741	(-2.48)	-0.834	(-2.80)
Snuggle	0.509	(2.19)	0.062	(0.23)	-0.316	(-1.08)	-0.574	(-1.93)
Sta-Puf	1.030	(3.82)	0.673	(2.38)	1.056	(3.37)	0.984	(3.15)
Toss n'Soft	0.196	(0.65)	-0.197	(-0.64)	0.046	(0.14)	-0.098	(-0.30)
Form:								
Re¯ll	1.381	(9.04)	1.679	(8.29)	2.544	(11.04)	2.636	(11.13)
Liquid	-1.160	-3.40)	-2.073	(-5.50)	-1.310	(-3.11)	-1.329	(-3.13)
Sheets	-0.107	(-1.03)	-0.132	(-1.18)	0.250	(1.88)	0.270	(2.06)
Formula:								
Regular	-0.076	(-0.77)	0.003	(0.035)	-0.464	(-3.93)	-0.451	(-3.85)
Staingard	-0.066	(-1.91)	-0.579	(-1.66)	-0.546	(-1.51)	-0.471	(-1.29)
Unscented	-0.053	(-2.20)	-0.186	(-0.78)	-0.140	(-0.55)	-0.320	(-0.12)
Size:								
Medium	0.861	(7.77)	1.476	(7.09)	1.097	(4.85)	1.070	(4.77)
Large	-0.066	(-0.44)	1.764	(5.29)	1.744	(4.89)	1.795	(5.04)
Extra Large	-0.265	(-1.37)	1.921	(4.50)	1.877	(3.98)	1.992	(4.25)
Marketing-mix Coef:								
Regular price	-	-	-0.957	(-7.58)	-0.910	(-6.72)	-0.916	(-6.84)
Price cut	-	-	1.665	(15.53)	1.752	(14.90)	1.739	(14.71)
Display	-	-	1.213	(9.20)	1.389	(9.56)	1.400	(9.56)
Feature	-	-	0.372	(2.31)	0.306	(1.71)	0.319	(1.77)
Stocking Variables:								
STKAT-brand	-	-	-		2.758	(18.67)	5.469	(8.48)
STKAT-form	-	-	-		2.078	(13.00)	4.527	(4.01)
STKAT-formula	-	-	-		1.435	(8.54)	4.415	(2.96)
STKAT-size	-	-	-		1.032	(6.31)	2.507	(2.64)
Non-linear terms:								
STKAT-brand^2	-	-	-		-		-3.267	(-4.41)
STKAT-form^2	-	-	-		-		-2.621	(-2.26)
STKAT-formula^2	-	-	-		-		-3.100	(-2.02)
STKAT-size^2	-	-	-		-		-1.576	(-1.55)
Log Likelihood	-2855.41		-2567.17		-2000.75		-1983.58	
Param. estimated	18		22		26		30	
Fit Statistic ½^2	0.073		0.166		0.350		0.356	
LL Ratio Test	-		576.48		1132.84		34.34	

All the extended models considered are statistically stronger than the previous constrained version, and all the groups of explanatory variables are statistically signi¯cant and intuitively correct. The F&H model ¯ts the observed choice data substantially better than the constrained state independence versions with or without the marketing variables, so we can conclude that the reference-dependence behavior constitutes an important source of variance for the category studied that is potentially missed in the zero-order choice models. When non-linearity is assumed, strong evidence of a hybrid behavior for the three attributes other than size is found. The non-linear parameters for the brand, form and formula attributes are all negative and signi¯cantly di®erent from 0 at a 5% level. These values imply strictly concave marginal utility functions as predicted. However, for a hybrid behavior to be presented, the estimated function must present a decreasing region in the domain (0,1). The estimated marginal utilities are depicted in ¯gure 6.



Figures 6a-6d: Estimated linear and quadratic marginal utility functions for the several attributes.

It is note worthy that the non-linear speci¯cation presents steeper slopes for the marginal utilities at a zero stock level than the F&H model. In line with our theoretical model, this suggest a strong inertial behavior for new or unstocked attributes. This inertia becomes weaker as the stock levels increase. Eventually, when the critical stock level r from ¯gure 6 is overcome the decreasing marginal utility implies that every new acquisition of the attribute category reduces the probability of acquiring an item containing it in the new purchase, so the consumer starts behaving variety seeking for this attribute category. To illustrate this variety-seeking e®ect, lets assume that a new consumer starts using fabric softeners departing from a zero level of attributes. According to the carry-over constant ß and the marginal utility function estimated (12), during the six ¯rst purchases of items containing the same formula the probability of acquiring a related item is every time higher. However, after the sixth purchase the stock variable has reached a value of 0.7569, surpassing the critical value r=0.712 from ¯gure 6c. The consumer is now less likely to buy a product with the same formula than in the previous purchase, seeking variety in this attribute.

6 Conclusions

The results from this dynamic choice model suggest the importance of modelling consumer preferences in the attribute space. Most models presented in the marketing literature consider the brand as the fundamental decision variable. However, the choices among the existing items reveal preferences not only for a brand, but also for several other underlying attributes, like sizes, formulas, °avors, etc. Our empirical results suggest that an attribute-based model should allow consumer preferences to vary among the constituent attributes, so the consumer may behave simultaneously inertial in some attributes and seeking variety in others depending on the consumption history. The model should also be able to capture mixtures of inertia and variety seeking as a more complex state-dependence pattern.

From a consumer-choice perspective, the major predictions of our theoretical framework are con¯rmed by the empirical analysis:

- The hybrid formulation has a superior ¯t compared to a linear state-dependence formulation.

- The non-linear coe±cients are statistically signi¯cant for most attributes, providing evidence of mixed behavior depending on the consumption history.

- The coe±cients support the theoretical assumptions that consumers behave strongly inertial when the attributes are new, or their stock levels are low. This trend is reversed and consumers seek variety once the attribute has been accumulated continuously for several consumption periods.

From a managerial perspective, the model o®ers a useful radiography of the competing products available in the market place for the category studied. Unlike the alternative-speci¯c choice models, estimating the attribute-speci¯c preference structure provides meaningful and interpretable parameters. The model may constitute a decision tool, allowing for evaluating the impact of several marketing strategies, such as launching new products or extending existing lines. The results of the hybrid model can also have implications for pricing and promotional activities, such as cross-promotional o®ers (e.g. for which groups of products will a joint promotion be more e®ective, depending on the variety-seeking levels shown for the implied combinations of attributes).

When consumers show a hybrid pattern, the frequency of consumption a®ects the average stock level of the implied attributes. If this is the case, frequent consumers will constantly seek for varied products, while sporadic consumers may show inertial patterns. A consumer who eats yogurt on a daily basis will reach high levels of the implied attributes, and will therefore seek varied products. On the contrary, a sporadic consumer may always choose the same product as the attribute stocks are almost fully depreciated on every new consumption occasion. In the previous examples the frequency of consumption determines the pattern, so the e®ective promotional activities aimed at frequent consumers will di®er from those aimed at sporadic ones (e.g. frequent consumers will confer a higher additional value to a multi-item joint package than sporadic consumers).

Some topics to be considered for further research are summarized as follows:

- An important source of heterogeneity among consumers is the purchase frequency. According to our theoretical results, in an heterogeneous market frequent consumers will accumulate a higher average stock level of attributes, and consequently will search for varied products, while sporadic consumers are more likely to behave inertial. This implication will be tested by extending the empirical analysis including the possibility of di®erent segments.

- The model can be further extended to consider a more general utility function that allows for interactions between attributes, in order to determine how this a®ects the consumption pattern. Later empirical analysis may be done to test for possible interaction e®ects among attributes in several markets.

References

Bawa, Kapil (1990), "Modeling Inertia and Variety Seeking Tendencies in Brand Choice Behavior," Marketing Science, 9 (Summer), 263-278.

Berlyne, D. C. (1963), "Motivational Problems Raised by Exploratory Behavior," in Psychology: A Study of a Science, (Ed.) S. Koch, New York: McGraw-Hill.

Berlyne, D.C. (1970), "Novelty, Complexity and Hedonic Value," Perception and Psychophysics, 8 (5A), 279-286.

Dawes, Robin M. and B. Corrigan (1974), "Linear Models in Decision Making," Psychological Bulletin, 81 (2) 95-106.

Dunn, R., S. Reader and N. Wrigley (1983), "An investigation of the Assumptions of the NBD Model as Applied to Purchasing at Individual Stores," Applied Statistics, 32, 249-259.

Fader, Peter S. and Bruce G. S. Hardie (1996), "Modeling Consumer Choice Among SKU's," Journal of Marketing Research, 23 (November), 442-452.

Givon, Moshe M. (1984), "Variety Seeking Through Brand Switching," Marketing Science, 3 (Winter), 1-22.

Green, Paul E. and V. Srinivasan (1978), "Conjoint Analysis in Consumer Research: Issues and Outlook," Journal of Consumer Research, 5 (September), 103-123.

Guadagni, Peter M. and John D.C. Little (1983), "A Logit Model of Brand Choice Calibrated on Scanner Data,"Marketing Science, 2 (Summer), 203-238.

Hardie, Bruce G. S., Eric J. Johnson and Peter S. Fader (1993), "Modeling Loss Aversion and Reference Dependence E®ects on Brand Choice," Marketing Science, 12 (Fall), 378-394.

Jeuland, Abel P. (1978), "Brand Preference Over Time: A Partially Deterministic Operationalization of the Notion of Variety Seeking," in Research Frontiers in Marketing, Dialogues and Directions, Nº43, AMA 1978 Educators' Proceedings, ed. Subashi Jain, Chicago: American Marketing Association.

Jeuland, Abel P. (1979), " Brand Choice Inertia as One Aspect of the Notion of Brand Loyalty," Management Science, 25 (July), 671-682.

Johnson, Eric J., Robert J. Meyer and Sanjoy Ghose (1989), "When Choice Models Fail: Compensatory Models in Negatively Correlated Environments," Journal of Marketing Research, 26 (August), 255-270.

Kahn, Barbara E., Manohar U. Kalwani and Donald G. Morrison (1986), "Measuring Variety-Seeking and Reinforcement Behaviors Using Panel Data," Journal of Marketing Research, 23, 89-100.

Kuehn, A.A. (1962), "Consumer Brand Choice - A Learning Process?," Journal of Advertising Research, 2, 10-17.

Lancaster, Kelvin (1971), Consumer Demand, A New Approach, New York: Columbia University Press.

Lattin, James M. (1987), "A Model of Balanced Choice Behavior," Marketing Science, 6 (Winter), 48-65.

Lattin, James M. and Leigh McAlister (1985), "Using A Variety-Seeking Model to Identify Substitute and Complementary Relationships Among Competing Products," Journal of Marketing Research, 22, 330-339.

McAlister Leigh (1982), "A Dynamic Attribute Satiation Model of Variety Seeking Behavior," Journal of Consumer Research, 9 (September), 141-150.

McAlister, Leigh and Edgar A. Pessemier (1982) , "Variety Seeking Behavior: An Inter-Disciplinary Review," Journal of Consumer Research, 9 (December), 311-322.

McFadden, D. (1974) "Conditional Logit Analysis of Qualitative Choice," in P. Zarembka (Ed.), Frontiers in Econometrics, New York: Academic Press.

Morrison, D. (1966), "Testing Brand-Switching Models," Journal of Marketing Research, 3 (November), 401-409.

Rogers, Robert D. (1979), "Commentary on 'The Neglected Variety Drive,"' Journal of Consumer Research, 6 (June), 88-91.

Thaler, Richard (1985), "Mental Accounting and Consumer Choice," Marketing Science, 4 (Summer), 199-214.

Tversky, A. and D. Kahneman (1991), "Loss Aversion and Riskless Choice: A Reference Dependent Model," Quarterly Journal of Economics, 106 (November), 1039-1061.

Wierenga, B. (1974), An Investigation of Brand Choice Processes, Rotterdam: Universitaire Pers Rotterdam.