A PROCESS APPROACH TO THE UTILITY FOR GAMBLING*

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Abstract

This paper argues that any specific utility or disutility for gambling must be excluded from expected utility because such a theory is consequential while a pleasure or displeasure for gambling is a matter of process, not of consequences. A (dis)utility for gambling is modeled as a process utility which monotonically combines with expected utility restricted to consequences. This allows for a process (dis)utility for gambling to be revealed. As an illustration, the model shows how empirical observations in the Allais paradox can reveal a process disutility of gambling. A more general model of rational behavior combining processes and consequences is then proposed and discussed.

Key Words: gambling, expected utility, process utility, rationality, irrationality, consequentialism.

JEL Classification: D81.

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1 Introduction

It has long been recognized that people can find pleasure in the mere act of gambling (Pascal, 1670). On the other hand, von Neumann and Morgenstern explicitly mention that their axiomatic treatment towards expected utility "eliminates the specific utility or disutility of gambling" (1953, pp. 28, 629, 632). Marschak (1950, p. 138) takes the following example:

> "[...] the following behavior is not rational: many men, not at all bent on suicide, are enthusiastic mountain climbers and are elated, not (or not only) by exercise and scenery but by the very danger, in the following sense. Suppose the probability of fatal accident is 5%. The climber may prefer a survival chance of 95% to one of, say, 80% *but also* to one of 100%!" (italics are his).

For Marschak, these climber's preferences violate monotonicity between probabilities and consequences, which is axiomatically required for the theory of expected utility. Should his behavior be considered irrational? A climber may like the risk associated with climbing while being prudent. An entrepreneur may launch a new and risky business while still attempting to reduce the probability of failure. In other contexts, individuals may have a specific displeasure for gambling, like in the Allais paradox (1953). Since only some individuals deviate from expected utility in the Allais paradox or like the thrill of risk in climbing, a pleasure or displeasure for gambling depends on individuals and on the context in which they act. The issue is whether a formal model of rational behavior can be sufficiently open to allow for such a dependence upon context while remaining sufficiently structured to support testable predictions.

In a paper about the normative validity and the meaning of von Neumann-Morgenstern utilities, Harsanyi (1993) argues that a specific utility or disutility for gambling is excluded from expected utility theory because such a theory is restricted to "outcome-oriented attitudes". A utility for gambling hence appears as a *process utility* necessarily excluded from expected utility theory and the relevance of a formal theory covering also process utility is emphasized (Harsanyi, 1993, p. 314; see also Sen, 1995, pp. 12 and 15). The consequential nature of expected utility theory has indeed been acknowledged since its axiomatization. However, it is often informally argued that consequences can be defined in order to encompass "everything", i.e. including process considerations (see for instance Hammond, 1988, 1996). Several authors have nevertheless pointed at the inherent difficulties of such extensions of the domain of utility functions. For instance, Harsanyi (1993) argues that the exclusion of process utility is necessary to ensure the cardinality of the expected utility function; Munier (1996) comments Hammond (1996) and shows that consequences cannot, within an expected utility framework, encompass the full memory of the process leading to them; Sen (1997) treats "comprehensive consequences"—which include procedural concerns, as necessarily incomplete, thus violating a basic axiomatic requirement; Pope (e.g. 1995, 1998) refers to "a pre-outcome" period that generates specific emotional considerations excluded by expected utility theory. The next section illustrates why such a specific (dis)utility for gambling cannot be treated by extending the domain of an expected utility function (section 2).

In order to reflect the influence of a (dis)utility for gambling, non-expected utility models have been proposed (Fishburn, 1980 for an axiomatic treatment; Diecidue et Al., 1999 for an extension and a review; see also Conlisk, 1993). Descriptive measures of the discrepancy between the utility of a sure outcome and the utility of the same outcome when composed with probabilities are formalized. By maintaining the characterization of choice solely in terms of consequences, these models thus loose the normative properties of expected utility theory. In contrast, this paper treats choice as composing both a process and a consequence and maintains an expected utility function restricted to consequences. By treating a specific pleasure (or displeasure) of gambling as a process (dis)utility, the model thus preserves part of the normative character of expected utility functions.

Besides this extension of the entities of choice, the main assumption of the model is that process utility combines monotonically with expected utility of consequences: a procedurally preferred process leading to a (consequentially) preferred consequence should be chosen. This assumption is shown to be sufficient for separating a (dis)utility for gambling and to support testable predictions, for instance in the Allais paradox (section 3). A more general framework of rational behavior combining processes and consequences is then proposed. The axioms of expected utility theory are embedded in a procedural context that formalizes a qualitative notion of process utility. We justify this qualitative approach by pointing to the necessary dependence between processes and consequences (section 4). To our knowledge, no other formal approaches combining process preferences with consequential preferences have been developed, despite an emerging body of empirical research on process utility (e.g. Shafir and Tversky, 1992; Gärling et Al., 1996; Donaldson and Shackley, 1997; Frey and Stutzer, 2000).

2 The Consequential Nature of Expected Utility

The consequential nature of expected utility stems from a "*reduction principle*" stating that the entities of choice need to be solely characterized by their probability distribution over potential outcomes (Fishburn, 1988, p. 27). Such probability distributions are here called *consequences*. For von Neumann and Morgenstern, this consequentialism is the main underlying hypothesis of expected utility theory (1953, p. 20; see also Fishburn, 1989, p. 138, 143; Fishburn and Wakker, 1995, p. 1132):

> "We have assumed only one thing—and for this there is good empirical evidence—namely that imagined events can be combined with probabilities. And therefore the same must be assumed for the utilities attached to them,—whatever they may be".

Implicit in this statement is that "anything" can be combined with probabilities. If something relevant to rational choice cannot be combined with probabilities, then it cannot be measured by expected utility theory. The question thus becomes: can we conceive something that cannot be combined with probabilities?

The answer is yes, and it suffices to provide an example. The attribute defined as "is *not* combined with probabilities", i.e. the attribute *Not Gambling*, cannot be combined with probabilities since combining *Not Gambling* with probabilities becomes *Gambling*. Therefore, expected utility theory cannot measure the attribute *Not Gambling* and cannot encompass everything relevant to rational choice.

For the attribute *Gambling,* this paradoxical construction occurs when using a multi-attribute version of expected utility (Keeney and Raiffa, 1976; see also Fishburn, 1982). Suppose we want to define consequences with an attribute specifying whether they are probabilistic (*Gambling*) or not (*Not Gambling*). Suppose further that there exists an expected utility function measuring preferences over such "comprehensive" consequences. As an example, consider the choice presented in Figure 1, which is one of the choice situation in the Allais paradox.

Observing a choice for the upper act and treating *Gambling* as an attribute (G) of consequences would be represented by the inequality

$$.10u(G,5M) + .90u(G,0) > .11u(G,1M) + .89u(G,0) \tag{1}$$

where u is an expected utility function. Hence, the above inequality is equivalent to

$$.10u(G, 5M) + .89u(G, 1M) + .01u(G, 0) > 1u(G, 1M). \quad (2)$$

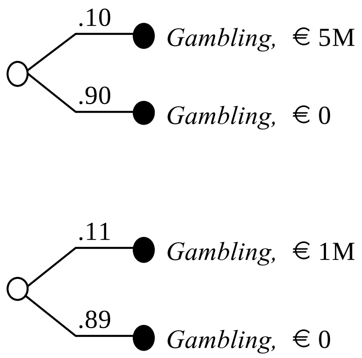

Figure 1

The entity $1u(G, 1\text{M})$ has, however, no possible empirical meaning. It states that the individual receives with probability 1 an amount of €1 Million with the attribute *Gambling*. But the meaning of *Gambling* is to characterize a *probabilistic* consequence, not a consequence attained with probability one. Therefore, expected utility theory cannot consider consequences such as the one assigning unit probability to $(G, 1\text{M})$ because such a consequence does not exist and will never be empirically observable. As a result, the attributes *Gambling* or *Not Gambling* cannot be measured by an expected utility function.

3 A Process (Dis)Utility for Gambling

In order to take account of *Gambling* or *Not Gambling* while preserving the expected utility function and its properties, we treat these attributes *outside* the expected utility function. The domain of the expected utility function is explicitly restricted to consequences and choice reveals a preference relation over combinations of processes and consequences. In this manner, consequences retain their interpretation as probability distributions over outcomes (degenerate or not) while any entity that remains under the control of the individual is interpreted as a *process.* A *behavior* that is empirically observable is assumed to comprise both a process and a consequence.

In order to study a specific pleasure or displeasure for gambling, processes are defined as taking the attribute *Gambling* (G) when they lead to a probabilistic consequence (i.e. a non-degenerate probability distribution) and as taking the attribute *Not Gambling* $(\overline{G})$ when they lead to a deterministic consequence (a degenerate probability distribution, i.e. a sure outcome).

For a consequence c, a behavior is thus written (G, c) or $(\overline{G}, c)$ depending on whether c is probabilistic or deterministic respectively. Choice, i.e. empirically observable preferences over behaviors, is reflected by a preference relation $\succsim^B$ over behaviors. The model further assumes a monotonicity condition between processes and consequences, i.e. that a behavior composed of a preferred process and a preferred consequence should be chosen. As shown below, such a structure allows for a specific utility or disutility for gambling to be separated from choice.

Consider the choice situations depicted in Figure 2. Compare them to Figure 1. The *individual* is now depicted and *Gambling* characterizes the *process.* This illustrates the modification of the entities of choice from consequences to combinations of processes and consequences.

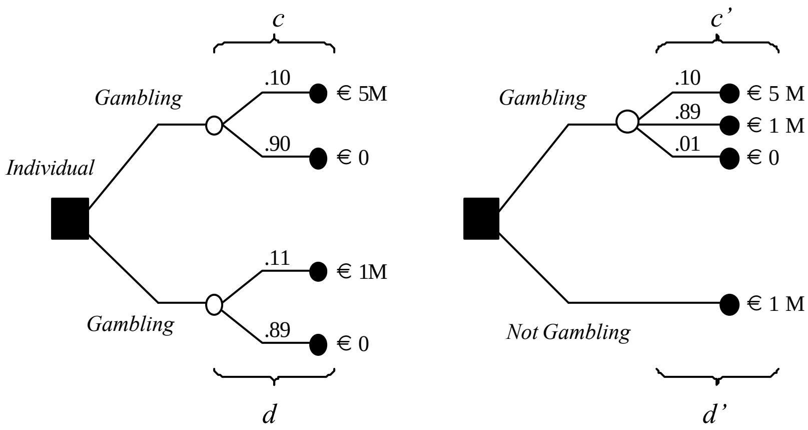

Figure 2

The empirical observation of a choice for the upper behavior in the first situation is written

$$(G, c) \succ^B (G, d), \tag{3}$$

which implies

$$c \succ^C d \tag{4}$$

where $\succ^C$ is the induced preference relation over consequences only. In this manner, the process attribute *Gambling* has been "abstracted" and the conventional expected utility reasoning applies over consequences. We have therefore

$$c \succ^C d \Longleftrightarrow u(c) > u(d) \Longleftrightarrow u(c') > u(d') \Leftrightarrow c' \succ^C d'. \tag{5}$$

where c' denotes the probabilistic consequence of receiving €5M with probability .1, €1M with probability .89 and nothing with probability .01, while the deterministic consequence of receiving €1M is denoted d'. Therefore, empirical observation of a choice for the upper behavior in the first situation of figure 2 implies that the *consequence* c' is *consequentially preferred* to the consequence d'. Because the individual may have a disutility for gambling, it does *not* follow that the upper behavior (G, c') should be preferred to the lower behavior $(\overline{G}, d')$. Besides, if $(G, c') \not\succsim^B (\overline{G}, d')$, a process disutility can be revealed. The reasoning follows.

Suppose the individual would have a process utility for gambling, that is $G \succ^A \overline{G}$ where $\succ^A$ is the induced preference relation over processes, then he *should* choose the upper behavior (G, c'). Such a behavior is indeed "optimal" in the sense it is composed of a preferred process and a preferred consequence. Thus, if the individual does *not* choose (G, c'), he must have a *process disutility for gambling.* We have:

$$\{(\overline{G}, d') \succ^B (G, c') \text{ and } c' \succsim^C d'\} \Longrightarrow \overline{G} \succ^A G. \tag{6}$$

A process disutility for gambling being revealed, testable predictions can be proposed. For instance, if a deterministic consequence $d'' \succ^C d'$ is substituted for d', we must have $(\overline{G}, d'') \succ^B (G, c')$. Observing the reverse would violate the explanation with a disutility for gambling and the individual would be considered "irrational". Another pattern is shown in Figure 3. If $(G, e) \succ^B (G, c)$ is observed, then $(\overline{G}, e') \succ^B (G, c'')$ *should* also be observed for the model not to be falsified. Such a pattern would not be explained by expected utility theory alone and illustrates how the consideration of process preferences allows for a larger class of behaviors to be treated as rational. Additional remarks further specify this process approach.

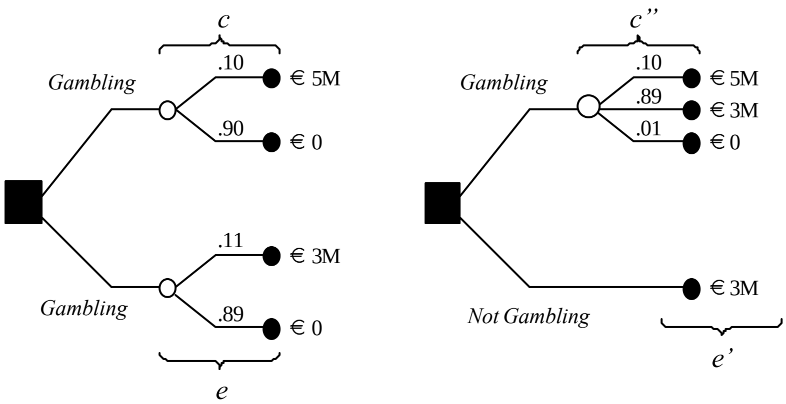

Figure 3

First, the observation of $(G, c) \succ^B (G, d)$ and $(G, c') \succ^B (\overline{G}, d')$ is consistent with a utility for gambling but also with a disutility for gambling. The individual may have a disutility for gambling that has too little "strength" to reverse expected utility of consequences and a specific (dis)utility for gambling would require additional empirical observations to be revealed.

Second, expected utility of consequences must remain valid for a process (dis)utility for gambling to be revealed. In particular, the independence condition between probabilities and consequences must hold for establishing expression (5). In this sense, this model builds on expected utility rather than invalidates it.

Third, the reasoning that reveals a process (dis)utility for gambling is counterfactual: because $(G, c') \succsim^B (\overline{G}, d')$ is *not* observed, we are able to infer that $\overline{G} \succ^A G$. This requires the empirical ordering $\succsim^B$ to be complete while it cannot encompass all combinations of processes and consequences (a choice for a behavior like (G, d') cannot be empirically observed since (G, d') does not exist). This requires a careful definition of the set of available behaviors and is treated in the next section.

Finally, and contrary to the interpretation of Marschak (see introduction), the monotonicity condition between probabilities and consequence is maintained. In Marschak's example, the climbing route with a survival chance of 100% is consequentially preferred to the climbing route with a survival chance of 95%, which is itself consequentially preferred to the climbing route with a survival chance of 80%. In terms of expected utility of consequences, these three routes monotonically rank in decreasing order. However, when the process is taken into account, it can be more fun to have *some* riskiness and a climber may rationally choose to climb the route with a survival chance of 95%, being prudent and moved by the thrill of risk at the same time. What the climber chooses is not merely a probability of survival but also comprises a certain process of climbing.

4 A Model Combining Processes and Consequences

This section presents a general model that qualifies an expected utility function over consequences by outside procedural considerations. It establishes the extent to which process preferences can be separated from consequential preferences represented by the expected utility function. To this aim, it first makes explicit our assumption that behavior is composed of both a process and a consequence.

A process is denoted by $a \in A$ and a consequence by $c \in C$. The set A is the set of *processes* and the set C is the set of *consequences*. Consequences depend on processes through a *consequence function* g with domain A and range C. This is written $g : A \rightarrow C$, $a \mapsto g(a) \in C$. A *situation of choice* then corresponds to the set of *available behaviors* $B = \{(a, g(a)) : a \in A\}$ and a binary relation $\succsim^B$ on B that is both complete and transitive reflects empirical observation of rational behavior. This is our first axiom:

Axiom 1 (Behavioral Preferences) *The binary relation $\succsim^B$ is a weak ordering of B.*

Rational individuals are then supposed to make *judgments* over processes and over consequences. These judgments are reflected by weak orderings $\succsim^A$ and $\succsim^C$of the sets A and C respectively.

Axiom 2 (Procedural Judgments) *The binary relation $\succsim^A$ is a weak ordering of A.*

Axiom 3 (Consequential Judgments) *The binary relation $\succsim^C$ is a weak ordering of C.*

These three orderings are now combined through a weak monotonicity condition. This axiom states that a behavior composed of a procedurally preferred process and a consequentially preferred consequence *should* be chosen. Called the *optimality* axiom, it provides the structure with normative implications.

Axiom 4 (Optimality) *If $a \succsim^A a'$ and $g(a) \succsim^C g(a')$ then $(a, g(a)) \succsim^B (a', g(a'))$.*

The following propositions are now derived. The first expresses a *dependent revelation of preferences over processes.*

Proposition 1 $\{(a', g(a')) \succ^B (a, g(a))$ *and* $g(a) \succsim^C g(a')\} \Longrightarrow a' \succ^A a.$

Proof. By contraposition of the optimality axiom, we have $not[(a, g(a)) \succsim^B (a', g(a'))] \Longrightarrow \{not[g(a) \succsim^C g(a')]$ *or* $not[a \succsim^A a']\}$. Since $not[(a, g(a)) \succsim^B (a', g(a'))] \Longleftrightarrow (a', g(a')) \succ^B (a, g(a))$ and $not[a \succsim^A a'] \Longleftrightarrow a' \succ^A a$, we have $\{(a', g(a')) \succ^B (a, g(a))$ and $g(a) \succsim^C g(a')\} \Longrightarrow a' \succ^A a$. ■

Similarly, *dependent revelation of preferences over consequences* is formulated as:

Proposition 2 $\{(a', g(a')) \succ^B (a, g(a))$ *and* $a \succsim^A a'\} \Longrightarrow g(a') \succ^C g(a)$.

Proof. Same as above. ■

Although behavioral preferences $\succsim^B$ are the only observable preferences, judgmental preferences $\succsim^A$ and $\succsim^C$ can be revealed dependently on each other. This formalizes the implication of expressions (4) and expression (6) in the previous section.

Axioms of expected utility are now assumed, here in the form proposed by Herstein and Milnor (1953) where C is a mixture set.

Axiom 5 (Independence) *If* $c, d \in C$ *and* $c \sim^C d$ *then* $\frac{1}{2}c + \frac{1}{2}e \sim^C \frac{1}{2}d + \frac{1}{2}e$ *for any* $e \in C$.

Axiom 6 (Continuity) *For any* $c, d, e \in C, \{\lambda c + (1-\lambda)d \succsim^C e\}$ *and* $\{e \succsim^C \lambda c + (1-\lambda)d\}$ *are closed subsets of* $[0, 1]$.

Theorem 3 (Expected Utility of Consequences) *Axioms 3, 5, 6 hold if and only if there exists an order-preserving linear functional* u *on* C *such that for all* $c, d \in C$, *we have* $c \succsim^C d \Leftrightarrow u(c) \geqslant u(d)$ *with* $u(\lambda c + (1-\lambda)d) = \lambda u(c) + (1-\lambda)u(d)$. *Such a functional is unique up to a positive affine transformation.*

Axioms 3, 5 and 6 allow for the derivation of expression (5) in the above section. For elicitation purposes, the bias from a utility or a disutility for gambling can be avoided through comparison of non-degenerate probability distributions (see e.g. Wakker and Deneffe, 1996). Remarks on the specificity of this model now conclude this section.

We have considered a complete empirical ordering $\succsim^B$ over a strict subset of the product set $A \times C$ so that $\succsim^B$ is complete over B and incomplete over $A \times C$. This subset $B = \{(a, g(a)) : a \in A\}$ is defined in a non-arbitrary manner because of the specification of the consequence function g. Without specifying g, $\succsim^B$would have been incompletely or arbitrarily defined and the counterfactual reasoning that reveals process preferences would not be properly justified (for $\succsim^B$ incomplete $b' \succ^B b \not\Leftrightarrow not[b \succsim^B b']$). Our model thus differs from Sen (1997) where procedural concerns are treated through an incomplete ordering over "comprehensive" consequences and where no separation of process preferences takes place.

On the other hand, specifying g reflects an empirical dependence between processes and consequences. For g being an application, some processes cannot be combined with some consequences and conjoint measurement does not apply because the independent realization of components is not fulfilled (see Krantz et Al., 1971, p. 246 for the role of this specific independence condition). In this framework, no continuous and monotonic function over behaviors is thus defined. Nevertheless, our analysis shows that process preferences can be revealed as a ranking. This reflects an ordinal approach to process utility (see Harsanyi, 1993). Since consequential utilities remain measured cardinally by the expected utility function, behavior is "measured" by the combination of an ordinal and a cardinal scale. Further research should investigate the specificity of such a formulation.

5 Concluding Remarks

Although expected utility has been widely criticized for its descriptive inadequacy, and that efforts have been dedicated to construct alternative approaches, the dominant view is that it remains the proper normative model (Wakker and Deneffe, 1996). Rather than modifying its axioms, this paper explicitly restricts expected utility to consequences and considers that observable behavior always implies a process that remains under the control of the individual. Assuming that such behavioral process does not—or should not, be taken into account in the modeling of rational behavior becomes a limit-case. In the general case, rational individuals can be motivated by both procedural and consequential considerations. Beside expected utility of consequences, the main assumption of the model is thus that rational individuals make judgments on the process by which they reach consequences and that a rational individual should prefer a behavior composed of a preferred process and a preferred consequence. Under these conditions, process preferences can be revealed and support testable implications. This model is introduced by an original treatment of the Allais paradox according to which expected utility, when properly restricted to consequences, is actually not violated by individuals having a specific process disutility for gambling. Beyond consequentialism, further work may interestingly study the combination of procedural and consequential considerations.

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