The HP-Filter in Cross-Country Comparisons[*]

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Abstract

Many empirical studies have applied the Hodrick-Prescott filter in cross-country comparisons of business cycle fluctuations. The Hodrick-Prescott filter involves the smoothing parameter, λ, and standard practise in the literature is to set this parameter equal to 1600 (in quarterly data) for all countries. We show that this choice might distort the results when the cyclical comoponent is highly serially correlated, and that care should be taken in checking if the results are "reasonable" in the light of common wisdom. For example, for Spanish data we show that, contrary to conventional wisdom, the results imply that the Spanish cycle is very smooth and that the period of 1975-1985 was one of macroeconomic tranquility. We propose to recast the HP-filter as a constrained minimization problem which selects endogenously a value of λ that imposes cross-country consistency of the imposed constraint. Our proposed method is easy to apply, retains all the virtues of the standard HP-filter. When applied to Spanish data the results imply a return to conventional wisdom and we find results in line with economic historian's views. We also examine data for a number of OECD countries and find that, with the exception of Spain, Italy and Japan, the standard choice of $\lambda = 1600$ is sensible.

JEL Classifications: C32, E32

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1   Introduction

This paper proposes a method that enhances the ability to make international comparisons of business cycle fluctuations. Following Lucas' (1977) concept of international business cycles, many researchers have attempted to document similarities and differences across countries in aggregate fluctuations at the business cycle frequencies. Studying business cycles is of interest for economic theory and economic policy alike. Economic policy may be adjusted to the state of the business cycle and sometimes policy is constrained by some measure of the business cycle. For example, a central bank may lower interest rates if the country is perceived to go into a recession, but not if growth is lower due to structural reasons; the nowadays popular Taylor rule needs a measure of the output gap, and while the output gap (a measure of the deviation from some equilibrium level of output) is a distinct concept from standard business cycle measures (the deviation from a trend perhaps also adjusted for irregular components of output), output gaps are in practise often approximated by business cycle measures. To cite a concrete example, the Growth and Stability pact of the EU calls for sanctions if a country is experiencing deficit higher than 3% or debt higher than 60%, but the sanctions do not take effect if the country is experiencing a temporary recession, thus, a measure of the business cycle in each country is needed in order to determine if the sanctions are to be imposed to that country.

The measurement of business cycles, however, relies upon a statistical measure of these types of fluctuations and many techniques are available for extracting the business cycle component from the data. Furthermore, each method has its advantages and disadvantages. A natural requirement in cross-country applications is that similar procedures are applied to data for different countries. Partly for that reason, and partly due to ease of computation and reproduction, many researchers have adopted the Hodrick and Prescott (1980, 1997) detrending method (the HP-filter from now on). Hodrick and Prescott originally applied this procedure to post-war US quarterly data and their findings have since been updated and extended in a number of papers including Kydland and Prescott (1990) and Cooley and Prescott (1995).

A large number of studies have applied the HP-filter to examine business cycle moments for other countries often comparing the statistics with those obtained for the US data. Blackburn and Ravn (1992) investigate UK business cycles, Brandner and Neusser (1992) study German and Austrian business cycles, Danthine and Girardin (1989) examine at Swiss data, Dolado, Sebastián and Vallés (1993), Puch and Licandro (1997) and Borondo, González and Rodríguez (1999) study Spanish data, and Kim, Buckle and Hall (1994) look at data from New Zealand. Other studies have directly looked at cross-country comparisons, see e.g. Fiorito and Kollintzas (1994) and Blackburn and Ravn (1991) for studies of post war business cycles in a cross section of OECD countries. Backus and Kehoe (1992) compare the business cycle features both across countries and across different periods of time. Yet other studies have looked at the relationship between business cycle fluctuations across countries (i.e. international correlations), see e.g. Backus, Kehoe and Kydland (1992), Ravn (1997) or Ambler, Cardia and Zimmerman (1999). This paper argues that the standard application of the HP-filter may not allow for a straightforward cross-country

comparison of the business cycle moments, and suggests a simple modification to the application of the filter that makes results more comparable across countries.

The HP-filter is implemented by minimizing an objective function that depends on the weighted average of two components: the squared sum of the business cycle component (the deviation from trend) and the squared sum of the acceleration of the trend component weighted by a parameter (usually denoted by λ). When choosing the value of λ most researchers have followed the suggestion of Hodrick and Prescott and set this parameter equal to 1600 for quarterly data.[1] Hodrick and Prescott's choice of $\lambda = 1600$ was guided partly by a prior on the variability of the trend and the cycle components of the US data and partly by the fact that it produces "reasonable results" in the sense that the implied cyclical component largely agrees with "conventional wisdom" about the US business cycle. This is a sensible strategy because the statistical measure of the business cycle thus captures key aspects identified by observers of the business cycle. However, most subsequent studies have simply adopted the value of 1600 without considering the sensibility of the results in the light of "conventional wisdom". This practice can under some circumstances be problematic. In particular, the HP-filter (as any other filter applied to finite samples) assigns parts of the low frequency fluctuations to the trend. Thus, while the HP-filter with $\lambda = 1600$ might work well for the U.S., if the trend component behaves markedly different in data for other countries, using the same value of the smoothing parameter for such countries may give rise to non-comparability of the measure of the cyclical components. For example, if a given country has experienced longer cycles than other countries, the trend component for this country will absorb a larger part of the cyclical component.

Another way to express this is that since the HP-filter is an approximation to a band pass filter, the quality of this approximation depends on the mass of the spectrum of the data that is subject to the approximation error. Therefore, since a larger part of the cycle will be assigned to the trend in those countries with more persistent fluctuations, cross-country comparisons of business cycles might be difficult. We will argue that such problems can easily be addressed by appropriately adjusting the value of the smoothing parameter in cross-country studies. Furthermore, we will argue that the proposed method gives rise to "reasonable" results in the sense that they agree with "conventional wisdom".

The fact that higher serial correlation of the cycle biases the results is not just an academically interesting issue for theoretical statisticians and econometricians to argue over. It can actually confuse (and it has confused) the interpretation of the cycle and the effect that various government policies can have over the cycle in certain countries. For example, consider a government that has a policy of absorbing external shocks at the cost of delaying reforms. This is likely to cause a deeper and long-lasting crisis, so it is likely to impart a higher serial correlation in the output fluctuations and to increase total volatility. We will show that in this case the unadjusted HP filter may say that economic fluctuations were small in that country. To take a concrete

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[1]Some studies explore the sensitivity of the results to the choice of λ. Hodrick and Prescott themselves examine this issue and conclude that the results are reasonably robust. Canova (1998) arrives at a more negative result.

example, it is commonly accepted that the Spanish government followed exactly this kind of policies during most of the 70's and that this caused the "economic crisis" of 1974 to 1984 in that country (see section 3 below). Indeed, the unadjusted HP filter does not capture this crisis in Spain but our adjusted measure does.

We initially illustrate the pitfalls of applying the HP-filter with $\lambda = 1600$ to all countries by examining quarterly data for Spain. We show that with this choice of the smoothing parameter, one arrives at conclusions at odds with the historians view on the modern history of economic fluctuations in Spain. The consensus among economic historians is that during and after the oil shocks of the 1970's, Spain experienced a very long recession, lasting from 74 to 84 of a size larger than what was experienced in most other countries. The cyclical component identified by the HP-filter with $\lambda = 1600$ in contrast delivers the result that the Spanish cycle was *less* affected by the oil shocks than most other countries. Furthermore, the HP filter with $\lambda = 1600$ arrives at the conclusion that the Spanish cycle is *less* volatile than most other countries, which is also at odds with informal observations of the economy.

Hence, a standard application of the HP-filter to Spanish data does not pass the "reasonability" test. To deal with this problem we suggest a simple method for choosing λ in a systematic way that will be valid in international comparisons. The idea is to select λ so that one generates a comparable level of volatility of the trend in each country. We show that, if we reinterpret the HP-filter as the solution to a *constrained* minimization problem, our procedure is consistent with imposing the same constraint on volatility of trend across countries, while the usual practice of keeping λ constant (and equal to 1600) amounts to changing the constraint across countries.[2] Furthermore applying our procedure to Spanish data gives rise to a cyclical component that agrees much better with the story that an economic historian would tell. In order to examine whether this result is special to Spanish data, we extend the analysis to data for other OECD countries. We find that the standard choice of $\lambda = 1600$ is sensible for most countries except for Japan for which we find results similar to those obtained for Spain.

The remainder of the paper is organized as follows. Section 2 reviews the HP-filter and the reasoning that lead several authors to use $\lambda = 1600$. Section 3 discusses the historian's view of output fluctuations in Spain and how they are not matched by the standard HP-filter. Section 4 discusses our reinterpretation of the filter and applies it to Spain. Section 4 provides more cross-country empirical evidence, and section 5 concludes.

2 The Hodrick-Prescott Filter and Its Implications

This section outlines the HP-filter, how the choice of the value of the smoothing parameter is usually made, and it shows that if the cycle is highly serially correlated the usual practice can give very bad results as the serial correlation of the cyclical

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[2]A similar problem relates to how one chooses to adjust the HP-filter when applied to data that are sampled at different frequences. Ravn and Uhlig (2002) discuss in detail how such adjustments to the frequency of observations should be carried out.

component increases. This will motivate our suggestion of how to modify this filter in the next section.

Let y_t denote the natural logarithm of a time-series observed over the sample from $t = 1$ to T. Consider decomposing this series into a trend component, denoted by y_t^{tr}, and a cyclical component, denoted by y_t^c, so that:

$$y_t = y_t^{tr} + y_t^c \tag{1}$$

Many methods are available for accomplishing such a decomposition, but much of the business cycle literature has applied the HP-filter, and this method shall be our concern in this paper.[3,4] Given the sample, the HP-filter involves the estimation of the trend component from the solution to the following minimization problem for fixed λ :

$$\min_{\left\{y_t^{tr}\right\}_{t=1}^T} \sum_{t=1}^{T} \left(y_t - y_t^{tr}\right)^2 + \lambda \sum_{t=2}^{T-1} \left(\left(y_{t+1}^{tr} - y_t^{tr}\right) - \left(y_t^{tr} - y_{t-1}^{tr}\right)\right)^2 \tag{2}$$

The first term in the objective function is a measure of the "goodness-of-fit". The second term punishes variations in the growth rate of the trend component. The parameter λ is key since it determines the trade-off between "goodness-of-fit" and the smoothness of the trend component. In the limit as $\lambda \to \infty$ the trend becomes linear thereby allowing for large fluctuations in the cyclical component. When $\lambda \to 0$ the trend component instead becomes equal to the data series y_t, and the cyclical component approaches zero.

Hodrick and Prescott take λ as a fixed parameter, which they set equal to 1600 for US quarterly data. Their choice of this value was based upon a prior about the variability of the cyclical part relative to the variability of the change in the trend component. Hodrick and Prescott (1997, p.4) state that:

> "If the cyclical components and the second differences of the growth components were *identically and independently distributed*, normal variables with means zero and variances σ_1^2 and σ_2^2 (which they are not), the conditional expectation of the y_t^{tr}, given the observations, would be the solution to program (2) when $\sqrt{\lambda} = \sigma_1/\sigma_2$", ... "Our prior view is that a 5 percent cyclical component is moderately large, as is a one-eight of 1 percent change in the growth rate in a quarter. This led us to select $\sqrt{\lambda} = 5/(1/8)$ or $\lambda = 1600$".

Kydland and Prescott (1990, p. 9) argue further in favor of the choice of $\lambda = 1600$ for quarterly post war US data because:

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[3] Baxter and King (1999) and Christiano and Fitzgerald (2003) provide alternative methods based on band-spectrum techniques. We show in the appendix that the standard Baxter-King filter used for quarterly data generates almost exactly the same problems as the HP(1600) for the Spanish data. Hence, this filter does not circumvent the problem that we highlight.

[4] Other popular methods for making this decomposition include polynomial trends, ARIMA decompositions (such as the Beveridge and Nelson, 1981, method), unobserved components methods (c.f. Harvey, 1985 or Watson, 1986) or multivariate methods. See Canova (1998) for a comprehensive discussion and evaluation.

"With this value, the implied trend path for the logarithm of real GNP is close to the one that students of the business cycle and growth would draw through a time plot of the series"

Judging the results against conventional wisdom is a very sensible strategy. Even if conventional wisdom may not be precise enough to allow one to make the statistical decomposition, a "good" detrending procedure should yield results consistent with facts argued for by economic historians and other students of business cycles. However, while the HP-filter with $\lambda = 1600$ may produce "reasonable" results for US data, there is no guarantee that similar good results are obtained when applied to data for other countries. The reason for is indicated by the first quote since the properties of the business cycle may differ across countries.

It is useful to make a connection to "optimal" linear filtering. A given linear filter is said to be optimal if it minimizes the mean square error (the mean of the squared difference between the "true" cyclical component and the *estimate* of the cyclical component). Moreover, any filter method is unlikely to be "optimal" under general assumptions about the processes that generate the data. In the case of the HP-filter, as indicated by the first quote above, one of the conditions for the filter being optimal is that the cyclical component is serially uncorrelated (which in general will not be true).[5] When the true cyclical component is serially correlated, the variance of the cyclical component as estimated by the HP-filter is likely to be underestimated. The reason for this is that the HP-filter associates low frequency fluctuations - including parts of the business cycle component - with changes in the trend.[6]

This is not too much of a problem when the HP-filter is applied to series that have similar cyclical properties. For example, if used to compare moments of key macroeconomic data with series generated by a calibrated macroeconomic model, applying the HP-filter to both the actual series and to the model generated data will, presumably, not generate any bias in favor or against the model (since the part of the cyclical component incorrectly assigned to the trend will be similar both in the model and in the data). However, in cross-country comparisons of business cycle features, if there are important cross-country differences in the persistence of the cyclical component, the usual practise of holding λ constant for all countries implies that a larger part of the cyclical component will be assigned to the trend in countries with larger serial correlation. Thus, countries with higher serial correlation will appear to have a less volatile cyclical component, even if the variance is the same in both countries.

The following proposition formalizes this argument. We show that for arbitrarily high auto-correlation, the cyclical component extracted by the HP-filter goes to zero and we stress that the proposition holds for any sample. Suppose that we observe a time-series $\{y_t\}_{t=0}^{T}$ and without loss of generality we set $y_0 = 0$, so that y_t denotes the increase between time 0 and time t. Denote the average growth rate in the sample

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[5]King and Rebelo (1993) provide an insightful of the conditions under which the HP-filter is an optimal linear filter.

[6]We stress that this is not a criticism of the HP-filter. Other filters are likely to suffer from the same problem.

by $\alpha_T \equiv \frac{y_T}{T}$. With this normalization, a linear trend at time can be defined as $\alpha_T\, t$.[7] Define the deviations from linear trend as $(y_t - \alpha_T\, t)$ and the *sample autocorrelation of deviations from linear trend* as

$$\rho_T \equiv \frac{\sum_{t=1}^{T}(y_t - \alpha_T t)\,(y_{t-1} - \alpha_T(t-1))}{\left(\sum_{t=1}^{T}(y_t - \alpha_T t)^2\ \sum_{t=1}^{T}(y_{t-1} - \alpha_T(t-1))^2\right)^{\frac{1}{2}}}$$

The following proposition says that the cyclical component extracted by HP goes to zero as the serial correlation increases.

Proposition 1 Consider any T, and any λ. Consider samples where the deviations from linear trend are uniformly bounded. Let $\left\{y_t^{HP}\right\}$ be the HP trend for each sample and for the given λ. Then

$$\sum_{t=1}^{T}\left(y_t - y_t^{HP}\right)^2 \to 0 \qquad \text{as } \rho_T \to 1,$$

uniformly in the sample.

More precisely, fix λ, T and choose some $K < \infty$. Consider samples such that $|y_t - \alpha_T t| \leq K$ for all t. Then, for any $\delta > 0$, we can find an $\varepsilon > 0$ such that, if $1 - \rho_T < \varepsilon$, then $\sum_{t=1}^{T}\left(y_t - y_t^{HP}\right)^2 < \delta$.

Proof

We have

$$\sum_{t=1}^{T}\left(y_t - y_{t-1} - \alpha_T\right)^2 = \sum_{t=1}^{T}(y_t - \alpha_T t)^2 + \sum_{t=1}^{T}(y_{t-1} - \alpha_T(t-1))^2$$
$$- 2\left(\sum_{t=1}^{T}(y_t - \alpha_T t)^2\ \sum_{t=1}^{T}(y_{t-1} - \alpha_T(t-1))^2\right)^{\frac{1}{2}}\rho_T$$

this follows from adding and subtracting $\alpha_T(t-1)$ inside the square of the left side, simple algebra and the definition of ρ_T. Now, using the normalizations $y_0 = 0$ and that $y_T = \alpha_T T$ we have that $\sum_{t=1}^{T}(y_t - \alpha_T t)^2 = \sum_{t=1}^{T}(y_{t-1} - \alpha_T(t-1))^2 = \sum_{t=1}^{T-1}(y_t - \alpha_T t)^2$. Plugging this into the previous equation we get

$$\sum_{t=1}^{T}\left(y_t - y_{t-1} - \alpha_T\right)^2 = 2\sum_{t=1}^{T-1}(y_t - \alpha_T t)^2 - 2\sum_{t=1}^{T-1}(y_t - \alpha_T t)^2\,\rho_T = 2(1-\rho_T)\sum_{t=1}^{T-1}(y_t - \alpha_T t)^2 \tag{3}$$

Simple algebra implies

$$\sum_{t=1}^{T}\left(y_t - y_t^{HP}\right)^2 \leq \sum_{t=1}^{T}\left(y_t - y_t^{HP}\right)^2 + \lambda\sum_{t=2}^{T-1}\left(\left(y_{t+1}^{HP} - y_t^{HP}\right) - \left(y_t^{HP} - y_{t-1}^{HP}\right)\right)^2$$
$$\leq \sum_{t=1}^{T}\left(y_t - y_t^{tr}\right)^2 + \lambda\sum_{t=2}^{T-1}\left(\left(y_{t+1}^{tr} - y_t^{tr}\right) - \left(y_t^{tr} - y_{t-1}^{tr}\right)\right)^2$$

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[7]To clarify, we are not proposing this as the best possible linear trend. We are just showing that, with this particular choice of linear trend, the proposition holds.

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for any candidate trend $\{y_t^{tr}\}_{t=0}^T$. The first inequality follows because we add a non-negative number, the second inequality follows from the fact that $\{y^{HP}\}_{t=1}^T$ is the minimizer of (2), so it achieves a smaller value than any alternative $\{y_t^{tr}\}$. Now plugging the (feasible) alternative $y_t^{tr} = y_t$ in the above relation gives the first inequality in

$$\sum_{t=1}^T \left(y_t - y_t^{HP}\right)^2 \leq \lambda \sum_{t=2}^{T-1} \left((y_{t+1} - y_t) - (y_t - y_{t-1})\right)^2 \leq 4\lambda \sum_{t=1}^T \left(y_t - y_{t-1} - \alpha_T\right)^2$$

$$= 8\lambda(1 - \rho_T)\sum_{t=1}^{T-1}(y_t - \alpha_T t)^2 \leq 8\lambda(1 - \rho_T)\,(T-1)K^2$$

where the second inequality follows from the fact that, given any two numbers a, b, we have $2(a^2 + b^2) \geq (a + b)^2$, the equality follows from (3) and the fourth inequality from boundedness of the deviations from trend.

Therefore, given any $\delta > 0$, taking $\varepsilon = \frac{\delta}{8\lambda(T-1)K^2}$ we have that for any sample with $\rho_T > 1 - \varepsilon$, then $\sum_{t=1}^T \left(y_t - y_t^{HP}\right)^2 < \delta.\blacksquare$

The proposition shows that as ρ_T approaches 1, the HP trend absorbs a larger and larger fraction of the cyclical component, the cyclical component becomes zero, and the trend becomes equal to the series. It is perhaps surprising that this holds true for any process and even any sample. Even in the extreme case that $\{y_t\}$ is a purely stationary process so that, in truth, $y_t^{tr} = 0$, the HP trend will indicate that $y_t^{tr} = y_t$ and the cyclical component is zero for sufficiently high serial correlation.

The implication of this proposition is that the standard application of the HP-filter in cross-country studies may not yield desirable results if the business cycle components behave markedly different across countries. In particular, for a fixed λ, a larger share of the business cycle component will be assigned to the trend component in countries with more persistent business cycle components.

3   The HP-Filter and the Business Cycle in Spain

Spanish quarterly data provides a good example of the problems that one might face. Here we will show that applying the standard value for λ would tell a story about the Spanish cycle quite different from the one that the most prestigious observers of this economy would tell. We will start by describing the consensus view about the Spanish cycle, supporting our claims with citations from some of the most prestigious students of this economy.[8]

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[8]We will cite works by E. Fuentes Quintana , Luis A. Rojo, J. Segura and on the yearly report of the Bank of Spain. L.A. Rojo was vice-governor of the Bank of Spain in 1988, and he was the governor of the Bank from 1992 until 2000. E. Fuentes Quintana was vicepresident and economics minister of the Spanish government from 1977 until 1978. For a more comprehensive description of this period in Spain, see García-Delgado (1990). All citations have been translated from Spanish by the authors.

The consensus view about the Spanish cycle is as follows. During the period 1974-1985 the economic performance of the Spanish economy was very bad. This period is referred to as one of "economic crisis".[9]

This crisis was worse and longer in Spain than in most other industrialized countries.[10] This is surprising because Spain grew very fast in the 1960's and in the early 1970's but there was still a lot of catching up to do in 1973. Spanish GDP per capita relative to the four main European economies (Germany, France, Italy, and the U.K.) rose from 40.9 percent to 54.3 percent in the period 1965-1975, but this ratio fell continuously to 48.9 percent by 1985. Inflation and unemployment were much higher than in the rest of Europe: yearly inflation reached 26% in 1977 and the unemployment rate reached 21% in 1985.

This deep and long crisis was in part fueled by a particular behavior of the economic authorities in Spain, who resisted taking the appropriate measures to react to the events.[11] The two oil shocks of the seventies had a large negative impact in Spain, possibly worse than in other Western economies, but the government implemented a "... delirious *compensatory policy*" and it "... maintained the price of energy products (in effect subsidizing their consumption)" (Fuentes Quintana, p. 38). This, in a still relatively poor, non-oil-producing country, caused a higher duration and deeper impact of oil crisis than in other countries.[12] Also, in a time of political unrest, the inflationary pressures faced by most countries at the beginning of the 70's and

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[9]Three citations to support this claim: the Bank of Spain (1989, p. 33) refers to, "the long and deep crisis that the economy experienced during the second half of the seventies and the first half of the eighties". The title of the essay by Rojo (1987) is: "La crisis de la economía española, 1973-1984". Fuentes Quintana (1993) says (p. 6): "The severity of the effect of the crisis of the 70's and the delay in implementing the appropriate adjustments explain the long duration of the crisis, which caused a divergence from the European Community in the ten years from 1975 to 1985".

[10]Says Rojo (p. 194) "(in 1975) ... a long period started that had very negative effects both for the Spanish economy as such and also relative to other countries - even comparing with the unsatisfactory performance of the European economies of those years". Fuentes Quintana (p. 36, 37) states that "The gravity of the effect of the crisis of the 70's on the Spanish economy would be hard to exagerate ... our crisis, in the 70's, was different because of the identical and maximal intensity of all the factors that played a role in the crisis". Further, García Delgado (1990) (p. 60): "It may be surprising to see that a decade after the first oil shock ... when Western countries had already absorbed the effects of that crisis, the behavior and unbalances found in Spain were those more commonly found in the middle of the 70's. This only reflects the enormous delay it took for our economy to adapt". Marimon and Zilibotti (1998) argue more formally for the presence of this long recession in Spain using a detrending method based on cross-country sectorally disaggregated data.

[11]Rojo (p. 193) states that "the policy of adjustment and restructuring, started in 1977, was less strict in Spain than in other countries". Similarly Fuentes Quintana (p. 38) writes that: "The first surprise when we analyze the arrival and the attitudes towards the crisis in Spain is the delay in approving and implementing a policy for its treatment."

[12]At the time, gasoline was supplied by monopolistically by CAMPSA, a government owned firm that absorbed losses with government funding, and the price of gasoline and other oil products was decided directly by the government. Therefore, the price of gas was seen as a fiscal instrument that could be used to stimulate aggregate demand. Rojo explains (p. 195) "The economic authorities in the last two years of the Franco regime ... tried, unsuccesfully, to stop the fall in internal demand and internal activity by *not* increasing the price of energy products despite the increase in the cost of oil". As a consequence "Between 1971 and 1979 the seven largest industrialized countries had reduced their final energy demand by 9%, while Spain had increased this demand by 10%".

the demands for salary increases from the trade unions were accepted and then turn into inflationary pressures.[13] Many government-owned firms producing manufactured goods that were no longer internationally competitive were having large losses and they were being subsidized, it was not until the mid 80's that these firms were sold or closed.

To summarize, the informal consensus is that Spain experienced one very long economic crisis from, roughly, 1975 to 1985. This crisis was worse than in most other countries. The economic authorities spent government funds trying to "stabilize" the economy, but this only postponed the reforms and it increased the depth and the length of the economic downturn. Furthermore, it is a commonheld view that Spanish business cycle fluctuations have been more severe than in other countries, so that one would expect the output volatility to be high in Spain.

We now examine whether the application of the HP-filter with $\lambda = 1600$ delivers results that are consistent with these views. We examine Spanish real GDP for the period 1970 quarter 1 to 1998 quarter 4 obtained from the Spanish National Institute of Statistics (downloaded from the web page of Instituto Nacional de Estadística at www.ine.es). This is a relatively short period but, unfortunately, quarterly Spanish national accounts data do not exist for a longer sample period. Furthermore, these data have been used in a number of previous studies of Spanish business cycles.

Panel A of Figure 1 illustrates graphically Spanish real GDP, the estimated Hodrick-Prescott trend component for $\lambda = 1600$, and the estimated cyclical component (Panel B).[14] This panel illustrates the large and prolonged fluctuations in the Spanish economy, especially during the period 1975-1985. Panel B of Figure 1 shows the cyclical component extracted by the standard value of λ and it indicates that, according to this filter, the Spanish business cycle has been relatively smooth. The years from 1975 to 1985 appear as a relatively calm period in the whole sample; indeed, panel B would suggest that instability in Spain was concentrated in the periods *before* 1975 or *after* 1985, so there is no sign of there having been the large "crisis económica" that so many observers talk about. Furthermore, compared to the US cycle in Figure 2, the oil shocks appear to have had little effect on the Spanish cycle, and the US cycle shows many more ups and downs within the years 75 to 85 than the Spanish economy. There is no sign that the crisis was deeper and longer in Spain, contradicting the informal discussion of so many observers of the Spanish economy. Perhaps that "delirious compensatory policy" was not such a bad idea after all?. In fact, we argue that precisely because this crisis was longer and deeper in Spain, the HP-filter associates the main part of the first oil shock and its aftermath to the trend

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[13]Says Rojo (p. 200) "International comparisons show our economy has one of the most rigid labor markets ... the real cost of labor per person has increased by 75% in the period 1973-84, the largest increase in industrialized countries". Fuentes Quintana (p. 40) talks about "a permissive policy which, with a clear subordination of the economic problems to the political situation, allowed for an overindexation of wages".

[14]Some authors have pointed out that there may be some shortcomings of this data set. The Spanish National Institute of Statistics does not report how the data have been constructed, and the GDP series appear to be very smooth at very high frequencies, suggesting that quarterly data perhaps have been constructed from interpolating annual data. We will discuss this in more detail later arguing that the procedure that we will propose is a good way to deal with such situations.

component, as can be seen from Panel A, Figure 1.

Table 1 reports the standard deviation for the cyclical component of Spanish and US real GDP together with the estimated autocorrelation coefficients of orders 1-5 quarters. As discussed by Dolado, Sebastian, and Vallés (1993), contrary to economic historians' view, the standard deviation of the cyclical component of Spanish GDP appears to be very low (around 30 percent lower than the corresponding US number). Several authors (e.g. Dolado, Sebastián and Vallés, 1993, Puch and Licandro, 1999, and Borondo et al., 1999) have interpreted these results as indicating that the Spanish economy has low output volatility.[15] We believe, however, that results with HP(1600) in this country (and others where the cyclical component is highly serially correlated) are a mere artifact of the way that the trend is estimated.

4   Choosing the Smoothing Parameter

This section suggests a method for calculating the trend component that mitigates into the sort of problems highlighted in the previous section, while keeping the method close to the original HP-filter, thereby hopefully retaining the attractive features of the standard HP-filter. Our approach is to select λ endogenously in order to maintain comparability across countries.[16]

4.1   The Adjustment Rules

The first step is to re-cast the HP-filter as the solution to the following constrained minimization problem:

$$\min_{\left\{y_t^{tr}\right\}_{t=1}^{T}} \quad \sum_{t=1}^{T}\left(y_t - y_t^{tr}\right)^2 \tag{4}$$

$$\text{s.t.} \quad : \quad \frac{\sum_{t=2}^{T-1}\left(\left(y_{t+1}^{tr} - y_t^{tr}\right) - \left(y_t^{tr} - y_{t-1}^{tr}\right)\right)^2}{\sum_{t=1}^{T}\left(y_t - y_t^{tr}\right)^2} \leq V \tag{5}$$

where $V \geq 0$ is a constant specified by the researcher computing the trend. V can be thought of as a "target value" for variability of the acceleration in the trend relative to the variability of the cyclical component. Setting V constant across countries ensures comparability across countries in the sense that the variability of the acceleration of the trend relative to the variability of cyclical component is common.

For appropriate choices of λ and V this problem and the HP-filter are equivalent. First, notice that if we set $V = 0$, the above problem results in a linear trend component, while letting V go to infinity implies that the trend becomes equal to the

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[15]Dolado et al., 1993, show calculations for various other values of λ, some of them close to the ones we will find for Spain in section 3.1, but in the conclusion they only discuss the values for $\lambda = 1600$.

[16]This general idea can be incorporated in other filters. For example, in the Baxter and King (1999) filter, the band width across countries could be chosen in a similar way.

series y_t. In other words, by changing V we have the same flexibility as changing λ in the standard formulation of the HP-filter. Second, if multiply both sides of (5) by $\sum_{t=1}^{T} (y_t - y_t^{tr})^2$, it is clear that the Lagrangian of the above minimization problem solves

$$\min_{\{y_t^{tr}\}_{t=1}^{T}} (1 - \bar{\lambda}V) \sum_{t=1}^{T} \left(y_t - y_t^{tr}\right)^2 + \bar{\lambda} \sum_{t=2}^{T-1} \left(\left(y_{t+1}^{tr} - y_t^{tr}\right) - \left(y_t^{tr} - y_{t-1}^{tr}\right)\right)^2 \qquad (6)$$

where $\bar{\lambda}$ is the Lagrange multiplier of the transformed constraint (5). The solution to this Lagrangian and the HP-filter are equivalent iff:

$$\lambda = \frac{\bar{\lambda}}{1 - \bar{\lambda}V} \qquad (7)$$

Thus, the constrained minimization problem will reproduce the results of the HP-filter with given value of λ if V is chosen to equal the ratio on the left hand side of (5) implied by the HP-filter's trend component. The usual value of $\lambda = 1600$ can then be interpreted as the value of λ that satisfies (7) when $\bar{\lambda}$ is the Lagrange multiplier of the rewritten constraint (5) for the US value of V.

Therefore, our approach is to impose a comparable level of variability of the acceleration of the trend and cyclical components across countries. The λ that will be applied for each country will be endogenously determined by solving for the Lagrange multiplier of constraint (5) for each country. It is more desirable to keep constant V rather than λ across countries, since V is a parameter that can be easily interpreted. We rarely have good reasons to expect that the ratio in (5), should be different across countries, while we do observe serial correlation of output to be different across countries, specially in short samples. Furthermore, various government policies may have precisely the effect of imparting a higher serial correlation in the cycle, and if (as in the example of Spain) government followed different policies to try and stabilize the cycle. Said differently, the usual practice of keeping λ constant amounts to changing the constraint (5) across countries arbitrarily.

We refer to this procedure (keeping V constants across countries) as "adjustment rule 1". Computation is straightforward and can be accomplished using a standard iterative scheme. Since the mapping between $\lambda \in [0, \infty)$ and $\bar{\lambda} \in [0, V^{-1})$, is one-to-one, solving for λ is equivalent to solving for $\bar{\lambda}$. For a given a value for λ we compute the trend on the basis of the HP-filter for each possible value of λ and define the function:

$$F(\lambda) \equiv \frac{\sum_{t=2}^{T-1} \left[\left(y_{t+1}^{tr}(\lambda) - y_t^{tr}(\lambda)\right) - \left(y_t^{tr}(\lambda) - y_{t-1}^{tr}(\lambda)\right) \right]^2}{\sum_{t=1}^{T} \left[y_t - y_t^{tr}(\lambda) \right]^2} \qquad (8)$$

where $y_t^{tr}(\lambda)$ is the trend component that relates to λ. Numerically, the problem is to find a value λ^{rule1} that solves the equation $F(\lambda^{rule1}) = V$. If there is no solution to this equation, the Kuhn-Tucker conditions to the problem (4) imply that the constraint is not binding so that the solution to the minimization problem is given by $\bar{\lambda} = 0$ and, therefore, $y_t = y_t^{tr}$.

A variety of iterative schemes can be used in order to solve the equation $F(\lambda^{rule1}) = V$. In practice we found no problems with multiplicity of solutions to the first order conditions, and it suffices to adjust λ upwards when $F(\lambda) - V$ is positive and vice versa. Thus, our proposed adjustment to the standard HP-filter allows for easy computation, and can be replicated in a straightforward manner by other researchers.

To study the robustness of the results to the precise form of the constraint (5), we explore a second method that replaces (5) by the following constraint

$$\frac{1}{T-2} \sum_{t=2}^{T-1} \left(\left(y_{t+1}^{tr} - y_t^{tr}\right) - \left(y_t^{tr} - y_{t-1}^{tr}\right) \right)^2 \leq W \tag{9}$$

The interpretation of this rule is clear: the constraint restricts the variability of the acceleration in the trend component directly and λ now has the interpretation as the Lagrange multiplier on (9). We refer to this problem (choosing W on the basis of the value implied by applying the standard HP-filter to e.g.. US data) as "adjustment rule 2". This can be computed analogously to the previous adjustment rule.

The difference between the two rules is that, while rule 2 imposes the same variability of the growth of the trend across countries, rule 1 allows for a larger variability of the growth rate in countries with a more volatile cyclical component. Rule 2 might be used if the researcher believes that the deviation of actual trend from a linear trend is similar across countries. Two countries that share common industrial structures and are subject to similar economic conditions (such as, say, the US and the UK) would be obvious candidates for imposing Rule 2. Rule 1 may be used instead, if the researcher believes that deviations from linear trend are larger in some of the countries considered. For example, if some of the countries considered had very different levels of initial wealth in the beginning and, (due to transitional growth as in a standard growth model) they grew faster in the first few periods as they were converging to a higher steady state income level, one would expected large deviations from linear trend in those countries, Rule 1 may be more appropriate. Rule 1 would be more appropriate also if some of the countries underwent larger changes in their environment, for example, if some countries went from a socialized economy to a market oriented economy.

4.2   An Application to Spanish Data

We now examine the consequences of applying the alternative filters to Spanish data. The results are reported in Table 1. We choose V and W on the basis of applying the standard HP-filter with $\lambda = 1600$ to US quarterly real GDP. For the US data (for the sample period corresponding to the Spanish data) we find that $V = 1.72 \times 10^{-4}$ and $W = 4.92 \times 10^{-8}$. Applying the HP-filter with $\lambda = 1600$ to the Spanish data implies that $V = 8.11 \times 10^{-4}$ and $W = 11.42 \times 10^{-8}$. Thus, the variance of the (acceleration in the) trend component relative to the variance of the cyclical component is around 5 times higher for the Spanish data than for the US data if both are computed with $\lambda = 1600$ and the variance of the acceleration in the trend component itself is more than twice as high in the Spanish data. Thus, to match V or W, it is clear that λ

needs to be increased in order to induce less variation in the Spanish trend component. We find that λ^{rule1} is equal to 5385 while λ^{rule2} increases further to 6369.

These adjustments have a large effect on the implied variability of the cyclical component. Recall that for $\lambda = 1600$, the standard deviation of the deviation from the trend is around 30 percent lower in Spain than in the US. With adjustment rule 1 the percentage standard deviation in Spain increases to 1.81 or 7 percent higher than in the US. For adjustment rule 2, the standard deviation of the cyclical component of 1.92 percent per quarter which is 13 percent higher than the corresponding US number. Thus, in line with common agreement among economic historians, both our adjustment rules imply that the Spanish business cycle has been more variable than the US business cycle.

It is also worth noticing that with HP(1600) we obtain much higher persistence of the cyclical component in Spain than in the US, and that this persistence increases even more with our adjustment rules. These results confirm the intuition provided by proposition 1 that, if λ is fixed, too much cyclical variability is assigned to the trend when the cyclical component is highly serially correlated.

Figure 3 illustrates the deviations from the trend. The cyclical components for the two adjustment rules are very similar but differ markedly from what is implied for $\lambda = 1600$. First, both adjustment rules lead to an increase in the magnitude of the Spanish business cycle as we have just discussed. Secondly, the general shape for the early 1970's and post 1987 are very similar for all three cases, the only difference being the magnitude of the cycle. But, for the period from 1974.2 to 1987 the results are rather different. As discussed in the previous section, $\lambda = 1600$ has the counterfactual implication that Spain had enjoyed quite a stable period around the oil shocks. When we adjust the smoothing parameter we find, instead, that output was going down continuously in the period 1974.2-1987. There was a small recovery around 1976-1977, but it is still a very small recovery compared to the recovery between the oil shocks in the US.

Hence, by imposing more comparable results across countries we obtain reasonable results for the implied business cycle components in Spain. This illustrates the usefulness of the approach that we suggest above - the results now pass the "reasonability" test much better with $\lambda = 1600$.

One might say that the cycle extracted from the adjustment rules in Figure 3 is far from showing such a big "economic crisis" one described by the authors quoted above. There is no way to prevent the HP-filter from assigning part of the "economic crisis" to the trend; to the extent that this filter (and any other flexible filter) works by the principle of signal extraction, *any* shock will be assigned in part to the trend and in part to the cycle. But it is clear that our adjustment rules improve the results considerably, they do give the conclusion that the cycle in Spain has been very volatile, they would suggest to the observer that the cycle was different and that perhaps policies played a role in this, and they do indicate that the period of 1974 to 1985 was a long downturn. The only way to guarantee that none of the economic crisis is assigned to the trend is by using a very tightly parameterized model for the trend.

4.3   Discussion

We have just shown how the application of our adjustment rules affect the estimate of the cyclical component and argued that the results fit better with economic historians view of the Spanish business cycle than the results implied by the HP-filter with $\lambda = 1600$. Our interpretation of this is that Spain has been through a period of catching-up coupled with "bad" policies which in combination imply a more persistent business cycle component than in other countries.

There are, however, other possible explanations for our results. It is evident from Figure 1 that the Spanish real GDP series is very smooth indeed. This raises the suspicion that the quarterly data perhaps have been constructed by interpolating between data sampled at lower frequencies of observation. We have no way of knowing if this is indeed the case, but we may still consider how such a possibility would affect our arguments and the results.

It turns out that our adjustment rules are very useful precisely if data has been interpolated. There are two reasons for this: *i)* it is well known that interpolated series are likely to be too highly serially correlated[17] and, as we have been arguing, this is precisely the situation where our adjustment rules work well; *ii)* the two adjustment rules we propose can be used to detect if the data have been indeed interpolated because, in this case, the two adjustment rules will in general cause the smoothing parameter to adjust in opposite directions.

To give some intuition for *ii)* notice that time-averaging reduces the variance of the original series. Applying the HP-filter (with constant λ) would generate even lower volatility of the cyclical component if applied to the interpolated data than to the "true" data, since the interpolated data is more persistent (and the HP-filter will therefore associate a larger fraction of the reported data with the trend). Hence unless λ is adjusted, the volatility of the trend component is too high *relative* to the cyclical component. Suppose that the data for Spain has been indeed interpolated but the US data has not, then the ratio in the left side of (5) is likely to go down. Adjustment Rule 1 is likely to indicate an *increase* in λ because the cyclical component will be very smooth relative to the trend component. Adjustment Rule 2 depends on the volatility of the acceleration in the trend component. Because of interpolation, the trend component is likely to be less volatile in the Spanish data than in the US data thus leading Adjustment Rule 2 to *lower* the value of λ applied to the Spanish data.

To make this more formal, we look at a Monte Carlo experiment. Suppose that the "true" data series, $\{y_t^{true}\}_{t=1}^T$ is generated at the quarterly frequency but only observed at the annual frequency. The reporting agency constructs quarterly data $\{y_t^{reported}\}_{t=1}^T$ from the observed annual data by interpolation. Such a procedure would remove high frequency movements from $\{y_t^{true}\}_{t=1}^T$ and introduce persistence

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[17]This is because temporally aggregated (more precisely, time-averaged) series are more serially correlated. Christiano and Eichenbaum (1987) show how if an ARMA process in continuous time is time-averaged and temporally aggregated this imparts an additional lag in the MA part. Marcet (1991), shows how time-averaging data in a general process shifts the weight of the Wold decomposition to low frequencies.

in $\left\{y_t^{reported}\right\}_{t=1}^{T}$ through standard temporal aggregation bias combined with persistence introduced by interpolation. We generate N artificial datasets each with 200 observations. We then use these generated data to produce another dataset in which the quarterly observations have been generated by linear interpolation between annual observations. To be precise, for each artificial dataset, we first averaged the original data over four consecutive and non-overlapping periods. We then linearly interpolated these data using the time-aggregated data at every fourth data point and using the linear interpolations for the points in between. With these two data sets in hand, each dataset was HP-filtered with the HP-filter for $\lambda = 1600$ and the adjustment rules were applied to the interpolated dataset. We assumed that the original ("true") data were generated by the process:

$$\begin{aligned} y_t &= \alpha_T + \beta t + x_t \\ x_t &= \gamma x_{t-1} + \varepsilon_t \end{aligned}$$

and we arbitrarily assumed that $\beta = 0.01$, $\gamma = 0.95$, $\varepsilon \sim NID(0, 0.02)$. Furthermore, we discharged the first 25 observations and set $N = 100$.

Table 2 reports the results. They indicate, as mentioned above, that for a fixed $\lambda = 1600$, the interpolated data display lower volatility and higher persistence than the original data. When we apply Adjustment Rule 1, we find that the mean value of the smoothing parameter increases to 2644 which, correctly, increases the volatility of the cyclical component (but at the cost of generating even higher persistence).[18] In contrast, Adjustment Rule 2, which depends only on the volatility of the acceleration in the trend component, leads to a mean value of the smoothing parameter of 1405 and applying this value further lowers the volatility of the business cycle component while decreasing persistence very slightly. This pattern is not consistent with the evidence we uncovered for the Spanish data leading us to believe that there is more to the results we showed in this section about Spain than simply "bad" data.

A perhaps more direct way to deal with this issue is to examine annual data since such data should not suffer from problems related to interpolation. Furthermore, measurement errors are likely to be less problematic. We apply our adjustment rules and the standard HP-filter to real US and Spanish GDP for the period 1970-2002. We use the value $\lambda = 6.25$ as the baseline value of the smoothing parameter since Ravn and Uhlig (2002) argue that with this value the HP-filter isolates business cycles in annual data similar to the business cycles that the HP-filter isolates in quarterly data for $\lambda = 1600$.

Table 3 reports the key moments of the deviations from trend of the annual data and Figure 4 illustrates the measures of the business cycle. The results are almost identical to those reported in the previous section: Using $\lambda = 6.25$ on the annual data implies that the Spanish cyclical component is approximately 30 percent less volatile than the US cyclical component. Furthermore, the results indicate that the period 1975-85 was a tranquil period in Spain. However, as for the quarterly data,

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[18] No method would be able to reconstruct the high frequency movements in the data since these have been removed by interpolation. Thus, the further increase in persistence should not be considered a problem in particular for our proposed adjustment.

both V and W are significantly higher in the Spanish data than in the US data when using a common value of the smoothing parameter. Thus, applying either adjustment method leads to an increase in the implied value of λ to approximately 18 and 20.5 for Adjustment Rule 1 and Adjustment Rule 2, respectively.[19] Quantitatively, the results are very similar for the two adjustment rules. In both cases, the volatility of the cyclical component rises to 7-10 percent above the corresponding US number and Figure 4 now indicates that the period 1975-85 was one long recession interrupted only by a very mild recovery. Hence, the results are very similar to those that we found for quarterly data. This indicates that the results are not simply due to low quality quarterly data.

Finally, one may wonder whether the problems that we identify are purely related to the use of the HP-filter. We doubt that this is the case. In particular, alternative methods such as the Baxter and King (1999) band-pass filter or the Christiano and Fitzgerald (2003) band-pass filter are likely to suffer from exactly the same problems. The reason is that these types of detrending techniques rely upon the extraction of fluctuations of a certain periodicity and the arguments that we have made argue that the periodicity may differ across countries. The appendix studies this in some detail for the Baxter and King (1999) filter.

5   International Evidence

This section extends the above analysis to a panel of OECD economies. We look at quarterly real GDP for Australia, Canada, France, Italy, Japan, Switzerland, and the United Kingdom.[20] For Australia, Japan and the UK the sample period is 1960.1 to 1998.4. For Canada the sample period is 1961.1-1998.4, for Italy and Switzerland it is 1980.1-1998.4, while for France it is 1985.1-1998.4.

As above we match V and W to the results obtained using the HP-filter with $\lambda = 1600$ to US data. The results are listed in Table 4. For Australia, Canada, France, Switzerland and the UK, both adjustment rules yield results that are very are close to $\lambda = 1600$ and the changes in λ that do occur, do not significantly affect the behavior of the cyclical component of GDP. In some cases the robustness of the choice of λ is rather remarkable; for France, for example, we find that $\lambda_1 = 1695$ and $\lambda_2 = 1615$, while for the UK we find $\lambda_1 = 1997$ and $\lambda_2 = 1989$. Thus, many previous studies of the business cycle properties of data from these countries based on the conventional choice of $\lambda = 1600$ have been consistent in the sense of imposing similar trend properties across countries when comparing the results with those of the US as reported by e.g. Kydland and Prescott (1990).

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[19]It may be of some interest to notice that the implied values of the smoothing parameter are very close to those argued by Ravn and Uhlig (2002). These authors argue that the measure of the business cycle component will be preserved at different frequencies of observation if the smoothing parameter is adjusted by the frequency change raised to the power of 4. For Adjustment Rule 1 this implies a value of the smoothing parameter at the annual frequency of just around 20 which is close to the value of 18 implied by the results of our adjustment rule.

[20]The data were all obtained from the OECD national accounts database and relate to GDP in constant prices.

For Italian data we find that the choice of the adjustment rule matters for λ but not so much for the implied volatility. Adjustment rule 1 leads to an increase in λ to 2479. Adjustment rule 2, however, leads to a drop in the value of the smoothing parameter to 1061. The reason for this difference is that the variabilities of both the trend and the cyclical component are quite small for the Italian data but more so as far as the cyclical component is concerned. However, regardless of whether ones uses $\lambda = 1600$ or either of the two adjustment rules, the variability of the cyclical component is significantly below what is observed for the US and we do not find major changes in the business cycle moments of Italian GDP. The results are in fact in line with what we would expect in case of the presence of interpolation errors, as discussed in the previous section, but we have no hard evidence that Italian quarterly data have been constructed from interpolating annual data.

The only other country for which we find large effects of using rules 1 or 2 is Japan. For $\lambda = 1600$ we find that the Japanese business cycle is slightly smoother than the US business cycle. For the US data we find that $V = 1.81 \times 10^{-4}$ and $W = 4.78 \times 10^{-8}$ for $\lambda = 1600$ while the Japanese numbers are $V = 3.56 \times 10^{-4}$ and $W = 8.78 \times 10^{-8}$.[21] Thus, our adjustment rules will lead to increases in λ. We find $\lambda^{rule1} = 4504$ and $\lambda^{rule2} = 8973$. When using these values of the smoothing parameter, we find that the Japanese business cycle is more volatile than the US business cycle with the standard deviation of the cyclical component being 13 percent higher than the corresponding US number for Adjustment Rule 1 and 32 percent higher for Adjustment Rule number 2. These results are similar to those obtained for Spain.

We find these results compelling in the light of the macroeconomic developments in Japan with the prolonged period of sustained growth in the 1960 and the 1970's and the lengthy period of macroeconomic turbulence of 1990's. With $\lambda = 1600$ these phenomena become almost exclusively attributed to the trend while the adjustment rules imply that there are also business cycle effects. Also, the case of Japan shows how the two rules differ. Japan grew much faster in the 60's than other countries while the prolonged recession of the Japanese economy in the 1990's now has reversed this picture. To the extent that the higher Japanese growth in the 60's should be attributed to transitional growth rule 1 seems more appropriate. Figure 5 illustrates graphically the cyclical components for $\lambda = 1600$ and for the two alternative values of the smoothing parameter. The figure reveals that the change in the value of the smoothing parameter mainly gives rise to an increase in the business cycle volatility. Thus, for Japan, the adjustment rules lead to an increase in business cycle volatility but leaves the business cycle dating unchanged.

6 Summary

This paper has proposed a simple method for adjusting the HP filter. We argue that the usual practice of setting $\lambda = 1600$ (when examining quarterly data) across countries may be inappropriate, especially if the panel under examination includes

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[21]The numbers for the US are slightly different from those we quoted in the previous section because the sample period is different.

countries expected to have either very low or very high persistence over the business cycle. We propose a way to choose λ in a systematic way across countries so as to make results comparable. Our suggestion is simply to re-interpret the HP-filter as a constrained optimization problem that keeps constant the variability of the acceleration in the trend. In this way the value of the smoothing parameter can be re-interpreted as relating to the multiplier on the imposed constraint. The standard procedure of setting $\lambda = 1600$ amounts to changing the constraint across countries, while our adjustment rule insure that the constraint is the same for all countries.

The standard choice of $\lambda = 1600$ leads to counterfactual results for Spanish data but application of the proposed adjustment rules, one would conclude that Spain indeed has large business cycles and that 1975-85 was, to a closer approximation, one long recession.[22] Thus, our adjustment rules make sense not only from a statistical point of view but also, from a practical point of view. We also showed that when applied to data for other OECD countries $\lambda = 1600$ approximates quite well the values that one would have chosen using either of our adjustment rules for most countries other than Spain and Japan.

We hope that these adjustment rules may be of use to researchers and policy makers alike. In particular, we believe that our arguments could be particular useful for researchers examining data for countries that have undergone economic transformation or for other reasons may display unusual high (or low) persistence over the business cycle.

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7   Appendix

In Section 2 we showed that as the estimated persistence of the data tends towards one, the HP-filter will incorrectly assign *all* the variation in the data to the trend component. Here we examine whether other filter suffer from similar types of problems. In particular, we examine the properties of the business cycle component isolated by the approximate band-pass filter of Baxter and King (1999). This filter is a finite sample approximation to an "optimal" band-pass filter. Baxter and King (1999) show how one can easily implement such filters in the time-domain by truncating an infinite-order moving average representation of the "optimal band-pass filter". Following Baxter and King (1999) let $BP_K(s_1, s_2)$ be an approximate band-pass filter truncated at lag K that admits frequencies between s_1 and s_2 periods. We will follow these authors in using $BP_{12}(6, 32)$ as a benchmark so that the business cycle duration is between 1.5 years and 8 years. We also experiment with the $BP_{12}(2, 32)$ which Baxter and King (1999) argue is a close approximation to the HP-filter.

Figure 6 illustrates the resulting measures of the business cycle component for the Spanish data discussed in section 2 together with the cyclical components isolated by the BP filter. The figure shows that the cyclical components of the band-pass filters look very similar to the cyclical components based on the HP(1600) filter and different from the cyclical component based on our alternative HP(5385) filter. To examine this further we report some statistical properties of the cyclical components in Table 5. We report the standard deviation of the cyclical components, the correlation between the cyclical components. The standard deviations of the cyclical components of HP(1600), $BP_{12}(2, 32)$, and $BP_{12}(6, 32)$ are all very similar and lower than those of HP(5385) and HP(6369). Furthermore, there is an almost perfect correlation between the HP(1600) cyclical component and the cyclical component of either of the band-pass filter while the correlation with the cyclical components of our alternative filters is substantially smaller.

Hence, the analysis indicates that application of band-pass filters lead to the same puzzles as the HP(1600) filter when applied to Spanish data. We are not certain if this occurs because the recession in Spain in the years 75 to 85 is excluded from $BP_{12}(6, 32)$ "by definition", (this would be problematic, since it would imply that a cycle spurred largely by the same oil shock as in other countries is excluded from the cyclical component just because it had a longer effect in Spain) or because of the approximation and short-sample effects in the filter proposed by Baxter and King. Nevertheless, the fact is that applying $BP_{12}(6, 32)$ would lead an observer to the odd conclusion that the oil shocks did not cause a recession in Spain and that output is less volatile in Spain than in other countries.

Table 1. Quarterly Data: 1970.1-1998.4

	Method	λ	Standard deviation		autocorrelation of order				
			$\sigma\left(y^{c}\right)(\%)$	$\sigma\left(y^{c}\right)/\sigma\left(y_{US}^{c}\right)$	1	2	3	4	5
US	-	1600	1.69	1	0.88	0.70	0.48	0.26	0.06
Spain	-	1600	1.19	0.70	0.94	0.80	0.62	0.43	0.25
Spain	1	5385	1.81	1.07	0.97	0.89	0.78	0.65	0.51
Spain	2	6369	1.92	1.13	0.97	0.89	0.79	0.67	0.54

Table 2. A Monte Carlo Experiment on the Effects of Interpolation

data	Method	$E(\lambda)$	$E(V)$	$E(W)$	standard deviation (%)	first order autocorrelation
Original data	-	1600	0.0176	$0.0120*10^4$	2.581	0.701
Interpolated data	-	1600	0.0317	$0.0108*10^4$	1.834	0.932
Interpolated data	1	2644.5	-	-	2.009	0.941
Interpolated data	2	1404.9	-	-	1.786	0.930

Notes: The results relate to the average of 100 simulations. $E(\lambda)$ denotes the average value
of the smoothing parameter over the 100 simulations, E(V) is the average value
of the variance ratio defined in (5) and E(W) is the average of the variance defined in (9).

Table 3. Annual Data, 1970-2002.

	Method	λ	Standard deviation		First order autocorrelation
			$\sigma\left(y^{c}\right)(\%)$	$\sigma\left(y^{c}\right)/\sigma\left(y_{US}^{c}\right)$	
US	-	6.25	1.50	1.00	0.35
Spain	-	6.25	1.06	0.71	0.44
Spain	1	17.93	1.57	1.05	0.68
Spain	2	20.46	1.65	1.11	0.70

Table 4. International Evidence

	Method	λ	Standard deviation (%) $\sigma(y^c)$	$\sigma(y^c)/\sigma(y^c_{US})$	autocorrelation of order 1	2	3	4	5
Australia	-	1600	1.63	1.00	0.58	0.30	0.17	0.10	-0.12
	1	1310	1.60	0.98	0.56	0.28	0.15	0.08	-0.14
	2	1255	1.60	0.98	0.56	0.27	0.14	0.07	-0.15
Canada	-	1600	1.45	0.91	0.84	0.62	0.43	0.23	0.07
	1	2147	1.53	0.96	0.85	0.66	0.47	0.28	0.13
	2	1959	1.50	0.94	0.85	0.65	0.46	0.27	0.11
France	-	1600	0.88	0.97	0.90	0.75	0.54	0.29	0.11
	1	1695	0.89	0.98	0.90	0.75	0.55	0.30	0.12
	2	1615	0.88	0.97	0.90	0.75	0.54	0.29	0.11
Italy	-	1600	0.89	0.64	0.83	0.63	0.43	0.24	0.10
	1	2479	1.00	0.72	0.86	0.69	0.52	0.35	0.21
	2	1061	0.80	0.58	0.80	0.57	0.34	0.13	-0.01
Japan	-	1600	1.57	0.96	0.83	0.66	0.45	0.20	-0.01
	1	4504	1.89	1.16	0.87	0.75	0.58	0.38	0.21
	2	8973	2.14	1.32	0.90	0.80	0.66	0.49	0.33
Switzerl.	-	1600	1.14	0.83	0.93	0.74	0.50	0.24	0.02
	1	2079	1.21	0.87	0.94	0.76	0.53	0.29	0.07
	2	1460	1.12	0.81	0.93	0.74	0.48	0.23	0.00
UK	-	1600	1.57	0.96	0.80	0.64	0.50	0.30	0.14
	1	1997	1.63	1.00	0.81	0.66	0.52	0.33	0.17
	2	1989	1.63	1.00	0.81	0.66	0.52	0.33	0.17

Table 5. Comparison of filters, Spanish data

	$\sigma(y^c)$ (%)	Correlations* $HP(1600)$	$HP(5385)$	$HP(6369)$	$BP_{12}(2,32)$
HP(1600)	1.189	1			
HP(5385)	1.812	0.945	1		
HP(6369)	1.919	0.931	0.999	1	
$BP_{12}(2,32)$	1.185	0.980	0.948	0.939	1
$BP_{12}(6,32)$	1.251	0.974	0.959	0.951	0.993

* This denotes the correlation among the business cycle components

Figure 1. Spanish real GDP, lambda=1600

A. Real GDP and it's trend

real gdp

HP(1600) trend
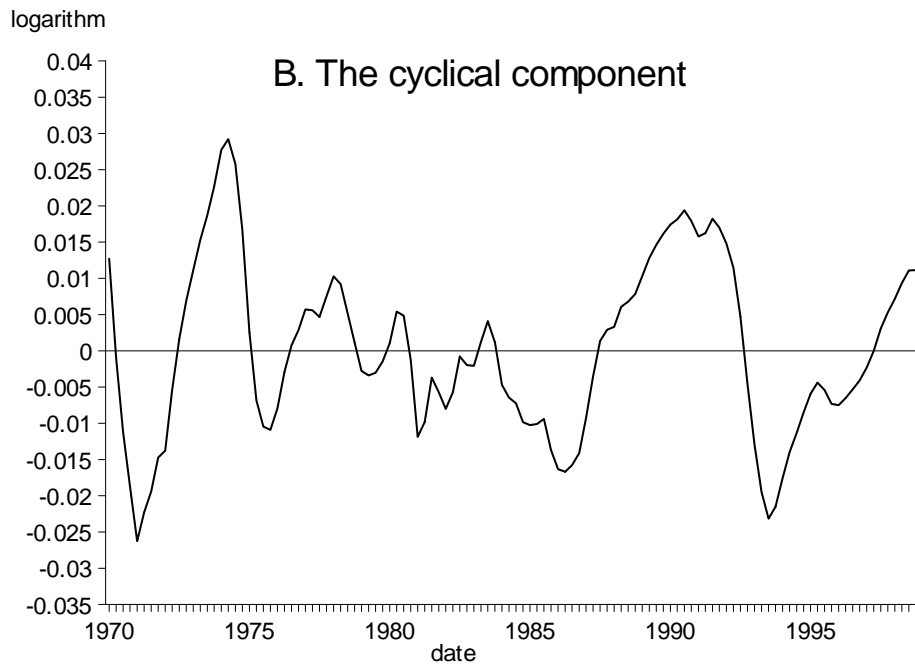
date

logarithm
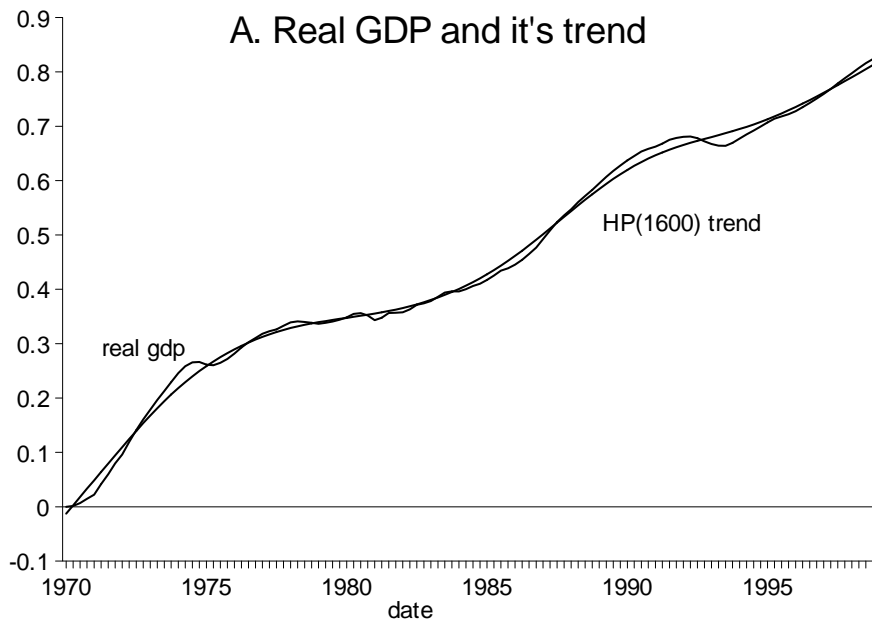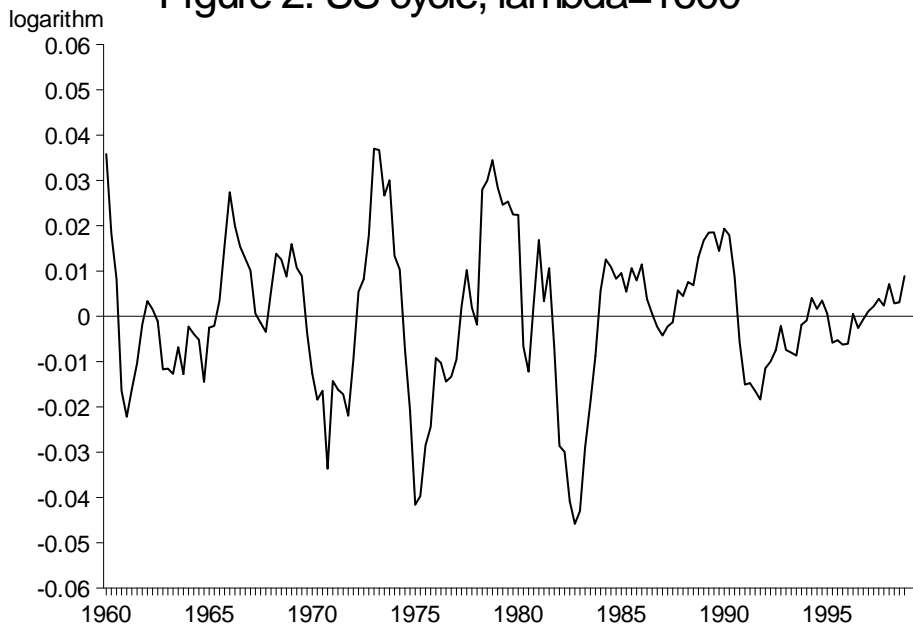
B. The cyclical component

date

Figure 2. US cycle, lambda=1600

Figure 3. Spanish Cyclical Components

Figure 4. Annual Data

logarithm

Cyclical components

Spain (Rule 1)

Spain (L=6.25)

Spain (Rule 2)

US(L=6.25)

date

Figure 5. Real GDP for Japan

logarithm

Cyclical components

hp2(l=8973)

1600

hp1(l=4504)

date

Figure 6. Spanish Data

Band-pass filters and HP-filters

Legend

— HP(1600)

— HP(5385)

— BP(2,32)

— BP(6,32)

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