ALLOCATING IDEAS: HORIZONTAL COMPETITION IN TOURNAMENTS.[*]

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January 2002

Abstract

We develop a stylized model of horizontal and vertical competition in tournaments with two competing firms. The sponsor cares about the quality of the design but also about the design location. A priori not even the sponsor knows his preferred design location, which is only discovered once he has seen the actual proposals. We show that the more efficient firm is more likely to be conservative when choosing the design location. Also, to get some differentiation in design locations, the cost difference between contestants can neither be too small nor too big. Therefore, if the sponsor mainly cares about the design location, participation in the tournaments by the two lowest cost contestants cannot be optimal for the sponsor.

Keywords: Horizontal and Vertical Competition, Tournaments.

JEL classification numbers: D44, D72, J31, L13.

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[*] We thank Antonio Cabrales and Arturo Kohatsu for helpful discussions. Ganuza gratefully acknowledges financial support from the Spanish Ministry of Science and Technology under DGES grant BEC 2000-1026. Hauk gratefully acknowledges financial support from the Spanish Ministry of Science and Technology under DGES grant BEC 2000-1029 and a Ramon y Cajal grant.

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1 Introduction

Tournaments are games in which players spend resources in order to win a prize. Tournaments are extensively used as allocation mechanisms, since they are easy to implement; only the relative performance of the participants has to be evaluated, which is usually less demanding than measuring the absolute performance. For example, tournaments are used in sports competitions and in procurement processes.[1] Promotion in labor markets, R&D races and lobbying are often disguised tournaments. So-called contests of ideas are also tournaments that are used to promote the generation of new ideas in particular in architecture, mechanical engineering, civil, transport and plant engineering, in animation and freeform/artistic expression.

A historical example of a contest of ideas is the tournament used by the Florentine Republic to choose the design of the second doors of the baptistery of the Duomo. This competition was announced in 1401 and artists were required to design a panel representing the sacri¯ce of Isaac. Some of the ¯nest sculptors in Florence took part in the contest. They were seven in total, among them the two ¯nalists Filippo Brunelleschi and Lorenzo Ghiberti. The panels of the two ¯nalists were considered equally good. While the interpretation of Ghiberti was still partly Gothic in style and hence fairly conservative for his time, Brunelleschi presented a more modern neoclassical design. Ghiberti's design won, not on the basis of quality but simply because it was easier to understand.[2] Ghiberti's victory was a pure matter of taste.[3] The two designs were horizontally di®erentiated, and it was this horizontal competition that was decisive in choosing the winner.

While working on their respective designs, the artists had to take two decisions, namely, (i) how to represent the sacri¯ce of Isaac and (ii) how much e®ort to put into this representation. Compared to the decision which interpretation to adopt the e®ort decision is costly and determines the ¯nal quality of the design. We therefore refer to the ¯rst decision as horizontal competition

[1] Fullerton and McAfee (1999) provide several examples of procurement process that have been carried out using tournaments. See Fullerton and McAfee (1999) for details.

[2] see http//www.mega.it/eng/egui/monu/bo.htm

[3] The two panels they presented for the competition are now exhibited beside each other in the Museum of the Bargello.

and to the competition in e®ort levels (quality) as vertical competition.

While the economic literature on tournaments is vast and has studied many relevant aspects like one prize versus multiple prizes,[4] complete information and incomplete information scenarios,[5] to our knowledge it has neglected the fact that in many tournaments participants do not only compete in e®ort and consequently in design quality but also in design location, i.e. there is some degree of horizontal competition. While this is especially relevant for contests of ideas, it might also a®ect other types of tournaments, e.g. labor market competitions. Lower level managers might compete for promotion by suggesting di®erent competitive strategies for the ¯rm. Which kind of strategy is chosen often depends on the preferences of the general manager.

The study of horizontal competition in tournaments is also important because contests of ideas are normally used to generate ideas, i.e. in situations where the sponsor does not really know what he wants. If the general managers had a clear idea about the best strategy for the ¯rm, he would simply order his subordinate managers to implement this strategy. However, he often needs them for generating new business strategies. Similarly, in architectural competitions the sponsor usually knows the type of building he wants, e.g. a museum, a concert hall or a bridge; he also knows that he wants good quality; but a priori he has no clear idea which type of design he would like most, simply because he cannot even imagine all possible types of design. He needs the actual design proposals to learn his ex post preferences.

The aim of this paper is to study a tournament in which the contest success function depends both on the e®ort exerted by the participants (vertical competition) and on the type of design chosen by the participants (horizontal competition). We present a very stylized model in which contest participants face some uncertainty on the sponsor's preferences. We do not attempt to build a general model of vertical and horizontal competition in tournaments, but use the simplest possible model with only two possible design locations and two competing ¯rms to show that some

[4] Most papers study tournaments with a single prize (e.g. Tullock (1980), Wright (1983), Dixit (1987),Baye et al (1993) and (1996), Amann and Leiniger (1996), Fullerton and McAfee (1999), Lizzeri and Perisco (2000). Glazer and Hassin (1988) and Moldovanu and Sela (2001) study a contest with multiple prizes.

[5] Compete information scenaries are among others: Tullock (1980), Dixit (1987), Glazer and Hassin (1988) and Baye et al (1993) and (1996). Amann and Leiniger (1996) and Lizzeri and Perisco (2000) study all-pay auctions with incompete information.

degree of horizontal competition in a classical model of tournaments can lead to qualitatively different results. In particular, it can change known results about optimal entry.

A well established result in tournaments is that limiting entry can be an optimal strategy for two reasons: on the one hand, limiting entry can rise the effort level of contestants, since it increases their probability of winning. On the other hand, it reduces costs; fewer offers have to be evaluated. Nalebuff and Stiglizt (1983) show that the overall effort in a labor contract can be decreasing in the number of workers participating in the contest. Taylor (1995) proves a similar result for research tournaments with homogenous contestants. Fullerton and McAfee (1999) study a research tournaments with heterogenous contestants and conclude that the optimal number of contestants is two, and that these two contestants have to be the lowest cost contestants. In this paper we show that Fullerton and McAfee's (1999) result may not be robust to the introduction of horizontal competition. In our model, in which the number of contestants is restricted to two, it might be beneficial for the sponsor if the contestants are differentiated in costs. If the two participants have similar costs (for example, if they were the lowest cost firms in the industry), they choose similar designs, too. But if the sponsor mainly cares about the type of design, it is worthy for him to get the lowest cost firm and a firm with high enough costs to compete in the contest. In this way, the less efficient contestant is willing to choose a different design location than the design chosen by the more efficient firm, and the sponsor has a higher probability to get a good match between one of the actual design proposals with his preferred design location.

While limiting the number of possible designs to two is obviously a restriction of our model, we are confident that a more general model would also modify standard results about optimal entry in tournaments. There is a very intuitive reason for this, which the present model does not capture. Increasing the number of participants, increases the number of proposals and diversity can have some value in a horizontal competition framework, especially in situations where design proposals help the sponsor to learn his ex post preferences.

A second result of the paper is that the more efficient contestant is more likely to choose the design with the higher ex ante probability for being the sponsor's preferred design. We call this design the conservative design and label the other design as radical. This result nicely ¯ts with the contest held by the Florentine Republic for the second doors of the baptistery. The result is also very much in line with those derived by Prendergast and Stole (1996) and Cabral (1999) in very di®erent (and dynamic) models. Prendergast and Stole (1996) show that youngsters who have no reputation to defend are more impetuous than old-timers who have to worry about the information that their new decision reveals concerning their past decisions. Cabral (1999) studies the important issue in which situation to choose an R&D project with a high variance versus a project with a low variance. He shows that the laggard has nothing to loose, i.e. the follower chooses a riskier project than the leader. However, unlike in Cabral (1999) in our model it is not always the ine±cient ¯rm that is impetuous/radical. If ¯rms are not too di®erent and the sponsor is not very likely to be conservative, multiple equilibria exist: it might be the e±cient ¯rm that chooses the radical design while the ine±cient ¯rm is more conservative.

The remainder of the paper is organized as follows. In Section 2 the model is introduced while Section 3 solves the second stage of the model: the e®ort decision. In Section 4 we solve the ¯rst stage of the model and present the main results of the paper. Section 5 discusses the scope and implications of the model and presents conclusions. All proofs are relegated to a technical appendix.

2 The model

Consider a sponsor (administration) who wants to undertake a public project but does not have a clear idea about the design of the project. To learn about possible designs, the sponsor organizes a contest of ideas. Two risk neutral ¯rms, ¯rm 1 and ¯rm 2, compete in the contest. The rules of the contest are simple: ¯rst the sponsor announces the prize P for the winner of the contest. Then participants submit design proposals and ¯nally the sponsor selects a design and thereby the winner of the contest.

We assume that the design competition has two dimensions: location of the design d and e®ort in developing the design e. We restrict the space of design locations to only two possible design locations: conservative (C), and radical (R). Location captures the type of design. A conservative design is a design that is most likely to be the preferred design of the sponsor, since for example it is close to one that won in a previous contest or since it is the current fashion; by radical designs we mean "vanguard" designs that are less likely close to the sponsor's preferences. The effort is a variable related to the quality of the design. The bigger the effort of the firm, the higher the expected quality of the design, where quality is an index whether or not a given design is well done: some conservative (radical) designs might be better than others.

Each firm has to choose first the design location d_i and then the amount of effort e_i it puts into developing the chosen type of design. The quality of the chosen design is linked to the firm's effort, but the relation between effort and actual produced quality is not deterministic. The bigger the effort of the firm is, the higher is the expected but not necessarily the actual quality of the design produced by this firm. There is no cost associated to choosing the design location. The cost of effort for firm i is $c_i e_i$. Without loss of generality we assume that firm 1 is more efficient than firm 2, i.e., $c_2 \geq c_1$.

The sponsor cares about the type of design and its quality. On the one hand, he wants to maximize the quality of the design location. On the other hand he wants to minimize the distance between the project design and his preferred design location. A priori the sponsor and the firms face some uncertainty about this preferred design location. With probability $\alpha > 0.5$ the sponsor prefers the conservative design while with probability $1 - \alpha$ he prefers the radical design. Once the sponsor sees the actual design proposals this uncertainty is resolved and the sponsor learns his preferred design location.

We do not state the exact form of the preferences of the sponsor but use the following contest success function instead which can be seen as a reduced form of the sponsor's maximization problem. The contest success function tells us the probability that firm i wins the contest given that it had submitted a design (d_i, e_i).

$$p_i(d_i, e_i, d_j, e_j, d_p, \lambda) = \begin{cases} \frac{e_i}{e_i + e_j} & \text{if } d_i = d_j \\ (1 - \lambda)\frac{e_i}{e_i + e_j} + \lambda h_i(d_i, d_j, d_p) & \text{if } d_i \neq d_j \end{cases}$$

where h_i is the comparative advantage of firm i due to horizontal competition

$$h_i(d_i; d_j; d_p) = \begin{cases} 1 & \text{if } d_i \neq d_j \text{ and } d_i = d_p \\ 0 & \text{Otherwise} \end{cases}$$

and $\lambda \in [0;1]$ is a measure of the transportation cost. We can interpret λ as the relative weight given to the design location with respect to quality in the sponsor's preferences. This contest success function captures both aims of the sponsor (to maximize quality and to get as close as possible to his preferred design location), his initial uncertainty about his preferred design location and the stochastic production of quality by firms.

Notice that for $\lambda = 0$ this contest success function coincides with the standard contest success function introduced by Tullock (1980). Fullerton and McAfee (1999) have shown that this contest success function can be derived from the following model of quality production: the choice of e_i determines the number of identical and independent draws[6] from some distribution function over the interval [0; 1]. The resulting quality of these draws is the maximum of these random draws.

The timing of the model is the following:

1. Nature choose the distribution of the preferences of the sponsor defined by α and the marginal cost c_i of effort for each firm i.
2. The sponsor announces the contest and the prize P for the winning firm.
3. The competing firms choose simultaneously the design location d_i.
4. The firms choose simultaneously the effort level e_i to develop the chosen type of design.
5. The sponsor's preferred design is determined by nature.
6. The winning firm is determined by nature according to the design proposals and the contest success function.

The game is solved by backward induction. All the proofs are relegated to the appendix.

[6] For convenience e_i is not restricted to be an integer.

3 The effort decision

When choosing how much effort to put into developing their design, firms already know the design locations which were chosen. The different situations that the firms can face, can be summarized by the following two main cases:

1. If both firms chose the same design, namely (C, C) or (R, R), the effort in developing the design will be decisive, since no firm has a locational advantage with respect to the other and the contest success function only depends on effort levels. Each firm's problem becomes

$$\begin{aligned} & \max_{e_i}[E_{d_p}\{p_i(d_i, e_i, d_j, e_j, d_p, \lambda)P - c_i e_i | e_i\}] \\ = \; & \max_{e_i}[\frac{e_i}{e_i + e_j}P - c_i e_i | e_i] \end{aligned}$$

It is now easy to show, that the solution to this Nash game is characterized by the following first order conditions:

$$\begin{aligned} \frac{Pe_2}{(e_1+e_2)^2} - c_1 &= 0 \\ \frac{Pe_1}{(e_1+e_2)^2} - c_2 &= 0. \end{aligned}$$

Therefore firms' optimal effort levels are:

$$\begin{aligned} e_1 &= \frac{Pc_2}{(c_1+c_2)^2} \\ e_2 &= \frac{Pc_1}{(c_1+c_2)^2}. \end{aligned}$$

In this case the firm with lower costs makes the higher effort and has a bigger chance to be the winner of the contest. The expected profits of the firms are:

$$\begin{aligned} \pi_1(C, C) &= \pi_1(R, R) = \frac{Pc_2}{(c_1+c_2)} - \frac{Pc_2c_1}{(c_1+c_2)^2} = \frac{Pc_2^2}{(c_1+c_2)^2} = P\delta^2 \\ \pi_2(C, C) &= \pi_2(R, R) = \frac{Pc_1}{(c_1+c_2)} - \frac{Pc_1c_2}{(c_1+c_2)^2} = \frac{Pc_1^2}{(c_1+c_2)^2} = P(1-\delta)^2 \end{aligned}$$

where $\delta = \frac{c_2}{c_1+c_2}$ represents the probability that firm 1 wins the contest if no firm has a comparative advantage in design location. Notice that $\delta \in [0.5, 1]$ since $c_2 \geq c_1$. Therefore, the profits of the more efficient firm are higher.

2. If one firm chooses the conservative location and the other firm a radical design, i.e. $(C;R)$ or $(R;C)$, effort levels will depend on the relative importance of design location and quality as captured by our parameter λ. Assuming that firm i chooses the conservative location C its problem becomes:

$$\begin{aligned}
&\max_{e_i}[E_{d_p}\{p_i(d_i;e_i;d_j;e_j;d_p;\lambda)P - c_i e_i | e_i\}] \\
= \; &\max_{e_i}\left[\left((1-\lambda)\frac{e_i}{e_i+e_j} + \lambda\alpha\right) P - c_i e_i | e_i\right]
\end{aligned}$$

while firm j's problem becomes:

$$\max_{e_j}\left[\left((1-\lambda)\frac{e_j}{e_i+e_j} + \lambda(1-\alpha)\right) P - c_i e_j | e_j\right]$$

Using the same arguments as in 1.) the optimal efforts for firm i and j are:

$$\begin{aligned}
e_i &= \frac{(1-\lambda)c_j P}{(c_1+c_2)^2} \\
e_j &= \frac{(1-\lambda)c_i P}{(c_1+c_2)^2}
\end{aligned}$$

Notice that the effort level is lower than under 1.) and it is decreasing in λ. The more weight the sponsor puts on design location, the lower is the competition in effort levels. The expected profits of the firms, when firm 1 chooses the conservative design, and firm 2 chooses the radical location, are:

$$\begin{aligned}
\pi_1(C;R) &= \frac{(1-\lambda)c_2^2 P}{(c_1+c_2)^2} + \lambda\alpha P = (1-\lambda)\delta^2 P + \lambda\alpha P \\
\pi_2(R;C) &= \frac{(1-\lambda)c_1^2 P}{(c_1+c_2)^2} + \lambda(1-\alpha)P = (1-\lambda)(1-\delta)^2 P + \lambda(1-\alpha)P
\end{aligned}$$

Finally, the expected profits of the firms, when firm 1 chooses the conservative design, and firm 2 chooses the radical location, are:

$$\begin{aligned}
\pi_1(R;C) &= \frac{(1-\lambda)c_2^2 P}{(c_1+c_2)^2} + \lambda(1-\alpha)P = (1-\lambda)\delta^2 P + \lambda(1-\alpha)P \\
\pi_2(C;R) &= \frac{(1-\lambda)c_1^2 P}{(c_1+c_2)^2} + \lambda\alpha P = (1-\lambda)(1-\delta)^2 P + \lambda\alpha P
\end{aligned}$$

4 The location decision

Now, we can define the payoff matrix of the first stage game as follows taking the second stage effort levels into account.

		F2 C	F2 R
F1	C	$\pi_1(C;C) = \delta^2 P$ $\pi_2(C;C) = (1-\delta)^2 P$	$\pi_1(C;R) = (1-\lambda)\delta^2 P + \lambda\alpha P$ $\pi_2(R;C) = (1-\lambda)(1-\delta)^2 P + \lambda(1-\alpha)P$
	R	$\pi_1(C;R) = (1-\lambda)\delta^2 P + \lambda(1-\alpha)P$ $\pi_2(R;C) = (1-\lambda)(1-\delta)^2 P + \lambda\alpha P$	$\pi_1(R;R) = \delta^2 P$ $\pi_2(R;R) = (1-\delta)^2 P$

Player 1 is the row player and player 2 is the column player. Player 1's payoffs are represented in the upper corner of each cell.

4.1 The equilibrium outcome

Proposition 1 characterizes the equilibrium outcome of the first stage

Proposition 1 The equilibrium in the first stage is:

1. If $(1-\delta)^2 > 1-\alpha$ firms locate at $(C;C)$.

2. If $(1-\delta)^2 < 1-\alpha$ and $\delta^2 > \alpha$ both firms use a mixed strategy. Firm 1 locates at C with probability β^* and firm 2 locates at C with probability σ^*, where β^* and σ^* are

$$\beta^* = \frac{\alpha - (1-\delta)^2}{1-2(1-\delta)^2}$$
$$\sigma^* = \frac{\delta^2 - \alpha}{2\delta^2 - 1}$$

3. If $(1-\delta)^2 < 1-\alpha$ and $\alpha > \delta^2 > 1-\alpha$, firms locate at $(C;R)$.

4. If $(1-\delta)^2 < 1-\alpha$ and $\delta^2 < 1-\alpha$, there is multiplicity of equilibria. The two pure equilibria are 1) firm 1 locates at C while firm 2 chooses location R, 2) firm 1 locates at R while firm

2 chooses location C. In the mixed strategy equilibrium firm 1 locates at C with probability β^{**} and firm 2 locates at C with probability σ^{**}, where β^{**} and σ^{**} are

$$\beta^{**} = \frac{\alpha - (1-\delta)^2}{1 - 2(1-\delta)^2}$$
$$\sigma^{**} = \frac{\alpha - \delta^2}{1 - 2\delta^2}$$

The equilibrium outcomes for all possible values of α and δ are illustrated in Figure 1.

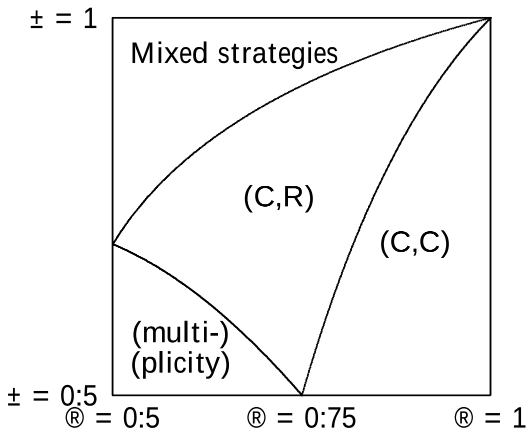

Figure 1 shows how the equilibrium outcomes vary with α and the cost differences (comparative (dis)advantage) of the two firms. First, notice that the solution of the location stage does not depend on the parameter λ; although effort decisions and profits do.

The solution of the game is quite intuitive. Consider α larger than 0.75. In this case the conservative design has a big advantage over the radical design. Therefore, if the comparative cost advantage of firm 1 is not very large, none of the firms wants to give up the privilege of being located at the conservative design: the equilibrium is $(C; C)$; firms only compete in effort levels. For intermediate levels of comparative advantage, the inefficient firm will give up the conservative design and locates at the radical design since the competition in effort levels is too costly for this firm. In this case there is maximal differentiation of design locations and the total effort exerted by firms is reduced. Finally, if the comparative advantage is large, the efficient firm has a very high probability of winning the competition in effort levels. This firm therefore tries to force this competition by choosing the same design as the inefficient firm while the latter tries to avoid the competition in e®ort levels by choosing a di®erent location than the e±cient ¯rm.[7] The equilibrium in this case is in mixed strategies.

If ® is lower than 0.75, the analysis of the equilibria is the same except for small values of ±. If the di®erence in competitive advantage is small the equilibrium changes from $(C; C)$ to multiplicity of equilibria. It is worthwhile for both ¯rms to avoid the costly competition in e®ort since the sponsor's preferences are more likely to be radical.

The next corollaries provide additional characterizations of this equilibrium.

Corollary 1 (i) Maximal di®erentiation in design locations is obtained for intermediate levels for comparative advantage. (ii) The e®ort level is non-monotonic in the comparative advantage of ¯rm 1.

Corollary 1 has important implications for a more general model with optimal entry into the tournament. It shows why well-established results of optimal entry into tournaments might be modi¯ed by the introduction of horizontal competition, in particular the results that the sponsor wants to induce the participation of the two lowest cost contestants and that any technological improvement which implies some cost reduction of the participating ¯rms always increases the overall e®ort exerted by ¯rms. Part (i) of Corollary 1 tells us that whether or not entry by the lowest cost contestants is optimal depends on the relative weight the sponsor gives to the design location. In particular, if ¸ is large, i.e. the sponsor mainly cares about design, this sponsor would like to induce participation in the contest of ¯rms that have not too di®erent and not too equal costs to achieve maximal di®erentiation in design locations which guarantees a better match of one of the design proposals with the sponsor's preferred design. Part (ii) of Corollary 1 states the unintuitive result that a reduction in costs of the most e±cient ¯rm can lead to a reduction in e®ort of both ¯rms and therefore in expected quality. To illustrate this point assume that before the cost reduction both ¯rms choose the central design. The resulting sum of e®ort levels is $e_1 + e_2 = \frac{P}{c_1 + c_2}$. A cost reduction of ¯rm 1 to c_1^0 might imply that it is bene¯cial for ¯rm 2 to

[7] This result is similar to Cabral (1999), where the laggard wants to di®erentiate from the leader whereas the leader wants to follow the follower.

move to the radical design location resulting in an overall effort level of $\frac{(1-\lambda)P}{c_1'+c_2}$. It is easy to see that there exist some parameter values of c_1' and λ such that the overall effort level is lower after the cost reduction of firm 1.

Corollary 2 For all parameter values with a unique equilibrium the firm with the cost advantage makes higher (expected) profits. The profits of the less efficient firm are increasing in λ.

The first part of the result is intuitive and does not need further explanations. When λ increases the competition in effort levels decreases since the horizontal competition becomes more important. This reduction in competition in effort levels increases the profits of both firms: less effort is exerted which reduces firms' cost, but also the comparative advantage of the most efficient firm is reduced. As both effects reinforce each other in case of the less efficient firm we can conclude that its profits are increasing in λ:

Corollary 3 For all parameter values with a unique equilibrium the disadvantaged firm is more likely to choose a radical design.

The intuition of this result is straightforward: given its comparative disadvantage when competing in effort levels, the inefficient firm has a bigger interest than the efficient firm in achieving an equilibrium with differentiated design locations. Since the radical design is the less attractive design, choosing the radical design is a way to get the differentiated equilibrium.

5 Conclusions

In this paper we have developed a very stylized model of horizontal and vertical competition in tournaments. We show that introducing horizontal competition in a standard research tournament can change some well known results. In particular, entry into the tournament by the lowest cost contestants may not be optimal for a sponsor who mainly cares about design, since some cost difference between firms can induce them to choose different design locations.

There are obviously many elements that should be included in a more full-fleshed model of contests of ideas. In particular, the set of design locations should be richer, e.g. it could be an entire Hotelling line. The analysis with the entire Hotelling line could be done using a similar technique than Aragones and Palfrey (2001) who study the policy choice in a Downsian model of two-candidate elections with one advantaged candidate and in which the location of the ideal point of the median voter is uncertain. The policy space is a grid, hence policy locations are discrete.[8] Similarly to our result, they show that the advantaged candidate adopts more moderate policies than the disadvantaged candidate. However, the decision of their agents is one-dimensional, while our agents' decision is two-dimensional: the design location and the e®ort put into developing the design. Therefore, it is unclear in how far their technique, namely to approximate continuous locations by letting the number of discrete designs in the grid go to in¯nite, could be applied to the present context.

In the industrial organization literature there are few papers using both horizontal and vertical competition and existing models - like the present article - tend to be very simple and fairly restrictive. In Motta and Polo (1997) TV channels are horizontally di®erentiated, but this is taken as an exogenous variable and it is analyzed how this variety a®ects the quality decisions of ¯rms. Irmen and Thisse (1998) extend Hotelling's analysis to a n-dimensional characteristic space and show that ¯rms will only compete severely among one dimension (maximum di®erentiation) and locate at the same point in all other dimensions. While working with n-dimensions, Irmen and Thisse (1998) only consider horizontal characteristics.[9] In contrast, our model considers very di®erent types of competition, one which is costly (e®ort), one which is not (design location) and is furthermore complicated by the existence of uncertainty.

We motivated this uncertainty by arguing that a contest of ideas is often used to create ideas and that the sponsor does not really know what he wants before seeing the proposals of the participants in the contest. We used a simple static approach to model this uncertainty. A priori only the distribution of the preferred design is known to both the sponsor and the ¯rms. Once

[8] The present model with two design locations could be translated into a model with a central design and two extreme designs that are equally likely to be the principal's preferred design. In a previous version of the present paper we analyzed this alternative model. We decided to present the current version of the model since the analysis is simpler and the results are qualitatively the same.

[9] Neven and Thisse (1990) ¯nd a min-max con¯guration for a model with a horizontal and a vertical characteristic.

the design proposals have been made the uncertainty is resolved according to this distribution. A more realistic model of learning about preferences would require the possibility of introducing new design locations of which the sponsor was initially unaware. This is left for future research.

Appendix

Proof of Proposition 1: Notice that for firm 2 the best strategy when firm 1 plays radical is to plays C, since $(1-\delta)^2 < \frac{1}{4}$ and $\alpha > \frac{1}{4}$. To solve the first stage game, we have to distinguish four cases.

Case 1: $(1-\delta)^2 > 1-\alpha$. In this case it is a dominant strategy for firm 2 to choose C, since $(1-\delta)^2 > 1-\alpha \Rightarrow (1-\delta)^2 P > (1-\lambda)(1-\delta)^2 P + \lambda(1-\alpha)P$. If firm 2 plays C, the best response of firm 1 is to play C, since $(1-\delta)^2 > 1-\alpha \Rightarrow \delta^2 > 1-\alpha \Rightarrow \delta^2 P > (1-\lambda)\delta^2 P + \lambda(1-\alpha)P$. Hence, the only Nash equilibrium is that both firms choose the central design $(C; C)$.

Case 2: $(1-\delta)^2 < 1-\alpha$ and $\delta^2 > \alpha$. In this case there is no equilibrium is pure strategies, since firm 1 wants to choose firm 2's location and firm 2 want to choose a different location than firm 1. To analyze the equilibrium in mixed strategies we compute the best response function of the firms. Let $\beta(\sigma)$ be the optimal probability with which firm 1 chooses C if firm 2 plays C with probability σ. Straightforward calculations show that

$$\beta(\sigma) = \begin{cases} 1 \text{ if } \sigma > \frac{\delta^2-\alpha}{2\delta^2-1} \\ \in [0;1] \text{ if } \sigma = \frac{\delta^2-\alpha}{2\delta^2-1} \\ 0 \text{ if } \sigma < \frac{\delta^2-\alpha}{2\delta^2-1} \end{cases}$$

Let $\sigma(\beta)$ be the optimal probability with which firm 2 chooses C if firm 1 plays C with probability β. Straightforward calculations show that

$$\sigma = \begin{cases} 0 \text{ if } \beta > \frac{\alpha-(1-\delta)^2}{1-2(1-\delta)^2} \\ \in [0;1] \text{ if } \beta = \frac{\alpha-(1-\delta)^2}{1-2(1-\delta)^2} \\ 1 \text{ if } \beta < \frac{\alpha-(1-\delta)^2}{1-2(1-\delta)^2} \end{cases}$$

Given these reaction functions the equilibrium in mixed strategies is that firm 1 plays C with probability $\beta^* = \frac{\alpha-(1-\delta)^2}{1-2(1-\delta)^2}$ and firm 2 plays C with probability $\sigma^* = \frac{\delta^2-\alpha}{2\delta^2-1}$.

Case 3: $(1-\delta)^2 < 1-\alpha$ and $\alpha > \delta^2 > 1-\alpha$. In this case it is a dominant strategy for firm 1 to choose C, since $\pm^2 > 1$ ¡ ®) $\pm^2$P > (1 ¡ ¸)$\pm^2$P + ¸(1 ¡ ®)P and $\pm^2$ < ®) $\pm^2$P < (1 ¡ ¸)$\pm^2$P + ¸®P. If ¯rm 1 plays C the best response of ¯rm 2 is to play R; since (1 ¡ ±)^2 < 1 ¡ ®) (1 ¡ ±)^{2}P < (1 ¡ ¸)(1 ¡ ±)^{2}P + ¸(1 ¡ ®)P: Hence, the only Nash equilibrium is (C; R).

Case 4: (1 ¡ ±)^2 < 1 ¡ ® and $\pm^2$ < 1 ¡ ®: On the one hand, the best response of ¯rm 1 if ¯rm 2 plays C is to play R; since $\pm^2$ < 1 ¡ ®) $\pm^2$P < (1 ¡ ¸)$\pm^2$P + ¸(1 ¡ ®)P and if ¯rm 2 plays R is to play C; since $\pm^2$ < 1 ¡ ®) $\pm^2$ < ®) $\pm^2$P < (1 ¡ ¸)$\pm^2$P + ¸®P On the other hand, the best response of ¯rm 2 if ¯rm 1 plays C is to play R; since (1 ¡ ±)^2 < 1 ¡ ®) (1 ¡ ±)^{2}P < (1 ¡ ¸)(1 ¡ ±)^{2}P + ¸(1 ¡ ®)P and if ¯rm 1 plays R is to play C; since (1 ¡ ±)^2 < 1 ¡ ®) (1 ¡ ±)^2 < ®) (1 ¡ ±)^{2}P < (1 ¡ ¸)(1 ¡ ±)^{2}P + ¸®P. Therefore, it is easy to see that we have two pure equilibria in this case: (R; C) and (C; R): Similar calculations as in Case 2 allow us to derive the mixed equilibrium. ■

Proof of Corollary 1: Without loss of generality we assume that $c_1 + c_2 = 1$ (this is a normalization). To show that the e®ort level is non-monotonic in the comparative advantage of ¯rm 1 we have to di®erentiated between two cases: Case 1) ® > 0:75 and case 2) ® < 0:75.

1. Let ® > 0:75: If (1 ¡ ±)^2 > 1 ¡ ® the equilibrium is (C; C) and the e®ort exerted by the ¯rms are

   $$e_1 = \frac{c_2 P}{(c_1 + c_2)^2} = \pm P \text{ and } e_2 = \frac{c_1 P}{(c_1 + c_2)^2} = (1 \text{ ¡ } \pm)P:$$

   If (1 ¡ ±)^2 < 1 ¡ ® and ® > $\pm^2$ > 1 ¡ ® the equilibrium is (C; R) and the e®ort exerted by the ¯rms are

   $$e_1 = \frac{(1 \text{ ¡ ¸})c_2 P}{(c_1 + c_2)^2} = (1 \text{ ¡ ¸})\pm P \text{ and } e_2 = \frac{(1 \text{ ¡ ¸})c_1 P}{(c_1 + c_2)^2} = (1 \text{ ¡ ¸})(1 \text{ ¡ } \pm)P$$

   Therefore, for a given ®, when the cost advantage of ¯rm 1 increases, the equilibrium changes from (C; C) to (C; R) and the ¯rm's e®ort decreases. But if ± increases more such that $\pm^2$ > ®; we know from proposition 1 that the location stage has a mixed strategy equilibrium and the expected e®ort exerted by the ¯rms are

   $$E\,\mathrm{f}e_1\mathrm{g} = (1 \text{ ¡ °})\pm P + \text{°}(1 \text{ ¡ ¸})\pm P \text{ and } E\,\mathrm{f}e_2\mathrm{g} = (1 \text{ ¡ °})(1 \text{ ¡ } \pm)P + \text{°}(1 \text{ ¡ } \pm)(1 \text{ ¡ ¸})(1 \text{ ¡ } \pm)P$$

where $\gamma = \beta^* + \sigma^* - 2\beta^*\sigma^*$. These effort levels are higher than those corresponding to the equilibrium (C,R).

2. Let $\alpha < 0.75$. We can use the same argument to show that the effort level can increase when δ increases and δ^2 becomes greater than α. ■

Proof of Corollary 2: If both firms choose the same design (Case 1 $(1-\delta)^2 > 1-\alpha$) $\pi_1 = \delta^2 P > \pi_2 = (1-\delta)^2 P$, since $\delta = \frac{c_2}{c_2+c_1} > \frac{1}{2}$. If firm 1 locates at C, while firm 2 chooses a radical design (Case 3: $(1-\delta)^2 < 1-\alpha$ and $\alpha > \delta^2 > 1-\alpha$), $\pi_1 = \delta^2 P > (1-\alpha)P > (1-\lambda)(1-\delta)^2 P + \lambda(1-\alpha)P = \pi_2$. Finally, if both firms mix over design locations (Case 2: $(1-\delta)^2 < 1-\alpha$ and $\delta^2 > \alpha$.), observe that $\pi_1 = \sigma^*\delta^2 P + (1-\sigma^*)((1-\lambda)\delta^2 P + \lambda\alpha P) > \alpha$ since $\delta^2 > \alpha$, and $\pi_2 = (1-\beta^*)(1-\delta)^2 P + \beta^*((1-\lambda)(1-\delta)^2 P + \lambda(1-\alpha)P) < (1-\alpha)$ since $(1-\delta)^2 < 1-\alpha$. Therefore, $\pi_1 > \alpha > 1-\alpha > \pi_2$. This conclude the proof. ■

Proof of Corollary 3: From proposition 1 it can be seen that for some parameter values the disadvantaged firm chooses the radical design while the advantaged firm chooses the conservative design. We only have to show that when both firms choose a completely mixed design location $\beta^* > \sigma^*$ always.

$$\beta^* - \sigma^* = \frac{(2\delta^2-1)(\alpha-(1-\delta)^2) - (1-2(1-\delta)^2)(\delta^2-\alpha)}{(1-2(1-\delta)^2)(2\delta^2-1)}$$

Straightforward calculations show that

$$\beta^* - \sigma^* = \frac{(2\alpha-1)(\delta^2-(1-\delta)^2)}{(1-2(1-\delta)^2)(2\delta^2-1)} > 0$$

■

References

[1] Amann, E. and Leiniger, W. (1996). “Asymmetric all-pay Auctions with Incompete Information: The Two-Player Case", Games and Economic Behavior 14: 1-18.

[2] Aragones, E. and T.R.Palfrey (2001). \Mixed Equilibrium in a Downsian Model with a Favored Candidate", forthcoming Journal of Economic Theory

[3] Baye, M., Kovenpck, D., and de Vries, C. (1993). \Rigging the Lobbying Process", American Economic Review, 83, 289-294.

[4] Baye, M., Kovenpck, D., and de Vries, C. (1996). \The All-Pay Auction with Complete Information", Economic Theory, 8: 291-305.

[5] Cabral, L.M.B. (1999). \Football, Sailing, and R&D: Dynamic Competition with Strategic Choice of Variance and Covariance", mimeo, London Business School.

[6] Dixit, A. (1987) \Strategic behavior in contest", American Economic Review, 77, 891{898.

[7] Fullerton, R.L. and McAfee, R.P. (1999). \Auctioning Entry into Tournaments", Journal of Political Economy 107(3): 573-605.

[8] Glazer, A. and Hassin, R. (1988). \Optimal Contests", Economic Inquiry 26: 133-143

[9] Hirshleifer, J. (1989) \Con°ict and rent-seeking success function: ratio vs di®erence models of relative success", Public Choice, 63, 101-112.

[10] Irmen, A. and J.-F. Thisse (1998). \Competition in Multi-characteristics Spaces: Hotelling Was Almost Right", Journal of Economic Theory, 78: 76-102.

[11] Lizzeri, A. and Periscon, N. (2000), \Uniqueness and Existence of Equilibrium in Auctions with a Reserve Price",. Games and Economic Behavior 30: 83-114.

[12] Moldovanu. B. and A. Sela (2001), \The Optimal Allocation of Prizes in Contests" American Economic Review, 91(3), 542-58.

[13] Motta, M. and M. Polo (1997), \Beyond the Spectrum Constraint: Concentration and Entry in the Broadcasting Industry", mimeo, Univeristat Pompeu Fabra, Barcelona.

[14] Nalebu®, B. and J. Stiglitz (1983). \Prizes and incentives: towards a general theory of compensation and competition", Bell Journal of Economics, 14: 21-43.

[15] Neven, D. and J.-F. Thisse (1990). On Quality and Variety Competition, in \Economic Decision Making: Games, Econometrics and Optimization. Contributions in the Honour of Jacques H. Drµeze" (J.J. Gabszewics, J.-F. Richard, and L. Wolsey, Eds.), pp.175-199, Noth-Holland, Amsterdam.

[16] Prendergast. C. and L. Stole (1996). \Impetuous Youngsters and Jaded Old-Timers: Acquiring a Reputation for Learning", Journal of Political Economy 104(6): 1105-1134.

[17] Taylor, C. (1995) \Digging for golden Carrots: An analysis of research tournaments", American Economic Review, 85, 872-890.

[18] Tullock, G. (1980) \E±cient rent seeking", in \Towards a theory of rent-seeking society" (Buchanan, J. M, Tollison, and R. D. Tullock, Eds.), pp.175-199, College Station: Texas A&M University Press.

[19] Wright, B. (1983). \The Economics of Invention Incentives: Patents, Prizes and Research Contracts", American Economic Review, 73: 691-707.