Correspondence analysis and categorical conjoint measurement[¤]

Anna Torres-Lacomba[y]

Abstract

We show the equivalence between the use of correspondence analysis (CA) of concadenated tables and the application of a particular version of conjoint analysis called categorical conjoint measurement (CCM). The connection is established using canonical correlation (CC). The second part introduces the interaction effects in all three variants of the analysis and shows how to pass between the results of each analysis.

Key words: Correspondence analysis, conjoint analysis, canonical correlation, categorical data.
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[¤]Requests for reprints should be sent to Anna Torres-Lacomba.

[y]Department of Economics and Business, Universitat Pompeu Fabra, Ramon Trias Fargas, 25-27, 08005 Barcelona (Spain). E-mail: anna.torres@econ.upf.es

1 Introduction

Conjoint analysis is used in marketing and other fields to quantify individuals' trade-offs when they can choose between multidimensional alternatives. Researchers ask the subjects to indicate their preferences for objects under a range of hypothetical situations. They use these judgements to estimate preference functions.

Conceptually, the researcher decomposes a respondent's overall preference judgements for objects defined on two or more attributes into part worths (partial utility values) for distinct attribute levels. With the resulting preference functions managers can predict the share of preference for any product under consideration, relative to other products. In other words, conjoint measurement procedures provide a methodological framework for the development of appropiate "psychophysical" transformations that can be used to ascertain the importance of classes of variables (e.g., color versus type of fragrance of soaps) as well as the scale values for various factor levels, (e.g. floral, lemon, medicinal) (Green and Wind, 1972).

The method has been applied in virtually every conceivable product category, including consumer durables (e.g., automobiles), nondurables (e.g., soft drinks), industrial products (e.g., copiers), financial services (e.g., checking accounts), and other services (e.g., hotel accommodation). Different algorithms are used, according to the three forms of conjoint measurement: categorical, additive or simple polynomial (Wittink, 1999).

The technique of correspondence analysis, also used in marketing research and in many other fields, can be understood as a method for finding the association between the categories of two or more categorical variables and presenting this relationship in an easy visual format. This general definition has had different extensions that have led to the technique being applied in a wide range of situations involving different types of categorical data, for example contingency tables, indicator matrices, preferences, paired comparisons and ratings.

The conjoint analysis process can be divided into four phases: data collection, measurement scale for respondent judgments, parameter estimation methods and market simulation. The present work is concerned with the parameter estimation method, or the third step in the process of a conjoint analysis. For the particular case of full profile collection method and rating scale in the measurement of respondent judgments, the nonmetric method used to analyze the data is known as Categorical Conjoint Measurement (CCM). Our objective is to show the equivalence between the CCM algorithm due to Carroll (1969) and a particular case of correspondence analysis (CA). Once the equivalence is shown, we try to see if the equivalence is maintained when interaction effects are included. Green and Wind (1972) say that one extension of CCM would be to include the interaction effects. The same idea is noted in Green (1973).

We shall start by describing the objective and the results of a CCM analysis followed by a brief introduction of canonical correlation analysis (CC). It is useful as an intermediate stage, since the equivalence between CCM and CC is already shown (Carroll, 1969). The equivalence between CC and CA has been shown for the particular case where there is one attribute being related to preference (see, for example, Greenacre 1984, chap.4). We will see what happens when two or more attributes are being related to preference and finally we will compare the results obtained from the analysis of the data using CCM, CC and CA. Later on, we will introduce the way to code the data so that CA can treat interactions effects. We will repeat the operation with CC as well as with CCM to demonstrate the equivalence empirically.

The first data set is from the paper of Rao (1977). It comes from the situation of an apartment-dweller planning to purchase a house that is already built in a college town. The decision-maker has isolated the attributes of the house considered most important in the decision. The attributes are three: size of the house (3 levels), price of the house (4 levels) and general condition of the house (3 levels) and the response variable has 4 levels. The second data set is from an airline company. The objective in this case is to know the trade-off value between the different attributes offered as well as possible interaction effects between them. The attributes are: airline company (5 levels), price (5 levels), service (3 levels) and timetable (3 levels). The response variable has 4 levels.

2 Methods

2.1 Categorical conjoint measurement (CCM)

We are interested in the analysis of a matrix of dummy variables, called an indicator matrix, of the following form:

Attrib.$_1$					Attrib.$_Q$				Response			
$j=1$	$j=2$	...	$j=m_1$	.....	$j=1$	$j=2$	...	$j=m_Q$	$k=1$	$k=2$	.....	$k=K$
1	0	...	0	......	1	0	...	0	1	0	.....	0
1	0	...	0	......	0	1	...	0	1	0	.....	0
1	0	...	0	......	0	0	...	1	0	0	.....	0
:	:	...	:	.		:	...	.	.	:	.....	:
:	:	...	:	.		:	...	.	.	:	.....	:
:	:	...	:	.		:	...	.	.	:	.....	:
0	0	...	1	......	0	0	...	1	0	0	.....	1

where

m_q: number of levels for the attributes $q = 1, \ldots, Q$.
K : number of response categories where $k = 1, \ldots, K$.

$M \equiv \prod_{q=1}^{Q} m_q$: number of all the possible combinations of the attribute levels.

The above indicator matrix is made up of a matrix Z_1 of dummy variables representing the full profile (i.e., the indicator matrix has M rows) and another matrix Z_2 of dummy variables indicating one subject's preferences for each combination. For the additive case, the objective is to find an optimal additive combination of scale values for the attribute levels $Z_1 a$ maximally correlated with the response categories assigned to each combination, $Z_2 b$.

Here $\underline{b}$ is the vector that collects the optimal scale values for the category k of the response variable, and $\underline{a}$ is the vector with the optimal scores for all $\sum_q m_q$ levels of the attributes.

Carroll (1969) has shown that this analysis is equivalent to canonical correlation analysis of the dummy variables. He also shows the equivalence between applying canonical correlation and the following formulation which is the one we are going to use to show the equivalence between CCM and CA.

First step Define:

$$s_{q;j_q;k} \equiv \sqrt{\frac{m_q}{n_k M}}\left(n_{q;j_q;k} - \frac{n_k}{m_q}\right) \tag{1}$$

where:

$n_{q;j_q;k} \equiv$ number of times k^{th} response category value occurs in j_q^{th} level of attribute q;

$n_k \equiv$ total number of times k^{th} response category value occurs $(k = 1, \ldots, K)$.

Second step Let S_q be the $m_q \times K$ matrix whose general entry is $s_{q;j_q;k}$. Define the $K \times K$ matrix R as:

$$R = \sum_{q=1}^{Q} S_q^T S_q \tag{2}$$

Third step Determine the eigenvectors v_l, for each dimension, of R and then the optimal scores w_l as:

$$w_l = \frac{v_l}{\sqrt{n_k}} \tag{3}$$

Notice that each attribute is initially treated separately, then combined in the matrix R, which is decomposed in order to find scores for the response categories only.

The data set used is from Rao (1977), and has three attributes ($Q = 3$):

1: Size of house (number of bedrooms), where $m_1 = 3$, with levels 2, 3 and 4.

2: Asking price (thousands of dollars), where $m_2 = 4$; with levels 25, 30, 35 and 40.

3: Condition of the house, where $m_3 = 3$; with levels E: Excellent, G: Good and P: Poor.

The number of attribute combinations is $M = 3 \times 4 \times 3 = 36$. The response has $K = 4$ categories: A: very high worth; B: just high worth; C: just low worth; D: very low worth. The data are given in appendix I.

2.2 Correspondence analysis (CA)

In the simple CA of a two-way table, we are interested in explaining the association between the row and column categories, representing the association in a low-dimensional space in the form of a map. The overall association is quantified by the chi-squared statistic divided by n_{++} (the total number of cases), i.e. χ^2/n_{++}; called total inertia:

$$\frac{\chi^2}{n_{++}} = \frac{1}{n_{++}} \sum_{j=1}^{J} \sum_{k=1}^{K} \frac{(n_{jk} - n_{j+}n_{+k}/n_{++})^2}{(n_{j+}n_{+k}/n_{++})} \tag{4}$$

where n_{jk} is the number of cases in a particular cell, n_{j+} the row total, n_{+k} the column total and n_{++} is the grand total.

The decomposition of this objective function is analogous to finding the largest principal component of a set of J observations on K variables with the generalization to accommodate different weights, called masses. If we analyze the row profile matrix, where each row of the table is divided by its total, the row masses $\mathbf{r}$, are the row totals divided by the grand total. Column profiles and the column masses $\mathbf{c}$ are similarly defined. The row profiles have centroid equal to $\mathbf{c}$ and the column profiles have centroid $\mathbf{r}$.

The row and column coordinates of the profiles with respect to their respective principal axes may be obtained from the singular value decomposition (SVD) of the matrix $\mathbf{N} = (n_{jk})$, transformed by double-centring and standaridzing:

$$\mathbf{D}_r^{-\frac{1}{2}}[(1/n_{++})\mathbf{N} - \mathbf{r}\mathbf{c}^T]\mathbf{D}_c^{-\frac{1}{2}} = \mathbf{U}\mathbf{D}_\alpha\mathbf{V}^T; \tag{5}$$

where $\mathbf{U}^T\mathbf{U} = \mathbf{V}^T\mathbf{V} = \mathbf{I}$. The singular values are the square roots of the principal inertias or eigenvalues: $\mathbf{D}_\alpha = \mathbf{D}_\lambda^{\frac{1}{2}}$ and $\mathbf{D}_r$ and $\mathbf{D}_c$ are diagonal matrices with the row and column masses, respectively, in their main diagonal.

The principal axes of the row and column pro…les are the column vectors of $D_r^{\frac{1}{2}}V$ and $D_c^{\frac{1}{2}}U$; respectively. For example, the two-dimensional coordinates of pro…le points (i.e., from the row pro…le matrix, each row becomes a point to be represented in the map) and vertex points (i.e., each column is a vertex point for the row pro…les points) in the dual problems are the rows of the …rst two columns of the following matrices:

Row problem : row pro…les, $D_r^{i\frac{1}{2}}UD_{®}$; column vertices, $D_c^{i\frac{1}{2}}V$: The rows will be the points projected in a map interpreted in terms of the columns, which have contributed the most to the orientation of the principal axes.

Column problem: column pro…les, $D_c^{i\frac{1}{2}}VD_{®}$; row vertices, $D_r^{i\frac{1}{2}}U$: The colums will be the points projected in a map interpreted in terms of the rows, which have contributed the most to the orientation of the principal axes.

2.3 Canonical correlation (CC).

The geometry of canonical correlation is given by Greenacre, (1984, section 4.4) and also its relationship to the geometry of the correspondence analysis of an indicator matrix, for the classical case where two categorical variables are treated. As we noted before, since categorical conjoint analysis can be applied to more than two attributes, the equivalence between this technique and the correspondence analysis of an indicator matrix, via canonical correlation, is not obvious. We will describe the basic geometry of CC and we will introduce the new de…nitions and operations that will let us establish the connection.

The objective of CC is to …nd strong linear relationships between two sets of variables (two-way table in CA) as observed across the sample of n_{++} cases. If Z_1 and Z_2 are the data matrices corresponding to the two sets of variables, this objective can be expressed formally as …nding linear combinations Z_1a and Z_2b, which have maximum correlation ½ :

$$½ = (a^T S_{12} b) = ((a^T S_{11} a)(b^T S_{22} b))^{\frac{1}{2}} \quad (6)$$

where $S_{12;}S_{11}$ and S_{22} are the covariance matrix between Z_1 and Z_2 and the covariance matrices of Z_1 and Z_2 respectively.

The vectors a_k and b_k of canonical weights can be obtained from the left and right singular vectors of the matrix $S_{11}^{i\frac{1}{2}}S_{12}S_{22}^{i\frac{1}{2}}$ (See for example, Greenacre 1984): The SVD of the matrix is:

$$S_{11}^{i\frac{1}{2}}S_{12}S_{22}^{i\frac{1}{2}} = UD_{½}V^T \quad \text{with} \quad U^TU = V^TV = I: \quad (7)$$

where $D_{½}$ is a diagonal matrix with the canonical correlations in the diagonal, U and V are the left and right singular vectors.

The matrices of canonical weights are:

$$A = S_{11}^{-\frac{1}{2}} U \qquad \text{and} \qquad B = S_{22}^{-\frac{1}{2}} V \tag{8}$$

The standarization of the singular vectors of U and V to be orthonormal as in (8) implies that A and B are standarized as follows:

$$A^T S_{11} A = B^T S_{22} B = I \tag{9}$$

The usual standarization of the vectors of canonical scores is that all of them have unit variance. At the same time it is also a set of identification conditions on the scale of the canonical weights and of the canonical scores. In order to identify the origins of the vectors of canonical scores, their means are conventionally set at zero, which is equivalent to each variable of Z_1 and Z_2 being centered with respect to its mean.

In this particular application, the first set of variables consists of 10 dummy variables for the attribute levels and the second set consists of 4 dummy variables for the response categories. The rows are the 36 possible combinations of attribute levels. Thus Z_1 and Z_2 have the following form:

$$Z_1 = \begin{bmatrix} s2 & s3 & s4 & p1 & p2 & p3 & p4 & CE & CG & CP \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \end{bmatrix}$$

$$Z_2 = \begin{bmatrix} A & B & C & D \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ . & . & . & . \\ . & . & . & . \end{bmatrix}$$

For example, the first combination of attributes, two bedrooms in the house, a price of $25.000 and excellent condition, gets a response of "very high worth". When an indicator matrix is analyzed, S_{11} and S_{22} are singular matrices, which implies that they cannot be inverted. To be able to do our computations, the last level for each attribute and the last response category will be eliminated. This operation lets us to estimate the canonical weights.

2.4 CA of a concatenated table and its connection with CC.

The operation of eliminating the last level for each attribute and the last category, lets us estimate the canonical weights.

The first important difference with respect to Greenacre (1984, section 4) is the data matrix to be analyzed. In this case, the matrix $Z_1^T Z_2$ is a concatenated table, whereas in previous work, $Q = 1$; that is there is only one categorical variable in the first set and then $Z_1^T Z_2$ is a single crosstabulation: for this simpler case Greenacre (1984) shows how the CA results can be obtained from the canonical weights by imposing the centring conditions of CA, using the masses in the weighted averaging procedure. In the present case where $Q > 1$, we again impose the CA condition to recover the standard coordinates for the categories and the principal coordinates for the levels of the different attributes. Finally we check that the coordinates obtained are identical to the ones obtained in the correspondence analysis of the concatenated table $T \equiv Z_1^T Z_2$.

Canonical correlation analysis gives solutions, for each axis, of the form:

Attributes:

$a_q^* = [a^*_{q,1} \dots a^*_{q,m_q-1}\ 0]^T$ where $q = 1, \dots, Q$

Response:

$b^* = [b^*_1 \dots b^*_{K-1}\ 0]^T$

The correspondence analysis results are obtained from these, as follows (for each dimension):

Attributes:

$q = 1, \dots, Q$

$$a_{q,m_q} = \sum_{j=1}^{m_q-1} r_{q,j_q} a^*_{q,j_q} \qquad a_{q,j_q} = a^*_{q,j_q} + a_{q,m_q} \qquad j = 1, \dots, m_q$$

Response:

$$b_K = \sum_{k=1}^{K-1} c_k b^*_k \qquad b_k = b^*_k + b_K \qquad k = 1, \dots, K \tag{10}$$

2.5 Results

From the CC numerical solution, which appears in appendix I, we recover the values of the standard coordinates of the categories in CA by applying the above transformation.

For example, for the first dimension, we obtain:

$$9(2.9321 + b_K) + 12(2.0246 + b_K) + 9(1.1488 + b_K) + 6b_K = 0$$

$$b_K = -1.6951$$

Then,

$$\begin{aligned} b_1 &= 1.237 \\ b_2 &= 0.3295 \\ b_3 &= -0.5463 \\ b_4 &= -1.6951 \end{aligned}$$

This operation is repeated for the second and the third principal axes, as well as all the process for the attributes.

To corroborate the results, we ran the SimCA program (Greenacre, 1986) to get the CA principal coordinates and then converted these to the following standard coordinates (see appendix I):

	Dim. 1	Dim. 2	Dim. 3
A	1.2545	0.7888	0.8966
B	0.3342	1.3029	-0.4367
C	-0.5540	1.0485	-1.2624
D	-1.7191	-0.1500	1.4220

The values agree with those recovered from CC once the correction factor $\sqrt{\frac{M-1}{M}} = \sqrt{\frac{35}{36}}$ is applied. This is due to the computation of unbiased variances in CC.

3 Equivalence of CCM and CA

So far, we have established the relationship between CA of concatenated tables and canonical correlation analysis where one set of variables is composed by several categorical variables. We now look at the relationship between Carroll's CCM and CA, showing that there are simple scaling factor differences in the eigenvalues (principal inertias in CA) and the response category scores (standard coordinates in CA). We show these relationships by detailing the CCM and CA theory side by side.

3.1 Relation between the eigenvalues

First Step

Categorical Conjoint Measurement

$$s_{q;j_q;k} \simeq \sqrt{\frac{m_q}{n_k M}} \left(n_{q;j_q;k} - \frac{n_k}{m_q}\right)$$

where k is the response category assigned to the level j_q of the factor q:

Correspondence Analysis.

From (5) the centered and standarized matrix can be written as:

$$t_{q;j_q;k} = \frac{\left(\frac{n_{q;j_q;k}}{QM} - \frac{1}{Qm_q} \times \frac{n_k}{M}\right)}{\sqrt{\frac{1}{Qm_q}} \times \sqrt{\frac{n_k}{M}}} =$$

$$\frac{1}{QM} \times \frac{\sqrt{Q}\sqrt{m_q}\sqrt{M}}{\sqrt{n_k}} \times \left(n_{q;j_q;k} - \frac{n_k}{m_q}\right) =$$

$$\frac{\sqrt{m_q}}{\sqrt{Q}\sqrt{M}\sqrt{n_k}} \times \left(n_{q;j_q;k} - \frac{n_k}{m_q}\right)$$

$$t_{q;j_q;k} = \frac{1}{\sqrt{Q}} \times \sqrt{\frac{m_q}{M n_k}} \times \left(n_{q;j_q;k} - \frac{n_k}{m_q}\right)$$

Thus, there is only a scaling factor equal to $\frac{1}{\sqrt{Q}}$; linking the two approaches, where Q is the number of attributes.

Second Step From the previous operations we obtain $S_{q;j_q;k}$ as well as $T_{q;j_q;k}$ which corresponds to the CCM and CA matrices that collect the centered and standarized data for each factor, for each level and for each one of the categories, calculated previously. Then we follow with the operation,

CCM	CA
$R = \sum_{q=1}^{Q} S_q^T S_q$	$T^T T = \frac{1}{Q} \sum_{q=1}^{Q} T_q^T T_q = \frac{1}{Q} R$:

which gives the relationship

$$\frac{1}{Q} \lambda_{CCM} = \lambda_{CA} \qquad (11)$$

where λ_{CCM} and λ_{CA} are the eigenvalues obtained applying CCM and CA respectively. This relation is corroborated with the data set. Thus the principal inertias in CA are equal to the eigenvalues from CCM divided by Q.

Since the CA of a concatenated table is the average of the inertias of the individual tables (Greenacre, 1994), the total variance in CCM is just the sum of the inertias of the Q tables.

3.2 Relationship between the coordinates of CCM and CA

We use the third step in Section 2.1 and the CA theory in Section 2.2. The standard coordinates in CA are defined as

$$Y = D_c^{-\frac{1}{2}} V \qquad (12)$$

where Y are the column standard coordinates, D_c are the diagonal matrix with the column masses in the main diagonal and V are the right singular vectors. Since $c_k = \frac{n_k}{M}$ it follows from (12) that:

$$y_l = \frac{\sqrt{M}}{\sqrt{n_k}} v_l$$

$$\sqrt{n_k} \frac{y_l}{\sqrt{M}} = v_l$$

Hence from (3), $\frac{y_l}{\sqrt{M}}$ are the response category scores in CCM.

Thus the response category scores obtained by CCM are the same as the standard coordinates obtained when we apply CA to the concatenated table but rescaled by the factor $\frac{1}{\sqrt{M}}$.

3.3 Results

For the first dimension, the values are the following:

	CA standard coordinates	CCM estimated coefficients
A	1.2545	−0.2091
B	0.3342	−0.0557
C	−0.5540	0.0923
D	−1.7191	0.2865

where the relation is equal to $\frac{1}{\sqrt{36}}$.

The relation between the eigenvalues is the following:

Eigenvalues	CA	CCM
λ_1	0.4755	0.9501
λ_2	0.2583	0.5166
λ_3	0.0440	0.0880

In this case $Q = 2$ since the variable “size” was not included in the analysis because it has zero inertia. We corroborate that the two sets of eigenvalues differ by a factor of 2.

4 Interaction E¤ects

As noted previously, Green and Wind (1973) have pointed out as future research the possibility to introduce interaction terms explicitly in categorical conjoint measurement. The same idea is re‡ected once again in Green (1973).

We now show how CA is able to handle interaction e¤ects. Furthermore, we show the connection again with the analysis of interactions using CC and CCM.

We illustrate our approach using a study designed by an airline company. At the beginning, the interest of the study was to examine if a particular respondent, with a particular pro…le, had di¤erent perceptions between di¤erent airline companies, di¤erent prices, service levels and timetables. The four attributes are the following:

1. Airline company, where $m_1 = 5$; with levels: TWA, IBERIA, KLM, British Airways and TAP.

2. Price, where $m_2 = 5$; with levels (in pesetas): Levels: P1 = 85.000, P2 = 100.000, P3 = 115.000, P4 = 130.000, and P5 = 145.000.

3. Service, where $m_3 = 3$; with levels: S1 = bellow the mean, S2 = in the mean and S3 = above the mean.

4. Timetable, where $m_4 = 3$; with levels: T1 = 3 hours before of your preferences, T2 = 2 hours before and T3 = 1 hour before.

4.1 Correspondence analysis. Results.

4.1.1 Analysis without interaction e¤ects

In this particular case, we have 5 £ 5 £ 3 £ 3 = 225 combinations. The analysis is applied at the individual-response level, getting the utility model for this person's pro…le. We use the answers from one of the PhD students in the Universitat Pompeu Fabra (Barcelona, Spain) as an illustration of the approach.

The output of the CA appears in appendix II. As shown in previous sections, the data are coded as a concatenated table, obtaining the same results as a conjoint analysis which has been suitably rescaled. The interpretation of the map is the following (see …gure 1 in appendix II):

Prices:

The …rst principal axis contains 70:8% of the total information (total inertia = 0:3123), and the …rst two principal axes contain 96:2%. This big concentration of variance in the …rst principal axis is because of the type of data we are analyzing. The response category points form a curve known as the "horseshoe" or "arch" which is common for data on an ordinal scale. One extreme of the …rst principal axis will indicate high levels of utility and the other extreme, low levels of utility. The second principal axis normally di¤erenciates the attribute levels depending on the distribution in the intermediate levels. Given the meaning in terms of utility of the principal axes, we can …nd that the cheapest prices, P_1 and P_2, very correlated with the …rst principal axis, are situated near the category which represents the higher utility level. On the other hand, while P_3

is highly correlated with the second principal axis and near the high utility B, P_4 and P_5, correlated also with the …rst principal axes, are situated near the lowest level of utility, D.

Companies:

TWA, KLM and B.A. are all associated with high levels of utility. The common characteristics among them are that they are non-national and have a well-known reputation. TAP is the one that is most disliked, maybe because it is not well known in Spain. IBERIA is di¤erenciated with respect to the others. It has an important correlation with the second principal axis because of the mixture of answers in the extreme categories. This is the only national company.

Services:

The extreme levels, S_1 and S_3, are more identi…ed with the …rst principal axis. The level representing "a service above the mean" is rated the best and the one which represents "a service below the mean" the worst rated. The intermediate level, S_2, has a strong correlation with the second principal axis, given a more equalized distribution of the answers between the di¤erent category levels.

Timetable:

With a similar pattern as the service attribute, the extreme levels, T_1 and T_3, have a high correlation with the …rst principal axis and so they are associated with the extreme utility category levels. The di¤erence with respect to the attribute of service is that the attribute level T_2; which re‡ects the mean level, totally explained by the second principal axis, is situated above the …rst principal axis because of a bigger concentration of answers in the extreme categories.

At this point the analysis o¤ers, for a particular subject, the utility of each attribute. But we do not stop here since further interests come with the following idea. When a subject has to evaluate attributes referring to a long ‡ight it may be possible that a combination of two variables generates a utility signi…catively bigger or smaller that the sum of the utilities previously obtained. In applied terms this can be translated as the possibility to sell special o¤ers that could contain the lower level of price with adjustments in other attributes like the service level (lower value) or the timetable (quite far from the preferences) or even changing the airline company. These considerations imply perceptions that are di¤erent from the linear combination of utilities previously estimated. Since "price" is the variable with bigger inertia in the previous analysis, we constructed an interaction variable composed of "price" and another variable. The di¤erent levels of "airline company" have similar utilities, since in terms of signals of quality, all of the companies are quite homogeneous. The exception is TAP, and the reason can be the ignorance. The levels of "service" have small inertia as well as their interactions with the rest of variables. The reason can be a small real di¤erence between the levels of this variable. But the situation is di¤erent with the "timetable" attribute. The total inertia increases more and the new results are adding interesting information (see appendix II).

4.1.2 Introduction of interaction e¤ects

The new variable, "price £ timetable" has 15 levels, which can be labelled as: PT_{11}, PT_{12}, PT_{13}, ... , PT_{51}, PT_{52}, PT_{53}, where the letters indicate the original variables and the subindices indicate the levels of the two attributes.

The data matrix to be analyzed and the results appear in appendix II. Once again we code the data as a concatenated table. It includes three active variables with their levels: the interaction variable (price £ timetable), airline company and service. Further, the original variables, price and timetable, will be added as supplementary points, being the centroids of the interactions. The reason is not to repeat information. The interaction variables include the within (eg., the utlilty of PT_{1j_2} versus the one of $PT_{1j_2'}$. The price label is the same in both variables, but the timetable levels are di¤erent: $j_2 \neq j_2'$) and the between inertia (eg., PT_{1j_2} and $PT_{2j_2'}$ where di¤erent levels for both attributes are taken) while the main variables contribute only with the between inertia.

We are going to compare the new inertia and the one previously obtained, to be able to quantify its increases and hence justify the inclusion of the interaction terms:

Inertia without interactions:	Between = 0:3123
Inertia with interactions:	Between + Within = 0:6188

With the introduction of the interaction e¤ects, we double the total inertia. We can interpret the results from the map that appears in appendix II (see …gure 2). The …rst principal axis still di¤erenciates between the most and the least preferred levels. The original levels (supplementary points) appear as the centroids of the interactions. The interactions show us that for the lowest level of the factor price, the variable timetable does not matter, in other words, if the price is the cheapest one, it does not matter if the timetable of the ‡ight is not adjusted to your preferences. The reason can be the fact that we are treating transatlantic ‡ights where the attribute timetable is less important than for shorter business travel where you might need to depart and return in the same day. On the other hand, if the price is the highest one, you only accept the timetable more adjusted to your preferences. We can see that once prices increase, the timetable attribute takes e¤ect again, in such a way that to mantain the same utility, a timetable more adjusted to ones preferences is required. These trade-o¤ e¤ects are collected on the second principal axis. We can conclude that as price increases, the interaction levels more correlated with the second principal axis re‡ect the timetable level you require to compensate the increase in price and to stay with an intermediate utility, B or C, and not going to the D one.

Finally we calculate the standard coordinates for the categories. Later we will compare them with the ones obtained in CC to check the equivalence.

	Dim.1	Dim.2	Dim.3
A	1.242	0.849	−0.506
B	0.427	−0.990	1.361
C	−0.520	−1.347	−1.553
D	−1.283	0.881	0.308

4.2 Canonical Correlation

When we include interaction effects, the relation between CA and CC is less obvious than in the previous case. We need one step more to understand the previous equivalence. The scheme of the explanation will be the following. First of all, we will describe the way to code the interaction in CC as a single dummy variable, in order to obtain identical results to those of CA. Secondly, we will show the equivalence between the results obtained with this approach and the results obtained with the more customary way of handling categorical variable interactions in linear models.

When we introduce an interaction variable, the type of data matrix to analyze in CA is still a concatenated table, in this case composed of 3 variables, one with 15 levels (price×timetable) and the others with 5 levels (company) and 3 levels (service). It suggests immediately that CC has to be computed as before and that we have to omit one of the PT levels, one of the company levels and one of the service levels. We drop the last level of each one. Finally we recover the CA results just as before.

We point out that the usual way of handling categorical variables plus their interactions in linear models would be to drop one category of each one of the main effects and all categories of the interactions involving these levels (see the example in the appendix II). In this case, the problem is how to be able to recover all the coefficients, especially those of the interaction terms which are not so obvious. The key point to realize is that even though this way of centering in the old way could bring to us to the conclusion that each interaction set is treated as a different variable, it is not like this in the calculations. All the interactions belong to the same variable and so all the omitted coefficients take the same value.

Once we consider the previous explanation, since the results should be the same, we know that the new way collects in its coefficients both main and differential effects, which should be obtained separately in the traditional way.

4.2.1 Operation of centering. New way.

We have 2 attributes composing the new interaction variable, where $j_1 = 1, \dots, m_1$ and $j_2 = 1, \dots, m_2$ are the levels for each attribute. To make the notation slilghtly easier and since we could be studying the interaction of any pair of variables, we set: $j_1 \equiv j$, $j_2 \equiv j'$ and $m_1 \equiv m$, $m_2 = m'$. The total number of levels for the interaction variable is equal to $m \times m'$. The operation of centering to recover the coefficients will be the following,

$$d_{mm'} = \sum\sum_{jj' \neq mm'} r_{jj'} d^*_{jj'}$$

We consider all the coefficients, which correspond to all the possible interaction combinations, except the one composed of the last level of each attribute. The mass $r_{jj'}$, is in this case the total number of cases for a particular interaction level, which is composed of the levels j for the first attribute and j' for the second, and $d^*_{jj'}$ is the coefficient obtained from the CC estimation in the new way, corresponding to level jj' of the interaction variable. Since in this case, $r_{jj'} = \frac{1}{m \times m'}$ for all jj',

$$\frac{1}{m \times m'} \sum\sum_{jj' \neq mm'} (d^*_{jj'} + d_{mm'}) + \frac{1}{m \times m'} d_{mm'} = 0$$

$$\sum\sum_{jj' \neq mm'} (d^*_{jj'} + d_{mm'}) + d_{mm'} = 0$$

$$\sum\sum_{jj' \neq mm'} d^*_{jj'} + (m \times m') d_{mm'} = 0$$

$$d_{mm'} = \frac{\sum\sum_{jj' \neq mm'} d^*_{jj'}}{m \times m'}$$

From here, we recover the coefficients as before:

$$d_{jj'} = d^*_{jj'} + d_{mm'}$$

4.2.2 Operation of centering. Traditional way.

$$\sum_{j=1}^{m-1} \sum_{j'=1}^{m'-1} r_{jj'} (e^*_{jj'} + c) + \frac{1}{m \times m'} (m - 1 + m')\, c = 0$$

where $e^*_{jj'}$ is the coefficient obtained from the CC estimation in the traditional way, corresponding to the level of the interaction variable composed of the levels j for the first attribute and j' for the second attribute and c is the estimated coefficient corresponding to the interaction levels: $(m1), (m2), \ldots, (1m'), (2m'), \ldots$.

Since $r_{jj'} = \frac{1}{m \times m'}$;

$$\begin{aligned}
\frac{1}{m \times m'} \sum_{j=1}^{m-1} \sum_{j'=1}^{m'-1} (e^*_{jj'} + c) + \frac{1}{m \times m'} (m - 1 + m')\, c &= 0 \\
(m-1)(m'-1)c + \sum_{j=1}^{m-1} \sum_{j'=1}^{m'-1} e^*_{jj'} + (m - 1 + m')\, c &= 0 \\
c[(m-1)(m'-1) + (m - 1 + m')] + \sum_{j=1}^{m-1} \sum_{j'=1}^{m'-1} e^*_{jj'} &= 0 \\
((m \times m') - m - m' + 1 + m - 1 + m')c + \sum_{j=1}^{m-1} \sum_{j'=1}^{m'-1} e^*_{jj'} &= 0
\end{aligned}$$

$$(m \times m')\, c + \sum_{j=1}^{m-1} \sum_{j'=1}^{m'-1} e^*_{jj'} = 0$$

$$c = \frac{-\sum_1^{m-1} \sum_1^{m'-1} e^*_{jj'}}{m \times m'}$$

From here, we recover the coefficients (in this case differentials with respect to the main effects) as always,

$$e_{jj'} = e^*_{jj'} + c$$

From the traditional way,

$$a_{1;m} + a_{2;m'} + c = f_{mm'}$$

where $a_{1;m}; a_{2;m'}$ are the coefficients of the main effects for two attributes and their last levels and $f_{mm'}$ is the interaction effect corresponding to the last level for each attribute.

From the centering of the main effects in the traditional way we get

$$\begin{aligned}
a_{1;m} &= -\frac{1}{m} \sum_1^{m-1} a^*_{1;j} \\
a_{2;m'} &= -\frac{1}{m'} \sum_1^{m'-1} a^*_{2;j'}
\end{aligned}$$

where $a^*_{1;j}; a^*_{2;j'}$ are the estimated coefficients of the main effects for two attributes and their levels.

Then

$$f_{mm'} = -\frac{1}{m}\sum_{1}^{m-1} a^{*}_{1;j} - \frac{1}{m'}\sum_{1}^{m'-1} a^{*}_{2;j'} + c$$

Since $d_{mm'} = f_{mm'}$; if we substitute this expression coming from the traditional way to code into the restriction of the new way, we obtain,

$$\sum\sum_{jj' \neq mm'} d^{*}_{jj'} + (m \times m')\left(-\frac{1}{m}\sum_{1}^{m-1} a^{*}_{1;j} - \frac{1}{m}\sum_{1}^{m'-1} a^{*}_{2;j'} + c\right) = 0$$

$$\sum\sum_{jj' \neq mm'} d^{*}_{jj'} - m'\sum_{1}^{m-1} a^{*}_{1;j} - m\sum_{1}^{m'-1} a^{*}_{2;j'} + (m \times m')c = 0$$

$$\sum_{j=1}^{m-1}\sum_{j'=1}^{m'-1}(d^{*}_{jj'} - a^{*}_{1;j} - a^{*}_{2;j'} + c) + \sum_{j'=1}^{m'-1}(d^{*}_{mj'} - a^{*}_{2;j'} + c) + \sum_{j=1}^{m-1}(d^{*}_{jm'} - a^{*}_{1;j} + c) + c = 0$$

and it is equivalent to the expression used in the traditional way,

$$\sum_{1}^{m-1}\sum_{1}^{m'-1}(e^{*}_{jj'} + c) + (m - 1 + m')\,c = 0$$

4.3 Results

In this particular example we have,

$$\frac{\lambda_{1CCM}}{\lambda_{1CA}} = \frac{0;9353}{0;3117} \approx 3$$

$$\frac{\lambda_{2CCM}}{\lambda_{2CA}} = \frac{0;5977}{0;1992} \approx 3$$

$$\frac{\lambda_{3CCM}}{\lambda_{3CA}} = \frac{0;3234}{0;1078} \approx 3$$

The canonical correlation coefficients for the new and the traditional way of coding the data are the following: 0:9671, 0:7731, 0:5687 and the standard coordinates for the categories:

	Dim:1	Dim:2	Dim:3
A	−1:238	0.846	−0:505
B	−0:426	−0:989	1:358
C	0.518	−1:343	−1:548
D	1:279	0.879	0.308

This agrees with the results reported in section 4.1.

5 Conclusions and discussions

This paper proposed that when our objective is to analyze categorical conjoint data to obtain the utility of different attribute levels, the results offered by CCM, which is the technique normally used in this context, are the same as those offered by the CA of a concatenated table. We have proved this idea analytically, using CC as a bridge between them, and illustrated the equivalence between the different methods.

A particular applied case, a study designed by an airline company, as well as the relevant literature, suggested that to include interaction effects could be useful. We did the extension of CA and CC to be able to treat interaction effects and finally to CCM. The equivalence was established theoretically and illustrated with the airline data.

Given the results we can conclude that CA should be used more often in this type of analysis since the results obtained are the same as those offered by CCM. Furthermore, CA offers a map and this allows us to interpret the results more rapidly and easily.

As future research, it could be interesting to apply the results of this paper to more than one subject. In this new case, the matrices of dummy variables should take the form of concatenated tables with all possible combination, as before, but for all the subjects, one below the other, for the attributes as well as for the response.

Another point to consider is what happens when we have more than one interaction effect. For the "new way" of coding, a problem appears when we want to analyze two interactions which involve one common attribute, eg. $P \times T$ and $P \times S$ since a double inclusion of the common attribute (in this case, price) occurs.

From an applied point of view, even though we are speaking all the time about one particular subject, from the first analysis without interactions, we could get information like the fact that, for long distance travel, price is the most relevant attribute. The company to travel with is not so important if it is well known, maybe because it implies the required safety. Timetable is more valued than service, maybe because it can be perceived as quite standard between the well known airline companies. Once interaction variables are included, the information obtained can help to establish particular offers from the travel agencies, establishing the right trade-off between the attributes, in other words, doing optimal combinations that generate different utilities than the first one obtained. If individual marketing was possible to do, to this particular person, the good offer could include, for example, the cheapest price, the company and the timetable that the travel agency prefers (except TAP), with flexilibity in the level of service.

6 References

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7 Appendix I. Housing data.

7.0.1 Conjoint analysis (Carroll, 1968). Data matrix

House no.(M)	SIZE (J_1)	PRICE (J_2)	CONDITION(J_3)	TASK(J_0)
1	2	25	E	A
2	2	25	G	A
3	2	25	P	C
4	2	30	E	A
5	2	30	G	B
6	2	30	P	C
7	2	35	E	B
8	2	35	G	B
9	2	35	P	D
10	2	40	E	B
11	2	40	G	C
12	2	40	P	D
13	3	25	E	A
14	3	25	G	A
15	3	25	P	C
16	3	30	E	A
17	3	30	G	B
18	3	30	P	C
19	3	35	E	B
20	3	35	G	B
21	3	35	P	D
22	3	40	E	B
23	3	40	G	C
24	3	40	P	D
25	4	25	E	A
26	4	25	G	A
27	4	25	P	C
28	4	30	E	A
29	4	30	G	B

7.0.2 Results

(a) Category Values

SOLUTION	1	2	3
EIGENVALUES	0; 9501	0; 5166	0; 0880
A	¡ 0; 2091	¡ 0; 1315	¡ 0; 1494
B	¡ 0; 0557	0; 2172	0; 0728
C	0; 0923	¡ 0; 1747	0; 2104
D	0; 2865	0; 0250	¡ 0; 2370

(b) Attribute Functions by Solution

		SOLUTION NUMBER		
ATTRIBUTE	LABEL	1	2	3
1: Size of house	S2	0	0	0
	S3	0	0	0
	S4	0	0	0
2:Asking Price	P1 = 25	¡ 0:1086	¡ 0:1459	¡ 0:0295
	P2 = 30	¡ 0:0575	¡ 0:0297	0:0446
	P3 = 35	0:0584	0:1531	¡ 0:0305
	P4 = 40	0:1077	0:0225	0:0154
3: Condition	CE	¡ 0:1324	0:0428	¡ 0:0383
	CG	¡ 0:0570	0:0320	0:0563
	CP	0:1894	¡ 0:0748	¡ 0:0133

7.1 Canonical correlation

7.1.1 Results

Categories (Y)

	Canonical weights 1^{st} axis	Canonical weights 2^{nd} axis	Canonical weights 3^{rd} axis
A	2.9320	-0.9256	0.5180
B	2.0246	1.1368	1.8327
C	1.1488	-1.1817	2.6469

Attributes (X)

	Canonical weights 1st axis	Canonical weights 2nd axis	Canonical weights 3rd axis
s2	0	0	0
s3	0	0	0
p1	1.3124	-1.3858	-0.8953
p2	1.0022	-0.4293	0.5822
p3	0.2994	1.0752	-0.9152
CE	1.9524	0.9689	-0.4994
CG	1.4952	0.8799	1.2952

Canonical correlations: 0.9752 0.7188 0.2966

7.2 Correspondence analysis

Concadenated table

	A	B	C	D
s2	3	4	3	2
s3	3	4	3	2
s4	3	4	3	2
p1	6	0	3	0
p2	3	3	3	0
p3	0	6	0	3
p4	0	3	3	3
CE	6	6	0	0
CG	3	6	3	0
CP	0	0	6	6

Inertia and percentages of inertia

```
1 0.4755   61.14% **************************************************
2 0.2583   33.21% ***************************
3 0.0440    5.65% *****
---------
0.7778
```

Row Contributions

```
I|Name| QLT MAS INR|   k=1  COR CTR|   k=2   COR CTR|   k=3  COR  CTR|
1| p1 |1000 125 196| 0.6517 347 112| 0.8753  627 371| 0.1769  26  89|
2| p2 |1000 125 36 | 0.3449 535  31| 0.17810 143  15|-0.2675 322 203|
3| p3 |1000 125 161|-0.3502 123  32|-0.91863 844 408| 0.1829  33  95|
4| p4 |1000 125 71 |-0.6463 940 110|-0.13482  41   9|-0.0923  19  24|
5| CE |1000 167 161| 0.7944 841 221|-0.25708  88  43| 0.2299  71 200|
```

```
6| CG |1000 167 54 | 0.3422 468  41|-0.19216 148  24|-0.3098 384 364|
7| CP |1000 167 321|-1.1366 861 453| 0.44924 135 130| 0.0798   4  24|
```

The attribute service does not appear given that its contribution is zero.

Column Contributions

```
Name|QLT  MAS INR|    k=1 COR CTR|   k=2  COR CTR|    k=3   COR CTR|
   A|1000 250 304| 0.8651 792 393| 0.4009 170 156| 0.18803  37 201|
   B|1000 333 214| 0.2304 106  37|-0.6622 877 566|-0.09157  17  64|
   C|1000 250 161|-0.3820 292  77| 0.5329 568 275|-0.26474 140 398|
   D|1000 167 321|-1.1854 937 493|-0.0762   4   4| 0.29820  59 337|
```

8 APPENDIX II. AIRLINE DATA

8.1 Correspondence analysis.

8.1.1 Data Matrix

	A	B	C	D
PT_{11}	12	3	0	0
PT_{12}	12	3	0	0
PT_{13}	12	3	0	0
PT_{21}	0	12	3	0
PT_{22}	12	3	0	0
PT_{23}	12	3	0	0
PT_{31}	0	0	14	1
PT_{32}	0	9	5	1
PT_{33}	4	10	1	0
PT_{41}	0	0	0	15
PT_{42}	0	0	7	8
PT_{43}	0	10	4	1
PT_{51}	0	0	0	15
PT_{52}	0	0	0	15
PT_{53}	0	0	7	8
TWA	16	10	8	11
IBERIA	16	8	6	15
KLM	16	10	8	11
B.A.	16	10	9	10
TAP	0	18	10	17
S_1	20	16	12	27
S_2	20	21	15	19
S_3	24	19	14	18

The previous attributes, price and timetable and their levels, appear in the analysis as supplementary points.

Row Contributions

I	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR
1	PT11	1000	22	51	1078	824	83	480	164	26	-133	12	4
2	PT12	1000	22	51	1078	824	83	480	164	26	-133	12	4
3	PT13	1000	22	51	1078	824	83	480	164	26	-133	12	4
4	PT21	1000	22	64	238	32	4	-1062	630	126	779	339	125
5	PT22	1000	22	51	1078	824	83	480	164	26	-133	12	4
6	PT23	1000	22	51	1078	824	83	480	164	26	-133	12	4
7	PT31	1000	22	136	-570	86	23	-1197	378	160	-1428	537	420
8	PT32	1000	22	38	-2	0	0	-985	904	108	320	96	21
9	PT33	1000	22	38	581	318	24	-524	259	31	669	422	92
10	PT41	1000	22	90	-1282	653	117	881	309	87	309	38	20
11	PT42	1000	22	43	-926	717	61	-158	21	3	-559	262	64
12	PT43	1000	22	43	61	3	0	-961	775	103	514	222	55
13	PT51	1000	22	90	-1282	653	117	881	309	87	309	38	20
14	PT52	1000	22	90	-1282	653	117	881	309	87	309	38	20
15	PT53	1000	22	43	-926	717	61	-158	21	3	-559	262	64
16	TWA	1000	67	3	130	645	4	57	125	1	-78	229	4
17	Iber	1000	67	6	21	7	0	240	964	19	-42	29	1
18	KLM	1000	67	3	130	645	4	57	125	1	-78	229	4
19	B.A.	1000	67	4	147	604	5	8	2	0	-119	394	9
20	Tap	1000	67	45	-429	443	39	-363	316	44	316	241	62
21	S1	1000	111	5	-123	519	5	117	470	8	18	12	0
22	S2	1000	111	2	22	47	0	-97	933	5	14	20	0
23	S3	1000	111	2	101	876	4	-19	32	0	-33	92	1

Supplementary points

IS	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR
24	P1	1000	67	152	1078	824	249	480	164	77	-133	12	11
25	P2	1000	67	72	798	954	136	-34	2	0	171	44	18
26	P3	1000	67	90	3	0	0	-902	974	272	-146	26	13
27	P4	1000	67	57	-716	973	110	-79	12	2	88	15	5
28	P5	1000	67	177	-1163	825	289	535	174	96	20	0	0
29	T1	1000	111	24	-364	992	47	-3	0	0	-33	8	1
30	T2	1000	111	4	-11	6	0	140	922	11	-39	72	2
31	T3	1000	111	29	374	855	50	-137	114	10	72	31	5

Column contributions

J	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR
1	A	1000	284	299	693	737	438	379	220	205	-166	42	73
2	B	1000	249	182	238	126	45	-442	433	245	447	442	461

```
 3|   C |1000 182 207| -290 119  49| -601 512 330| -510 369  439|
 4|   D |1000 284 311| -716 756 468|  393 228 221|  101  15   27|
```

8.2 Canonical Correlation. New way of coding.

Linear combinations for first canonical correlation. Number of obsservations = 225.

Response categories,

	Coef.	Std. Error	t	p> |t|
A	-2.5174	.04683	-53.756	0.000
B	-1.7052	.04847	-35.179	0.000
C	-.7614	.05230	-14.369	0.000

Attribute levels,

	Coef.	Std. Error	t	p> |t|
TWA	-.5770	.0581	-9.928	0.000
IBERIA	-.4637	.0581	-7.977	0.000
KLM	-.5771	.0581	-9.928	0.000
B.A.	-.5946	.0581	-10.229	0.000
S_1	.2303	.0450	5.116	0.000
S_2	.0813	.0450	1.806	0.072
PT_{11}	-2.0677	.1007	-20.537	0.000
PT_{12}	-2.0677	.1007	-20.537	0.000
PT_{13}	-2.0677	.1007	-20.537	0.000
PT_{21}	-1.2007	.1007	-11.925	0.000
PT_{22}	-2.0677	.1007	-20.537	0.000
PT_{23}	-2.0677	.1007	-20.537	0.000
PT_{31}	-.3674	.1007	-3.649	0.000
PT_{32}	-.9530	.1007	-9.465	0.000
PT_{33}	-1.5547	.1007	-15.442	0.000
PT_{41}	.3674	.1007	3.649	0.000
PT_{42}	0	.1007	0.000	1.000
PT_{43}	-1.0180	.1007	-10.112	0.000
PT_{51}	.3674	.1007	3.649	0.000
PT_{52}	.3674	.1007	3.649	0.000

```
Canonical correlations:
 0.9671 0.7731 0.5687
```

We recover the missing coefficients by centering:

Companies:

$$(a^*_{TWA} + a_{TAP}) + (a^*_{IBERIA} + a_{TAP}) + (a^*_{KLM} + a_{TAP}) + (a^*_{B.A.} + a_{TAP}) + a_{TAP} = 0$$

$$a_{TAP} = 0.443$$
$$a_{TWA} = -0.134$$
$$a_{IBERIA} = -0.021$$
$$a_{KLM} = -0.134$$
$$a_{B.A.} = -0.152$$

Services:

$$(a^{*}_{S_1} + a_{S_3}) + (a^{*}_{S_2} + a_{S_3}) + a_{S_3} = 0$$

$$a_{S_1} = 0.126$$
$$a_{S_2} = -0.023$$
$$a_{S_3} = -0.104$$

Interaction variable PT:

$$(a^{*}_{PT_{11}} + a_{PT_{53}}) + (a^{*}_{PT_{12}} + a_{PT_{53}}) + \ldots + a_{PT_{53}} = 0$$

$$a_{PT_{53}} = 0.955$$

We recover the true values:

$$\begin{array}{lllll}
a_{PT_{11}} = -1.113 & a_{PT_{21}} = -0.245 & a_{PT_{31}} = 0.588 & a_{PT_{41}} = 1.322 & a_{PT_{51}} = 1.322 \\
a_{PT_{12}} = -1.113 & a_{PT_{22}} = -1.113 & a_{PT_{32}} = 0.002 & a_{PT_{42}} = 0.955 & a_{PT_{52}} = 1.322 \\
a_{PT_{13}} = -1.113 & a_{PT_{23}} = -1.113 & a_{PT_{33}} = -0.595 & a_{PT_{43}} = -0.063 & a_{PT_{53}} = 0.955
\end{array}$$

8.3 Canonical Correlation. Traditional way of coding.

Linear combinations for first canonical correlation. Number of observations = 225.

Response categories,

	Coef.	Std. Error	t	p> |t|
A	−2.5174	.04683	−53.756	0.000
B	−1.7052	.04847	−35.179	0.000
C	−.7614	.05230	−14.369	0.000

Attribute levels,

	Coef:	Std:Error	t	P > jtj
P_1	2:0677	:10068	20:537	0:000
P_2	2:0677	:10068	20:537	0:000
P_3	1:5547	:10068	15:442	0:000
P_4	1:0180	:10068	10:112	0:000
TWA	:5771	:05813	9:928	0:000
IBERIA	:4637	:05813	7:977	0:000
KLM	:5771	:05813	9:928	0:000
B.A.	:5946	:05813	10:229	0:000
T_1	¡ :3674	:10068	¡ 3:649	0:000
T_2	¡ :3674	:10068	¡ 3:649	0:000
S_1	¡ :2305	:04503	¡ 5:116	0:000
S_2	¡ :0813	:04503	¡ 1:806	0:072
PT_{11}	:3674	:14238	2:581	0:011
PT_{12}	:3674	:14238	2:581	0:011
PT_{21}	¡ :5000	:14238	¡ 3:509	0:001
PT_{22}	:3674	:14238	2:581	0:011
PT_{31}	¡ :8199	:14238	¡ 5:758	0:000
PT_{32}	¡ :2343	:14238	¡ 1:646	0:101
PT_{41}	¡ 1:0180	:14238	¡ 7:150	0:000
PT_{42}	¡ :6506	:14238	¡ 4:569	0:000

Canonical correlations: 0.9671 0.7731 0.5687

From the results we corroborate that the canonical correlation coe¢cients are the same if we code the data in the new way and if we do it in the traditional form. Further, the coe¢cients for the main e¤ects are also the same.

The remaining work is to …nd the values for the interactions in the new form.

The restriction to be applied in this case is the following:

$$(a'_{PT_{11}} + c) + (a'_{PT_{12}} + c) + c + (a'_{PT_{21}} + c) + (a'_{PT_{22}} + c) + c + ::: + 3c = 0$$

$$c = 0:141$$

Then, the true coe¢cients are recovered in the following way:

$$a_{PT_{11}} = a_{P_1} + a_{T_1} + (a'_{PT_{11}} + c)$$
$$a_{PT_{11}} = 1:113$$

which di¤er from the previous recovered values only in their sign. The operation is repeated for all the other coe¢cients obtaining the following solutions:

$a_{PT_{21}} = 0:245$ $a_{PT_{31}} =$ ¡ 0:588 $a_{PT_{41}} =$ ¡ 1:322 $a_{PT_{51}} =$ ¡ 1:322
$a_{PT_{22}} = 1:113$ $a_{PT_{32}} =$ ¡ 0:002 $a_{PT_{42}} =$ ¡ 0:955 $a_{PT_{52}} =$ ¡ 1:322
$a_{PT_{23}} = 1:113$ $a_{PT_{33}} = 0:595$ $a_{PT_{43}} = 0:063$ $a_{PT_{53}} =$ ¡ 0:955

```
                        .
                        .
  *D                    .                     *A
                        .
                        .
    *P5                 .
                        .                  *P1
                        .
                  IBERIA*KLM
                    S1* * *TWA
. . . . . . . . . *T1 .T2**BA. . . . . .*P2. . . . .
            *P4       S2*S3   *T3
                        .
                *TAP    .
                        .
                        .
                        .
                        .
                        *P3
                        .
                        .     *B
                *C      .
```

Figure 1:

Correspondence Analysis ASCII Map by SimCA
Horizontal axis is dimension 1 with inertia = 0.2206 (70.5%)
Vertical axis is dimension 2 with inertia = 0.0821 (26.2%)
96.7% of total inertia is represented in the above map.

```
   PT52                 .
PT51*PT41               .                   *A
    D                   .
                        .           PT13
      *P5               .         PT12*PT11
                        .         PT23 P1
                        .*IBERIA
                      T2*KLM
                    S1* TWA**B.A.
  . . . . . . .*P4. .*T1.S2**S3 . . . . .*P2. . . .
      PT53*PT42         .     *T3
                        .
                  *TAP  .
                        .         *PT33
                        .
                        .
                        *P3
                    PT32**PT43  *B
                        .   *PT21
               *PT31    .
                *C      .
```

Figure 2:

Correspondence Analysis ASCII Map by SimCA

Horizontal axis is dimension 1 with inertia = 0.3117 (50.4%)
Vertical axis is dimension 2 with inertia = 0.1992 (32.2%)
82.6% of total inertia is represented in the above map