Failure to Collude in the Presence of Asymmetric Information*

Doh-Shin Jeon (Universitat Pompeu Fabra, CREA)†

June 22, 2005

Abstract

In this paper, we design the optimal contract when two agents can collude under asymmetric information. They have correlated types, produce complementary inputs and are protected by limited liability. Therefore, a joint manipulation of reports allows them to internalize informational and productive externalities. We show that by taking advantage of the transaction costs created by asymmetric information, even though they collude, the principal can achieve the outcome without collusion regardless of the sign and the degree of correlation. In particular, the principal can implement a non-monotonic quantity schedule in a collusion-proof way while this is impossible if collusion occurs under complete information.

Key words: Contract, Asymmetric information, Transaction Costs, Collusion.

JEL Classification: D8, L2

*I thank Antonio Cabrales, Jos Jansen, David Martimort, Jean-Charles Rochet and Joel Shapiro for helpful comments. I am very indebted to Jean-Jacques Laffont for his advice. I gratefully acknowledge the financial support from the Spanish Ministry of Science and Technology under BEC2003-00412 and the Ramon y Cajal grant.

†Department of Economics and Business, Universitat Pompeu Fabra, Ramon Trias Fargas, 25-27, 08005 Barcelona, Spain. Tel: (34) 93 542 1657. Fax: (34) 93 542 1746. doh-shin.jeon@upf.edu

1 Introduction

Collusion is an old and recurring phenomenon in organizations and has been a main theme of research for organizational sociologists.[1] Their common view is that behavior in organizations is often best predicted by the analysis of group as well as individual incentives. Hence, they think that incentive schemes must account for the possibility that members form coalitions to manipulate their functioning. However, the design of such incentive schemes necessitates a good understanding of coalition formation. In particular, understanding the transaction costs in coalition formation is fundamental since it helps us to predict under what circumstances agents can successfully form coalitions and to design incentive schemes which take advantage of the transaction costs in dealing with collusions.

In this paper, we study mechanism design when the agents can collude under asymmetric information. We identify the transaction costs in coalition formation generated by asymmetric information and derive the optimal mechanism which fully exploits the transaction costs to deter collusion. In particular, we show that asymmetric information can effectively create barriers to coalition formation making the agents fail to realize gains from collusion. Indeed, this result was conjectured by Pfeffer (1981):

"it is probably the lack of knowledge about the preferences and beliefs of others within the organization that constitutes a major barrier to the formation of coalitions (p. 166)."

More precisely, we study collusion between two units (agents) in a multi-divisional firm. The principal is the owner of the firm and the agents produce perfectly complementary inputs: to produce one unit of the final good, the firm needs a unit from each agent. Each agent has private information about his cost parameter (type) and is protected by limited liability. An agent can have either an L-type (low cost type) or an H-type (high cost type) and an agent's type is correlated with the other's type. Despite simple and specific features of out setting, it allows us to capture two general externalities which make collusion have bites. Precisely, an agent's report about his type affects the other agent's payoff through two channels: correlation creates information externalities and complementarity creates production externalities.[2] Therefore, the agents have the incentive to coordinate their

[1] See Crozier (1967) and Dalton (1959).

[2] Complementatiry is not enough to make collusion have a bite in our setting since Laffont and Martimort (1997) show that when the types are independent, collusion can be deterred at no cost even if there is no transaction cost in coalition formation.

reports in order to internalize these externalities. Although we apply the model to the internal organization of a firm, our model can also be applied to the regulation of firms producing complementary inputs (Gilbert and Riordan, 1995, Laffont and Martimort, 1997).

Before we study collusion, we characterize the optimal mechanism without side-contracting (i.e. the optimal mechanism when the agents do not collude). In our framework, full rent extraction, a result well-known from Crémer and McLean (1985), cannot be achieved since limited liability imposes a bound on the principal's capacity to penalize the agents. Hence, the optimal mechanism is derived from a trade-off between efficiency and rent extraction. The optimal quantity schedule, defined with respect to the sum of the agents' costs, is decreasing in the case of negative or weak positive correlation while the schedule is non-monotonic in the case of strong positive correlation. In the latter case, the rent obtained by an H-type is mainly determined by the quantity produced when one agent has an H-type and the other has an L-type. Therefore, it is optimal to introduce a large downward distortion in the quantity produced at this state of nature such that the quantity is smaller than the quantity produced when both agents have an H-type.

After analyzing the optimal mechanism without side-contracting, we study the collusion between the two agents. Drawing on the methodology developed by Laffont and Martimort (1997, 2000), we model the collusion by a side-contract offered by a benevolent and *uninformed* third-party who maximizes the sum of the agents' payoffs. The third-party uses the side-contract to implement joint manipulations of reports. We show that the collusion-proofness principle holds in our model and characterize the set of collusion-proof mechanisms. A mechanism is collusion-proof if it satisfies the coalition incentive constraints. In the presence of asymmetric information between the agents, the constraints are written in terms of the *virtual costs* instead of the real costs.

Before we analyze the impact of asymmetric information on collusion, we investigate whether the optimal mechanism without side-contracting exhibits any room for collusion by considering the case in which collusion takes place under complete information between the agents. It turns out that, under Bayesian implementation, there exists room for collusion only when the optimal quantity schedule is not monotonic (equivalently, when there is strong positive correlation). However, when we require dominant-strategy implementation,[3] there exists room for collusion when there is negative correlation as well.

As the main result, we show that under Bayesian implementation, the optimal mech-

[3] For the distinction between Bayesian implementation and dominant-strategy implementation, see Fudenberg and Tirole (1992, pp. 270-271).

anism without side-contracting can be implemented in a collusion-proof way without any loss regardless of the sign and the degree of correlation if collusion takes place under asymmetric information.[4] We also show that in the case of weak negative correlation, the principal can implement the optimal quantity schedule in dominant strategies and in a collusion-proof way without loss if collusion takes place under asymmetric information.

To give an intuition of the main result, we consider the case of strong positive correlation. We first note that, in this case, there exists room for collusion only when one agent has an L-type and the other has an H-type. In this state of nature, the agents have an incentive to report that both of them have an H-type since, given the non-monotonic quantity schedule, this manipulation of reports makes each of them to produce more quantity and this increase in quantity allows an L-type to obtain more rent while it does not hurt an H-type since the latter's ex post participation constraint always binds. However, the fact that the manipulation makes an H-type produce more quantity creates an incentive problem within the coalition; an L-type's incentive to pretend to have an H-type to the third-party is larger in the presence of the manipulation than in its absence since his rent is increasing in the quantity produced by an H-type. Therefore, in order to implement the collusion, the third-party has to give an L-type more rent than he would obtain in the absence of collusion. This additional rent is the transaction costs created by asymmetric information. Since the transaction costs are larger than the gains from collusion, the agents fail to collude.

From a theoretical point of view, our paper extends Laffont and Martimort (2000) who study collusion in a framework of public good provision when agents' types are correlated.[5] The main difference in terms of the setting is that they do not assume limited liability and therefore the principal can fully extract the rent in the absence of collusion as in Crémer and McLean (1985). They furthermore limit their analysis to the case of weak positive correlation and the two polar cases of almost perfect correlation and no correlation. As their main result, they show that collusion prevents the principal from achieving the first-best outcome and restores the continuity in the principal's payoff. However, in their optimal collusion-proof mechanism, asymmetric information does not generate any transaction cost except in the limit case of almost perfect correlation. We show that when the agents have limited liability, collusion is irrelevant in that the principal can imple-

[4] This means that, in the case of strong positive correlation, the principal can implement the optimal non-monotonic schedule in a collusion-proof way without additional cost. This result holds as long as the probability of having two L-types is larger than the probability of having one L-type and one H-type, which is satisfied if the probability of having two L-types is close to the probability having two H-types.

[5] They also study a two-type setting.

ment the optimal mechanism without side-contracting in a collusion-proof way without loss regardless of the sign and the degree of correlation by exploiting the transaction costs.[6]

Collusion under dominant-strategy implementation was first studied by Laffont and Maskin (1980). Using a differential approach, they show that there is a strong tension between individual and coalition incentives when dominant-strategy implementation is required. Laffont and Martimort (1997) show that in the case of no correlation, the optimal contract without side-contracting can be implemented in a collusion-proof way and in dominant strategies. Hence, we generalize their result to the case of weak positive and weak negative correlation.

The result that asymmetric information can make collusion inefficient was also obtained by Jeon (forthcoming) and Jeon and Menicucci (2005). However, in both papers, the agents' types are independently distributed. In the first paper, which extends Laffont and Martimort (1997)'s adverse selection framework by adding moral hazard (effort), the principal is constrained to use uniform transfers and this generates room for collusion. In the second paper, they consider a setting of monopolistic screening and room for collusion arises since buyers can engage in arbitrage by signing a side-contract.

Mailath and Postlewaite (1990) show that when each worker has private information on his non-wage benefit, the workers may receive a total compensation less than their total contribution since they fail to agree on a division of surplus should they leave for a new firm. Although they analyze coalition formation under asymmetric information, they do not study the principal's mechanism design. In contrast, we first characterize the coalition incentive constraints under asymmetric information and then find the transfer scheme which allows the principal to implement the second-best quantity profile.

Dana (1993) and Jansen (1999) characterize the optimal mechanism under correlated types and limited liability in a two-type setting. Dana studies the case in which each agent produces a final product while Jansen studies the case in which each agent produces a complementary input. None of them studies collusion.[7]

Our paper is related to the literature analyzing contracting between a principal and privately informed suppliers of complementary inputs (Baron and Besanko 1992, 1999 and Gilbert and Riordan). They compare the case of informational decentralization in

[6]We note that the principal's payoff in our optimal collusion-proof mechanism is continuous since limited liability restores continuity, as is shown by Robert (1991).

[7]Our framework is more general than Jansen's one since, in our paper, the principal decides the quantity to produce while, in his paper, the principal decides solely whether to produce or to shutdown.

which each agent knows only the cost of producing his own input with the case of informational consolidation in which one consolidated agent knows the costs of producing both inputs without specifying the mechanism which induces the agents to share their private information. By contrast, in our model, we specify this mechanism and identify the transaction costs generated by asymmetric information which, in turn, can be exploited by the principal.[8]

The paper is organized as follows. In Section 2, we present the model. In Section 3, we study as a benchmark the optimal grand-mechanism without side-contracting. In Section 4, we prove the collusion-proofness principle and characterize the set of collusion-proof grand-mechanisms. In Sections 5 and 6, we study collusion under asymmetric information distinguishing Bayesian implementation (Section 5) from dominant-strategy implementation (Section 6). Section 7 concludes. All the proofs which are not presented in the main texts are relegated to Appendix.

2 The Model

2.1 The Basic Setting

We consider the production decision of a firm composed of two divisions (agents). The production process consists of two stages, intermediary and final stage. Each agent is charged with one stage. The production technology of the agents is Leontief and one-to-one: to produce a unit of output, the firm needs a unit from each stage. The principal (the owner) has to decide the quantity of outputs to produce, which is denoted by q. The principal cannot observe realized costs. She uses transfers to induce the agents to produce.

Agent i's utility, with $i \in \{1, 2\}$, is given by:

$$U_i = t_i - \theta_i q,$$

where θ_i represents agent i's cost parameter and t_i the transfer from the principal to agent i. Agent i's cost parameter (type) θ_i is his private information. θ_1 and θ_2 are drawn from a joint distribution with the common support $\Theta \equiv \{\theta_L, \theta_H\}$. The joint distribution is supposed to be common knowledge. Let $\Delta\theta \equiv \theta_H - \theta_L > 0$. The agent with $\theta_i = \theta_L$ is called an L-type and the agent with $\theta_i = \theta_H$ is called an H-type. Let $p(\theta_i, \theta_j)$ denote

[8] Furthermore, in their models, consolidation changes the participation constraint from the individual one to a group participation constraint. and types are independently distributed.

the probability of having a state of nature (θ_i, θ_j) for $(i, j) \in \{1, 2\}$. For expositional simplicity, we introduce the following notation:

$$p(\theta_L, \theta_L) = p_{LL},\ p(\theta_L, \theta_H) = p(\theta_H, \theta_L) = p_{LH},\ p(\theta_H, \theta_H) = p_{HH}.$$

We denote conditional probability by $p(\cdot\,|\cdot)$. We assume that θ_1 and θ_2 can be positively or negatively correlated.[9] When θ_1 and θ_2 are positively correlated, the following inequality holds:

$$\frac{p_{LL}}{p_{LH}} > \frac{p_{LH}}{p_{HH}}.$$

When θ_1 and θ_2 are negatively correlated, the reverse of the above inequality holds. Let $\rho \equiv p_{LL}p_{HH} - (p_{LH})^2$ denote the degree of correlation.

The principal maximizes her profit, which consists of revenue from selling outputs minus transfers. Her objective function is given by:

$$\pi = S(q) - \sum_{i=1}^{2} t_i,$$

where $S'(\cdot) > 0$, $S'(0) = \infty$ and $S''(\cdot) < 0$. We assume that the revenue from selling outputs is large enough to employ both agents for any realization of cost parameter.

According to the revelation principle, a grand-mechanism, M, between the principal and the agents takes the following form:

$$\{t_1(\widehat{\theta}_1, \widehat{\theta}_2), t_2(\widehat{\theta}_1, \widehat{\theta}_2), q(\widehat{\theta}_1, \widehat{\theta}_2)\},$$

where $\widehat{\theta}_i$ is agent i's report about his cost parameter to the principal.

Each agent's reservation utility is normalized to zero regardless of his type. Each agent is protected by limited liability in that he has the option of terminating his relationship with the principal at any time before incurring the production cost[10]. Therefore, each agent's participation constraints must be satisfied ex post:

$$t_i(\theta_1, \theta_2) - \theta_i q(\theta_1, \theta_2) \geq 0,\ \forall(\theta_1, \theta_2) \in \Theta^2.$$

[9] The independent case is included in our framework as a particular case with measure zero.

[10] We assume limited termination penalties, which are common in practice. See Sappington (1983), Dana (1993) and Lewis and Sappington (1997). If termination pemalities are large enough, the principal can achieve the first-best outcome as in Crémer and McLean (1985).

2.2 Collusion

We model collusion between the two agents by a side-contract offered by a benevolent and *uninformed* third-party denoted by T as in Laffont and Martimort (1997, 2000).[11] The third-party uses the side-contract to implement joint manipulations of reports. He maximizes the sum of the agents' rents subject to a set of incentive compatibility, acceptance, budget balance and ex post participation constraints.

A side-contract takes the following form:

$$\{\phi(\widetilde{\theta}_1, \widetilde{\theta}_2), y_1(\widetilde{\theta}_1, \widetilde{\theta}_2), y_2(\widetilde{\theta}_1, \widetilde{\theta}_2)\},$$

where $\widetilde{\theta}_i$ is agent i's report about his cost parameter to the third-party. $\phi(\cdot)$ is the manipulation of report function. This maps any pair of reports made by the agents to the third-party into the set of (possibly stochastic) reports to the principal. $y_i(\cdot)$ is the monetary transfer from agent i to the third-party.

We assume that the third-party is not a source of money and therefore require that the following ex post budget balance constraint be satisfied for all states of nature:

$$\sum_{i=1}^{2} y_i(\theta_1, \theta_2) = 0, \ \forall(\theta_1, \theta_2) \in \Theta^2.$$

We note that there is no loss of generality in restricting the set of feasible side-contracts to direct revelation mechanisms since the revelation principle applies at this stage of the game. We assume as in Laffont and Martimort (1997, 2000) that the side-contract is enforceable even though the secrecy of this contract implies that there is no court of justice available to enforce it.[12]

2.3 Timing

The timing is as follows.

1. Nature draws each agent's cost parameter. Each agent learns only his own parameter.

[11] The readers might wonder why we do not use an extensive form of bargaining between the agents to describe collusion. However, any outcome of an extensive form of bargaining can be achieved by a side-contract designed by the third-party. Hence, the modelling strategy of using the third-party as a side-contract designer is a shortcut which allows us to characterize the highest bound of what can be achieved by collusion.

[12] This assumption allows us to focus on the highest bound of what can be achieved by collusion. It is a shortcut to capture in a static-context the reputations of the third-party and the agents which guarantee that the self-enforceability of these contracts would emerge in repeated relationship.

2. The principal proposes a grand-mechanism M.

3. Each agent accepts or refuses it. If at least one agent refuses, each agent gets the reservation utility and the following sequences do not occur.

4. The third-party offers the side-contract S.

5. Each agent accepts or refuses it. If at least one agent refuses, the grand-mechanism is played non-cooperatively. In this case, reports are directly made in the grand-mechanism and the next two stages do not occur.

6. If the side-contract has been accepted, reports in the side-contract are made.

7. The corresponding side-transfers and the reports in the grand-mechanism are made.

8. Production and transfers are enforced.

After the third-party proposes S, a two-stage game is played: first, the agents accept or refuse S and then they send their messages either to the principal or to the third-party depending on their acceptance decisions at the previous stage. We call this two-stage game the game of coalition formation. We assume that after agent i rejected the side-contract offered by the third-party, the other agent j (with $j \neq i$) does not change his own beliefs about i's type.

3 Benchmark: Optimal grand-mechanism without side-contracting

In this section, we analyze, as a benchmark, the optimal grand-mechanism when there is no side-contracting. Since the two agents are perfectly symmetric, there is no loss of generality in looking for the optimal contract within the class of mechanisms which are symmetric. For expositional simplicity, we introduce the following notation: for the transfers,

$$t_{LL} = t_1(\theta_L, \theta_L) = t_2(\theta_L, \theta_L),\ t_{LH} = t_1(\theta_L, \theta_H) = t_2(\theta_H, \theta_L),$$
$$t_{HL} = t_1(\theta_H, \theta_L) = t_2(\theta_L, \theta_H),\ t_{HH} = t_1(\theta_H, \theta_H) = t_2(\theta_H, \theta_H);$$

and for the quantity of outputs to produce,

$$\underline{q} = q(\theta_L, \theta_L),\ \widehat{q} = q(\theta_L, \theta_H) = q(\theta_H, \theta_L),\ \overline{q} = q(\theta_H, \theta_H).$$

The optimal grand-mechanism should satisfy the following incentive compatibility constraints to induce truth-telling: for an L-type,

$$p_{LL}(t_{LL} - \theta_L \underline{q}) + p_{LH}(t_{LH} - \theta_L \widehat{q}) \geq p_{LL}(t_{HL} - \theta_L \widehat{q}) + p_{LH}(t_{HH} - \theta_L \overline{q});[13] \qquad (1)$$

[13] In fact, the original expression for the incentive constraint is given by: $\frac{p_{LL}}{p_{LL}+p_{LH}}(t_{LL} - \underline{\theta q}) +$

and for an H-type,

$$p_{LH}(t_{HL} - \theta_H \widehat{q}) + p_{HH}(t_{HH} - \theta_H \overline{q}) \geq p_{LH}(t_{LL} - \theta_H \underline{q}) + p_{HH}(t_{LH} - \theta_H \widehat{q}). \quad (2)$$

The grand-mechanism should satisfy the following ex post participation constraints:

$$t_{LL} - \theta_L \underline{q} \equiv u_{LL} \geq 0, \quad (3)$$

$$t_{LH} - \theta_L \widehat{q} \equiv u_{LH} \geq 0, \quad (4)$$

$$t_{HL} - \theta_H \widehat{q} \equiv u_{HL} \geq 0, \quad (5)$$

$$t_{HH} - \theta_H \overline{q} \equiv u_{HH} \geq 0, \quad (6)$$

where, for instance, u_{LL} represents the utility that an L-type obtains when the other agent reports that he has an L-type.

The principal maximizes expected profit, given below, subject to the constraints (1) to (6):

$$E(\pi) = p_{LL}[S(\underline{q}) - 2t_{LL}]$$

$$+2p_{LH}[S(\widehat{q}) - t_{LH} - t_{HL}] + p_{HH}[S(\overline{q}) - 2t_{HH}].$$

The optimal grand-mechanism without side-contracting is characterized in the next proposition.

Proposition 1 *We assume that for $\rho > \rho^* \equiv \frac{p_{LH}^2}{p_{LL}}$, $\Delta\theta$ is small enough relative to $\underline{\theta}$*[14]*. The optimal grand-mechanism without side-contracting is characterized as follows:*

(i) Only the L-type's incentive constraint and the H-type's ex post participation constraints are binding.

(ii) The optimal quantity schedule is given by:

$$S'(\underline{q}^*) = 2\theta_L,$$

$$S'(\widehat{q}^*) = \theta_L + \theta_H + \frac{p_{LL}}{p_{LH}}\Delta\theta,$$

$$S'(\overline{q}^*) = 2\theta_H + 2\frac{p_{LH}}{p_{HH}}\Delta\theta.$$

$\frac{p_{LH}}{p_{LL}+p_{LH}}(t_{LH} - \underline{\theta}\widehat{q}) \geq \frac{p_{LL}}{p_{LL}+p_{LH}}(t_{HL} - \underline{\theta}\widehat{q}) + \frac{p_{LH}}{p_{LL}+p_{LH}}(t_{HH} - \underline{\theta}\overline{q})$. After multiplying both sides of the inequality by $p_{LL} + p_{LH}$, we obtain (1).

[14]This condition allows us to focus on the case in which the H-type's incentive constraint is slack: for more details, see the proof of proposition 1.

It is decreasing ($\underline{q}^* > \widehat{q}^* \geq \overline{q}^*$) *for* $\rho \leq \rho^*$ *but non-monotonic* ($\underline{q}^* > \overline{q}^* > \widehat{q}^*$) *for* $\rho > \rho^*$.[15]

(iii) The following transfer scheme implements the optimal quantity schedule:

(a) for an H-type: $t^*_{HL} = \theta_H \widehat{q}^*,\ t^*_{HH} = \theta_H \overline{q}^*$.

(b) for an L-type:

$$t^*_{LL} = \theta_L \underline{q}^* + \Delta\theta \widehat{q}^*,\ t^*_{LH} = \theta_L \widehat{q}^* + \Delta\theta \overline{q}^*,\ \textit{for } \rho \leq \rho^*,$$

$$t^*_{LL} = \theta_L \underline{q}^* + \Delta\theta \widehat{q}^* + \frac{p_{LH}}{p_{LL}} \Delta\theta \overline{q}^*,\ t^*_{LH} = \theta_L \widehat{q}^*,\ \textit{for } \rho > \rho^*.$$

In both cases, the principal has a residual degree of freedom in choosing the transfers (t_{LL}, t_{LH}).[16]

When the ex post participation constraints have to be satisfied, an H-type has to be given at least zero rent for every state of nature and this makes the principal concede a positive rent to an L-type.[17] Therefore, the optimal quantity schedule is determined by the trade-off between efficiency and rent extraction. In this trade-off, as usual, the relevant cost for an agent is his virtual cost. An L-type's virtual cost is equal to his real cost while an H-type's one is larger than his real cost. The H-type's virtual cost changes depending upon the sign and the degree of correlation. To give an intuition about how the virtual cost is determined, we consider the intermediate state of nature (θ_L, θ_H) or (θ_H, θ_L). Suppose that $\rho \leq \rho^*$ holds and that the principal increases $\widehat{q}$ by $d\widehat{q}$. This will increase, when the state of nature is (θ_L, θ_L), from t^*_{LL} in proposition 1(iii)(b), the rent abandoned to each L-type by $\Delta\theta d\widehat{q}$. The total increase in cost is given by the sum of the increase in the production cost $2p_{LH}(\theta_L + \theta_H)d\widehat{q}$ and the increase in the rent $2p_{LL}\Delta\theta d\widehat{q}$ while the increase in revenue is given by $2p_{LH}S'(\widehat{q})d\widehat{q}$. Therefore, an H-type's virtual cost is given by $\theta_H + \frac{p_{LL}}{p_{LH}}\Delta\theta$.

The optimal quantity schedule is decreasing in the sum of the two agents' cost parameters ($\underline{q}^* > \widehat{q}^* \geq \overline{q}^*$) if there is negative or weak positive correlation ($\rho \leq \rho^*$). In the case of strong positive correlation ($\rho > \rho^*$), the sum of the agents' virtual costs is smaller when both agents have an H-type than when one has an L-type and the other has an H-type. This makes the schedule non-monotonic: $\underline{q}^* > \overline{q}^* > \widehat{q}^*$. Intuitively, in this case, when an

[15] An interesting thing to note is that our case of strong positive correlation $\rho > \rho^*$ exactly corresponds to the case of strong positive correlation in Armstrong and Rochet (1999) in which they characterize the solution of a two-dimensional screening problem. We are grateful to Jean-Charles Rochet for this remark.

[16] See Appendix 1 for the range of the transfers.

[17] In terms of the binding constraints, our characterization of the optimal mechanism is similar to those obtained by Dana (1993) and Jansen (1997).

L-type pretends to have an H-type, the probability for him to produce $\widehat{q}$ is much higher than the probability to produce $\overline{q}$. Since his rent is mainly determined by $\widehat{q}$, it is optimal for the principal to introduce a large downward distortion in $\widehat{q}$.

The principal has a residual degree of freedom in choosing the transfers (t_{LL}, t_{LH}) which satisfy the Bayesian incentive constraints. When the optimal quantity schedule is decreasing, by using the degree of freedom, she can satisfy the incentive compatibility constraints in dominant strategies, given as follows: for an L-type,

$$t_{LL} - \theta_L \underline{q} \geq t_{HL} - \theta_L \widehat{q}, \tag{7}$$

$$t_{LH} - \theta_L \widehat{q} \geq t_{HH} - \theta_L \overline{q}; \tag{8}$$

for an H-type,

$$t_{HL} - \theta_H \widehat{q} \geq t_{LL} - \theta_H \underline{q}, \tag{9}$$

$$t_{HH} - \theta_H \overline{q} \geq t_{LH} - \theta_H \widehat{q}. \tag{10}$$

Let M^D denote the optimal grand-mechanism implemented in dominant strategies. Under M^D, transfers are given as follows: $t_{LL} = \theta_L \underline{q}^* + \Delta\theta \widehat{q}^*$, $t_{LH} = \theta_L \widehat{q}^* + \Delta\theta \overline{q}^*$, $t_{HL} = \theta_H \widehat{q}^*$, $t_{HH} = \theta_H \overline{q}^*$.

When the optimal quantity schedule is not monotonic, the H-type's Bayesian incentive constraint may not be satisfied if the principal proposes M^D. In this case, using a transfer scheme in which the principal gives a rent only when both agents have an L-type maximizes the chance to satisfy the H-type's Bayesian incentive constraint.[18] In the proof of proposition 1, we show that when $\Delta\theta$ is small enough, the principal can strictly satisfy the H-type's Bayesian incentive constraint by using this scheme. Thus, in this case, she still has a residual degree of freedom.

4 Collusion-proof grand-mechanisms under asymmetric information

In this section, we assume that there is asymmetric information between the agents. We define the collusion-proof grand-mechanism, prove the collusion-proofness principle and characterize the set of collusion-proof grand-mechanisms.

[18] Given the thee binding constraints, the principal has only one degree of freedom in choosing the transfers. Transforming all the transfers as a function of t_{LH} from the binding constraints and injecting them into the H-type's incentive constraint reveals that minimizing t_{LH} maximizes the chance to satisfy the constraint.

4.1 Definition

In order to define the collusion-proof grand mechanism, we need to introduce some definitions.

Definition 1 *A side-contract* $S^* = \{\phi^*(\cdot), y_1^*(\cdot), y_2^*(\cdot)\}$ *is coalition-interim-efficient with respect to a grand-mechanism* $M = \{t_1(\cdot), t_2(\cdot), q(\cdot)\}$ *providing the reservation utilities* $V_i(\theta_i)$ *if and only if it is the solution of the following program (thereafter denoted by* (T)*):*

$$\max_{\phi, y_1, y_2} p_{LL}[t_1(\phi_{LL}) + t_2(\phi_{LL}) - 2\theta_L q(\phi_{LL})]$$

$$+p_{LH}[t_1(\phi_{LH}) + t_2(\phi_{LH}) - (\theta_L + \theta_H)q(\phi_{LH})]$$

$$+p_{LH}[t_1(\phi_{HL}) + t_2(\phi_{HL}) - (\theta_H + \theta_L)q(\phi_{HL})]$$

$$+p_{HH}[t_1(\phi_{HH}) + t_2(\phi_{HH}) - 2\theta_H q(\phi_{HH})]$$

subject to

$$U_i(\theta_i) = \sum_{\Theta_j} p(\theta_j|\theta_i)[t_i(\phi(\theta_i, \theta_j)) - y_i(\theta_i, \theta_j) - \theta_i q(\phi(\theta_i, \theta_j))], \ \forall \theta_i \in \Theta;$$

$$(BIC) \ U_i(\theta_i) \geq \sum_{\Theta_j} p(\theta_j|\theta_i)[t_i(\phi(\widehat{\theta}_i, \theta_j)) - y_i(\widehat{\theta}_i, \theta_j) - \theta_i q(\phi(\widehat{\theta}_i, \theta_j))], \ \forall (\theta_i, \widehat{\theta}_i) \in \Theta^2;$$

$$(BIR) \ U_i(\theta_i) \geq V_i(\theta_i), \ \forall \theta_i \in \Theta;$$

$$(BB) \ \sum_{k=1}^{2} y_k(\theta_i, \theta_j) = 0, \ \forall (\theta_i, \theta_j) \in \Theta^2;$$

$$(Ex\ post\ IR) \ t_i(\phi(\theta_i, \theta_j)) - y_i(\theta_i, \theta_j) - \theta_i q(\phi(\theta_i, \theta_j)) \geq 0, \ \forall (\theta_i, \theta_j) \in \Theta^2.$$

A side-contract is coalition-interim-efficient if it maximizes the sum of the agents' expected utilities subject to incentive, acceptance, budget balance and ex post participation constraints. We note below the difference between the acceptance constraint and the ex post participation constraint. The first is defined in Bayesian terms with respect to the reservation utility $V_i(\theta_i)$ that agent i can obtain when he plays non-cooperatively the grand-mechanism after rejecting the side-contract.[19] The second is defined in ex post terms with respect to zero reservation utility. Therefore, both the principal and the third-party have equal standing in that the agents are protected by limited liability.

[19] We note that $V_i(\theta_i)$ represents i's utility when j $(j \neq i)$ does not change his beliefs on i's type after observing i's rejection of the side-contract.

Definition 2 *The null side-contract is the side-contract where there is no manipulation of report* $(\phi(\cdot) = I_d(\cdot))$ *and no transfer between agents* $(y_1(\cdot) = y_2(\cdot))$.

We now define the collusion-proof grand-mechanism.

Definition 3 *A grand-mechanism* $M = \{t_1(\cdot), t_2(\cdot), q(\cdot)\}$ *providing the reservation utilities* $V_i(\theta_i)$ *is collusion-proof when the null-side contract is coalition-interim-efficient with respect to this mechanism.*

4.2 Characterization

We first show that the collusion-proofness principle is valid in our model.

Proposition 2 *There is no loss of generality in restricting the principal to offer collusion-proof mechanisms to characterize the outcome of any perfect Bayesian equilibria of the game of grand-mechanism offer cum coalition formation.*

In general, if the third-party has an informational advantage over the principal or can use finer instruments than the principal can, the collusion-proofness principle may not hold. However, in our framework, the third-party has no informational or instrumental advantage and is subject to the incentive, acceptance, budget balance and ex post participation constraints. Hence, all the outcomes that can be implemented by allowing collusion to happen can be mimicked by the principal in a collusion-proof way without any loss. This collusion-proofness principle simplifies our analysis, since what can be achieved by the principal is contained in the set of collusion-proof grand-mechanisms.

In the next proposition, we characterize the set of symmetric collusion-proof grand-mechanisms. We focus on the subset of these collusion-proof grand-mechanisms where the H-type's incentive constraint is not binding.[20]

Proposition 3 *A grand-mechanism is collusion-proof if and only if there exist* δ (≥ 0), $\epsilon = \epsilon(\delta) > 0$ *(with* $\frac{p_{LL}}{p_{LH}} > \frac{\delta}{\epsilon} \geq 0$*) and* $\epsilon' = \epsilon'(\delta)$ *(with* $\epsilon'(0) > 0$*) such that:*

$$2t_{LL} - 2\theta_L \underline{q} \geq t_1(\widehat{\theta}_1, \widehat{\theta}_2) + t_2(\widehat{\theta}_1, \widehat{\theta}_2) - 2\theta_L q(\widehat{\theta}_1, \widehat{\theta}_2), \ \forall (\widehat{\theta}_1, \widehat{\theta}_2) \in \Theta^2;$$

$$t_{LH} + t_{HL} - (\theta_L + \theta_H + \frac{\delta}{\epsilon}\Delta\theta)\widehat{q} \geq t_1(\widehat{\theta}_1, \widehat{\theta}_2) + t_2(\widehat{\theta}_1, \widehat{\theta}_2) - (\theta_L + \theta_H + \frac{\delta}{\epsilon}\Delta\theta) q(\widehat{\theta}_1, \widehat{\theta}_2),$$

[20] In fact, it will be shown later on in section 5.2 that the H-type's incentive constraint is slack in the optimal collusion-proof grand-mechanism

$$\forall(\widehat{\theta}_1,\widehat{\theta}_2)\in\Theta^2;$$

$$\textit{When } \epsilon' \neq 0,\ \epsilon'[2t_{HH} - 2(\theta_H + \frac{\delta}{\epsilon'}\Delta\theta)\overline{q}]$$

$$\geq \epsilon'[t_1(\widehat{\theta}_1,\widehat{\theta}_2) + t_2(\widehat{\theta}_1,\widehat{\theta}_2) - 2(\theta_H + \frac{\delta}{\epsilon'}\Delta\theta)q(\widehat{\theta}_1,\widehat{\theta}_2)],$$

$$\forall(\widehat{\theta}_1,\widehat{\theta}_2)\in\Theta^2;$$

$$\textit{When } \epsilon' = 0,\ -\delta\overline{q} \geq -\delta q(\widehat{\theta}_1,\widehat{\theta}_2),\ \forall(\widehat{\theta}_1,\widehat{\theta}_2)\in\Theta^2;$$

where δ is the multiplier associated with the L-type's Bayesian incentive constraint in the third-party's program.

In Proposition 3, we characterized the coalition incentive constraints under asymmetric information. If the constraints are satisfied, the agents have no incentive to jointly manipulate their reports made to the principal. We note that if δ is equal to zero, the coalition incentive constraints under asymmetric information are equivalent to the constraints derived when there is complete information between the agents. The constraints under asymmetric information are written in terms of the *virtual costs* instead of the real costs. An L-type's virtual cost is equal to his real cost since the H-type's Bayesian incentive constraint is slack while an H-type's virtual cost differs from his real cost as long as δ is strictly positive. δ/ϵ (or δ/ϵ') represents the cost of giving a rent to an L-type. δ is equal to zero if the L-type's Bayesian incentive constraint is slack in the third-party's program while δ can be positive when the constraint is binding. Giving a rent to an L-type can be costly when there exists a tension between incentive, budget balance and participation constraints. Since an L-type's rent depends on the quantity produced by an H-type, an H-type's virtual cost can be different from his real cost,[21] which in turn creates distortions in the third-party's decisions to manipulate reports compared to the case

[21] When the state of nature is (θ_L,θ_H) or (θ_H,θ_L), as usual, the H-type's virtual cost, $\theta_H + \frac{\delta}{\epsilon}\Delta\theta$, is larger than his real cost. However, when the state of nature is (θ_H,θ_H), the H-type's virtual cost, $\theta_H + \frac{\delta}{\epsilon'}\Delta\theta$, can be either larger or smaller than his real cost since ϵ' can be positive or negative. ϵ' can be negative, for instance, if there is strong negative correlation. In this case, since the state of nature is likely to be (θ_L,θ_H) or (θ_H,θ_L), the utility that an L-type can obtain in the side-contract by pretending to be an H-type to the third-party is essentially determined by ϕ_{HH}, the manipulation of report when both agents have an H-type. This makes it very costly for the third-party to give a large utility to the coalition composed of two H-types; the higher is the utility given to this coalition, the higher is the rent abandoned to an L-type. When this negative effect dominates the other benefits from the manipulation, the third-party will maximize with respect to ϕ_{HH} an objective having a negative sign and therefore ϵ' is negative.

in which the decisions are taken under complete information. We note that the principal has some flexibility in choosing δ/ϵ (or δ/ϵ') because the null-side contract satisfies the necessary and sufficient conditions for optimality in the third-party's problem for any (δ, ϵ, ϵ') satisfying $\delta \geq 0$, $p_{LL}/p_{LH} > \delta/\epsilon \geq 0$ and $\epsilon'(0) > 0$.[22]

One might argue that the principal might ask the agents for the information that they may have learned during the course of coalition formation. But then the third-party could react by inducing further manipulations of those reports of the learned information. These reactions and counter-reactions lead naturally to a problem of infinite regress. By restricting the principal to use grand-mechanisms only contingent on the agents' types, we cut arbitrarily this process in favor of colluding agents and give collusive behavior its best chance. The restriction strengthens our main result since we show that the optimal mechanism without side-contracting can be implemented in a collusion-proof way.

5 Failure to collude under asymmetric information: Bayesian implementation

We study, in this section, the impact of asymmetric information on collusion under Bayesian implementation (i.e. when the grand-mechanism offered by the principal should satisfy the Bayesian individual incentive constraints (1) and (2)). For this purpose, we first examine whether the optimal grand-mechanism without side-contracting exhibits any room for collusion when collusion takes place under complete information between the agents. After identifying the case in which the agents can realize gains from collusion in the absence of transaction costs, we analyze how the principal can exploit asymmetric information between the agents to make the collusion inefficient.

5.1 Collusion under complete information

Suppose that collusion takes place under complete information between the agents. We examine below whether or not the optimal grand-mechanism without side-contracting exhibits any room for collusion. When collusion takes place under complete information, the grand-mechanism should satisfy the following coalition incentive constraints:

$$(CIC_{LL,LH}) \quad 2u_{LL} \geq u_{LH} + \Delta\theta\widehat{q}^*; \tag{11}$$

$$(CIC_{LL,HH}) \quad u_{LL} \geq \Delta\theta\overline{q}^*; \tag{12}$$

[22]See the proof of proposition 3.

$$(CIC_{LH,LL}) \quad u_{LH} \geq 2u_{LL} - \Delta\theta \underline{q}^*; \tag{13}$$

$$(CIC_{LH,HH}) \quad u_{LH} \geq \Delta\theta \overline{q}^*; \tag{14}$$

$$(CIC_{HH,LL}) \quad \Delta\theta \underline{q}^* \geq u_{LL}; \tag{15}$$

$$(CIC_{HH,LH}) \quad \Delta\theta \widehat{q}^* \geq u_{LH}. \tag{16}$$

We distinguish below two cases: when the optimal quantity schedule without side-contracting is monotonic ($\rho \leq \rho^*$) and non-monotonic ($\rho > \rho^*$).

Consider first the case in which the optimal quantity schedule without side-contracting is monotonic. In this case, the principal can satisfy these coalition incentive constraints without additional loss. For example, consider the following rent scheme:

$$u_{LL} = \frac{(p_{LL} + p_{LH})\widehat{q}^* + p_{LH}\overline{q}^*}{p_{LL} + 2p_{LH}} \Delta\theta, u_{LH} = \frac{p_{LL}\widehat{q}^* + 2p_{LH}\overline{q}^*}{p_{LL} + 2p_{LH}} \Delta\theta, u_{HL} = u_{HH} = 0.$$

Under the scheme, the L-type's incentive compatibility constraint is binding and all the coalition incentive constraints are satisfied.

Consider now the case in which the optimal quantity schedule without side-contracting is not monotonic: $\underline{q}^* > \overline{q}^* > \widehat{q}^*$. In this case, the principal cannot implement the schedule in a collusion-proof way. To show this, we sum up (14) and (16) and obtain $\widehat{q}^* \geq \overline{q}^*$, which is contradictory. Hence, the optimal grand-mechanism exhibits room for collusion. More generally, proposition 4(ii) below states that whatever the sign and the degree of correlation, the principal can never implement a non-monotonic quantity schedule in a collusion-proof way if collusion takes place under complete information. The intuition for the result is simple. Since, in the absence of asymmetric information, there exists no transaction cost in side-contracting, the situation is similar to the case in which the principal deals with one consolidated agent who has three different types: $(\theta_L + \theta_L)$, $(\theta_L + \theta_H)$, $(\theta_H + \theta_H)$. Thus, the monotonicity condition has to be satisfied as it does in the one-agent case. Summarizing, we have:

Proposition 4 *If collusion takes place under complete information,*

(i) When there is negative or weak positive correlation ($\rho \leq \rho^*$), the principal can implement the optimal grand-mechanism without side-contracting in a collusion-proof way

(ii) The principal can never implement a non-monotonic quantity schedule in a collusion-proof way. Therefore, when there is strong positive correlation ($\rho > \rho^*$), the principal cannot implement the optimal grand-mechanism without side-contracting in a collusion-proof way.

5.2 Collusion under asymmetric information

In this section, we assume that collusion takes place under asymmetric information between the agents. Since we have seen in the previous section that there is no room for collusion when the optimal quantity schedule is monotonic and the third-party can always implement the null side-contract, we focus on the case of strong positive correlation in which the optimal schedule is not monotonic.

From Proposition 1, the set of the rent schemes which implement the optimal mechanism without side-contracting is given by:

$$\{u_{LL} \geq 0,\ u_{LH} \geq 0,\ u_{HL} = 0,\ u_{HH} = 0,$$

$$p_{LL}u_{LL} + p_{LH}u_{LH} = p_{LL}\Delta\theta\widehat{q}^* + p_{LH}\Delta\theta\overline{q}^*,$$

$$p_{LH}\Delta\theta\underline{q}^* + p_{HH}\Delta\theta\widehat{q}^* \geq p_{LH}u_{LL} + p_{HH}u_{LH}\Big\}.$$

When collusion takes place under asymmetric information, from proposition 3, the coalition incentive constraints are given as follows:

$$(CIC_{LL,LH}) \quad 2u_{LL} \geq u_{LH} + \Delta\theta\widehat{q}^*; \tag{17}$$

$$(CIC_{LL,HH}) \quad u_{LL} \geq \Delta\theta\overline{q}^*; \tag{18}$$

$$(CIC_{LH,LL}) \quad u_{LH} \geq 2u_{LL} - \Delta\theta\underline{q}^* - \frac{\delta}{\epsilon}\Delta\theta(\underline{q}^* - \widehat{q}^*); \tag{19}$$

$$(CIC_{LH,HH}) \quad u_{LH} \geq \Delta\theta\overline{q}^* - \frac{\delta}{\epsilon}\Delta\theta(\overline{q}^* - \widehat{q}^*); \tag{20}$$

$$(CIC_{HH,LL}) \quad \Delta\theta\underline{q}^* \geq u_{LL} - \frac{\delta}{\epsilon'}\Delta\theta(\underline{q}^* - \overline{q}^*); \tag{21}$$

$$(CIC_{HH,LH}) \quad \Delta\theta\widehat{q}^* \geq u_{LH} + 2\frac{\delta}{\epsilon'}\Delta\theta(\overline{q}^* - \widehat{q}^*); \tag{22}$$

where for the last two constraints, we suppose that $\epsilon' > 0$.[23] We note in particular that a positive ϵ relaxes the two incentive constraints for the coalition composed of one L-type and one H-type (i.e. (19) and (20)).

From the incentive constraint which prevents the coalition composed of two L-types from pretending to be the coalition composed of two H-types (18), we obtain a lower bound for u_{LL}. From the incentive constraint which prevents the coalition composed of

[23] As explained in the proof of proposition 3, when the grand-mechanism is collusion-proof, the principal has certain degree of freedom in choosing the values of some multipliers in the third-party's program such that she can make ϵ' strictly positive.

one L-type and one H-type from pretending to be the coalition composed of two H-types (20), we obtain a lower bound for u_{LH}. The lower bound for u_{LH} is the smallest when δ/ϵ is the largest. Since we want to obtain the supremum of the principal's payoff, we allow δ/ϵ to take p_{LL}/p_{LH}.[24] Therefore, we obtain the following rent scheme from the two lower bounds:

$$u_{LL} = \Delta\theta\overline{q}^*, u_{LH} = \Delta\theta\overline{q}^* - \frac{p_{LL}}{p_{LH}}\Delta\theta(\overline{q}^* - \widehat{q}^*), u_{HL} = u_{HH} = 0.$$

Under this scheme, the L-type's incentive compatibility constraint is binding and the H-type's one is slack if $p_{LL} > p_{LH}$ holds and if $\Delta\theta$ is small enough. The L-type's ex post participation constraint when the other reports H-type is slack if $\Delta\theta$ is small enough. We assume in what follows that $p_{LL} > p_{LH}$ holds and $\Delta\theta$ is small enough.[25]

When the principal offers the above rent scheme, there exists room for collusion only for the coalition composed of one L-type and one H-type. First, the coalition always has the incentive to manipulate its report to (θ_H, θ_H). The manipulation does not affect the H-type's rent since his ex post participation constraint always binds while it increases the L-type's rent since the following inequality holds:

$$u_{LH} < \Delta\theta\overline{q}^*. \tag{23}$$

Second, the coalition may have the incentive to manipulate its report to (θ_L, θ_L) since, when correlation is very strong, the following inequality can hold:

$$u_{LH} < 2u_{LL} - \Delta\theta\underline{q}^*. \tag{24}$$

However, in the presence of asymmetric information, when δ/ϵ is equal to p_{LL}/p_{LH}, the above rent scheme satisfies the two incentive constraints for the coalition composed of one L-type and one H-type:

$$u_{LH} \geq \Delta\theta\overline{q}^* - \frac{p_{LL}}{p_{LH}}\Delta\theta(\overline{q}^* - \widehat{q}^*). \tag{25}$$

$$u_{LH} \geq 2u_{LL} - \Delta\theta\underline{q}^* - \frac{p_{LL}}{p_{LH}}\Delta\theta(\underline{q}^* - \widehat{q}^*), \tag{26}$$

Hence, asymmetric information allows the principal to implement the optimal grand-mechanism without side-contracting in a collusion-proof way. Therefore, we have:

[24] In the proof of proposition 5, for any given $\varepsilon > 0$, we find a collusion-proof grand-mechanism which gives the principal a payoff which is ε-close to the payoff obtained by taking δ/ϵ equal to p_{LL}/p_{LH}.

[25] We note that we already assumed that $\Delta\theta$ is small enough for $\rho > \rho^*$ in Proposition 1.

Proposition 5 *Suppose that for $\rho > \rho^*$, $p_{LL} > p_{LH}$ holds and $\Delta\theta$ is small enough and collusion takes place under asymmetric information between the agents.*

(i) In the case of strong positive correlation ($\rho > \rho^*$),the principal can implement the optimal grand-mechanism without side-contracting which exhibits a non-monotonic quantity schedule in a collusion-proof way without additional loss.

(ii) Therefore, the principal can implement the optimal grand-mechanism without side-contracting in a collusion-proof way without additional loss regardless of the sign and the degree of correlation.

Proposition 5(ii) results from proposition 4(i)and proposition 5(ii). The intuition of the result that asymmetric information can make collusion fail can be given as follows. We first note that in order to realize the gains from collusion, the third-party should require an H-type to produce more quantity than the quantity he would produce in the absence of collusion. Since the rent that an L-type can obtain by pretending to have an H-type to the third-party is increasing in the quantity produced by an H-type, an L-type' incentive to pretend to have an H-type is higher in the presence of collusion than in the absence of collusion. As a consequence, the third-party must concede to an L-type a rent larger than the one he would obtain in the absence of collusion. This increase in the rent is the transaction costs in side-contracting generated by asymmetric information, which makes collusion fail.

To illustrate our point, we consider the case in which the third-party implements the manipulation of report from (θ_L, θ_H) and (θ_H, θ_L) to (θ_H, θ_H) given that the principal offers the rent scheme previously described. The expected gains from this manipulation of report is given by $p_{LL}\Delta\theta(\overline{q}^* - \widehat{q}^*)$. Suppose now that agent i has the L-type and pretends to have an H-type when he reports to the third-party while j (with $j \neq i$) reports truthfully regardless of his type. Then, regardless of j's type, the third-party will ask the agents to report (θ_H, θ_H) to the principal and consequently the agents will always produce $\overline{q}^*$. Hence, an L-type can have a rent equal to $\Delta\theta\overline{q}^*$ by pretending to have an H-type, which is strictly larger than the rent he would obtain in the absence of the manipulation of report, $\frac{p_{LL}}{p_{LL}+p_{LH}}\Delta\theta\widehat{q}^* + \frac{p_{LH}}{p_{LL}+p_{LH}}\Delta\theta\overline{q}^*$. Hence, the expected increase in the L-type's rent is given by $p_{LL}\Delta\theta(\overline{q}^* - \widehat{q}^*)$, which represents the transaction costs in side-contracting generated by asymmetric information. Therefore, the transaction costs are as large as the gains from collusion. In the proof of proposition 5, we show that the principal can make the transaction costs strictly larger than the gains from collusion by increasing u_{LL} and u_{HL} by ε (> 0) which is small enough. Increasing u_{LL} and u_{HL} by the same amount is necessary to keep the L-type's Bayesian incentive constraint binding. Under the modified scheme, the expected gains from collusion are reduced by $\varepsilon 2p_{LH}$ while the transaction costs are not affected by ε. Therefore, the third-party cannot implement the manipulation of report. Since ε can be chosen close to zero, the principal can implement the optimal schedule in a collusion-proof way without additional cost. We note that in the case of manipulation of report from (θ_L, θ_H) and (θ_H, θ_L) to (θ_L, θ_L), the transaction costs are much lager since an L-type can have an informational rent equal to $\frac{p_{LL}}{p_{LL}+p_{LH}}\Delta\theta\underline{q}^* + \frac{p_{LH}}{p_{LL}+p_{LH}}\Delta\theta\overline{q}^*$ by pretending to have an H-type.[26]

In summary, asymmetric information puts restrictions on the plausible rules to share the gains from collusion such that only the rules that give an L-type more than the gains from collusion are incentive compatible, which makes collusion fail. The sufficient condition $p_{LL} > p_{LH}$ is easily satisfied for $\rho > \rho^*$ if the probability of having two H-types is close to the probability of having two L-types since, when $p_{LL} = p_{HH}$, the condition holds if $\rho > 0$. Laffont and Martimort (2000) show that, in the polar case of almost perfect correlation, the principal can achieve an almost first-best outcome by implementing a non-monotonic quantity schedule. Therefore, proposition 5 extends their result to the case of $\rho > \rho^*$.

6 Failure to collude under asymmetric information: dominant-strategy implementation

In this section, we study the impact of asymmetric information on collusion under dominant-strategy implementation (i.e. each agent is induced to tell the truth whatever the other agent's report). Since dominant-strategy mechanisms are not sensitive to beliefs that agents have about each other, the principal might prefer these mechanisms to Bayesian mechanisms. However, focusing on dominant-strategy mechanisms considerably restricts the set of mechanisms. This restriction in turn can create room for collusive behavior. Here, we study how asymmetric information helps the principal to implement the optimal mechanism without side-contracting in dominant strategies and in a collusion-proof way. As in the previous section, we first examine whether the optimal mechanism without side-contracting exhibits any room for collusion when collusion takes place under complete information. Second, we study how asymmetric information affects collusion. We

[26] Since the H-type's ex post participation constraint has to be satisfied in a side-contract, the third-party has to give at least a transfer equal to $\theta_H\underline{q}^*$ to an H-type when he implements the manipulation from (θ_L, θ_H) and (θ_H, θ_L) to (θ_L, θ_L). Hence, by pretending to have an H-type, an L-type will have at least a rent equal to $\Delta\theta\underline{q}^*$ with probability $\frac{p_{LL}}{p_{LL}+p_{LH}}$.

continue to assume that the third-party can use Bayesian mechanisms.[27]

The transfer scheme which implements the optimal grand-mechanism without side-contracting in dominant strategies is uniquely given by:

$$\left\{t_{LL} = \theta_L \underline{q}^* + \Delta\theta \widehat{q}^*,\ t_{LH} = \theta_L \widehat{q}^* + \Delta\theta \overline{q}^*,\ t_{HL} = \theta_H \widehat{q}^*,\ t_{HH} = \theta_H \overline{q}^*\right\}.$$

When the optimal schedule is not monotonic, the above transfer scheme does not satisfy the H-type's incentive compatibility constraint when the other agent reports θ_H (10). Therefore, in this section, we focus on the case in which the schedule is monotonic. For expositional facility, we introduce some definitions.

Definition 4 *The quantity schedule is called I-decreasing if* $\underline{q}^* > \widehat{q}^* \geq \overline{q}^*$ *and* $\underline{q}^* - \widehat{q}^* < \widehat{q}^* - \overline{q}^*$ *hold and D-decreasing if* $\underline{q}^* > \widehat{q}^* \geq \overline{q}^*$ *and* $\underline{q}^* - \widehat{q}^* \geq \widehat{q}^* - \overline{q}^*$ *hold.*

The above distinction turns out to be useful when we study collusion.

6.1 Collusion under complete information

Suppose that collusion takes place under complete information between the agents. When the optimal quantity schedule is monotonic, it is easy to see that only one manipulation of report can be profitable under the above transfer scheme: the coalition composed of one L-type and one H-type will have the incentive to pretend to be the coalition composed of two L-types if the optimal quantity schedule is I-decreasing: $\underline{q}^* - \widehat{q}^* < \widehat{q}^* - \overline{q}^*$. To give the intuition about why this manipulation generates gains from collusion, we consider the case in which the degree of negative correlation is so strong (for example, p_{LL} and p_{HH} are very small) that $\underline{q}^*$ is close to $\widehat{q}^*$ while $\overline{q}^*$ is almost equal to zero. Then, if the coalition composed of one L-type and one H-type announces the truth or pretends to be the coalition composed of two H-types, its rent is equal to $\Delta\theta\overline{q}^* \approx 0$. On the contrary, if the coalition pretends to be the coalition composed of two L-types, its rent is equal to $\Delta\theta(2\widehat{q}^* - \underline{q}^*) \approx \Delta\theta\widehat{q}^* \gg 0$. Summarizing, we have:

Proposition 6 *Suppose that collusion takes place under complete information between the agents. The optimal grand-mechanism without side-contracting can be implemented in a collusion-proof way and in dominant strategies if and only if the optimal quantity schedule is D-decreasing.*

Example 1 *When* $\Delta\theta$ *is small enough, the optimal quantity schedule is D-decreasing for* ρ *with* $0 \leq \rho \leq \rho^*$.

[27] We can also require the third-party to use dominant-strategy mechanisms. Then, collusion will be even more inefficient.

6.2 Failure to collude under asymmetric information

We now consider the case in which collusion takes place under asymmetric information between the agents. The incentive constraint for the coalition made of one L-type and one H-type not to pretend the coalition made of two L-types is given as follows:

$$(\underline{q}^* - \widehat{q}^*) - (\widehat{q}^* - \overline{q}^*) \geq -\frac{\delta}{\epsilon}(\underline{q}^* - \overline{q}^*). \tag{27}$$

Since the L-type's Bayesian incentive constraint is binding when the principal offers M^D, we know that the H-type's virtual cost can be superior to his real cost. In other words, δ/ϵ can be strictly positive. A positive δ/ϵ relaxes the above coalition incentive constraint. The constraint is the most relaxed when δ/ϵ is equal to p_{LL}/p_{LH}.[28] Summarizing, we have:

Proposition 7 *In the presence of asymmetric information between the agents, the optimal grand-mechanism without side-contracting can be implemented in a collusion-proof way and in dominant strategies if the quantity schedule is decreasing and satisfies the following inequality:*

$$(\underline{q}^* - \widehat{q}^*) - (\widehat{q}^* - \overline{q}^*) > -\frac{p_{LL}}{p_{LH}}(\underline{q}^* - \widehat{q}^*).$$

Example 2 *Suppose that $\Delta\theta$ is small enough. Then, in the presence of asymmetric information, the optimal grand-mechanism without side-contracting can be implemented in a collusion-proof way and in dominant strategies for ρ with $-\rho^0 < \rho \leq \rho^*$ where ρ^0 is given by:*

$$\rho^0 = \frac{p_{LL}p_{HH}}{2}[1 + \frac{p_{LL}}{p_{LH}}].$$

We know from example 1 that when $\Delta\theta$ is small enough, there exists room for collusion whenever correlation is negative. Example 2 states that when the degree of negative correlation is not very strong, asymmetric information makes the agents unable to realize the gains from collusion.

The intuition for the result derived in this section is similar to the one given in the previous section. What is important in both sections is that in order to realize the gains from collusion, the third-party has to require an H-type to produce more quantity than he would produce in the absence of collusion. Laffont and Martimort (1997) show that in the case of no correlation, the optimal contract can be implemented in a collusion-proof way and in dominant strategies. We generalize their result to the case of weak positive and weak negative correlation.

[28] Again, to focus on the supremum of the principal's payoff, we allow δ/ϵ to be equal to p_{LL}/p_{LH}. In the proof of proposition 7, we use a different approach to obtain the result that we find by taking δ/ϵ equal to p_{LL}/p_{LH}.

7 Concluding remarks

We studied collusion in a setting in which the agents have correlated types and produce complementary inputs such that an agent inflicts information and production externalities on the other agent when he makes a report to the principal. Hence, the agents have the incentive to coordinate their reports in order to internalize the externalities. We found that when the agents collude under asymmetric information, they fail to realize the gains from collusion because of the incentive problem within the coalition and that the principal can implement the optimal mechanism without side-contracting in a collusion-proof way by judiciously exploiting the transaction costs in coalition formation.

In our model, participation constraints should be satisfied ex post since we assume that the principal cannot force the agents to bear losses. Therefore, our result implies that when participation constraints should be satisfied ex post, only the individual incentive constraints matter and the coalition incentive constraints are irrelevant. In contrast, Laffont and Martimort (2000) show that when participation constraints should be satisfied in expected terms, only the coalition incentive constraints matter and the individual incentive constraints are irrelevant. Since zero-liability and unlimited liability represent two extreme cases, it would be interesting to study the intermediate case.

Although our setting is simple[29], the insight we derive about the transaction costs in coalition formation has general implication. What is crucial for asymmetric information to create the transaction costs in our model is that in order to realize the gains from collusion, the third-party has to require an H-type to produce more quantity than he would produce in the absence of collusion. This makes the third-party, in order to induce truth-telling, give an L-type more rent than he would obtain in the absence of collusion and therefore creates the transaction costs. In contrast, if we interpret Laffont and Martimort (2000)'s weak correlation case from the point of view of our setting,[30] in order to realize the gains from collusion, the third-party should require an H-type to produce less quantity than he would have produced in the absence of collusion. Thus, an L-type would have less incentive to pretend to be an H-type to the third-party in the presence of collusion than in its absence. This is why they find asymmetric information does not create any

[29] Extention to the case with more than two agents raises the question of how to deal with subcoalitions. Extention to the case with more than two types can be done at the cost of complexity since in this case, collusion problem becomes a sort of multi-dimensional screening problem in terms of the coaltion incentive constraints.

[30] In fact, they consider a public good provision problem. Therefore, their high-valuation type (low-valuation type) corresponds to the L-Type (H-type) in our model.

transaction cost.

The above comparison shows that, from the point of view of collusion, asymmetric information can create either conflict of interest between the agents as in our case or congruence of interest as in their case. With future research, we hope to find the fundamental factors that make asymmetric information create either conflict or congruence of interest between the agents.

Finally, the stark contrast between complete information case (proposition 4(ii)) and asymmetric information case (proposition 5(i)) suggests a question for future research: it would be interesting to study the intermediate information structure in terms of each agent' knowledge about the other's type in order to see whether there is a continuity of the optimal collusion-proof mechanism with respect to the superiority of the agents' internal information.

References

Armstrong, M. and J.-C. Rochet, 1999, "Multidimensional Screening: A User's guide", *European Economic Review*, 43: 959-979.

Baron, D. and D. Besanko. 1992, "Information, Control, and Organizational Structure." *Journal of Economics and Management Strategy.* 1 (2): 237-275.

Baron, D. and D. Besanko. 1999, "Informational Alliances." *Review of Economic Studies.* 66 (4): 743-768.

Crémer, J. and R. McLean, 1985, "Optimal Selling Strategies under Uncertainty for a Discriminating Monopolist when Demands are interdependent", *Econometrica*, 53: 345-361.

Crozier, M. 1967, *The Bureaucratic Phenomenon.* Chicago, University of Chicago Press.

Dalton, M. 1959, *Men Who Manage: Fusions of Feeling and Theory in Administration*, New York, Wiley.

Dana, J. D. 1993, "The Organization and Scope of Agents: Regulating Multiproduct Industries", *Journal of Economic Theory*, 59: 288-310.

Fudenberg, D. and J. Tirole. 1992, *Game Theory*, MIT Press.

Gilbert, R. and M. Riordan. 1995, "Regulating Complementary Products: A Comparative Institutional Analysis." *Rand Journal of Economics.* 26 (2): 243-256.

Jansen, J, 1999, "Regulating Complementary Input Supply: Cost Correlation and Limited Liability", mimeo.

Jeon, D.-S., forthcoming, “Mechanism Design under Collusion and Uniform Transfers” *Journal of Public Economic Theory.*

Jeon, D.-S. and D. Menicucci, 2005, ”Optimal Second-degree Price Discrimination and Arbitrage: On the Role of Asymmetric Information among Buyers,” *Rand Journal of Economics,* 36(2).

Laffont, J.J. and D. Martimort, 1997, “Collusion under Asymmetric Information”, *Econometrica*, 65: 875-911.

Laffont, J.J. and D. Martimort, 2000, “Mechanism Design with Collusion and Correlation”, *Econometrica,* 68: 309-342.

Laffont, J.J. and E. Maskin, 1980, “A Differential Approach to Dominant Strategy Mechanisms”, *Econometrica,* 48: 1507-1520.

Laffont, J.J., and J. Tirole, 1991, “The Politics of Government Decision Making: a Theory of Regulatory Capture,” *Quarterly Journal of Economics,* 106: 1089-1127.

Lewis, T and D. Sappington. 1997. “Information Management in Incentive Problems.” *Journal of Political Economy.* 105 (4): 796-821.

Mailath, G. and A. Postlewaite, 1990, “Asymmetric Information Bargaining Problems with Many Agents”, *Review of Economic Studies*, 57: 351-368.

Pfeffer, J. 1981. *Power in Organizations.* Marshfield, MA: Pitman Publishing.

Robert, J. 1991, “Continuity in Auction Design”, *Journal of Economic Theory,* 55: 169–179.

Sappington, D, 1983, “Limited Liability Contracts between Principal and Agent”, *Journal of Economic Theory* 29: 1-21.

Appendix
Proof of proposition 1

Let

$$f(\cdot) = p_{LL}[S(\underline{q}) - 2t_{LL}] + 2p_{LH}[S(\widehat{q}) - t_{LH} - t_{HL}] + p_{HH}[S(\overline{q}) - 2t_{HH}],$$

$$g_1(\cdot) = p_{LL}(t_{LL} - \theta_L \ \underline{q}) + p_{LH}(t_{LH} - \theta_L\widehat{q}) - p_{LL}(t_{HL} - \theta_L\widehat{q}) - p_{LH}(t_{HH} - \theta_L\overline{q}),$$

$$g_2(\cdot) = p_{LH}(t_{HL} - \theta_H\widehat{q}) + p_{HH}(t_{HH} - \theta_H\overline{q}) - p_{LH}(t_{LL} - \theta_H\underline{q}) - p_{HH}(t_{LH} - \theta_H\widehat{q}),$$

$$g_3(\cdot) = t_{LL} - \theta_L \ \underline{q},$$

$$g_4(\cdot) = t_{LH} - \theta_L \widehat{q},$$
$$g_5(\cdot) = t_{HL} - \theta_H \widehat{q},$$
$$g_6(\cdot) = t_{HH} - \theta_H \overline{q}.$$

Then, the principal's problem is given by:

$$\max \ f(\cdot)$$
$$\text{subject to}$$
$$g_i(\cdot) \geq 0, \ \text{for } i = 1, \ldots, 6.$$

Define the Lagrangian as follows:

$$L(\cdot : \mu_1, \ldots, \mu_6) = f(\cdot) + \sum_{i=1}^{6} \mu_i g_i(\cdot).$$

Since $f(\cdot)$ is concave and $g_i(\cdot)$ is linear, the Kuhn-Tucker condition is sufficient. It is easy to verify that the following candidate satisfies the Kuhn-Tucker condition:

$$t^*_{HL} = \theta_H \widehat{q}^*, \ t^*_{HH} = \theta_H \overline{q}^*,$$
$$\text{when } \rho \leq \rho^*, \ t^*_{LL} = \theta_L \ \underline{q}^* + \Delta\theta \widehat{q}^*_{12}, \ t^*_{LH} = \theta_L \widehat{q}^* + \Delta\theta \overline{q}^*,$$
$$\text{when } \rho > \rho^*, \ t^*_{LL} = \theta_L \ \underline{q}^* + \Delta\theta \widehat{q}^*_{12} + \frac{p_{LH}}{p_{LL}} \Delta\theta \overline{q}^*, \ t^*_{LH} = \theta_L \widehat{q}^*,$$
$$S'(\underline{q}^*) = 2\theta_L,$$
$$S'(\widehat{q}^*) = \theta_L + \theta_H + \frac{p_{LL}}{p_{LH}} \Delta\theta,$$
$$S'(\overline{q}^*) = 2\theta_H + 2\frac{p_{LH}}{p_{HH}} \Delta\theta,$$
$$\mu_1 = 2, \mu_2 = \mu_3 = \mu_4 = 0, \mu_5 = 2(p_{LL} + p_{LH}), \mu_6 = 2(p_{LH} + p_{HH}).$$

When $\rho \leq \rho^*$, the optimal quantity schedule is monotonic in the sum of the agents' cost parameters ($\underline{q}^* > \ \widehat{q}^* \geq \ \overline{q}^*$) and when $\rho > \rho^*$, it is not monotonic ($\underline{q}^* > \ \ \overline{q}^* > \ \widehat{q}^*$).

Our candidate is the solution if $(t^*_{LL}, t^*_{LH}, t^*_{HL}, t^*_{HH}, \underline{q}^*, \widehat{q}^*, \overline{q}^*)$ satisfies the H-type's Bayesian incentive constraint. First, when the optimal quantity schedule is monotonic (i.e. $\rho \leq \rho^*$), it is manifest that the candidate satisfies strictly the H-type's Bayesian incentive constraint. Hence, the principal has a residual degree of freedom in choosing (t_{LL}, t_{LH}): a small change from the transfer scheme specified in the above candidate, keeping the L-type's Bayesian incentive constraint binding, can satisfy the H-type's Bayesian incentive constraint.

Second, when the optimal quantity schedule is not monotonic (i.e. $\rho > \rho^*$), the transfer scheme in which the principal gives rent only when both agents have an L-type maximizes the chance to satisfy the H-type's Bayesian incentive constraint. We show below that when $\Delta\theta$ is small enough, the above candidate strictly satisfies the H-type's Bayesian incentive constraint.

The H-type's Bayesian incentive constraint is satisfied when the following inequality holds:

$$p_{LH}(\Delta\theta\widehat{q} + \frac{p_{LH}}{p_{LL}}\Delta\theta\overline{q} - \Delta\theta\underline{q}) - p_{HH}\Delta\theta\widehat{q} \leq 0.$$

It can be equivalently written as follows:

$$-(p_{LL}p_{HH} - (p_{LH})^2)\widehat{q} - p_{LL}p_{LH}(\underline{q} - \widehat{q}) + (p_{LH})^2(\overline{q} - \widehat{q}) \leq 0,$$

which holds when $\Delta\theta$ is small enough, since the first term dominates the last two terms. Since the inequality is strictly satisfied, the principal has a residual degree of freedom in choosing (t_{LL}, t_{LH}).

Regardless of the sign and the degree of correlation, the set of the transfers which implement the optimal mechanism without side-contracting is given by:

$$\Big\{t_{LL} \geq \theta_L\underline{q}^*,\ t_{LH} \geq \theta_L\widehat{q}^*,\ t_{HL} = \theta_H\widehat{q}^*,\ t_{HH} = \theta_H\overline{q}^*,$$

$$p_{LL}\left(t_{LL} - \theta_L\underline{q}^*\right) + p_{LH}\left(t_{LH} - \theta_L\widehat{q}^*\right) = p_{LL}\Delta\theta\widehat{q}^* + p_{LH}\Delta\theta\overline{q}^*,$$

$$p_{LH}\Delta\theta\underline{q}^* + p_{HH}\Delta\theta\widehat{q}^* \geq p_{LH}\left(t_{LL} - \theta_L\underline{q}^*\right) + p_{HH}\left(t_{LH} - \theta_L\widehat{q}^*\right)\Big\}.$$

Proof of proposition 2

Let M^* be an initial grand-mechanism offered by the principal, which satisfies the incentive compatibility and ex post participation constraints. Let S^* be a coalition-interim-efficient side-contract with regard to the reservation utilities given by $V_i(\theta_i)$, the payoff of agent i when each agent plays non-cooperatively M^*. Suppose that the side-contract S^* contingent on the offer of the grand-mechanism M^* gives a payoff $U_i(\theta_i)$ for each agent. Then, from the interim-efficiency of the side-contract S^*, we know that it satisfies the incentive compatibility, acceptance, budget balance and ex post participation constraints.

Define now a new grand-mechanism M^{**} by $M^* \circ S^*$. We note that from the interim-efficiency of the side-contract S^*, the new grand-mechanism satisfies the incentive compatibility and ex post participation constraints. We can show that this grand-mechanism is collusion-proof. Equivalently, it is optimal for the third-party to offer the null side-contract.

Suppose not, then there exists an interim-efficient side-contract S' different from the null side-contract, which gives a sum of expected utilities strictly higher than the one achieved by the null side-contract. But this contradicts the coalition-interim-efficiency of S^*. Suppose that the side-contract S' contingent on the offer of the grand-mechanism M^{**} gives a payoff $U_i'(\theta_i)$ for each agent. Then, from the interim-efficiency of the side-contract S', it satisfies the incentive compatibility, acceptance, budget balance and ex post participation constraints. In particular, the following inequality should hold for each agent: $U_i'(\theta_i) \geq U_i(\theta_i)$. Consider now the side-contract $S^* \circ S'$ contingent on the offer of the grand-mechanism M^*. Since we have that $U_i'(\theta_i) \geq U_i(\theta_i) \geq V_i(\theta_i)$, $S^* \circ S'$ can be implemented by the third-party. Moreover, it should guarantee strictly higher utility at least for one agent without reducing the other's utility. This contradicts the interim-efficiency of the side-contract S^*.

Proof of proposition 3

We are interested in grand-mechanisms such that the H-type's incentive constraint is not binding. The third-party's problem is given by:

$$\max_{\phi, y_1, y_2} p_{LL}[t_1(\phi_{LL}) + t_2(\phi_{LL}) - 2\theta_L q(\phi_{LL})] + p_{LH}[t_1(\phi_{LH}) + t_2(\phi_{LH}) - (\theta_L + \theta_H) q(\phi_{LH})]$$

$$+ p_{LH}[t_1(\phi_{HL}) + t_2(\phi_{HL}) - (\theta_H + \theta_L) q(\phi_{HL})] + p_{HH}[t_1(\phi_{HH}) + t_2(\phi_{HH}) - 2\theta_H q(\phi_{HH})]$$

subject to

- Budget balance constraint:

$$\sum_{k=1}^{2} y_k(\theta_i, \theta_j) = 0, \ \forall (\theta_i, \theta_j) \in \Theta^2, \tag{28}$$

- L-type's Bayesian incentive constraint for agent 1:

$$p_{LL}[t_1(\phi_{LL}) - y_1(\theta_L, \theta_L) - \theta_L q(\phi_{LL})] + p_{LH}[t_1(\phi_{LH}) - y_1(\theta_L, \theta_H) - \theta_L q(\phi_{LH})]$$

$$\geq p_{LL}[t_1(\phi_{HL}) - y_1(\theta_H, \theta_L) - \theta_L q(\phi_{HL})] + p_{LH}[t_1(\phi_{HH}) - y_1(\theta_H, \theta_H) - \theta_L q(\phi_{HH})], \tag{29}$$

- L-type's Bayesian incentive constraint for agent 2 :

$$p_{LL}[t_2(\phi_{LL}) - y_2(\theta_L, \theta_L) - \theta_L q(\phi_{LL})] + p_{LH}[t_2(\phi_{HL}) - y_2(\theta_H, \theta_L) - \theta_L q(\phi_{HL})]$$

$$\geq p_{LL}[t_2(\phi_{LH}) - y_2(\theta_L, \theta_H) - \theta_L q(\phi_{LH})] + p_{LH}[t_2(\phi_{HH}) - y_2(\theta_H, \theta_H) - \theta_L q(\phi_{HH})], \tag{30}$$

• L-type's acceptance constraint for agent 1:

$$p_{LL}[t_1(\phi_{LL})-y_1(\theta_L,\theta_L)-\theta_L q(\phi_{LL})]+p_{LH}[t_1(\phi_{LH})-y_1(\theta_L,\theta_H)-\theta_L q(\phi_{LH})] \geq (p_{LL}+p_{LH})V(\theta_L), \tag{31}$$

• L-type's acceptance constraint for agent 2 :

$$p_{LL}[t_2(\phi_{LL})-y_2(\theta_L,\theta_L)-\theta_L q(\phi_{LL})]+p_{LH}[t_2(\phi_{HL})-y_2(\theta_H,\theta_L)-\theta_L q(\phi_{HL})] \geq (p_{LL}+p_{LH})V(\theta_L), \tag{32}$$

• H-type's acceptance constraint for agent 1:

$$p_{LH}[t_1(\phi_{HL})-y_1(\theta_H,\theta_L)-\theta_H q(\phi_{HL})]+p_{HH}[t_1(\phi_{HH})-y_1(\theta_H,\theta_H)-\theta_H q(\phi_{HH})] \geq (p_{LH}+p_{HH})V(\theta_H), \tag{33}$$

• H-type's acceptance constraint for agent 2 :

$$p_{LH}[t_2(\phi_{LH})-y_2(\theta_L,\theta_H)-\theta_H q(\phi_{LH})]+p_{HH}[t_2(\phi_{HH})-y_2(\theta_H,\theta_H)-\theta_H q(\phi_{HH})] \geq (p_{LH}+p_{HH})V(\theta_H), \tag{34}$$

• Ex post participation constraints for agent 1: first, when he has an L-type and the other also has an L-type,

$$t_1(\phi_{LL}) - y_1(\theta_L,\theta_L) - \theta_L q(\phi_{LL}) \geq 0, \tag{35}$$

second, when he has an L-type while the other has an H-type,

$$t_1(\phi_{LH}) - y_1(\theta_L,\theta_H) - \theta_L q(\phi_{LH}) \geq 0, \tag{36}$$

third, when he has an H-type while the other has an L-type,

$$t_1(\phi_{HL}) - y_1(\theta_H,\theta_L) - \theta_H q(\phi_{HL}) \geq 0, \tag{37}$$

last, when he has an H-type and the other also has an H-type,

$$t_1(\phi_{HH}) - y_1(\theta_H,\theta_H) - \theta_H q(\phi_{HH}) \geq 0, \tag{38}$$

• Ex post participation constraints for agent 2:

$$t_2(\phi_{LL}) - y_2(\theta_L,\theta_L) - \theta_L q(\phi_{LL}) \geq 0, \tag{39}$$

$$t_2(\phi_{HL}) - y_2(\theta_H,\theta_L) - \theta_L q(\phi_{HL}) \geq 0, \tag{40}$$

$$t_2(\phi_{LH}) - y_2(\theta_L,\theta_H) - \theta_H q(\phi_{LH}) \geq 0, \tag{41}$$

$$t_2(\phi_{HH}) - y_2(\theta_H,\theta_H) - \theta_H q(\phi_{HH}) \geq 0. \tag{42}$$

We introduce the following multipliers:

- $\rho(\theta_1, \theta_2)$ for the budget-balance constraint in state (θ_1, θ_2),
- δ_i for the L-type's Bayesian incentive constraint concerning agent i,
- v_{Li} for the L-type's acceptance constraint concerning agent i,
- v_{Hi} for the H-type's acceptance constraint concerning agent i,
- $\lambda_i(\theta_1, \theta_2)$ for the ex post participation constraint in state (θ_1, θ_2) concerning agent i.

We define the Lagrangian as follows:

$$L = E(U_1 + U_2) + \sum_{i=1,2} \delta_i (BIC)_i(\theta_L) + \sum_{i=1,2} v_{Li}(BIR)_i(\theta_L) + \sum_{i=1,2} v_{Hi}(BIR)_i(\theta_H)$$

$$+ \sum_{\theta_1,\theta_2} \rho(\theta_1, \theta_2)(BB)(\theta_1, \theta_2) + \sum_{\theta_1,\theta_2} \sum_{i=1,2} \lambda_i(\theta_1, \theta_2)(ExPostIR)(\theta_1, \theta_2).$$

Starting from a symmetric equilibrium of the grand-mechanism, the solution to (T) is symmetric and we have:

$$\delta_i = \delta, v_{Li} = v_L, v_{Hi} = v_H,$$

$$\lambda_{LL} = \lambda_1(\theta_L, \theta_L) = \lambda_2(\theta_L, \theta_L),\ \lambda_{LH} = \lambda_1(\theta_L, \theta_H) = \lambda_2(\theta_H, \theta_L),$$

$$\lambda_{HL} = \lambda_1(\theta_H, \theta_L) = \lambda_2(\theta_L, \theta_H), \lambda_{HH} = \lambda_1(\theta_H, \theta_H) = \lambda_2(\theta_H, \theta_H).$$

- Optimizing with respect to ϕ_{LL} yields:

$$\phi_{LL}^* \in \arg\max_{\phi_{LL}} [t_1(\phi_{LL}) + t_2(\phi_{LL}) - 2\theta_L q(\phi_{LL})].$$

- Optimizing with respect to ϕ_{LH} yields:

$$\phi_{LH}^* \in \arg\max_{\phi_{LH}} [t_1(\phi_{LH}) + t_2(\phi_{LH}) - (\theta_L + \theta_H + \frac{\delta}{\epsilon}\Delta\theta) q(\phi_{LH})],$$

where

$$\frac{1}{\epsilon} = \frac{p_{LL}}{p_{LH}(1 + \delta + v_L) + \lambda_{LH}}.$$

- Optimizing with respect to ϕ_{HH} yields:

When $p_{HH}(1 + v_H) + \lambda_{HH} > \delta p_{LH}$,

$$\phi_{HH}^* \in \arg\max_{\phi_{HH}} [t_1(\phi_{HH}) + t_2(\phi_{HH}) - 2(\theta_H + \frac{\delta}{\epsilon'}\Delta\theta) q(\phi_{HH})],$$

where

$$\frac{1}{\epsilon'} = \frac{p_{LH}}{p_{HH}(1 + v_H) + \lambda_{HH} - \delta p_{LH}}.$$

When $p_{HH}(1+v_H)+\lambda_{22} < \delta p_{LH}$,

$$\phi^*_{HH} \in \arg\min_{\phi_{HH}} \; [t_1(\phi_{HH}) + t_2(\phi_{HH}) - 2(\theta_H + \frac{\delta}{\epsilon'}\Delta\theta)q(\phi_{HH})].$$

When $p_{HH}(1+v_H)+\lambda_{22} = \delta p_{LH}$,

$$\phi^*_{HH} \in \arg\max_{\phi_{HH}} \; -\delta q(\phi_{HH}).$$

• After optimizing with respect to side-transfers, we obtain some relationships between multipliers, which allow us to write ϵ' equivalently as follows:

$$\frac{1}{\epsilon'} = \frac{p_{LH}}{p_{HH}(1+\delta+v_L) + \delta(\frac{p_{LL}p_{HH}-p^2_{LH}}{p_{LH}}) + \frac{p_{HH}}{p_{LH}}(\lambda_{LH}-\lambda_{HL}) + \lambda_{HH}}.$$

• Note that (31) to (34) are binding for a collusion-proof mechanism. Hence, for such a mechanism, the slackness conditions obtained from the Lagrangian optimization do not give any information on the multipliers v_L and v_H. Furthermore, if the H-type's incentive constraint and the L-type's ex post participation constraints are binding in a grand-mechanism which is collusion-proof, the slackness conditions do not give any information on the multipliers δ, λ_{HL} and λ_{HH} either. Hence, the principal has some flexibility in choosing $\frac{\delta}{\epsilon} \in \left[0, \frac{p_{LL}}{p_{LH}}\right)$ and $\frac{\delta}{\epsilon'}$.[31]

Proof of proposition 4

Since (i) is proved in the main texts, we only need to prove (ii). The proof is based on a standard revealed preference argument. Let ϕ_{LH} (respectively, ϕ_{HH}) the optimal manipulation of report for the coalition (θ_L, θ_H) or (θ_H, θ_L) (respectively, (θ_H, θ_H)). Then, the following inequalities should be satisfied: for the coalition composed of one L-type and one H-type,

$$t_1(\phi_{LH}) + t_2(\phi_{LH}) - (\theta_L+\theta_H)q(\phi_{LH}) \geq t_1(\phi_{HH}) + t_2(\phi_{HH}) - (\theta_L+\theta_H)q(\phi_{HH});$$

for the coalition composed of two H-types,

$$t_1(\phi_{HH}) + t_2(\phi_{HH}) - 2\theta_H q(\phi_{HH}) \geq t_1(\phi_{LH}) + t_2(\phi_{LH}) - 2\theta_H q(\phi_{LH}).$$

After summing the two inequalities, we obtain $q(\phi_{LH}) \geq q(\phi_{HH})$.

Using the same kind of argument, we can obtain $q(\phi_{LL}) \geq q(\phi_{LH})$. Hence, only a monotonic quantity schedule $(q(\phi_{LL}) \geq q(\phi_{LH}) \geq q(\phi_{HH}))$ can be implemented in a collusion-proof way when collusion takes place under complete information.

[31] To obtain our results, we just need that the principal can choose the value of some multipliers to make $\frac{\delta}{\epsilon'}$ strictly positive.

Proof of proposition 5

We only need to prove (i). Suppose that the principal offers the optimal grand-mechanism without side-contracting with the following transfer scheme:

$$t_{LL} = \theta_L \underline{q}^* + \Delta\theta\overline{q}^*, t_{LH} = \theta_L \widehat{q}^* + \Delta\theta\overline{q}^* - \frac{p_{LL}}{p_{LH}}\Delta\theta(\overline{q}^* - \widehat{q}^*),$$

$$t_{HL} = \theta_H \widehat{q}^*, \ t_{HH} = \theta_H \overline{q}^*.$$

First, we show that the coalition composed of one L-type and one H-type has the incentive to manipulate its reports in the absence of transaction costs. The coalition has the incentive to announce (θ_H, θ_H) if the following inequality holds:

$$u_{LH} < \Delta\theta\overline{q}^*,$$

which is the case since we have $u_{LH} = \Delta\theta\overline{q}^* - \frac{p_{LL}}{p_{LH}}\Delta\theta(\overline{q}^* - \widehat{q}^*)$.

The coalition has the incentive to announce (θ_L, θ_L), if the following inequality holds:

$$u_{LH} < 2u_{LL} - \Delta\theta\underline{q}^*,$$

which is the case if $\rho \geq (\frac{2p_{LH}}{p_{LL}} + 1)\rho^*$.

It is easy to see that there does not exist any room for collusion for the other coalitions.

Second, we examine the third-party's problem: whether or not the third-party can successfully implement the manipulation of report from (θ_L, θ_H) and (θ_H, θ_L) to (θ_L, θ_L) or (θ_H, θ_H). Since the two agents are perfectly symmetric, without loss of generality, we focus on the set of symmetric side-contracts. Consider the stochastic manipulation from (θ_L, θ_H) and (θ_H, θ_L) to (θ_L, θ_L) with probability p ($0 \leq p \leq 1$) and from (θ_L, θ_H) and (θ_H, θ_L) to (θ_H, θ_H) with probability $1-p$. Let $\widehat{y}$ the transfer from the L-type to the H-type when the agents announce (θ_L, θ_H) or (θ_H, θ_L). To show that it is strictly impossible for the third-party to implement the above stochastic manipulation, we consider the following ε-close optimal scheme:

$$u_{LL} = \Delta\theta\overline{q}^* + \varepsilon, u_{LH} = \Delta\theta\overline{q}^* - \frac{p_{LL}}{p_{LH}}\Delta\theta(\overline{q}^* - \widehat{q}^*), \ u_{HL} = \varepsilon, \ u_{HH} = 0,$$

where $\varepsilon(> 0)$ is small enough in order not to create any other room for collusion. It is easy to check, under the above scheme, that the individual incentive constraints and the ex post participation constraints are satisfied and that there exists no other stake of collusion except the manipulation of report from (θ_L, θ_H) and (θ_H, θ_L) to (θ_L, θ_L) or (θ_H, θ_H).

Suppose that the principal proposes the above scheme and the third-party wants to implement the stochastic manipulation using $\widehat{y}$.

Then the side-contract should satisfy the L-type's Bayesian incentive constraint, which is given by:

$$p_{LL}u_{LL} + p_{LH}[pu_{LL} + (1-p)\Delta\theta\overline{q}^* - \widehat{y}] \geq p_{LL}[pu_{LL} + (1-p)\Delta\theta\overline{q}^* + \widehat{y}] + p_{LH}\Delta\theta\overline{q}^*. \quad (43)$$

It gives an upper bound for $\widehat{y}$:

$$\frac{p_{LL}(1-p) + p_{LH}p}{p_{LL} + p_{LH}}(u_{LL} - \Delta\theta\overline{q}^*) \geq \widehat{y}. \quad (44)$$

The side-contract should satisfy the H-type's acceptance constraint, which is given by:

$$p_{LH}[p(u_{LL} - \Delta\theta\underline{q}^*) + (1-p)u_{HH} + \widehat{y}] + p_{HH}u_{HH}$$
$$\geq p_{LH}u_{HL} + p_{HH}u_{HH}.$$

It gives a lower bound for $\widehat{y}$:

$$\widehat{y} \geq u_{HL} + p(\Delta\theta\underline{q}^* - u_{LL}). \quad (45)$$

Hence, for the third-party to implement the stochastic manipulation, there should be a transfer $\widehat{y}$ which satisfies the following constraints:

$$\frac{p_{LL}(1-p) + p_{LH}p}{p_{LL} + p_{LH}}\varepsilon \geq \widehat{y} \geq \varepsilon + p(\Delta\theta\underline{q}^* - u_{LL}).$$

Since the L.H.S. is strictly smaller than ε while the R.H.S. is greater than ε, we have shown that the third-party cannot implement the stochastic manipulation.

The same kind of argument can be applied when we consider only the manipulation from (θ_L, θ_H) and (θ_H, θ_L) to (θ_L, θ_L) (or from (θ_L, θ_H) and (θ_H, θ_L) to (θ_H, θ_H)). Since we can choose ε arbitrarily small, the principal can implement the optimal grand-mechanism without side-contracting in a collusion-proof way without additional loss.

Proof of proposition 7

Here, we use a different approach to obtain the result that we found by taking $\frac{\delta}{\epsilon}$ equal to $\frac{p_{LL}}{p_{LH}}$. Suppose that the principal proposes M^D. We investigate whether or not the third-party can successfully implement the manipulation from (θ_L, θ_H) and (θ_H, θ_L) to (θ_L, θ_L) by examining the third-party's problem. Hence, when the agents announce to the third-party either (θ_L, θ_L) or (θ_H, θ_H), there is no manipulation of report and no side-transfer. Because the two agents are perfectly symmetric, without loss of generality, we focus on the set of symmetric side-contracts. Let $\widehat{y}$ the transfer from the L-type to the H-type when the agents announce (θ_L, θ_H) or (θ_H, θ_L).

The L-type's Bayesian incentive constraint is given by:

$$p_{LL}(t_{LL} - \underline{\theta}\ \underline{q}^*) + p_{LH}(t_{LL} - \widehat{y} - \underline{\theta}\underline{q}^*) \geq p_{LL}(t_{LL} + \widehat{y} - \underline{\theta}\underline{q}^*) + p_{LH}(t_{HH} - \underline{\theta}\overline{q}^*). \quad (46)$$

After some calculations, it can be simplified as follows:

$$\frac{p_{LH}}{p_{LL} + p_{LH}}(\widehat{q}^* - \overline{q}^*)\Delta\theta \geq \widehat{y}. \quad (47)$$

The H-type's ex post participation constraint when the other agent has an L-type is given by:

$$t_{LL} + \widehat{y} - \theta_H \underline{q}^* \geq 0. \quad (48)$$

After some calculations, it can be simplified as follows:

$$\widehat{y} \geq (\underline{q}^* - \widehat{q}^*)\Delta\theta. \quad (49)$$

Hence, the third-party cannot successfully implement the manipulation of report, if the following inequality holds:

$$(\underline{q}^* - \widehat{q}^*)\Delta\theta > \frac{p_{LH}}{p_{LL} + p_{LH}}(\widehat{q}^* - \overline{q}^*)\Delta\theta, \quad (50)$$

which is equivalent to the following inequality:

$$(\underline{q}^* - \widehat{q}^*) - (\widehat{q}^* - \overline{q}^*) > -\frac{p_{LL}}{p_{LH}}(\underline{q}^* - \widehat{q}^*). \quad (51)$$