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General equilibrium, coordination and multiplicity on spot markets.

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General equilibrium, coordination and multiplicity on spot markets.

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Résumé : Ce texte constitue une version anglaise légèrement révisée d'un article paru dans la "Revue d'Economie Politique" 112 (5) sept-oct 2002. Il passe en revue des travaux récents concernant la coordination des anticipations dans les modèles d'équilibre général de la tradition walrassienne. Il évoque de façon cursive les problèmes de multiplicité de l'équilibre général inter-temporel dans les modèles à horizon infini et les questions d'apprentissage « divinatoire » de l'équilibre.   Il traite plus systématiquement des difficultés créées par la multiplicité des équilibres au comptant dans les modèles à horizon fini et des relations entre coordination et incomplétude.

Equilibre général, coordination, multiplicité.

Abstract : This is a slightly revised English version of a paper published in the "Revue d'Economie Politique" 112 (5) sept-oct 2002. The text reviews recent work on expectational coordination in general equilibrium models of the Walrasian tradition. It evokes briefly the multiplicity questions associated with infinite horizon models and the issues associated with "eductive learning". It examines in a more systematic way the coordination difficulties that would arise in finite horizon models with spot multiplicity and discusses the relationship between coordination and incompleteness.

General equilibrium models, coordination, multiplicity.

JEL : D5,E3

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[1] This is a text to appear in "Beliefs, Knowledge and Economics" Eward Edgar.   I thank the translator for careful work.

1 Introduction

Before embarking into our main argument, we should point out that any economic or social theory, based on the assumption that individuals enjoy a degree of autonomy, must explain both their motivation and their cognitive representations of the world. Economic theory often postulates the rationality of motives, to which it associates preferences described through utility functions. Representations or beliefs in this context take the form of expectations, a functional reduction that comes naturally into general equilibrium theory, the subject to which this contribution is mainly devoted.

The world of static general equilibrium may be imaginary but it is a world whose understanding has proved surprisingly instructive in deciphering the complexity of interactions between real markets. In this world, the only information affecting agents's decisions, apart from any private information they may have, is embodied in equilibrium prices, supposing that the mechanisms leading to their realisation are in place. In a sequential context, the relevant beliefs concern future prices. Such beliefs on future prices are captured in formal models by estimates in the form of probabilistic expectations. In this context, a reflection on beliefs is therefore tantamount to a reflection on price expectations.

The objective of this introductory article is not to present an exhaustive retrospective of the history of research on expectations in Walrasian general equilibrium models. While attempting to sketch out a general overview, what follows touches upon on certain aspects of this history, focussing on finite horizon general equilibrium models and stressing the difficulties associated with the multiplicity that can affect spot market equilibria. These aspects of the work on expectations in general equilibrium models are not amongst the best known in economics but, I believe, are of unquestionable relevance to both economic theory and economic history.

Section 2 of this paper presents the inter-temporal version, à la Radner, of the Arrow-Debreu general equilibrium model; it discusses the role that the related concepts of completeness and of 'rationality of expectations' have for allocative efficiency. In this

context, section 3 introduces the at times contradictory but often complementary positions taken by research on expectations (subsection 3-1), then goes on to evoke the problem of inter-temporal equilibrium multiplicity in infinite horizon models (subsection 3-2).; particularly in models that placed the analysis of expectations at centre stage (overlapping generation models) in Section 4 describes the problems of expectation coordination that follows from the possibility of multiple 'spot' equilibria over time, as discussed in subsection 4-1. Attention is then turned to the analysis of stochastic equilibria by focussing on the effects of <u>ex-ante</u> and <u>ex-post</u> insurance (subsection 4-2) and the possible roles of redundant assets (subsection 4-3). Section 5 returns to the theme of uniqueness versus spot multiplicity (section 5-1) and introduces incompleteness into the analysis. In section 6, in order to get a quick glimpse of the rest of the panorama, the focus moves from multiplicity to learning and particularly to eductive learning which emphasises the 'cognitive' stability of expectations, setting aside the problem of multiplicity.

2 From the Arrow-Debreu equilibrium to the equilibrium of plans, prices and price expectations

Let us start from a retrospective on the emergence of the theme of expectations and more specifically of rational expectations in general equilibrium theory after the 1970s.

2.1 The Arrow-Debreu model

The mathematical economics of the 1950s and 1960s transformed the Walrasian general equilibrium model into the Arrow-Debreu model. There is little point in emphasizing the merits of this work in this context. The rigour and the greater generality of the argument in particular allow an at times considerable extension of the interpretations. By distinguishing

goods by date or even by the state of nature in which they become available, this formally static theory takes both time and uncertainty into account.

An Arrow-Debreu inter-temporal equilibrium specifies prices for dated goods: these are discounted prices and the evolution of the prices of a physical good over time gives rise to interest rates that are specific to this good. In this model, the prices are announced and the decisions to exchange and consume are taken at the beginning of time, even if transactions only take place at the moment in which the goods become available.

In order to illustrate the model, in what follows I shall examine an exchange economy with two goods in each period and two types of agents (to simplify, I assume that each type is constituted by a continuum of identical infinitesimal agents whose total mass comes to 1/2, so that everything takes place as if there were two representative agents). I shall at times refer to this model as the 2x2x2 model.[1] The reader will note that this simple economy has the advantage that it can be graphically represented within an Edgeworth box.
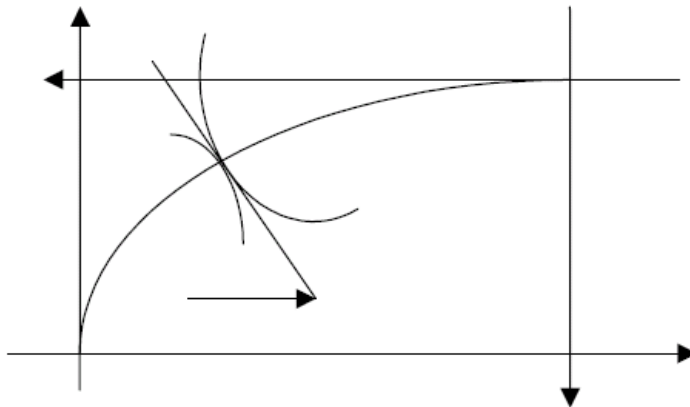

Figure 1: The 2x2x2 model

The markets for dated future goods are forward markets and the Arrow-Debreu model therefore describes a system in which, in traditional language, spot markets for

goods available today (in period 1 for the two-period model) coexist with forward markets for each of the goods available tomorrow.

2.2 The sequential model

We can substitute a purely sequential organisation to the type of forward market organisation just introduced. Now goods are only negotiated on spot markets, in other words, when they become available; the only forward markets - where a trade and the actual realisation of the transaction occur at different dates - are financial markets. In the simple 2x2x2 model used to illustrate the argument, during the first period, the sequential organisation means spot markets for available goods coexist with a financial market that operates as a medium of exchange between income (or the numeraire) in period one and income (or the numeraire) in period two. In period two, there exist only spot markets where the available goods are exchanged.

We can formalise the competitive equilibrium of the sequential organisation in a variety of ways. In the logic of the old temporary equilibrium, expectations are treated, similarly to preferences, as exogenous. Preferences can be made endogenous by using rational expectations: in the 2x2x2 model this would consist in making them perfect in the sense of assuming perfect foresight.

This is exactly what the concept of equilibrium of prices, plans and price expectations (EPPPE) proposed by Radner (1972) does in this context.[2]

The new formalisation recognises the sequential character of the exchanges so that in this sense it appears to be more 'realistic' than the Arrow-Debreu story.

The idea of the superiority of realism would however be more convincing if it were supported by a theory explaining the formation of markets. Outlining such a theory goes beyond the scope of this article, even though, as we shall see later on, a complete answer to the issues that will be raised, would require a more satisfactory analysis of the conditions for and obstacles to the creation of markets.

2.3 Efficiency and the sequential organisation of exchange

In the framework we have just outlined, the allocations of the forward market model are identical to those of the spot market, as long as the system of financial markets is sufficiently 'deep'. In the 2x2x2 model the proof of the identity of the Arrow-Debreu and Radner equilibrium allocations is particularly simple. However, the property is more general. As long as markets are essentially complete in the Hahn (1973) sense, as Arrow (1953) had suggested and as Guesnerie and Jaffray (1974) proved quite generally in a context combining both time and uncertainty, the equilibrium allocations of the two finite horizon models coincide.[3]

The sequential organization of exchange actually allows the emergence of an Arrow-Debreu (Pareto-optimal) equilibrium: Muth's rational expectations hypothesis, the fact that agents do not make systematic errors in predicting the future state of the economy, which in this context leads to a Radner type EPPPE, is in some ways the missing link between the static competition model and the dynamic competition model. Once again, we find rational expectations going hand in hand with an optimistic vision of the functioning of markets, as it was already the case when the rational expectations hypothesis was introduced into monetary macroeconomics.

We should of course emphasise the conditions under which the above report is true. Intuitively, the rationality of expectations, which guarantees the absence of forecast errors in the sense adopted by the EPPPE should be a necessary condition for efficient allocation. It is a sufficient condition only when combined with the assumption that markets are essentially complete.

The required 'depth' of the financial markets is easy to analyse in the context of the 2x2x2 model: a securities market in period one is sufficient. This can be a lot more difficult to assess in other contexts: such an assessment inevitably raises the issue of "incompleteness", the subject of a new chapter in general equilibrium research in the 1980s.

Moreover, the coordination of expectations inherent in an EPPPE equilibrium, which is 'perfect', because the expectations are identical and accurate, raises issues relating

to the realisation of equilibrium that are a lot more delicate than may at first appear. It is to these that we shall now turn.

3 The coordination of expectations in EPPPE

3.1 Foreword

The <u>first difficulty</u> reflects the most obvious form of the multiplicity problem. Let us imagine that there are many intertemporal Arrow-Debreu equilibria and therefore many sequential PPPE equilibria. The assumption of perfect foresight and rational expectations does not lead to a unique prediction. Neither the external observer, nor the model's agents can get a univocal idea of the evolution of the economy by simply calculating the equilibrium.

The <u>second difficulty</u> is still related to multiplicity but in a less direct way. Let us suppose that only one intertemporal Arrow-Debreu equilibrium exists and, therefore, that there is only one sequential PPPE equilibrium. This sequential equilibrium defines a vector of equilibrium prices in each set of spot markets. But this vector of equilibrium prices is not necessarily, and generally has no reason to be, the only vector of spot equilibrium prices! There may well be 'spot' multiplicity along the intertemporal path of the (unique) equilibrium. Figure 2 illustrates the phenomenon in the case of a 2x2x2 model. The arrow shows the change in initial endowments associated with the exchange of numeraire-denominated securities during the first period. The unique intertemporal equilibrium consists in two price vectors, p_1, p_2^a. However, if we take the security transactions (equilibrium transactions) into account, there are three spot equilibrium vectors p_2^a, p_2^b, p_2^c
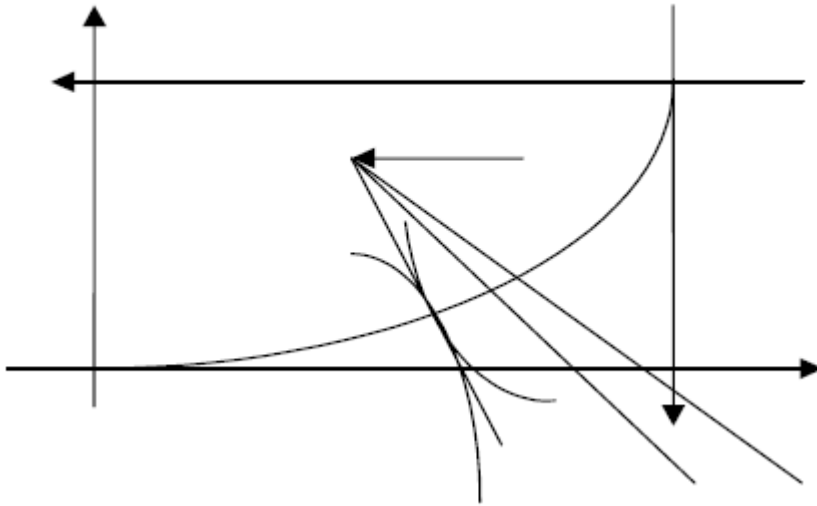
Figure 2 : Spot multiplicity, second period. The equilibrium price vectors are $p_2^a, p_2^b, p_2^c,$ *(budget lines from the left to the right)*

The third difficulty is not related to multiplicity: how can we ensure that agents will coordinate along the spot equilibrium price vector, even when this happens to be unique?

Let us try to make a certain number of polar conditions on coordination more explicit in order to place the problems that we have just informally mentioned into a better-defined perspective.

Hypothesis 1: The intertemporal 'Scarf agency'

Assume the information on preferences and allocations is public. If so, a control agency endowed with adequate computational capabilities, can calculate the Arrow-Debreu equilibrium, deduce the PPPE equilibria, choose one (if there is more than one) and announce the prices to the markets in each period. These announcements, whether they refer to effective prices or to price forecasts, will be self-fulfilling. In other words, they will become true if they are believed. According to this interpretation, all the coordination difficulties will be resolved due to the activity of a central agency – which is informed and

endowed with adequate and credible computational capabilities – and not thanks to the cognitive activity of the agents. We refer to an intertemporal Scarf agency rather than to an intertemporal Walrasian auctioneer in order to emphasise the fact that the agency is endowed with computational and information gathering capabilities that exceed those that Walras attributed to his auctioneer.[4]

Hypothesis 2: 'Common knowledge' equilibrium coordination

Let us imagine that this central coordination agency no longer exists, but that all the information on endowments and preferences is public and therefore accessible to each agent, who, moreover, is endowed with infinite computational capabilities. Let us also imagine that every agent knows that each coordinates on a specific equilibrium, or that coordination on a particular equilibrium is common knowledge. This strong cognitive hypothesis ensures, in the case in which the intertemporal equilibrium is unique, that the agents will compute it and coordinate around it. However, in the case of multiple equilibria, a further intervention is necessary, in other words, there needs to be at least a public signal highlighting one of the equilibria.

Hypothesis 3: Potential common knowledge of the equilibrium coordination with independent 'Scarf agencies'

Although the agents are potentially capable of computing the equilibria, the calculations and announcements are made by independent agencies acting sequentially. Thus, in the 2x2x2 model, the second period equilibrium prices will be announced by an agency whose rules of operation are known, but which is not necessarily synchronized with the agency operating in period one. The prices announced in the second period will of course take into account the transactions carried out in the first period. In line with the previous example, the agency in the second period will have three possible <u>a priori</u> choices, p_2^a, p_2^b, p_2^c. Without being too precise at this stage as to the underlying formal model, let us say that the independence of the second period agency means the first period agency is unable to make

unique predictions on the prices of the second period. Despite this fact, the agents are assumed to anticipate prices correctly.

<u>Hypothesis 4: Common knowledge of rationality</u>

In this hypothesis, the equilibrium in the first period depends on the activity of an agency whose operation does not interfere with the process of agents' expectations formation. The agents' forecast is the result of a collective, though private, cognitive activity which rests on two hypotheses, a sufficient knowledge (in the sense of common knowledge) of rationality, preferences and endowments of the other agents and the expectation that the operations carried out in financial markets, the second period spot equilibrium,[5] will be competitive.

This list of possibilities, which has been cursorily reviewed and whose description remains deliberately informal, does not exhaust all the possibilities. However, it has the advantage of setting out the limits of this exposition (essentially the research based on Hypothesis 3 and set out in sections 4 and 5), while also briefly presenting the difficulties associated with Hypotheses 1 and 2, the subject of what follows, and those relating to Hypothesis 4, discussed in section 6.

3.2 A parenthesis: coordination problems due to intertemporal equilibrium multiplicity

A standard finite horizon general equilibrium model with complete markets can have many PPPE intertemporal equilibria. This multiplicity raises a coordination problem that an intertemporal Scarf agency as outlined in Hypothesis 1 could resolve, but which destroys the predictive ability associated with the common knowledge of equilibrium coordination associated with Hypothesis 2. A move to infinite horizon models would exacerbate the coordination difficulties due to intertemporal multiplicity. Without delving too much into

the details of the literature that has focused on this problem, let us review some of the most pertinent results for our purposes in order to place this text into a wider perspective.

Difficulties do not mainly originate from the infinite horizon hypothesis (though we should point out that the difficulties examined in the following paragraphs increase with the length of the horizon and, in a way, do so more than proportionally), but from the passage from a finite to an infinite number of agents.[6] This move calls into question both the efficiency of the PPPE inter-temporal competitive equilibrium and its 'determinacy', in other words the fact that the equilibria are generally finite and topologically 'isolated'. These two phenomena can be easily observed in the simple overlapping generation models put forward by Allais and Samuelson (Allais 1947, Samuelson 1958). In the model without money, where agents have (too) poor endowments when they are 'old', autarchy becomes an inefficient equilibrium. In the model with money, which can be viewed either as a bubble or a kind of coordinated sequence of lump-sum transfers, we know that the stationary monetary equilibrium can be indeterminate: in that case there is a continuum of perfect forecast equilibria which converge towards this stationary equilibrium as time tends to infinity. Each of these phenomena (indeterminacy, inefficiency) is robust, as is clearly shown in a context of intermediate generality (n goods, one step forward-looking, memory 1) by the work of Kehoe and Levine (1985).

Indeterminacy poses a problem of coordination of expectations that is a lot more complicated than that posed by finite horizon intertemporal multiplicity. And this is not the only problem: in the neighbourhood of a given equilibrium (which is stationary and deterministic in the overlapping generations model referred to earlier) there can be infinite neighbouring equilibria (which are not stationary but are deterministic in the same example). Also, sunspot equilibria in the sense of Cass and Shell (1983), which are regular in the sense of being stationary, can arise founded on extrinsic uncertainty. For example, in an overlapping generations model with money, stationary sunspot equilibria (Azariadis 1981, Azariadis and Guesnerie 1982) exist once the deterministic stationary equilibrium is 'indeterminate'. All these equilibria are of course rational expectations equilibria or equilibria that are based on perfectly indexed forecasts on the sunspots (note the semantic ambiguity). De facto, these equilibria are stochastic quasi-cycles and coexist with solutions

of another class, the deterministic cycles à la Grandmont (1985).[7] Although the structure of the set of rational expectations equilibria, in particular the so-called sunspot equilibria, in finite horizon models is not fully elucidated except in the one-dimensional , one step forward looking overlapping generations model (see Grandmont 1989, Chiappori and Guesnerie 1991, Guesnerie and Woodford 1992, Guesnerie 2001a), the potential generality of the phenomena outlined here is not questionable and is largely illustrated by existing research.

4. Spot multiplicity and coordination

Let us revert to our more limited finite horizon outlook and go back to Figure 2 which provides a graphic representation of a case of spot market equilibrium multiplicity.

Again, the unique intertemporal equilibrium consists in two price vectors, p_1, p_2^a. However, taking into account the securities trades (equilibrium trades) there are three spot equilibrium vectors, p_2^a, p_2^b, p_2^c.

How should we translate the idea that coordination does not result from a perfectly centralized mechanism and that the selection of the spot equilibrium is carried out by an independent agency?

4.1 On the operation of the independent agency

To start with, we can imagine that the agency makes use of a deterministic choice mechanism, a mechanism that the logic of independent agencies suggests should depend on nothing else but the characteristics of the second period 'problem', for example the vector of initial endowments modified by the realisation of trades associated with the securities market (the starting point of the budget lines on Figure 1 and Figure 2).

However, a greater difficulty appears here: it is well known that the rule that has just been referred to, i.e. that the spot market equilibrium price is a function of the

endowments (modified by previous financial commitments), cannot be continuous if there are multiple equilibria in the domain of initial endowments under consideration.[8] This remark suggests that the independent functioning of tomorrow's market agency (in the 2x2x2 model) might be incompatible (for certain configurations of the intertemporal economy) with the existence of an equilibrium. This conjecture is not as mysterious as it might appear at first sight. If the 2x2x2 model illustrated here has only a single intertemporal equilibrium and if the rule of the independent coordination agency specifies that $p_2 = p_2^B$ for the configuration of initial endowments W', then there is no (perfect foresight) equilibrium that is compatible with the choice of the agency.

Another way we could try to compensate for the difficulty that arises due to the independence of the second period Scarf agency, would consist in allowing for complete insurance against the uncertainty of the announcement of a further equilibrium. For example, it is tempting to see the choice of the second period coordinator in our 2x2x2 model as the operation of a sunspot that takes the three values r, g and v and to make the assumption that the signal is not only observable but also verifiable and contractible. An insurance market would then allow income exchanges, arising from previous trades on contingent securities, that would precede the choice of equilibrium. But if all contingent securities exist, the market is complete and the Pareto optimal equilibrium vis-à-vis the set of random (and not only deterministic) allocations coincides, in a convex economy, with the initial equilibrium. This is the Cass and Shell (1983) 'ineffectivity" theorem. For example in our 2x2x2 model, the new equilibrium allocations with the new securities coincide with the initial equilibrium allocations. Still, have we solved our problem? No, the new securities are not exchanged at equilibrium and, paradoxically in view of our objective, the number of spot equilibria remains unchanged. We have not advanced by an iota towards the solution of the multiplicity problem. The ineffectivity theorem, contrary to what a superficial reading might suggest, does not imply that the problem of multiplicity can be resolved by an insurance that proceeds ex-ante to the choice of the spot equilibrium.

What lessons can we learn from this first analysis? That in a 'complex' sequential market the deus ex machina of coordination on the unique intertemporal equilibrium

operates with less force. Short of coordinating the coordinators (the independent agencies in this case) the emergence of a fully coordinated and efficient equilibrium is problematic.

It would appear inevitable, under the independence hypothesis we envisage, to make the choice of the agency random in the case of spot multiplicity. It would then be described by probability distributions over the equilibrium prices that would variy continuously with the vector of modified initial endowments. This assumption would allow enough flexibility in the choice of probabilities to make the distribution continuous and it should also succeed in restoring the existence of an intertemporal rational expectations equilibrium (with a random , but "rationally expected, choice of tomorrow's spot equilibrium prices). Despite the fact that the construction put forward by Mas-Colell (1991) and generalised by Kajii and Gottardi (1999) cannot be directly transposed[9] to the 2x2x2 model, it suggests that, in the case of Figure 2, we can find a stochastic equilibrium of the type I have just described.[10] We note that an equilibrium of this kind is a rational expectations equilibrium (coordination is maintained) though it ceases to be efficient!

4.2 Should we insure against spot multiplicity?

Let us admit that in the case of spot multiplicity, the independent Scarf agency chooses randomly. Let us call stochastic equilibrium or sunspot equilibrium the equilibrium that corresponds to such a choice. The noise introduced, which destroys the first best efficiency justifies the introduction of insurance. How does it perform?

Let us call ex ante insurance a system of income transfers that, conditional on the equilibrium choice, would occur at the same time as the equilibrium choice. This ex ante insurance,[11] which, as we have already seen, is not in a position to solve the multiplicity problem, would in principle attenuate the effects of a random choice of prices. It is, however, more reasonable to think that in this case, ex post insurance would be a better theoretical choice. The introduction of such an insurance mechanism would require us to complete our 2x2x2 model by introducing a third period: in this third period, after having undone all the previous trades that were contingent on the equilibrium prices, the spot markets would be reopened. We note that the securities that we have just introduced are

reminiscent of options. As the analyses of Chichilnisky, Dutta and Heal (1992) have shown, the ex post options considered here play the role of imperfect insurance vis-à-vis the choice of a coordinator à la Scarf.

4.3 Multiplicity and redundant assets

There is perhaps another solution to the problem of insuring against the hazards of the choice of equilibrium in the case of spot multiplicity.

Returning to the 2x2x2 model, we observe that the initial allocations that determine the problem of the computation of the equilibrium in the second period are not independent, for a given intertemporal equilibrium, of the securities actually present on the financial market. For example, Figure 3 shows security trades when the security assigns rights on good 2 in the subsequent period.
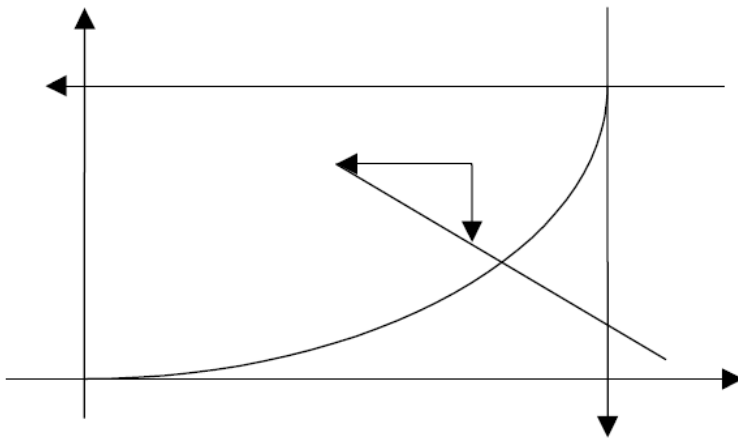
Figure 3 : Visualisation of the exchange of securities, depending on whether they are labelled in good 1 (horizontal arrow) or in good 2 (vertical arrow).

As the figure shows, although the spot equilibrium remains the same, the excess demand functions change. This remark suggests that the choice of financial assets affects

our problem and that redundant assets, in other words the assets in excess of those required to make the market essentially complete, could play a favourable role in the coordination of expectations. Indeed, Mas-Colell (1991) has shown that the introduction of a redundant asset makes the stochastic equilibria disappear in a framework that is not too distant from the model we are considering here. His argument applies here and more generally to the *n* goods version of the two period model considered here. It shows that if the number of independent assets allowing for the forward exchange of independent baskets of goods is equal to the number of goods, then a stochastic equilibrium does not exist. Since the hypothesis on the assets thus introduced goes back to assuming that the market is complete in an Arrow-Debreu sense and not only essentially complete, the intuition of the result is, in a certain sense, immediate: the addition of Radner-type assets does not have any effect on the intertemporal Arrow-Debreu allocation. Despite this, the careful Mas-Colell (1991) reader will note that, setting aside the lack of realism of the assumption on the number of assets, the result does not fully satisfactorily resolve the coordination problem arising from spot multiplicity (when this is present). At the equilibrium he describes, the agents are indifferent between trading securities in the first period or engaging in further trades of goods and coordination, which assumes equilibrium of security trades today, must, in one way or another, have some centralised aspect.

5. Uniqueness, multiplicity, incompleteness.

The stochastic equilibrium that we have just described and whose existence we have briefly discussed has the following characteristics: after the trades have taken place there are three spot equilibria and one of these is chosen randomly. We can see this, using the commonly accepted terminology, as a sunspot equilibrium. In this particular case though, the sunspot equilibrium is tied to spot multiplicity. Is this necessary?

5.1 Sunspots and spot uniqueness

Let us imagine, for example in the 2.2.2 model, that there exists only one spot equilibrium in quite a large domain of initial allocations. Is the uniqueness of the spot equilibrium compatible with the existence of a stochastic sunspot equilibrium correlated to an extrinsic random variable $s \in S$? The answer is clearly no in the two extreme cases:

- No assets exist allowing insurance against sunspots. For example, with the single security on the numeraire postulated at the beginning of this paper for the 2.2.2 model, the uniqueness of the spot equilibrium makes any stochastic allocation impossible in the second period and therefore any intertemporal stochastic equilibrium impossible as well.

- Enough assets $(\succ S)$ exist to be fully insured against sunspots. In this case the allocation is Pareto optimal and therefore, if the economy is convex, deterministic. It should moreover be noted that one consequence of the redundancy argument mentioned earlier, despite the limitations we highlighted, is that it is not necessary to have an infinite number of insurance markets in order to be fully insured against sunspots, even if the potential number of sunspots is infinite. Along similar lines, the argument put forward by Kajii (1999) shows that the addition of a sufficiently large number of 'call' options to a financial security can eliminate extrinsic uncertainty, whatever the number of sunspots considered.

The answer to the question posed (stochastic equilibrium despite spot uniqueness) cannot therefore be positive unless the number of assets is intermediate, for example, if our 2x2x2 model has $S = 3$ and two negotiable assets in the first period. The example provided by Hens (2000a) is set against this kind of framework. Although the example entails an error (Barnett and Fisher 2002), it can be modified in such a way as to save the conclusion (Hens 2000b): when insurance markets against sunspots are incomplete, despite spot uniqueness, a stochastic intertemporal equilibrium can exist.

5.2 Multiplicity, incomplete markets and sunspots

Our argument is set out in the context of the 2.2.2 model in which the financial market renders the economy essentially complete at least vis-à-vis intrinsic uncertainty. What happens when this is no longer the case? Without attempting to answer the question at a very general level, we can tackle the issue within the maintained framework of the 2x2x2 model by supposing that there is intrinsic uncertainty in the second period (applying to either preferences or endowments) and that it is associated to two signals. If the signals are not verifiable, then no contingent asset is viable and, for example, only the initial financial market is open. Figure 4 shows the equilibrium allocation with such an incomplete market.
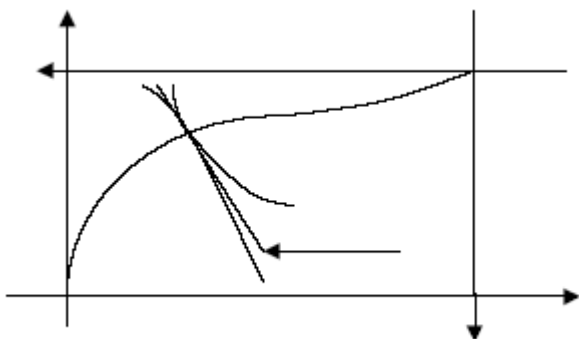

Figure 4 : Spot equilibrium, (second period), intrinsic, (and unverifiable) state of nature s

In this case, the PPPE equilibria are no longer efficient in terms of first best Pareto efficiency. However, in the case of second period spot multiplicity, a further hurdle is added to the difficulty of coordinating equilibrium choices -the subject of the preceding paragraph. Paradoxically, this difficulty appears when extrinsic uncertainty (the one tied to sunspots etc.) is, as in the paragraph that follows, contractible and can therefore act as a support to a security of the following type: a numeraire unit if the sunspot is in that state. If the securities are traded in the first period and are realised before the second period spot equilibrium has been reached, then if the number of spot equilibria in the model without extrinsic uncertainty is p, the number of equilibria in the model with extrinsic uncertainty, and insurance against it, is $2p-1$: n other words, there exist necessarily $p-1$ stochastic equilibria. This is the result obtained by Guesnerie and Laffont (1988) in a context of n goods, which holds for the 2x2x2 model with intrinsic uncertainty referred to here. It

implies there that, in the absence of endowments in the first period, so that the non contingent security is not tradable in the absence of non-contingent securities, if there were initially 3 spot equilibria in each non insurable intrinsic state, (hence 9 Radner equilibria), the number of stochastic equilibria when all susnspot contingent securities are available is, in the case suggested by Figure 4, of 8 ! One must also understand that the just suggested increased intertemporal equilibrium multiplicity will not, at least in general, reduce the multiplicity of the spot equilibria contingent on the states of an extrinsic nature. In this case the coordination difficulties are exacerbated: in our example, the number of intertemporal equilibria goes to p-1, but in each case, it is plausible that the number of spot equilibria is equal to 3. Paradoxically, the new markets created to take the sunspots into account are responsible for this exacerbation of the difficulty. Their creation increases price volatility and is an additional source of inefficiency.

The analysis put forward by Bowman and Faust (1997) is even more intriguing in a certain sense. In their analysis, in a (three period) model where we cannot insure against intrinsic uncertainty (a shock on preferences) but where there is spot uniqueness, the introduction of a 'call option' leads to the emergence of an Pareto optimal equilibrium, a point is in line with the results of traditional analysis (Ross 1976). But the introduction also gives rise to new sunspot equilibria that can be 'less good' than the equilibrium without the option. What's more, although the market is initially essentially complete, the addition of a redundant option, and hence one that can be priced using Black-Scholes type techniques, can introduce, in addition to the efficient equilibrium that remains attainable, sunspot equilibria.

The conclusion of these exploratory thoughts is not crystal clear. Let us say that if the phenomenon of a sunspot stochastic equilibrium in finite horizon general equilibrium models is not the necessary consequence of spot multiplicity, this makes it more formidable. The multiplication and / or redundancy of assets has ambiguous effects on coordination.[12] The subject deserves to be examined in greater depth, so that the results of Hens (2000b), Guesnerie and Laffont (1988), Bowman and Faust (1997) can be placed into a unified and improved framework.

6 Cognitive coordination processes

We have seen how the problem of multiplicity in finite horizon general equilibrium models that could initially appear minor takes a more complex turn once we allow for spot equilibrium multiplicity, even when the intertemporal equilibrium is unique. By concentrating on this difficulty and associating it to the problem of coordination between agencies charged with computing the equilibrium in each period, we implicitly admitted that the only difficulty in the forecast of the second period price depended on the hazards of the choice of the Scarf agency and not on the uncertainty of the conditions in which it operated. In other words, in the 2x2x2 model, we assumed that the vector of modified initial endowments, which was the starting point of the analysis, was perfectly predictable. We now abandon this hypothesis and assume that, in line with the logic of what we called Hypothesis 4, individual agents have to forecast all the data of the second period. This problem is clearly independent from the multiplicity problem, be it intertemporal or spot.[13] The analysis will therefore be carried out in relation to models with both intertemporal and spot uniqueness and which include learning.

6.1 Learning the equilibrium: evolutionary and eductive points of view

Standard evolutionary procedures (those of evolutionary learning) are based on the application of mechanical rules, that start from a limited understanding of the functioning of the system or from a limited rationality, and lead to a correction of expectations when these are found to be inaccurate. For example, in this case the agents would revise the second period price expectations that turned out to be wrong and the process would continue until convergence, for each repetition of the two period economy considered. The revision mechanisms studied are generally simple (e.g. adaptive rules) and such that their repeated application can allow agents to learn the equilibrium, in the sense that the only

resting state of the system that is compatible with the revision rule is the equilibrium state itself. There is a considerable number of works on evolutionary learning. [14]

Work on eductive learning starts from a different idea, the iterated elimination of the strategies that are not best responses, a process that leads to the set of rationalisable solutions.[15] Alternatively, this procedure can be seen as stemming from the model's assumptions of common knowledge and individual rationality.[16] I have suggested associating the success of a local eductive procedure, which, in my words, establishes the strong rationality of expectations, to the convergence of the elimination process triggered by an assumed common knowledge of a local restriction on the state of the system. Research of this kind affects both models where decisions taken independently today affect the state of the system tomorrow and infinite horizon models where the eductive expectation stability criterion is confronted with alternative criteria.[17] A detailed review of this work is presented in Guesnerie (2002).

6.2 Eductive learning in standard general equilibrium models: a brief introduction

I would like to refer to some of the studies on the eductive stability of expectations in the general equilibrium model (a two period exchange economy) a little less briefly. In this model, the agents in the first period must 'educe' the second period equilibrium price before providing the necessary information to the agency charged with computing the first period equilibrium. However, assuming the second period equilibrium prices are unique for the sake of simplicity, these depend in a way on the ability of agents to calculate the wealth transfers (through saving between the first and second period) that are decided simultaneously by the agents. The collective eductive process affects both the savings forecast and the equilibrium price simultaneously. This occurs because in the 2x2x2 model with spot uniqueness on the point expectation of initial endowments modified by the financial market's transactions (in Figure 1) the plausibility of the rational expectations hypothesis (in the sense of the aforementioned eductive stability criterion) increases with the insensitivity of tomorrow's equilibrium price to the distribution of purchasing power or

the insensitivity of savings to the goods' interest rates. Both these properties depend on the agents' preferences in their intertemporal and static aspects (see Ghosal 2000, Guesnerie and Hens 1999).

The last two models I would like to refer to are situated within a 'macroeconomic' framework where firms today make hiring decisions (Guesnerie 2001b). These decisions determine the volume of product brought to market, a volume that will be sold at the competitive price, given the total revenue distributed during production. It can be shown that the eductive stability of the Keynesian equilibrium (fixed salary) is favoured by a weak price elasticity of supply of the producers (which makes errors in the cross forecast of each agent's price expectations less significant), a strong demand elasticity (which makes tomorrow's price less uncertain) and a high value for the elementary Keynesian multiplier (the inverse of the marginal propensity to consume), which gives a positive role to the strategic complementarities associated with the increased revenue and new job opportunities by reducing the strategic substitutabilities. Curiously enough the same factors favour the eductive stability of the flexible wage Walrasian equilibrium. Despite this, in this last case, the generalisation of the argument to an n good model shows that the stabilising force of the Keynesian multiplier is reduced (Guesnerie 2001c).

7 Conclusion.

This brief review of contemporary thinking on expectations is both subjective and incomplete. It is subjective because it assigns a privileged place to the works I know best and therefore, among others, my own contributions and in particular those in sections 3.2 and 5 which only incompletely touch important subjects and considerable areas.

It is incomplete because I have only summarily treated one of the least known chapters of theoretical developments, the one relating to spot equilibrium multiplicity in finite horizon models, in this case in two period models. This choice, even if we have often gone beyond the strict problems of spot equilibrium multiplicity, is clearly debateable, given that the degree of pertinence of the phenomenon of spot equilibrium multiplicity for

the understanding of economic phenomena is an open question: which stylised facts are or could be explained by spot equilibrium multiplicity? Even if Hildebrand's work (1994) on the plausibility of the law of demand is an important exception, too few works allow us to approach this question with the seriousness it deserves in relation to the analyses referred to in this article.

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Notes

[1] In the 2x2x2 model, an Arrow-Debreu equilibrium consists in p_1, p_2, two price vectors of R^n (good 1 of the first period being the numeraire whose price equals 1), an intertemporal consumption plan for each of the agents, $\left(\overline{x_1^i}, \overline{x_2^i} \right)$, $i=1,2$, such that:

1. $\left(\overline{x_1^i}, \overline{x_2^i} \right)$ is the solution of the following programme:

$Max\ u\left(x_1^i, x_2^i \right)$

$p_1 x_1^i + p_2 x_2^i \le p_1 \omega_1^i + p_2 \omega_2^i$

1. $\sum \overline{x_1^i} \le \sum \omega_1^i, \sum \overline{x_2^i} \le \sum \omega_2^i$

[2] In the 2x2x2 model, an EPPPE consists in two given price vectors, p_1, p_2, of R^n (good 1 being in each period the numeraire whose price equals 1), an interest rate i, an intertemporal consumption plan for each of the agents, $\left(\overline{x_1^i}, \overline{x_2^i} \right)$, $i=1,2$, and positions $\overline{y^i}$, $i=1,2$, on the financial market such that:

1. $\left(\overline{x_1^i}, \overline{x_2^i} \right)$ is the solution of the following programme:

$Max\ u\left(x_1^i, x_2^i \right)$

$p_1 x_1^i + y^i \le p_1 \omega_1^i$

$p_2 x_2^1 \le p_2 \omega_2^i + (1+i) y^i$

1. $\sum \overline{x_1^i} \le \sum \omega_1^i, \sum \overline{x_2^i} \le \sum \omega_2^i$

[3] In an infinite horizon model with a finite number of participants (a fact that implies a certain number of them has an infinite life span) and under the hypothesis that they remain small, the identity of allocations also obtains (see Kehoe and Levine 1985).

[4] We could indeed see behind it the ideal "Commissariat du Plan" as set out by Massé (1971)

[5] Which we could, in the first instance, consider to be unique.

[6] Which in a certain sense affects the power of the transversality conditions.

[7] In the simple overlapping generations model, these two classes of solution are intimately related to each other, as shown by Azariadis and Guesnerie (1986): (binary) sunspot equilibria exist if and only if cycles of order 2 exist.

[8] See for example Balasko (1981).

[9] Mas-Collel's argument requires three goods and therefore is valid in the 2x3x2 version of our model.

[10] However, it is not fully enlightening on the conditions under which variations of the probability distributions would make the existence of an equilibrium with a stochastic agency probable.

[11] Whose formal definition if not realisation, the reader will verify, is not trivial.

[12] A different argument, in a different model, is put forward by Guesnerie and Rochet (1992): in this case redundancy contributes to the destabilization of expectations.

[13] We reexamine the problem in a different way, that is more modest and more ambitious at the same time. It is more modest, because, considering the recurrent character of multiplicity, we look amongst all equilibria for those that are the most credible candidates from the point of view of expectation coordination. It is more ambitious because the criteria that we adopt can lead to the rejection of a model's unique equilibrium.

[14] The majority of studies on learning do not take place within a general equilibrium context, with the exception of those that fall within the framework of overlapping generations models. In this context, many studies have been made on evolutionary learning of the stationary state (de Canio 1979, Marcet and Sargent 1989a, 1989b), of cycles (Grandmont and Laroque 1986, Guesnerie-Woodford 1992), or of sunspot equilibria

(Woodford 1990, Desgranges and Negroni 2003). The interested reader can usefully consult the review and recent work of (Evans and Honkaphoja 1997, 2001).

[15] In the framework of a strategic complementarity case, this set has a simple structure and, if the equilibrium is unique, it is reduced to it (see Cooper and John 1989, Milgrom and Roberts 1990).

[16] See Bernheim (1984), Pearce (1984), Tan and Werlang (1988).

[17] See for example Evans and Guesnerie (2003), Gauthier (2003) on the restrictions, the support but also the reasons that the eductive viewpoint provides for the choice of saddle point solutions.