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### **Strategic Substitutabilities versus Strategic Complementarities: Towards a General Theory of Expectational Coordination?**

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# Strategic Substitutabilities versus Strategic Complementarities: Towards a General Theory of Expectational Coordination?

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**Abstract:** This paper contrasts the views of expectational coordination in a stylised economic model under two polar assumptions: Strategic Complementarities (StCo) dominate or on the contrary are dominated by Strategic Substitutabilities (StSu). Although in the StCo case, 'uniqueness' often 'buys' 'eductive stability', the two issues are strikingly different in the second case. Furthermore if, in the first case, incomplete information often improves 'expectational coordination', it may have the converse effect in the StSu case. It is finally argued that, in macroeconomic contexts, StSu often unambiguously dominate StCo, even in a large class of models with Keynesian features, and even in an aggregate framework that magnifies the StCo effects. The 'remains' of StCo in general cases are discussed.

**Résumé :** Cet article compare l'analyse de la stabilité des anticipations dans un modèle économique stylisé dans deux cas polaires : Les Complémentarités Stratégiques (CoSt) dominent ou sont dominées par les Substituabilités Stratégiques (SuSt). Alors que, dans le premier cas, l'unicité va souvent de pair avec la "stabilité divinatoire", les deux questions sont largement indépendantes sous SuSt. Qui plus est, l'information incomplète, qui souvent contribue positivement à la coordination sous hypothèse de SuSt, a souvent l'effet inverse sous CoSt. Enfin, l'examen d'une classe de modèles macroéconomiques, même dans un cadre agrégé qui amplifie les CoSt, et même sous des hypothèses néo-keynésiennes, suggère que les SuSt y dominent les CoSt. Les "vestiges" des CoSt subsistant cependant dans les modèles désagrégés sont finalement discutés.

# 1 Introduction<sup>1</sup>

For a time, economic theory was tempted to view expectations as fixed data (this is the view taken in the so-called 'temporary equilibrium' literature, where expectations, like preferences, are unexplained) or moving data (adaptive expectations), but in both cases as partly exogenous data. The Rational Expectations Hypothesis, which has become dominant in formal economic theory, views expectations as endogenous: expectations, conforming to the predictions of the so-called 'relevant economic theory', tend to be 'self-fulfilling'. Although the Rational Expectations Hypothesis has brought a major and fruitful conceptual breakthrough in the way we think about expectations, only zealots of the Hypothesis would believe that the problem of expectational coordination has disappeared from the agenda of economic theory. Indeed, the title of this conference, 'Coordination failures', where coordination, if I infer from the program, refers mainly to expectational coordination, echoes this remark.

When and why expectational coordination succeeds or fails (success or failure being associated with conformity to, or divergence from, the rational expectations predictions)? In order to think about this problem, a line of reflection, broadly represented in the program of the conference, has attracted a lot of attention and produced a number of related and consistent results that have generated a clear advance in our understanding. This line of research has exploited successfully the analytical regularities generated by a certain type of assumptions concerning economic interactions, the so-called Strategic Complementarities (from now on, StCo) assumptions (a subject pioneered by Topkis, 1979). Early influential contributions are associated with the names of Cooper and John (1988), Milgrom and Roberts (1990) and Vives (1990). Carlsson and Van Damme (1993) have introduced new ideas on the role of incomplete information in StCo contexts, that will be scrutinized later.<sup>2</sup>

The reflection on expectational coordination is however far from been confined to the Strategic Complementarities literature. As an example, my own main subjects of interest over the last twenty years, endogenous fluctuations and 'eductive stability' of expectations,<sup>3</sup> designate two different lines of attack

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<sup>1</sup>This article is based on my talk at the Conference Cofail (Coordination Failures) held at Nanterre in June 2004. I thank the discussant Patrick Pintus, and Antoine d'Autume, Jean-Charles Rochet and Olivier Tercieux for their useful remarks on a preliminary version. My understanding of the issues has greatly benefited from conversation with Christophe Chamley within the framework of a 2004 EHESS seminar that we jointly organised. I am also grateful to Steve Morris who found the analysis of noisy strategic substitutabilities intriguing. The computations he then undertook with Hyun Song Shin (personal correspondence : "notes on strategic substitutes and complements in global games" (January 2005)) indeed support and shed additional light on the argument of Section 4-2. Finally, I am particularly indebted to Christian Bidard who made many suggestions : most of them have been implemented and have improved the text. I remain responsible of any shortcomings.

<sup>2</sup>These ideas have been exploited in economic contexts by (without claim to exhaustivity) Morris-Shin (1998), Morris (2000), Chamley (1999), Frankel-Pauzner (2000) and Rochet-Vives (2003).

<sup>3</sup>A number of papers I authored or co-authored are reproduced in *Assessing Rational Expectations* (MIT Press), which has two volumes with subtitles: *Sunspot Multiplicity and*

for understanding expectational coordination. The present paper is particularly concerned with the last line of research, that relative to 'eductive stability' or, to follow my previous terminology on the topic, 'strong rationality' (Guesnerie, 1988, 1992; this terminology is adopted in my work with co-authors: for example, Desgranges-Geoffard-Guesnerie, 2003, Evans-Guesnerie, 1993, 2003, Guesnerie-Rochet, 1992).<sup>4</sup>

These two lines of research, let us call them strategic complementarities and eductive stability, have common references, such as iterated dominance.<sup>5</sup> They superficially exploit similar types of argument, based on 'eductive reasoning', and refer to Common Knowledge (from now on CK) considerations. However, and quite surprisingly, they seem to have few connections. When comparing the two approaches, one might argue that the first evokes a highway whereas the second is rather a mountain trail. The comfort of the highway benefits from the power and generality of well developed analytical tools in situations where they are appropriate. The diversity of situations met in the second line of research requires, like strolling on a mountain trail, hesitating and winding approaches, and stresses solutions of doubtful universality. However, the experience from the mountain trail that will be described later suggests that, if the view of the landscape from the highway is neater and more organized, it concerns a small part of the whole landscape.

The purpose of the present paper is to give some content to the above image by revisiting the connections of the two lines of approach. The ordered landscape that the dominance of strategic complementarities allows us to discover will be first appraised (Section 2): the emphasis will successively be put on the connections that StCo establish between uniqueness and expectational stability and, then, on the now well established role of incomplete information in triggering both properties. This may shed some light on why the first one has developed in a rather autonomous way.

When strategic substitutabilities dominate (Section 3), the previous two messages are basically affected: uniqueness and expectational stability as assessed through the 'eductive stability' criterion evoked above are distinct properties; and incomplete information is no longer a powerful *deus ex machina* for the analysis of expectational stability (Section 4).

Finally (Section 5), it will be argued by means of examples that in many macroeconomic contexts, strategic substitutabilities tend to dominate strategic complementarities. The 'remains' of strategic complementarities in the explanation of expectational coordination will be discussed.

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*Economic Fluctuations* (2001) and *Eductive Stability in Economics* (2005).

<sup>4</sup>Contributions along similar lines includes Borgers (1992), Dekel-Wolinsky (2003), Desgranges (1999), Heinemann F. (1995), Heinemann M. (2002) and Gauthier (2003).

<sup>5</sup>The game-theoretical ideas of 'iterative dominance' are already present in Luce-Raiffa (1957). Early economic applications are found in Gabay-Moulin (1980) and Bernheim (1984).

## 2 Strategic Complementarities

### 2.1 The grammar of the argument

The framework considered here is rather specific, when compared to an abstract game-theoretical framework (see Topkis, 1979, Milgrom-Roberts, 1990, or Vives, 1990) in which the most general version of the properties I am describing can be expressed. Let there be a continuum of agents  $i \in [0, 1]$ , each one being concerned only with his own action  $a(i)$  and with aggregate data, denoted  $A$ , a vector of  $R^p$ , on which everybody has only an infinitesimal influence. Each individual's best response depends on the subjective probability distribution of the aggregate data, denoted  $p(i)$  (with  $p(i)$  belonging to the set  $\Delta(R^p)$  of probability distributions over the space of aggregate data). In the reduced form version of the model (see Evans-Guesnerie, 1993, for a general discussion of reduced forms of this type), agent  $i$ 's best response only depends on the probability distribution of the aggregate. It is then denoted  $b(i, p(i))$ . The aggregate situation  $B$  depends itself on the collection of actions taken by individual agents. The outcome can be formally written as  $B(\cdot) = \Phi[\Pi_i[b(i, \cdot)]]$ ,<sup>6</sup> where  $\Phi$  is some aggregation map, so that  $B$  is a mapping which associates an aggregate situation  $B(P) \in R^p$  with a profile of individual distributions (each of them is over  $R^p$ ) denoted  $P = \Pi_i p(i)$ . When agents have similar point-expectations on the aggregate state  $A$ , the outcome is denoted  $B(\mathbf{A})$  where, by abuse of notation,  $\mathbf{A}$  denotes the collection of point-expectations (Dirac measures) concentrated on  $A$ . With these notations, in **equilibrium**, *beliefs on the aggregate state are self-fulfilling*, i.e.  $A^* = B(\mathbf{A}^*)$ .<sup>7</sup> Here, the strategic complementarity (StCo) assumption means that each individual best response function increases monotonically in the sense that, given  $b(i, A)$ , we have  $b(i, \mathbf{A}') > b(i, A)$ ,<sup>8</sup> whenever  $\mathbf{A}'$  is either a deterministic point-expectation with  $A' > A$  or a stochastic aggregate with a support  $S(\mathbf{A}')$  such that  $S(\mathbf{A}') > A$ . In this basic framework, let us consider the equilibria and the 'rationalizable' outcomes. We will provide below some simple examples illustrating this abstract framework.

### 2.2 The basic StCo framework: equilibria and rationalizable outcomes

#### A one-dimension basic story

Consider the simplest version of the problem, with a one-dimension aggregate here denoted  $a$  (instead of  $A$  in the general version of the model just sketched). The graph of  $B(\mathbf{a})$ , which describes the aggregate state of the system as a function of point expectations  $\mathbf{a}$ , is increasing (Fig. 1). If  $B(0)$  is positive and

<sup>6</sup>A local approximation of the type  $B(\cdot) = \int B(i, \cdot) di$  is often used in the analysis (Evans-

Guesnerie, 1993, 2003).

<sup>7</sup>Equilibria in the present framework are 'deterministic', at least at the aggregate level.

<sup>8</sup>Symbol  $>$  designates the vector inequality:  $x > y \iff x_h \geq y_h \forall h$

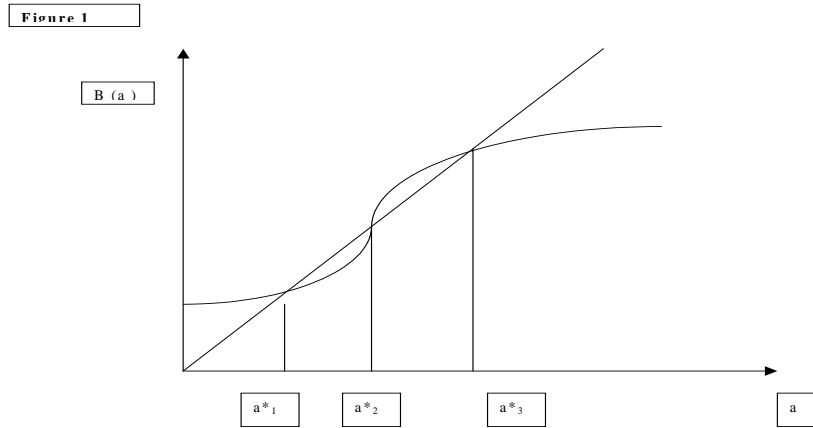


Figure 1:

$B(+\infty)$  finite, then the number of fixed points  $\{\bar{a}; B(\bar{a}) = \bar{a}\}$ , i.e. of equilibria, is 'normally' odd (three in Figure 1).

What was called earlier, in the introduction, the 'eductive' viewpoint, puts emphasis on game-theoretical 'rationalizability' arguments, the logic of which in the present context is explained in Guesnerie (2002). The 'eductive' viewpoint leads here to select the set of rationalizable outcomes. In Figure 1, this set simply consists of the segment  $[\bar{a}_1, \bar{a}_3]$ . The eductive argument sustaining the conclusion is extremely simple:

- Step 0: From the StCo assumption and the fact that  $a$  cannot be negative, the final state of the system, whatever the individual expectations may be, will be such that  $a > B(0)$ . We assume  $B(0) > 0$ .

- Step 1: Everybody knowing that  $a > B(0)$  (we start using the Common Knowledge assumption), and given StCo, the final state of the system  $a$  will be such that :  $a > B \circ B(0)$  ( $B \circ B$  or  $B^2$  denotes the second iterate of  $B$ ).

- Step 2: Everybody taking into account the outcome of step 1 (they know and know that the others know), the situation will be such that  $a > B \circ B \circ B(0)$ .

- The argument goes on, with step  $n$  taking into account the conclusion of step  $n - 1$ , and so on. Then, as illustrated by Figure 1, the iterative argument eliminates the whole segment  $[0, \bar{a}_1[$ . A symmetric argument, starting from the CK fact that  $B(+\infty)$  is finite, eliminates  $]\bar{a}_3, +\infty[$ . Hence, the conclusion.

It is worthwhile noting that, as it is implicit in the 'rationalizability' construct,<sup>9</sup> the argument relies on the fact that the model is CK and that Bayesian rationality is also CK.

<sup>9</sup>See Tan and Werlang (1988), who discuss rationalizability à la Bernheim (1984) and Pearce (1984). Similar analyses apply to dominance solvability à la Luce-Raiffa (1957), see Moulin (1984).

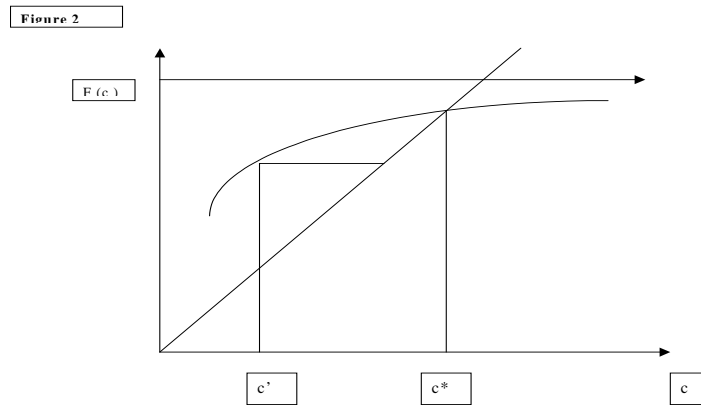


Figure 2:

### A simple one-dimension example

A simple example of a StCo situation, used now and later, has been analyzed systematically by Chamley (1999, 2004). In this model, agents either join or do not join. The state of the system is characterized by the mass  $m$  of joining agents ( $0 \leq m \leq 1$ ); the profitability of joining is supposed to be equal to  $m$ . At equilibrium, an agent whose cost  $c$  is smaller than  $m$  joins. Let  $F$  be the cumulative distribution of costs, associated with density  $f$ . If the collective belief on the mass of joining agents is  $m$ , all agents of cost smaller than  $m$ , i.e.  $F(m)$  agents, would like to join.

The model displays StCo: the more, the better for everybody. Figure 2, with the cost parameter on the horizontal axis, shows the graph of the cumulative distribution. At equilibrium, the cost  $\bar{c}$  of the highest cost joining agent is such that  $\bar{c} = F(\bar{c})$ . We now argue that  $F$  plays the role of the aggregate best response function of previous theory. Indeed, there are one (Figure 2a) or three equilibria (Figure 2b), such that  $\bar{c} = F(\bar{c})$ .<sup>10</sup> Moreover, the rationalizable set follows from an iterative argument exploiting the same simple intuition as above: The zero-cost people join anyway;<sup>11</sup> the highest-cost people ( $c$  higher than one) never join. This triggers the eductive process: at intermediate levels, agents whose cost is strictly below  $\bar{c}_1$  join if they believe that all agents below them have joined; symmetrically, agents whose cost strictly exceeds  $\bar{c}_3$  do not join if they believe that all agents above them do not join.

### A flavor of the general argument

Under StCo, the above line of argument extends to the case where  $A$  is  $n$ -dimensional. This is visualized in Figure 3, where  $A = [a_1, a_2]$  is a 2-dimension aggregate and where the aggregate best response (vector) function

<sup>10</sup>It is assumed that  $F(0) > 0$ , i.e. there is an 'atom' of zero-cost agents.

<sup>11</sup>We assume that some zero-cost people, with a positive global weight though as small as desired, do join.

$B(a_1, a_2)$  is increasing in  $(a_1, a_2)$ .

The iterative dominance argument starts by considering the South-West boundary:  $(a_1, a_2) > (0, 0) \implies B(a_1, a_2) > B(0, 0) = A_1$ . Whatever the initial beliefs of the agents may be, the aggregate state will have coordinates above those of  $A_1$ . Next, as in step 1 above, we use the fact that the previous fact is known. We derive the implication:  $\mathbf{A} > A_1 \implies B(\mathbf{A}) > B(A_1) = B \circ B(0) = A_2$ . As  $A_1 > 0$ , StCo implies that  $B(A_1) > B(0)$ , i.e.  $A_2 > A_1$ . But this new inequality is known by everybody, because it is known that the previous inequality is known. Pursuing the argument, and using further implications of the CK assumption, generates an increasing sequence  $A_1, A_2, \dots, A_n$  which, if  $B(R_+^2)$  is bounded from above, converges. Symmetrically, an iterative argument triggered by the fact that  $B(R_+^2)$  has an upper bound generates a decreasing sequence  $A'_1, A'_2, \dots, A'_n$  which converges.

Hopefully, the argument and its diagrammatic support have made clear the following key properties of the eductive coordination process leading to the set of rationalizable outcomes (those that survive the consequences of CK, as progressively derived in the above iterative process):

- *The limit points of the process are equilibria, i.e. they are states such that  $A^* = B(\mathbf{A}^*)$  (this was obvious in the one-dimension case);*
- *The set of rationalizable outcomes is a product set (here, a square) which contains all equilibria.*

### 2.3 StCo with incomplete information

It has been repeatedly argued in the literature on StCo that incomplete information, i.e. the fact that people do not know exactly the game they are playing, makes the analysis both more convincing and in a sense neater. This idea originates in the work of Carlsson-Van Damme (1993) and has been exploited by a number of authors (see footnote 2). I will introduce the flavor of the argument by relying on Chamley's model. The idea has two distinct components.

#### **Incomplete information helps 'buying' uniqueness of equilibrium**

Consider a model in which: (i) each agent observes his true cost; (ii) the costs are taken from a distribution with a known variance but an unknown mean, which is taken from a random distribution.<sup>12</sup>

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<sup>12</sup>One could also consider a variant of this model in which the agents do not know exactly their cost  $c$  (although the distribution of costs is known) but receive a noisy signal  $s$ . At a symmetric equilibrium (which is *a priori* plausible since every agent uses the same criterion), the cumulative distribution to be taken into account for the equilibrium analysis is the distribution of signals, that depends on the noise and the objective distribution of costs. The signal distribution is obtained as a mean preserving spread of the cost distribution. Is it 'flatter' than the non-noisy distribution of costs? Assume that the noisy signal is symmetric and of zero mean. The left-side cumulative distribution of the signals will be above that of the costs. This phenomenon often involves that, in the middle area, the derivative of the cumulative distribution of signals is smaller than that of the costs. As the equilibria correspond to the intersections of the cumulative distribution curve with the diagonal, the uniqueness of equilibrium is somewhat more likely. However the argument remains loose and, in general, is not sufficient to 'buy' uniqueness.



Assume that joining is determined by a trigger strategy depending on the signal received: join if  $c < \bar{c}$ , do not join if not. In these conditions, the mass of agents whose cost is smaller than yours is a random variable, whose mean is a function of the observed cost. Such a function is the cumulative of a new distribution which, under a normality hypothesis on the two underlying distributions, has a higher variance than the initial distribution of costs (Chamley, 2004, chapter 11-2). This property is not generally sufficient to conclude to uniqueness. However, if the distribution of costs, conditional on the mean, is symmetric and close to a Dirac measure, everybody thinks that a proportion close to 1/2 has received a signal lower than his own signal, therefore the relevant distribution is almost flat, and a flat enough distribution involves a unique equilibrium.

**Incomplete information does not preclude iterative elimination**

Take the last version of Chamley’s model. It is a dominant strategy to join if your cost is very low. Assume that all agents who have received a (true) cost signal less than  $c$  have joined, and that their expected mass is  $F(c) > c$ . If, being risk neutral, you receive  $c$  or  $c + e$ ,  $e$  being very small, you still join. But if  $e$  is not very small, and even if  $c + e > F(c)$ , you will rationally infer from your signal  $c + e$  that the mass of people who have received a signal smaller than  $c$  is smaller than  $F(c)$  (because your subjective evaluation of the actual mean is below the evaluation of somebody who has received the signal  $c$ ). Therefore, the profitability of joining, as considered in the mental process of elimination (under the assumption that everybody below  $c$  has joined) will not stay as flat as in Figure 1, but will drop more quickly as suggested in Figure 3. The process will still be convergent, although in longer virtual time (see Chamley, 2004, Figure 11-7).

### 3 Strategic Substitutabilities

#### 3.1 A one-dimension view of StSu

Let us come back to the one-dimension version of the model of section 2.2, but assume that  $B$  satisfies the property  $dB/da < 0$ , as in Figure 5: We have strategic substitutabilities. The Muthian model, considered in Guesnerie (1988, 1992) for assessing expectational coordination in a partial equilibrium context, provides such an example of one-dimension strategic substitutabilities. Let the variable  $a$  denote the size of the crop. As the price  $P(a)$  decreases with  $a$ , the size  $B(a)$  of the supplied crop, associated with expectations  $a$ , decreases with  $a$ . There is a unique equilibrium that can be ‘educated’ through arguments of the following type: If some farmers with high costs do not join, then the crop has a maximal size for which the price is rather low, but not so low as to prevent a few efficient enough farmers to be willing to plant their land (the argument is the

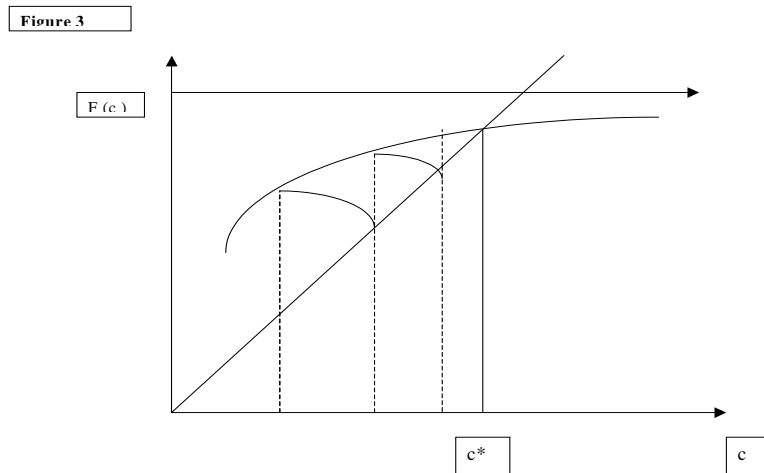


Figure 3:

same as in step one of the previous argument). But, when only these low-cost farmers participate, the size of the crop is such that the price will be lower than some (high) threshold. As a consequence, some farmers, who *a priori* would have liked to join initially, drop. Therefore, the price will be higher than assumed in the previous step, so that more low-cost farmers will join, etc. The process is described more formally in Guesnerie (1992) and its steps are illustrated in Figure 4.

Let us reformulate the model in order to make it directly comparable to the above StCo cost model: Each farmer produces only one unit at cost  $c$ . I denote  $B(u)$  the mass of farmers that produce if they expect that the mass of active farmers is  $u$ . Clearly,  $B(u) = F(P(u))$ , where  $F$  is the cumulative distribution of costs;  $B(u)$  is also the mass of farmers who want to produce if all the farmers of cost smaller than  $u$ , and only them, do produce. It is left to the reader to rewrite the just sketched 'eductive' argument in the modified setting. The reader will also verify that the analysis (either the equilibrium analysis or the 'eductive' one) is not basically modified in the presence of risk, with risk neutral farmers ( $u$  being a signal, define  $B(u)$  as  $B(u) = F(E(P(u)))$ , where  $E$  designates the expectation operator).

Note that, in this model:

- Equilibrium is unique.
- When the eductive process can be started (this condition may require some hypothesis on CK at the boundary), the set of rationalizable equilibria generally consists of a segment  $[u_1, u_2]$  such that  $u_1 = B(u_2), u_2 = B(u_1)$ , i.e. its limits are a 2-cycle of  $B$ .
- The unique equilibrium is globally eductively stable if  $B$  has no cycle. It is locally eductively stable iff  $|dB/du(u)| = |F'(P'(u))| < 1$ , i.e. if the price

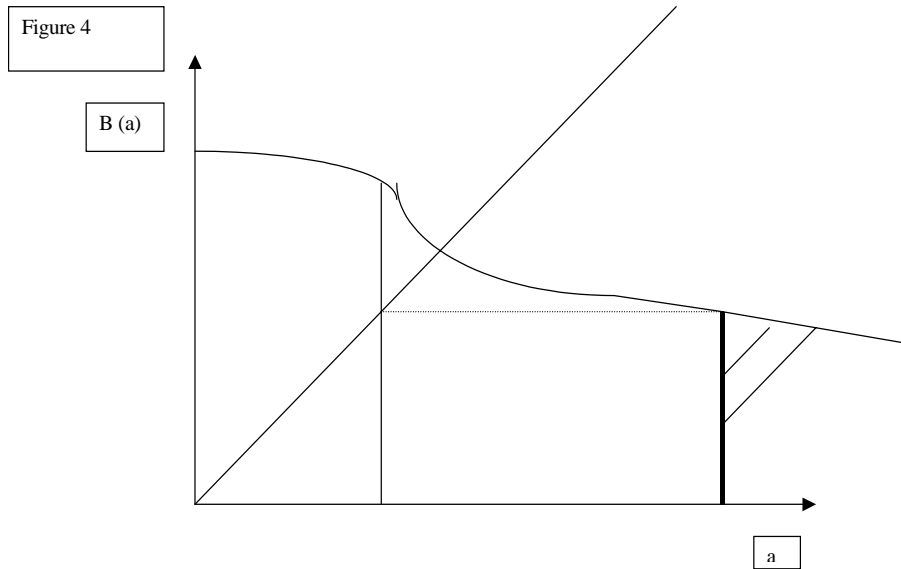


Figure 4:

elasticity of aggregate supply is smaller than the price elasticity of aggregate demand.

### 3.2 The $n$ -dimension version of the story

Figure 6 visualizes the two-dimension counterpart of Figure 2 when StSu replaces StCo. Now  $B(a_1, a_2)$  is (coordinate by coordinate) decreasing in  $[a_1, a_2]$ . We also have  $B(a_1, a_2) < B(\mathbf{A}')$  if  $\mathbf{A}'$  is any distribution the support of which is 'above'  $[a_1, a_2]$ , and  $B(a_1, a_2) > B(\mathbf{A}')$  if  $\mathbf{A}'$  is any distribution the support of which is 'below'  $[a_1, a_2]$ .<sup>13</sup>

- Assume that  $B(0) = A'_1$  and (this is not entirely innocuous)  $A_1 = B(A'_1) > 0$ . The iterative dominance argument can be started, at the North-East boundary, whenever it is CK that  $(a_1, a_2) < A'_1$ . This implies  $B(a_1, a_2) > B(A'_1) = A_1$ , where  $A_1$  is as in Figure 6.

- As in step 1 above, the CK belief  $\mathbf{A} > A_1$  implies that  $B(\mathbf{A}) < \Lambda B(A_1) = B \circ B(A'_1) = A'_2$ .

- Suppose (that is indeed an assumption) that  $A'_2 < A'_1$ . Pursuing the argument generates both an decreasing sequence  $A'_1, A'_2, \dots, A'_n$  such that  $A'_n = B \circ B(A'_{n-1})$  and an increasing sequence  $A_1, A_2, \dots, A_n$  such that  $A_n = B \circ B(A_{n-1})$ .

The procedure suggests the following key properties of the eductive coordination process leading to the set of rationalizable equilibria:

<sup>13</sup>'above', in the sense that the support belongs to  $[a_1, a_2] + R_+^2$ , 'below' when the support belongs to  $[a_1, a_2] - R_+^2$

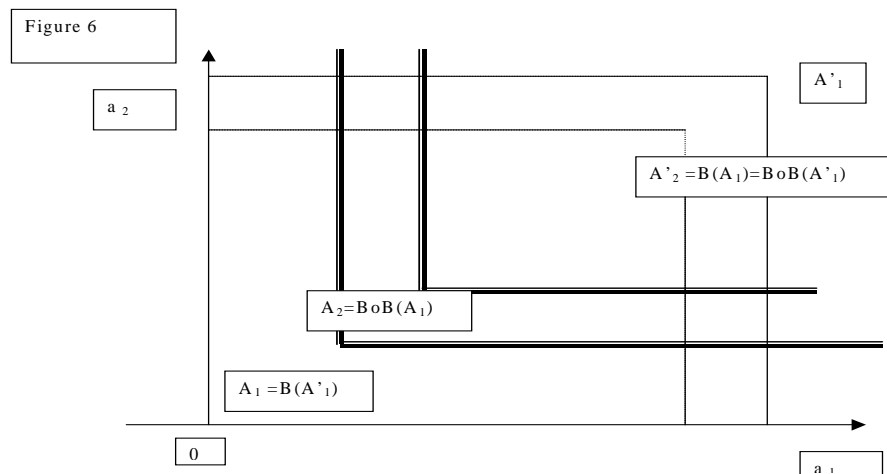


Figure 5:

- The set of rationalizable outcomes is a product set (at least, under the assumptions introduced in the above argument).
- This set contains all equilibria.<sup>14</sup> As in the one-dimension case, the limit points of the processes are cycles of order 2 of  $B$ : they are not equilibria, unless they coincide.

## 4 Comparing worlds: StCo versus StSu

We are now in a position to assess the somewhat striking differences in the conditions of expectational coordination in the polar contexts of StCo and StSu. We first consider the complete information case, in which our theory is rather general.

### 4.1 Key differences and implications

#### Main insights from the complete information model

The results obtained in the complete information framework sustain different views on expectational coordination.

Under StCo, uniqueness is not guaranteed but uniqueness implies the success of the elimination process. In my terminology, a unique equilibrium is 'strongly rational' (Guesnerie, 1992) or 'eductively stable' (Evans-Guesnerie, 1993). And stability is global (in case of multiplicity, local eductive stability may however obtain: it is the case for the extreme two equilibria of Figure 1). In short,

<sup>14</sup>In the one-dimension model, uniqueness obtains, but not necessarily in this 2-dimension version of the model.

uniqueness is the Graal: the unique equilibrium is a focal point which is furthermore singled out by the eductive mental process. Rational expectations are not, as some initial enthusiasts argued, the consequence of the rationality hypothesis. But they follow from the Common Knowledge of both the model and rationality.

Under StSu, uniqueness is still obtained in the one-dimension case, but it does not necessarily hold in the  $n$ -dimension version of the model. But uniqueness does not solve the expectational coordination problem neatly: the equilibrium may not be globally eductively stable or 'strongly rational'; the convergence property may not even hold locally (in the sense that no hypothetical CK conjecture that restricts the outcome of the situation to a close neighbourhood of the equilibrium may exist, so that there is no way to trigger, even locally, the mental process towards the equilibrium, see Guesnerie (2002)). For example, in the Muth model, local eductive stability requires that supply is less elastic than demand.

If one considers that the case for expectational coordination is serious whenever the equilibrium under consideration is locally eductively stable (which I tend to believe), uniqueness is neither a sufficient condition nor a necessary one for creating favourable conditions to 'expectational coordination'.

In a sense, the views suggested by the analysis of StSu models are embarrassing to the dominant intellectual sensitivity of the profession that adheres to the precepts of philosophical determinism, according to which the ideal of scientificity is a univoque prediction. The strong rationality criterion advocated here is at variance with such a view. The criterion may reject a unique equilibrium and, in the presence of several equilibria, it may pick none, one or several of them!<sup>15</sup>

### Mixed worlds

Let me now stress, although this point is rarely noted, that the opposition just raised between the orderly world of StCo and the anarchist outside world is not as clear cut as suggested above. The reason is that StCo in the aggregate may coexist with some elements of StSu at the micro-level. Let us adopt the notation of footnote 5, i.e.  $B(\cdot) = \int B(i, \cdot)$ , but assume  $dB/da > 0$  (this amounts to saying that  $\int [dB(i, \cdot)/da] di > 0$  does not rule out  $dB(i, a)/da < 0$  for a subset  $I_{SS}$ ). In this mixed world, uniqueness obtains when the aggregate reaction is flat enough. But, now, even uniqueness does not buy 'eductive stability': Indeed, the argument developed in Evans-Guesnerie (1993) and the posterior discussion in Guesnerie (2002), show that local 'eductive stability' requires  $\int_{I_{SS}} [-dB(i, a)/da] di + \int_{I \setminus I_{SS}} [dB(i, a)] di < 1$ . Pessimism in  $I_{SS}$  combined with optimism in  $I_{NSS}$  may perturb considerably the coordination of beliefs.<sup>16</sup> This example leads us to wonder if there is any basis for the Graal!

<sup>15</sup>In terms of the metaphor sketched in the introduction, the highway displays an orderly landscape, the mountain trail is synonym of disorder.

<sup>16</sup>This is likely to have consequences for the model of crisis à la Morris-Shin, where the homogeneity of agents, in the expectatioanal terms introduced here, is doubtful.

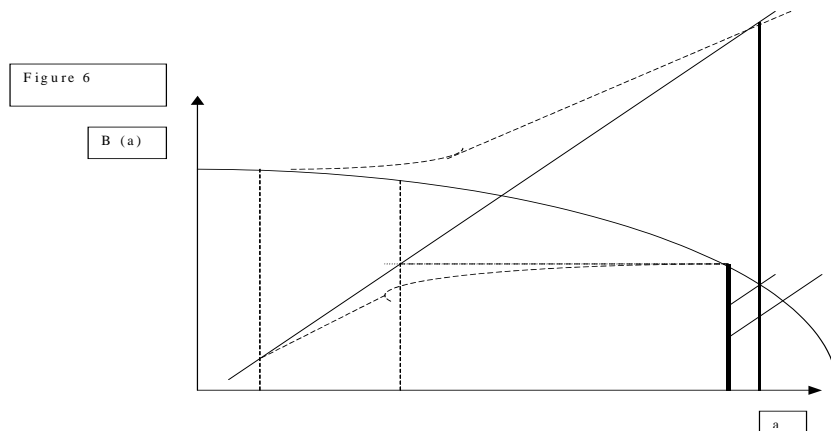


Figure 6:

## 4.2 The case of incomplete information

Up to now, complete information has been assumed. To examine the case of incomplete information, let us return to the above version of the Muth problem, but suppose now that the agents have a known cost drawn from an unknown distribution. Under the same assumptions as above, the relevant distribution of costs for the analysis has a higher variance than the actual distribution of costs. Naturally, as uniqueness still holds, a flatter distribution of costs does not make the equilibrium more unique. Does it make it more eductively stable? Consider the limit case where the mean of the distribution is unknown but the distribution of costs very concentrated (remind this is the favourable case for eductive stability in the presence of StCo). Is it still true that, if they know that some high-cost agents drop, some low-cost agents are induced to join, so that other high-cost agents may drop? It is certainly true that a Bayesian rational agent receiving his cost signal will believe that about half of the agents have received a higher signal. But the hypothesis, similar to the one used in the triggering argument of the StCo iterative process, that all agents with higher cost signals have dropped, has no much grasp on the other agents' actions, in the present case. Assume indeed that high-cost agents (i.e., above some cut-off cost  $\bar{c}$ ) drop. Because you believe that the distribution of costs is highly concentrated, you can only infer, if you have, say, average cost (and, *a fortiori*, if you are a low-cost agent), that only a small number of high-cost people will drop: then only a small number of lower-cost agents are induced to join. This is the first iterate of process depicted by the dotted curve of Figure 6. The process goes on: knowing the first part of the argument, high-costs agents will know that only very-low-cost agents join under the initial conjecture; but they rationally believe that a very small number of low cost agents are induced to join, and this weakens the rationale for the initial drop decision (as suggested

by the upper dotted curve). This makes reasonably clear why the equilibrium will not be eductively stable.<sup>17</sup> Note that the graphical argument that extends the one used in the StCo case suggests a rather general "destabilizing" role for incomplete information!

The conclusion is: The circumstances in which incomplete information generates the Graal (uniqueness plus eductive stability) do not contribute to destroying uniqueness but tend to destroy eductive stability.<sup>18</sup>

## 5 Is macroeconomics dominated by StCo or StSu?

The question that arises from the previous analysis is the plausibility of StCo in economic analysis. I do not deny that the topics studied in the StCo literature (crisis, agglomeration) are relevant. However, in the field of macroeconomics, where the StCo tools have been seen as potentially useful by some authors, the case is disputable. Let me indeed consider a set of macroeconomic models for which the StCo apparatus seems relevant (they are aggregate models in which the role of StCo is *a priori* magnified, thus having a Keynesian flavour). I want to argue that, for these models, StSu are likely to dominate StCo, even if the analysis will also point out to 'remains' of StCo.

### 5.1 One-dimension macroeconomics with Keynesian flavour

It has been argued that macroeconomic situations give rise to strategic complementarities. The argument goes back at least to Diamond (1982): in his model, trade results from random meetings *and* trade frequency increases with the number of active participants (this is the source of standard StCo). The incentive to participate increases with the number of active participants, therefore the corresponding diagram is similar to the one of Figure 1 or 2, with one or several equilibria. However, this story leaves in the shadow some key features of exchange such as price competition.<sup>19</sup> In order to stress the limits of the StCo story in a macroeconomic context, let us take a variant of the model I have investigated in Guesnerie (2001a).

Let there be a large number of firms, each one producing one unit of good, so that, in the limit, total production is  $Q = \int_I di$ , the integral being taken on the set of active firms  $I$ . But firms are more or less productive, so that firm  $i$  needs to hire  $\theta(i)$  workers (in other words, its cost is  $c(i) = \theta(i)\bar{w}$ , where  $\bar{w}$  is the wage). All firms have to decide on production, in the morning of the first day. Income in the afternoon is  $\bar{p}Q + M$ ,<sup>20</sup> where  $M$  is the money held. If the afternoon

<sup>17</sup>S. Morris found this argument striking, but has developed a formal calculation with H. Shin, in one of the models they previously studied (personal correspondence). These calculations clearly support the above analysis. Our analysis has not the status of a formal proof but is likely to provide the basic ingredients for it.

<sup>18</sup>Walking on the mountain trail requires a behaviour at odds with driving on the highway...

<sup>19</sup>Although the specific story told by Diamond make the price issue actually irrelevant.

<sup>20</sup>I assume rather unrealistically that agents receive not only wage income but also the ex-post realized profit. This simplifies the algebra without changing the conclusions.

price  $\bar{p}$  and the morning wage  $\bar{w}$  are fixed, all firms  $i \in I(\bar{p})$  such that  $\theta(i)\bar{w} < \bar{p}$  would like to produce. However, effective demand is  $D(\bar{p}, \bar{p}Q + M)$ , notional supply is  $Q = \int_{I(\bar{p})} di$ , and supply is supposed to exceed demand when  $\bar{p}$  is large (a 'Keynesian' situation). In that case there is rationing, assumed to be such that the most efficient firms remain in operation. Hence, there is a relationship between the expected production  $Q^e$  and the actual production  $D(\bar{p}, \bar{p}Q^e + M)$ . At equilibrium  $Q^* = D(\bar{p}, \bar{p}Q^* + M)$ . We are in a somewhat standard StCo case: actual production increases with anticipated production and the slope of the function is  $\bar{p}(\partial D/\partial R)$ , i.e. the marginal propensity to consume  $\pi$  governs the speed of the eductive process. An exogenous shock of 1 on demand would push production by  $1/(1 - \pi)$ , the standard Keynesian multiplier. There is no doubt that this type of Keynesian macroeconomics enters the class of standard one-dimension StCo models.

However, assume now, as in my paper (Guesnerie, 2001a), that the afternoon price is the market clearing price. The model remains Keynesian, with the morning wage fixed at a level that generates an excess supply of labour. However, the equilibrium price  $\psi(Q)$  satisfies  $Q = D(\psi(Q), \psi(Q)Q + M)$ , so that<sup>21</sup>  $\psi'(Q) = (\partial D^C)/ (1 - \pi(Q))$  where  $\partial D^C$  is a 'compensated elasticity' of price demand and  $\pi(Q)$  is the marginal propensity to consume, calculated at the point  $(\psi(Q), \psi(Q)Q + M)$ . The (Keynesian ?) fixed wage equilibrium  $Q^*$  satisfies  $S(\psi(Q^*)) = D(\psi(Q^*), \psi(Q^*)Q^* + M)$  where  $S$  is the 'notional' competitive supply.

The point to be stressed is that the limited-price-flexibility hypothesis has led us away from the StCo world. Indeed, the model under consideration displays strategic substitutabilities. Hence, it has a unique equilibrium which turns out to be locally eductively stable whenever  $(S'/D')_* < m_*$  where  $S'/D'$  is, as in the Muth model, the ratio of supply over demand elasticities, and  $m_*$  is the Keynesian multiplier (all evaluated at equilibrium). The set of rationalizable equilibria is included in the segment  $(\psi(Q_0), Q_0)$ , where  $Q_0$  is the full employment activity level, and only in good cases may coincide with the equilibrium... And there is no clear sense in which incomplete information would make things better.<sup>22</sup>

Introducing now wage flexibility in the morning brings us back to a Walrasian setting closer to the presently standard macroeconomic models, and further away from the StCo world of fully fixed price models. In the simplest setting considered in section 5 of Guesnerie (2001a), equilibrium is unique and, in a sense, 'often' eductively stable: the reason is that wage flexibility guarantees full employment and this is a powerful anchor to expectation coordination. Still, the set of rationalizable equilibria is not always reduced to the Walrasian equilibrium. It may happen, for example, that low-cost firms (those whose cost is smaller than the Walrasian cut-off cost) do not produce and are replaced by less efficient firms (those whose cost exceeds the Walrasian cut-off cost). These latter firms are optimistic in the sense they expect a price higher than the Wal-

<sup>21</sup>This is a simple computation left to the reader who will check that  $\partial D^c = \partial D + Q\partial D/\partial R$ .

<sup>22</sup>StSu may even worsen the result.



rasian price; however, since they are less efficient than the Walrasian firms, the market-clearing wage will be mechanically higher than the Walrasian wage. However, the expectations associated with this situation are rationalizable in the following sense: price expectations of the most efficient firms (the producing firms at the Walrasian equilibrium) coincide with the Walrasian price and, both in the intuitive sense and in the formal sense, this is 'rationalizable', even if it induces some of them to drop at the actual market-clearing wage. The price expectations of the optimistic firms are also rationalizable, for the simple reason that the price they expect does finally prevail! The fact that this price is above the Walrasian price is due to the lower productivity of these firms, which also leads to a lower actual production.

Can we conclude that the StCo story is entirely irrelevant when one leaves the fully rigid price world? In fact, strategic complementarities associated with income effects generated by increased economic activity are still present in the two models I have briefly presented. Although they are dominated by strategic substitutabilities, strategic complementarities alleviate strategic substitutabilities and play a positive role in improving expectational coordination. This is illustrated by the role of the Keynesian multiplier in the local eductive stability of the equilibrium: roughly speaking, as suggested by the above formula, a higher Keynesian multiplier makes eductive stability more plausible. In a more surprising way, the Keynesian multiplier turns out to be an ingredient of some conditions that are sufficient for eductive stability of the Walrasian equilibrium (formula (4-13) in Guesnerie (2001a)): The intuitive explanation is that a high Keynesian multiplier favours the expectational stability of the Walrasian equilibrium!

To sum up, the basic insights of the standard StCo models disappear, but with significant remains of the StCo story. The next subsection attempts to evaluate the importance of these remains.

## 5.2 $n$ -dimension world: the remains of StCo in macroeconomics?

In a loose sense, the previous analysis suggests that strategic complementarities, although they are dominated by StSu (outside the extreme polar world we first considered) still coexist with them and alleviate StSu in a sense favourable to expectational coordination, even in a fully Walrasian world.<sup>23</sup> However, the level of aggregation of the model leaves a doubt on the robustness of the analysis: are the 'remains' of StCo an artefact due to the aggregation of the model?

In order to examine this question, I have considered in Guesnerie (2001b), an  $n$ -dimension ('Keynesian') version of the three-goods model of the previous subsection. There is a market clearing price for each of these goods, but the morning wage is fixed at a 'Keynesian' level. Now, local eductive stability of the equilibrium is obtained whenever the eigenvalue of highest modulus of

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<sup>23</sup>Note, however, that when going towards a world closer to the RBC-like model, the multiplier we have pointed at would become smaller, as a result of the decrease of the marginal propensity to consume present income, if a life-cycle model of consumption is adopted.

the matrix  $\{(\partial S)(\partial Z)^{-1}(I - A)\}$  is smaller than 1, where  $\partial S$  is the Jacobian matrix of supply (as a function of prices),  $(\partial Z)^{-1}$  is the inverse of the Jacobian matrix of demand, and  $A$  is the matrix of marginal propensities to consume. The reader will note that this formula generalizes the previous one-dimension condition. Is it the case that a 'bigger' matrix  $A$  might improve expectational coordination? In that case, there are unambiguous StCo remains in the problem: higher propensities to consume, i.e. a higher (Keynesian-like) income multiplier, would be, in a reasonable sense, good for expectational coordination. Such a positive answer can only be given when  $Z$  satisfies the gross substitutability hypothesis. This is the case where the 'remains' of the StCo intuitions, with the positive role of income effects they stress, are significant and identifiable. In the general case, the intuition drawn from the StCo approach is difficult to disentangle from the other aspects of the problem.

## 6 Conclusion

This paper has made, hopefully convincingly, three main points:

- It has compared and opposed two analyses of expectational coordination. In the StCo paradigm, a neat route consists in searching for uniqueness. In the opposite polar framework (StSu), global or even local stability involves a variety of tools and approaches. However, StSu is a more relevant framework for economic analysis.

- It has suggested that the interplay of StCo and incomplete information opens only a rather tiny territory of reflection. In particular, incomplete information does not seem to facilitate the analysis of expectational coordination in StSu situations.

- It has finally argued, from the examination of a class of models, that the StCo paradigm is likely to account for a very limited set of phenomena in the field of macroeconomics. However, even when StCo are dominated by StSu, detecting the 'remains' of StCo may help understanding expectational coordination.

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