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The real effect of inflation in liquidity constrained models

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Keywords : Inflation, credit constraints, heterogeneous agents

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Abstract

This article identifies a new channel through which inflation affects the real economy. In a simple monetary model where agents face heterogenous income flows, it is proven that credit constraints create heterogeneity in money demand. Because of this heterogeneity, long run inflation affects the real interest rate and real variables, even when there are no redistributive effects, no distorting fiscal policy, no substitution between leisure and working time, and when prices are flexible. For realistic utility functions, inflation is found to raise the capital stock, but to decrease welfare.

Keywords : Inflation, Credit Constraints, Heterogenous Agents

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1 Introduction

This paper shows the existence of a new channel through which inflation affects the real economy based only on financial market imperfections. It is shown that inflation affects capital accumulation and welfare inequality in the long run if credit constraints are binding for some agents. The reason for this result is that credit constraints create an heterogeneity in agents' reaction facing a change in inflation. If money is held because it provides some liquidity services, unconstrained agents substitute money by financial assets when inflation increases. However, constrained agents, who also hold money because of liquidity services, can not change their demand for financial assets when inflation changes. Thus, the change in their real money holdings is different from the one of unconstrained agents. Hence, financial market imperfections are enough to yield heterogeneity in money demand, without any assumptions on heterogeneity in preferences or in transaction technologies. Because of this heterogeneity, inflation has a real effect in the long run. This effect is different from previous effects stemming from redistribution between generations as in Weiss [15] or Weil [14] or between households as in Kehoe et al. [10], or because of an effect of inflation on distorting taxes (e.g. Phelps [11] Chari et al., [5]) or on labor supply.

This result is obtained within a simple liquidity constrained model. In such models, heterogenous agents face idiosyncratic income shocks and are unable to borrow as much as they would like in the loan markets. Contrary to previous general equilibrium of money, such as Gandmont and Younes [6], this type of model drastically simplifies the heterogeneity across agents to be able to derive analytical properties of the equilibrium. Such models have been used to study the demand for fiat money in Bewley [2], the effect of public debt in Woodford [16], the redistributive effect of inflation Kehoe et al. [10] and the property of the stochastic steady states (e.g. Kehoe and Levine [9]). However, they have not been used to study substitution between money and financial titles, which is at the core of this paper. Indeed, I assume that money yields liquidity services and enters the utility function. Because of this, both money and interest bearing financial titles are held in equilibrium.

First, it is shown that even if the new money is distributed to private agents by lump-sum transfers proportionally to their money holdings, money is not superneutral and inflation affects the real interest rate. This result is obtained with an inelastic labor supply and without distorting taxes, to ensure that other mechanisms through which inflation could affect the real equilibrium are absent. Second, it is shown that inflation increases capital accumulation for realistic values of the elasticity of substitution between money and consumption, what is consistent with empirical evidence on the effect of low inflation (e.g. Bullard and Keating [3]). As a consequence, as binding credit constraints are a well established fact (e.g. Jappelli

[8] among others) the effect of inflation stemming from this channel may be crucial to assess the long run effect of inflation. In a simple calibration exercise, an increase in inflation from 2% to 3% is found to increase the capital stock, but to decrease welfare.

The paper is presented in 5 other sections. Section 2 introduces the model. Section 3 derives the stationary equilibria. Section 4 derives analytically the equilibrium. Section 5 provides additional results with further specifications of the utility function.

2 The Environment

There is an infinite number of discrete time periods $t = 1, \dots$. In each period, there is a continuum of length 1 of two types of households $i = 1, 2$. The households can be in two states, H or L , and they switch deterministically from state H to state L and from state L to state H . They sell inelastically one unit of labor in state H and they sell no labor in state L . Type 1 households are in state H and type 2 households are in state L in period 1. As a consequence, if e_t^i denotes the quantity of labor sold by type i households in period t , e_t^i follows the simple law for $t = 1, 2, \dots$, $e_{t+1}^i = 0$ if $e_t^i = 1$ and $e_{t+1}^i = 1$ if $e_t^i = 0$, the initial states being $e_1^1 = 1$ and $e_1^2 = 0$.

The commodities in this economy are labor, a consumption-capital good and money. Money is assumed to yield liquidity services and it is thus demanded although it is dominated by interest bearing financial assets. I follow a long tradition by assuming that money enters the utility function. As a consequence, the utility of a type i household in period t , depends on the quantity of final goods consumed, c_t^i , and of the real quantity of money held at the end of period t , denoted m_t^i . The households have a common additively separable utility function u such that the total utility derived from the vector of positive values of consumption and money holdings $\{(c_1^i, m_1^i), (c_2^i, m_2^i), \dots\}$ is $\sum_{t=1}^{\infty} \beta^{t-1} u(c_t^i, m_t^i)$ with $0 < \beta < 1$. To provide analytical results on the existence of credit constrained equilibrium in this monetary framework, I assume that the period utility function has a constant elasticity of substitution between consumption and money:

$$u(c, m) = \frac{1}{1-\sigma} \left[\left(\omega c^{\frac{\eta-1}{\eta}} + (1-\omega) m^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \right]^{1-\sigma} \quad 0 < \omega < 1 \quad (1)$$

$u(c, m)$ is twice continuously differentiable for $\sigma > 0, \sigma \neq 1$ and for $\eta > 0, \eta \neq 1$. But, using the standard assumption that the term inside the bracket is equal to $c^\omega m^{1-\omega}$ when $\eta = 1$ and that $\frac{1}{1-\sigma} (\cdot)^{1-\sigma} \equiv \ln(\cdot)$ when $\sigma = 1$, the previous utility function can be defined for $\sigma, \eta > 0$.

In each period t , P_t denotes the monetary price of the final good in period t , and Π_{t+1} is the gross inflation rate between period t and period $t+1$, that is $\Pi_{t+1} = P_{t+1}/P_t$. With

their revenue in period $t \geq 1$, each type i household buys an amount c_t^i of final goods, he buys an amount a_{t+1}^i of financial titles, which yield $R_{t+1}a_{t+1}^i$ in period $t + 1$, where R_{t+1} is the gross real interest rate between period t and period $t + 1$. A borrowing constraint is introduced in its simplest form, and I assume that households can not borrow, $a_t^i \geq 0$. Finally, type i household buys a nominal quantity of money M_t^i , which corresponds to real balances $m_t^i = M_t^i/P_t$. It yields a revenue m_t^i/Π_{t+1} in period $t + 1$. Indeed, the nominal value of money transferred to period $t + 1$ is $P_t m_t^i$ and its period $t + 1$ value is $P_t m_t^i/P_{t+1}$. Labor income of household i in period t is $w_t e_t^i$, where w_t denotes the real wage expressed in final good. In addition to labor and capital income, each household receives by helicopter drops a monetary transfer from the State, denoted μ_t^i in nominal terms. The problem of the type i household, $i = 1, 2$, is

$$\max_{\{c_t^i, m_t^i, a_{t+1}^i\}_{t=1.. \infty}} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t^i, m_t^i) \quad \text{with } 0 < \beta < 1 \quad (2)$$

$$\text{s.t. } c_t^i + m_t^i + a_{t+1}^i = R_t a_t^i + \frac{m_{t-1}^i}{\Pi_t} + w_t e_t^i + \frac{\mu_t^i}{P_t} \quad \text{with } a_t^i, c_t^i, m_t^i \geq 0 \quad (3)$$

with a_1^i and $M_0^i = P_0 m_0^i$ given, and subject to the standard transversality conditions for a_t^i and m_t^i .

The production function of the representative firm has a simple Cobb-Douglas form $K^\alpha L^{1-\alpha}$ where L is total labor and K is total capital which fully depreciates in production, and which must be installed one period before production. Profit maximization is $\max_{K_t, L_t} K^\alpha L^{1-\alpha} - R_t K_t - w_t L_t$ and the standard first order conditions are

$$R_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha}, \quad w_t = (1 - \alpha) K_t^\alpha L_t^{-\alpha} \quad (4)$$

In period $t \geq 1$, the financial market equilibrium is $K_{t+1} = a_{t+1}^1 + a_{t+1}^2$. The labor market equilibrium is $L_t = e_t^1 + e_t^2 = 1$. The goods market equilibrium is $F(K_t, L_t) = K_{t+1} + c_t^1 + c_t^2$.

Finally, I denote \bar{M}_t the nominal quantity of money in circulation and Σ_t the real quantity of money in circulation at the end of period t , $\Sigma_t = \bar{M}_t/P_t$. The money market equilibrium is thus $m_t^1 + m_t^2 = \Sigma_t$ in real terms and $M_t^1 + M_t^2 = \bar{M}_t$ in nominal terms.

Monetary authorities give a new nominal quantity of money in period t , which is proportional to the nominal quantity of money in circulation at the end of period $t - 1$. As a consequence, $\mu_t^1 + \mu_t^2 = \pi \bar{M}_{t-1}$ where the initial nominal quantity of money, $\bar{M}_0 = M_0^1 + M_0^2$ is given. The law of motion of the nominal quantity of money is thus

$$\bar{M}_t = (1 + \pi) \bar{M}_{t-1} \quad (5)$$

Throughout the paper, it is assumed that monetary authorities follow the ‘‘most’’ neutral rule, which is to distribute by lump sum transfer the exact amount of resources paid by

private agents because of the inflation tax. As a consequence, the new money is distributed proportionally to the beginning of period money balances. In period t , type i agents have a beginning of period quantity of money M_{t-1}^i . hence, I assume that $\mu_t^i = \pi M_{t-1}^i$, and the real transfer is

$$\frac{\mu_t^i}{P_t} = \frac{\pi}{\Pi_t} m_{t-1}^i \quad (6)$$

Given initial conditions a_1^1, a_1^2, M_0^1 , and M_1^2 , and given π , an equilibrium of this economy is a sequence $\{c_t^1, c_t^2, m_t^1, m_t^2, a_{t+1}^1, a_{t+1}^2, P_t, R_t, w_t\}_{t=1 \dots \infty}$ which satisfies the problem of households (2), the first order condition of the problem of the firms (4), and the different market equilibria. More precisely, I focus on symmetric stationary equilibria¹, where all real variables are constant, and where all households in each state H and L have the same consumption and savings levels. The variables describing households in state H will be denoted m^H, c^H, a^H , and households in state L will be described by m^L, c^L, a^L . As a consequence, as the real quantity of money in circulation $\Sigma = \bar{M}_t/P_t$ is constant in a stationary equilibrium, equality (5) implies that the price of the final goods grow at a rate π , and hence $\Pi = 1 + \pi$.

3 Stationary Equilibrium

With the budget constraint (3), and the amount μ_t^i/P_t given by (6), one finds that the budget constraint of H and L households is respectively

$$c^H + m^H + a^H = Ra^L + m^L + w \quad (7)$$

$$c^L + m^L + a^L = Ra^H + m^H \quad (8)$$

Note that the inflation rate does not appear in these equations because the creation of new money does not introduce any transfer between the two types of households.

Using standard dynamic programming arguments, the problem of the households can be solved easily. This is done in appendix A. For H agents, one finds the following optimal conditions

$$u'_c(c^H, m^H) = \beta R u'_c(c^L, m^L) \quad (9)$$

$$u'_c(c^H, m^H) - u'_m(c^H, m^H) = \frac{\beta}{\Pi} u'_c(c^L, m^L) \quad (10)$$

The first equation is the Euler equation for H agents, who can smooth their utility thanks to positive savings. Indeed, H agents are the high income agents and are never credit constrained. The second equality is the arbitrage equation, which determines the demand

¹In liquidity constraint models, the path of the economy converges toward a steady state, or even begins at a steady state if a period 1 transfer is made to households consistently with steady state values (Kehoe and Levine, 2001)

for real money balances. H agents equalize the marginal cost of holding money in the current period, (i.e. the left hand side of equation 10), to the marginal gain of transferring one unit of money to the following period where they are in state L , (i.e. the right hand side of equation 10). The marginal utility of money appears here as a decrease in the opportunity cost of holding money, and the gain of money holdings takes into account the real return on money $1/\Pi$.

The solution of the program of L households depends on the credit constraints being binding or not. If credit constraints are binding, the solution is $a^L = 0$ and

$$u'_c(c^L, m^L) > \beta R u'_c(c^H, m^H) \quad (11)$$

$$u'_c(c^L, m^L) - u'_m(c^L, m^L) = \frac{\beta}{\Pi} u'_c(c^H, m^H) \quad (12)$$

The first inequality stipulates that L agents would be better off if they could transfer some income from the next period toward the current one. The second equality is the same trade-off as the one of H households. Finally, if credit constraints do not bind for L households, the inequality (11) becomes an equality and $a^L > 0$.

Using expression (9) together with condition (11), one finds that credit constraints are binding if and only if $R < 1/\beta$. If credit constraints do not bind, equalities (9) and (11) with equality directly yield $R = 1/\beta$. The following proposition summarizes this standard result.

Proposition 1 *Credit constraints are binding for L agents if and only if $R < 1/\beta$. If credit constraints do not bind then $R = 1/\beta$.*

When credit constraints are binding, the gross real interest rate R is lower than the inverse of the discount factor. As a consequence, there is always capital over-accumulation because of the precautionary motive to save, which is a standard result in this type of liquidity constrained models (e.g. Woodford [15]; Kehoe and Levine [8]). When credit constraints do not bind, the inflation rate does not affect the long run real interest rate. In this case, monetary variables only are affected by inflation.

Conditions on the parameters of the model

R can not be lower than $1/\Pi$ in equilibrium, otherwise the return on money would be higher than the return on financial titles and the financial market could not clear. As a consequence, an equilibrium with binding credit constraints can exist only if $1/\Pi < 1/\beta$. Moreover, I assume that the surplus left for consumption, $F(K) - K$ is positive at the Friedman rule, that is when $R = 1/\Pi$. With the Cobb-Douglas production function, this condition implies $\alpha < 1/\Pi$. Thus, I assume that the following inequalities, which are fulfilled

for realistic values of the parameters, are satisfied.

$$\alpha < \frac{1}{\Pi} < \frac{1}{\beta}$$

4 Credit Constrained Equilibria

This section presents sufficient condition for a credit constrained equilibrium to exist and proves that inflation is not neutral in such an equilibrium. When credit constraints bind, that is when $R < \frac{1}{\beta}$, one finds, dividing (10) by (9) and using (9) with (12), together with the expression of the utility function:

$$\frac{m^H}{c^H} = \left(\frac{1-\omega}{\omega}\right)^\eta \left(1 - \frac{1}{\Pi R}\right)^{-\eta} \quad \text{and} \quad \frac{m^L}{c^L} = \left(\frac{1-\omega}{\omega}\right)^\eta \left(1 - \frac{\beta^2 R}{\Pi}\right)^{-\eta} \quad (13)$$

For the H households, the ratio of money over consumption is determined by preference parameters and by the opportunity cost to hold money. To see this, assume that R and Π are close to unity, then $1 - 1/\Pi R \simeq (R - 1) - (1 - \Pi)$, which is the difference between the real net return on financial titles and the real net return on money or, in other words, which is the nominal interest rate.

The equilibrium ratio for L agents is not simply determined by the opportunity cost to hold money, but by the difference between consumption in the current period and the return on money holdings two periods ahead. Indeed, the ratio $\beta^2 R/\Pi$ is the discounted value of one unit of money held in state L , transferred in state H , and then saved on financial market to the next period, where the household is in state L again. When this ratio increases, L households increase the ratio of their money holdings over their consumption. As a consequence, state L households increase the relative demand for money when the real interest increases, contrary to state H households. Indeed, the real interest rate appears as the remuneration of future savings and not as the opportunity cost to hold money.

It is now possible to derive the conditions for the existence of a stationary equilibrium with binding credit constraints. The proof of the following propositions are left in appendix.

Proposition 2 *An equilibrium with binding credit constraints exists if 1) $\alpha < \frac{1}{2+\beta}$ and 2) $\eta \neq 1$ and either $\eta < 1$ or $\eta < \frac{1}{\sigma}$. In such an equilibrium inflation has an effect on real variables.*

Condition 1) stipulates that the capital share in production α must not be too high or, conversely, that the labor income earned by H agents must be high enough. This condition ensures that H agents have the incentives to smooth consumption in equilibrium. This condition is fulfilled for the standard value $\alpha = \frac{1}{3}$ and $\beta < 1$. Condition 2) states that the

elasticity of substitution between money and consumption must not be too high. Otherwise, H agents would demand a little amount of final goods, and would use money both to derive utility and to smooth consumption. In this case, they could transfer enough resources in state L , such that they would never face binding credit constraints. This proposition does not guarantee uniqueness, and does not consider the Cobb-Douglas case which is analyzed below.

The second part of the proposition proves that inflation is not neutral when credit constraints are binding. The reason for this result is that one can show that $\frac{\partial m^H}{\partial \Pi} \neq \frac{\partial m^L}{\partial \Pi}$ under the conditions of proposition 2. Indeed, money is the only store of value for L agents, whereas H agents can substitute money for financial assets when inflation varies. This yields a difference in the reaction of agents' facing a change in inflation.

When credit constraints do not bind, the result of Sidrauski [12] is obtained because agents react symmetrically to a change in inflation. Indeed, in this case, $\frac{\partial m^H}{\partial \Pi} = \frac{\partial m^L}{\partial \Pi}$.

5 Further Specifications

5.1 The Cobb-Douglas Utility function

The Cobb-Douglas utility function $u(c, m) = (c^\omega m^{1-\omega})^{1-\sigma} / (1-\sigma)$ is often used in macroeconomics and it can find some empirical support (Holman [7]). In this case it is possible to determine the effect of inflation on capital accumulation.

Proposition 3 *If $\alpha < \frac{1}{2+\beta}$, there is a unique equilibrium with credit constraints and $\frac{\partial R}{\partial \Pi} < 0$.*

Inflation favors capital accumulation and output. Indeed, the L households lower less rapidly their money holdings m^L than H households, because money is their only store of value. As a consequence, H households have more resources to save and consume when inflation increases. Indeed, their budget constraints yields $c^H + a^H = w + m^L - m^H$. As an indirect effect, capital accumulation raises w and the incentives to save to smooth consumption. This effect of inflation on capital accumulation is consistent with the data for low values of inflation as proven in Bullard and Keating [3]. But, unfortunately, welfare analyzes can not be performed analytically because of the various general equilibrium effects at stake. Instead, I provide a simple calibration.

5.2 Calibrated CES Utility function

I simulate the model with a standard calibration. $\alpha = 0.33; \beta = 0.96; \sigma = 1; \omega = 0.99; \eta = 0.39$. These parameters are standard values taken from Chari et al. [4]. The only difference

is that the coefficient ω is equal to 0.94 in their simulation whereas I take the value 0.99 to obtain a smaller and more realistic quantity of money on GDP (it is around 30% here). Fig. 1 plots the equilibrium real interest rate in percent as a function of the net inflation rate in percent. It has been checked that the condition $\frac{1}{\Pi} < R < \frac{1}{\beta}$ is fulfilled for the whole range of parameters.

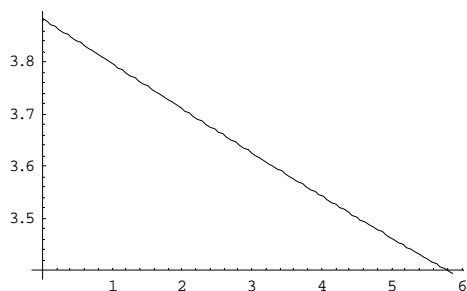


Fig.1 : Real interest rate as a function of the net inflation rate, both in percent

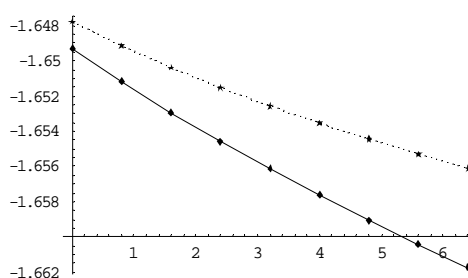


Fig. 2 : Utility of H and L households as a function of inflation

Fig. 1 shows that the real interest rate is a decreasing function of the inflation rate. One can check that $m^L - m^H$ is increasing in Π , what increases the resources of H households to save. When inflation increases from 2% to 3%, the capital stock increases by 0.12%. Fig. 2 plots $u(c^H, m^H)$ and $u(c^L, m^L)$ as a function of inflation, the solid line and the dashed line respectively². The utility of both agents decreases because of the decrease in money demand induced by higher inflation. The utility of H agents decreases more sharply because of the higher decrease in money holdings. Hence, although inflation increases capital accumulation, it decreases welfare.

²Intertemporal welfare of H households is simply $(u(c^H, m^H) + \beta u(c^L, m^L)) / (1 - \beta^2)$ and the intertemporal welfare of L households is defined the same way. The discussion of period utility exhibits more sharply the various effects.

A Solution to the Problem of the Households

Using the Bellman equations, the problem of the households can be written in a recursive form. Stationary solutions satisfy, of course, the usual transversality conditions. As a consequence, one can focus on the first order condition of the problem of the households. This one is

$$V(q_t^i, e_t^i) = \max_{\{c_t^i, m_t^i, a_{t+1}^i\}} u(c_t^i, m_t^i) + \beta V(q_{t+1}^i, e_{t+1}^i)$$

$$c_t^i + m_t^i + a_{t+1}^i = q_t^i + w_t e_t^i + \frac{\mu_t^i}{P_t} \quad (14)$$

$$q_{t+1}^i = R_{t+1} a_{t+1}^i + \frac{m_t^i}{\Pi_{t+1}} \quad (15)$$

$$c_t^i, m_t^i, a_{t+1}^i \geq 0 \quad (16)$$

with q_1^1, q_1^2 given and with the deterministic change of state $e_{t+1}^i = 0$ if $e_t^i = 1$, and $e_{t+1}^i = 1$ if $e_t^i = 0$. Using (14) and (15) to substitute for c_t^i and q_{t+1}^i , one can maximize only on a_t^i and m_t^i . Using the first order conditions, together with the envelop theorem (which yields in all cases $V'(q_t^i, e_{t+1}^i) = u'_c(c_t^i, m_t^i)$), one finds

$$u'_c(c_t^i, m_t^i) = \beta R_{t+1} u'_c(c_{t+1}^i, m_{t+1}^i) \quad (17)$$

$$u'_c(c_t^i, m_t^i) - u'_m(c_t^i, m_t^i) = \frac{\beta}{\Pi_{t+1}} u'_c(c_{t+1}^i, m_{t+1}^i) \quad (18)$$

If the previous equations yield a quantity $a_{t+1}^i < 0$, then the borrowing constraint is binding and the solution is given by $a_{t+1}^i = 0$ and $u'_c(c_t^i, m_t^i) > \beta R_{t+1} u'_c(c_{t+1}^i, m_{t+1}^i)$ together with (18). In a stationary equilibrium, all H agents become L agents the next period, and the reverse. The H agents are the high revenue agents, and their savings are always higher than the ones of L agents, who have no labor income. As a consequence, credit constraints never bind for H agents. One can rewrite the previous equations using the state of the households instead of their type. In a stationary equilibrium it yields the expressions given in section 3.

B Proof of Proposition 2

I first assume that credit constraints are binding and I exhibit the condition under which it is effectively the case. First the equalities (9) and (13) imply that $c^H/c^L = \psi(R, \Pi)$ with

$$\psi(R, \Pi) \equiv \beta^{-\frac{1}{\sigma}} R^{-\frac{1}{\sigma}} \left(\frac{1 + \left(\frac{1-\omega}{\omega}\right)^\eta \left(1 - \frac{\beta^2 R}{\Pi}\right)^{1-\eta}}{1 + \left(\frac{1-\omega}{\omega}\right)^\eta \left(1 - \frac{1}{\Pi R}\right)^{1-\eta}} \right)^{1 - \frac{1-\sigma}{\sigma} \frac{1}{\eta-1}}$$

For a given Π the function $\psi(R, \Pi)$ is continuous for $R \in (\frac{1}{\Pi}; \frac{1}{\beta}]$. And $\psi(\frac{1}{\beta}, \Pi) = 1$. If $\eta < 1$ then $\lim_{R \rightarrow \frac{1}{\Pi}} \psi(\frac{1}{\Pi}, \Pi) = \beta^{-\frac{1}{\sigma}} \Pi^{\frac{1}{\sigma}} \left(1 + \left(\frac{1-\omega}{\omega}\right)^\eta \left(1 - \frac{\beta^2}{\Pi^2}\right)^{1-\eta}\right)^{1 - \frac{1-\sigma}{\sigma} \frac{1}{\eta-1}} > 0$. If $1 < \eta < \frac{1}{\sigma}$

then $1 - \frac{1-\sigma}{\sigma} \frac{1}{\eta-1} < 0$ and $\lim_{R \rightarrow \frac{1}{\Pi}} \psi\left(\frac{1}{R}, \Pi\right) = +\infty$. As a consequence, under the condition of proposition 2, the $\psi(R, \Pi)$ is positive and bounded away from 0 when R approaches $\frac{1}{\Pi}$.

Second, using the budget constraints (7) and (8) to substitute for $1/c^L$, one finds that (recall that $a^L = 0$)

$$\frac{c^H}{c^L} + \frac{m^H}{c^H} \frac{c^H}{c^L} - \frac{m^L}{c^L} = \frac{w - a^H}{Ra^H} \left(1 + \frac{m^L}{c^L} - \frac{m^H}{c^H} \frac{c^H}{c^L}\right)$$

with the Cobb-Douglas production function and as $K = a^H$, one finds $\frac{w-a^H}{Ra^H} = \alpha^{-1} - R^{-1} - 1$. Then, substituting m^H/c^H , m^L/c^L and c^H/c^L by their expressions given respectively by (13) and $c^H/c^L = \psi(R, \Pi)$ one finds the implicit relationship between R and Π : $\Delta(R, \Pi) = \Theta(R)$ where $\Theta(R) \equiv \alpha^{-1} - R^{-1} - 1$ and where

$$\begin{aligned} \Delta(R, \Pi) &= \left(1 + \left(\frac{1}{\alpha} - \frac{1}{R}\right) \left(\frac{1-\omega}{\omega}\right)^\eta \left(1 - \frac{1}{\Pi R}\right)^{-\eta}\right) \psi(R, \Pi) \\ &\quad - \left(\frac{1}{\alpha} - \frac{1}{R}\right) \left(\frac{1-\omega}{\omega}\right)^\eta \left(1 - \frac{\beta^2 R}{\Pi}\right)^{-\eta} \end{aligned}$$

For a given Π , the function $\Delta(R, \Pi)$ and $\Theta(R)$ are continuous as a function of $R \in (\frac{1}{\Pi}; \frac{1}{\beta}]$. One can check that $\Delta\left(\frac{1}{\beta}, \Pi\right) = 1$. As $\left(\frac{1}{\alpha} - \Pi\right) > 0$ and as $\psi(R, \Pi)$ is positive and bounded away from 0 when $R \rightarrow \frac{1}{\Pi}$, one finds that $\lim_{R \rightarrow \frac{1}{\Pi}} \Delta\left(\frac{1}{R}, \Pi\right) = +\infty$.

The definition of Θ yields $\Theta\left(\frac{1}{\beta}\right) = \frac{1}{\alpha} - \beta - 1$ and $\Theta\left(\frac{1}{\Pi}\right) = \frac{1}{\alpha} - \Pi - 1$. Under the condition $\alpha < \frac{1}{2+\beta}$ one finds that $\Theta\left(\frac{1}{\beta}\right) > 1$. To summarize these findings $\lim_{R \rightarrow \frac{1}{\Pi}} \Delta(R, \Pi) > \lim_{R \rightarrow \frac{1}{\Pi}} \Theta(R)$ and $\Delta\left(\frac{1}{\beta}, \Pi\right) < \Theta\left(\frac{1}{\beta}\right)$. By continuity of the function of R , $\Delta(R, \Pi)$ and $\Theta(R)$ one finds that there is at least one value $\frac{1}{\Pi} < R^* < \frac{1}{\beta}$ such that $\Delta(R^*, \Pi) = \Theta(R^*)$. R^* is an equilibrium interest rate of the credit constrained economy.

It is easy to prove that inflation affects real variables and that the result of Sidrauski [12] does not hold when credit constraints are binding. The proof is made by contradiction. Assume that c^H, c^L, a^H, R and w are not affected by inflation. Then, equality (8) yields $m^H - m^L = c^L - Ra^H$. As a consequence, if inflation is neutral then the right hand side is constant when inflation varies, and so is the left hand side. Thus, one must have $\frac{\partial m^H}{\partial \Pi} = \frac{\partial m^L}{\partial \Pi}$: Inflation must affect symmetrically the money demand of all agents. But, using (13), one can substitute for m^H and m^L and one finds that if inflation is neutral, then

$$\left(\frac{1 - \beta^2 R^2}{z} + \beta^2 R^2\right)^{-(1+\eta)} = \frac{1}{\beta^2 R^2} \frac{c^H}{c^L}$$

where the new variable $z \equiv 1 - \frac{1}{\Pi R}$ is increasing in Π . The left hand side is decreasing in z and hence in Π , because $\beta R < 1$ and the right hand side is constant, what is a contradiction (the previous equality can not be true for two different values of Π). As a consequence, inflation can not be neutral or, in other words, money is not superneutral.

C The Cobb-Douglas Case

The ratios $\frac{m^H}{c^H}$ and $\frac{m^L}{c^L}$ are given by (13) with $\eta = 1$. The value of $\frac{c^H}{c^L}$ is given by $\frac{c^H}{c^L} = \tilde{\psi}(R, \Pi)$ where

$$\tilde{\psi}(R, \Pi) \equiv (\beta R)^{-\frac{1}{\sigma}} \left(\frac{1 - \frac{\beta^2 R}{\Pi}}{1 - \frac{1}{\Pi R}} \right)^{(1-\omega)\frac{1-\alpha}{\sigma}}$$

One easily gets $\tilde{\psi}\left(\frac{1}{\beta}, \Pi\right) = 1$ and $\lim_{R \rightarrow \frac{1}{\Pi}^+} \tilde{\psi}(R, \Pi) = +\infty$. One can show that the derivative $\tilde{\psi}'_1(R, \Pi) < 0$ and that $\tilde{\psi}'_2(R, \Pi) < 0$. Moreover, $\tilde{\psi}(R, \Pi) > 1$ for $R \in (\frac{1}{\Pi}; \frac{1}{\beta}]$.

The implicit relationship between Π and R can be written as $\tilde{\Delta}(R, \Pi) = \Theta(R)$, and where

$$\tilde{\Delta}(R, \Pi) = \left(1 + \frac{1-\omega}{\omega} \frac{1}{\alpha} \frac{R-\alpha}{R-\frac{1}{\Pi}} \right) \tilde{\psi}(R, \Pi) - \frac{1-\omega}{\omega} \frac{1}{\alpha} \frac{1-\frac{\alpha}{R}}{1-\frac{\beta^2 R}{\Pi}}$$

As before, $\tilde{\Delta}(R, \Pi)$ is a continuous function of R for $R \in (\frac{1}{\Pi}; \frac{1}{\beta}]$. One can show that $\lim_{R \rightarrow \frac{1}{\Pi}^+} \tilde{\Delta}(R, \Pi) > \lim_{R \rightarrow \frac{1}{\Pi}^+} \Theta(R)$ and that $\tilde{\Delta}\left(\frac{1}{\beta}, \Pi\right) < \Theta\left(\frac{1}{\beta}\right)$ under the condition of proposition 3. As a consequence, there is a $R^* \in (\frac{1}{\Pi}; \frac{1}{\beta}]$ such that $\tilde{\Delta}(R^*, \Pi) = \Theta(R^*)$.

Uniqueness stems from the variation of $\tilde{\Delta}(R, \Pi)$ when $R \in (\frac{1}{\Pi}; \frac{1}{\beta}]$. First, one can prove that $\tilde{\Delta}'_1(R, \Pi) < 0$. Indeed, the fraction $(1 - \frac{\alpha}{R}) / (1 - \frac{\beta^2 R}{\Pi})$ is increasing in R . and the fraction $\frac{R-\alpha}{R-\frac{1}{\Pi}}$ is positive and decreasing in R because it has been assumed that $R > 1/\Pi > \alpha$. As a consequence, the function $\tilde{\Delta}(R, \Pi)$ is unambiguously decreasing in R , because $\tilde{\psi}(R, \Pi)$ is decreasing in R . As the function $\Theta(R)$ is increasing in R , the equation $\tilde{\Delta}(R, \Pi) = \Theta(R)$ has at most one solution, what proves uniqueness.

Then, one can prove that $\tilde{\Delta}'_2(R, \Pi) < 0$. Indeed,

$$\tilde{\Delta}'_2(R, \Pi) = \frac{1-\omega}{\omega} \frac{1}{\alpha} (R-\alpha) \left(\frac{\beta^2}{(\Pi - \beta^2 R)^2} - \frac{\tilde{\psi}(R, \Pi)}{(R\Pi - 1)^2} \right) + \left(1 + \frac{1-\omega}{\omega} \frac{1}{\alpha} \frac{R-\alpha}{R-\frac{1}{\Pi}} \right) \tilde{\psi}'_2(R, \Pi)$$

As $\tilde{\psi}'_2(R, \Pi) < 0$ and as $R > \alpha$, a sufficient condition to get the result is $\frac{\beta^2}{(\Pi - \beta^2 R)^2} - \frac{\tilde{\psi}(R, \Pi)}{(R\Pi - 1)^2} < 0$. But as $\tilde{\psi}(R, \Pi) > 1$, this is always true because $R\beta < 1$. As a consequence, by the theorem of the implicit function one gets $\frac{\partial R^*}{\partial \Pi} = \frac{\tilde{\Delta}'_2(R, \Pi)}{\Theta'(R) - \tilde{\Delta}'_1(R, \Pi)} < 0$, which concludes the proof of the proposition.

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