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### **Liquidity effects in non Ricardian economies**

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# Liquidity Effects in non-Ricardian Economies

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## Abstract

It has often been found difficult to generate a liquidity effect (i.e. a negative effect of monetary injections on the nominal interest rate) in the traditional “Ricardian” stochastic dynamic model with a single infinitely lived household. We show that moving to a non-Ricardian environment where new agents enter the economy in each period allows to generate such a liquidity effect.

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# 1 Introduction

The purpose of this article is to investigate a new mechanism through which liquidity effects can be introduced into dynamic stochastic general equilibrium (DSGE) models.

What we call here “liquidity effects” is the negative response of the nominal interest rate to monetary injections. That liquidity effect was already present in the famous IS-LM model, and it appears to be found in the data (see, for example, Christiano, Eichenbaum and Evans, 1997). As it turns out, this liquidity effect has been found difficult to obtain in standard monetary DSGE models. The reason is an “inflationary expectations effect” which tends to actually raise the nominal interest rate in response to a monetary injection.

We now briefly outline the mechanism behind this inflationary expectations effect<sup>1</sup>. It is found in the data that money increases are positively correlated in time. So when an unexpected money injection occurs, this creates the expectation of further money increases in the future, which will itself create the expectation of future inflation. Now from Fisher’s equation the nominal interest rate is the sum of expected inflation and the real interest rate, so that, *ceteris paribus*, this will tend to raise the nominal interest rate.

In spite of this “inflationary expectations effect” one can find in the literature a few models and mechanisms which introduce a liquidity effect in DSGE models. Two prominent ones are:

- Models of limited participation (Lucas, 1990, Christiano and Eichenbaum, 1992, Fuerst, 1992), where households cannot adapt immediately their financial portfolios when a monetary policy shock occurs.

- Models of sticky prices (Jeanne, 1994, Christiano, Eichenbaum and Evans, 1997), where prices are preset in advance. Actually the liquidity effect occurs if the intertemporal elasticity of substitution in consumption is sufficiently low.

What we want to do here is to explore a third avenue, which we will call “non-Ricardian”. By non-Ricardian we mean models, like the overlapping generations model of Samuelson (1958), where new agents are born every period, and where “Ricardian equivalence” (Barro, 1974) does not hold, by opposition to the traditional “Ricardian” model where the same agent or dynasty lives from the beginning to the end of time.

We shall argue that the non-Ricardian character of these economies is conducive to a liquidity effect. In a nutshell, the channel is the following: the existence of future, yet unborn, generations creates, as we shall see below,

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<sup>1</sup>This effect appears more formally in section 5 below.

a “wealth effect” akin to those studied by Pigou (1943) and Patinkin (1956). This wealth effect itself produces a liquidity effect. We shall now proceed to demonstrate this formally.

The plan of the article is the following: section 2 presents the non-Ricardian model. Section 3 shows how a wealth effect arises in it. Section 4 derives the dynamic equations. Section 5 shows how the non-Ricardian economy displays a liquidity effect. Section 6 gives a generalization. Section 7 proves that the liquidity effect can be quite persistent. Section 8 concludes and the appendix gives an intuitive derivation of the results in an IS-LM model.

## 2 A non-Ricardian model

In order to have a non-Ricardian structure that “nests” the usual Ricardian model, we shall use a model adapted from that of Weil (1987, 1991). Households never die, but new “generations” are born each period. Call  $N_t$  the number of households alive at time  $t$ . Since households do not die,  $N_{t+1} \geq N_t$ . We will actually mainly work below with the case where the population grows at the constant rate  $n \geq 0$ , so that  $N_t = (1 + n)^t$ .

### 2.1 Households

Consider a household  $j$  (i.e. a household born in period  $j$ ). We denote by  $c_{jt}$  and  $m_{jt}$  his consumption and money holdings at time  $t \geq j$ . This household receives in period  $t \geq j$  an endowment  $y_{jt}$  and maximizes the expected value of the following utility function:

$$U_{jt} = \sum_{s=t}^{\infty} \beta^{s-t} \text{Log } c_{js} \quad (1)$$

Household  $j$  is submitted in period  $t$  to a “cash in advance” constraint:

$$P_t c_{jt} \leq m_{jt} \quad (2)$$

Household  $j$  enters period  $t$  with a financial wealth  $\omega_{jt}$ . He then receives from the government a lump sum monetary transfer  $\tau_{jt}$  (if negative this will be called taxes), so that his available wealth is  $\omega_{jt} + \tau_{jt}$ . Then there are two successive “subperiods” where the bonds and goods markets open sequentially.

In the first subperiod the bonds market opens. All agents can exchange freely money against bonds at the nominal interest rate  $i_t$ . So at this stage

household  $j$  divides his available wealth  $\omega_{jt} + \tau_{jt}$  between bonds  $b_{jt}$  and money  $m_{jt}$ :

$$m_{jt} + b_{jt} = \omega_{jt} + \tau_{jt} \quad (3)$$

Then in the second subperiod the goods market opens. Household  $j$  sells his endowment  $y_{jt}$  and consumes  $c_{jt}$ , subject to the cash constraint (2). Consequently his budget constraint is:

$$\begin{aligned} \omega_{jt+1} &= (1 + i_t) b_{jt} + m_{jt} + P_t y_{jt} - P_t c_{jt} \\ &= (1 + i_t) (\omega_{jt} + \tau_{jt}) - i_t m_{jt} + P_t y_{jt} - P_t c_{jt} \end{aligned} \quad (4)$$

## 2.2 Aggregation

Aggregate quantities are obtained by summing the various individual variables. There are  $N_j - N_{j-1}$  agents born in period  $j$ , so for example:

$$T_t = \sum_{j \leq t} (N_j - N_{j-1}) \tau_{jt} \quad \Omega_t = \sum_{j \leq t} (N_j - N_{j-1}) \omega_{jt} \quad (5)$$

Similar formulas apply to aggregate money  $M_t$ , bonds  $B_t$ , consumption  $C_t$  and output  $Y_t$ .

## 2.3 Endowments and transfers

Now we describe how endowments and transfers are distributed among households. In the main body of the article we shall assume that all households have the same income and transfers, so that:

$$y_{jt} = y_t = \frac{Y_t}{N_t} \quad \tau_{jt} = \tau_t = \frac{T_t}{N_t} \quad (6)$$

In section 6 we explore the more general assumption that households' resources diminish at a rate  $\xi < 1$ . We shall see that this still reinforces our results.

## 2.4 Government

Now an other important part of the model is the government. The households' aggregate financial wealth  $\Omega_t$  has as a counterpart an identical amount  $\Omega_t$  of financial liabilities of the government. The evolution of these liabilities is described by the government's budget constraint:

$$\Omega_{t+1} = (1 + i_t) B_t + M_t = (1 + i_t) (\Omega_t + T_t) - i_t M_t \quad (7)$$

## 2.5 Monetary policy

If we aggregate equation (3) across all generations we obtain:

$$M_t + B_t = \Omega_t + T_t \quad (8)$$

Monetary policy is an “open market” policy whereby all agents, including government, freely exchange bonds against money on the bonds market. As in all studies on the “liquidity effect” we shall assume that the government uses the quantity of money  $M_t$  as the policy variable, and that consequently the nominal interest rate  $i_t$  is endogenously determined through the equilibrium on the bonds market. A positive shock on money  $M_t$  corresponds to a purchase of bonds by the government.

Following the literature we shall make the assumption that  $M_t$  is a stochastic process. As an example, it is often assumed in the literature that money increases are autocorrelated in time:

$$\text{Log} \left( \frac{M_t}{M_{t-1}} \right) = \frac{\varepsilon_t}{1 - \rho L} \quad 0 \leq \rho < 1 \quad (9)$$

where  $L$  is the lag operator, and  $\varepsilon_t$  is i.i.d.

As indicated above, we shall say there is a liquidity effect if a positive shock on  $M_t$  leads to a decrease in  $i_t$ .

## 3 The wealth effect<sup>2</sup>

As we indicated in the introduction an important part of the story is the “wealth effect” through which financial wealth influences consumption and the dynamic equations.

One may of course wonder why part of financial assets, now and in the future, represents actual purchasing power in the non-Ricardian model, whereas it does not in the Ricardian model. The reason is simple: some of the future taxes that are the counterpart of current nominal wealth will not be paid by the currently alive agents, but by future, yet unborn, generations, so that this part of  $\Omega_t$  represents actual purchasing power. We shall now make this intuition more formal. As the intertemporal government budget constraint will play an important role in the reasoning, we start with it.

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<sup>2</sup>The existence of a wealth effect for financial assets in a non-Ricardian economy was first uncovered by Weil (1991).

### 3.1 The government intertemporal budget constraint

In what follows we shall repeatedly aggregate discounted values. It is convenient to compute in monetary terms, and we shall thus use the following discount factors:

$$R_t = \prod_{s=0}^{t-1} \frac{1}{1+i_s} \quad R_0 = 1 \quad (10)$$

Let us consider the government's budget constraint (7) in period  $s$  multiplied by  $R_{s+1}$ :

$$R_{s+1}\Omega_{s+1} = R_s\Omega_s + R_sT_s - (R_s - R_{s+1})P_sY_s \quad (11)$$

Now let us define total taxes in period  $s$ ,  $\Gamma_s$ , as:

$$R_s\Gamma_s = R_s \left( \frac{i_s M_s}{1+i_s} - T_s \right) = (R_s - R_{s+1})P_sY_s - R_sT_s \quad (12)$$

Total taxes consist of proper taxes  $-T_s$  and the money economized by the state because of the cash in advance constraint  $i_s M_s / (1+i_s)$ , the "money tax". Using this definition, (11) can be rewritten:

$$R_s\Gamma_s = R_s\Omega_s - R_{s+1}\Omega_{s+1} \quad (13)$$

Taking expectation of (13) as of time  $t$ , and summing from time  $t$  to infinity we get:

$$R_t\Omega_t = E_t \sum_{s=t}^{\infty} R_s\Gamma_s \quad (14)$$

We see that every single dollar of financial wealth is matched by discounted current and future taxes.

### 3.2 The consumption function

Let us now consider household  $j$ 's budget equation (4). We assume that  $i_t$  is strictly positive, so that the cash in advance constraint is always satisfied exactly and  $m_{js} = P_s c_{js}$ . Applying the discount factor  $R_{s+1}$  to this budget constraint, it becomes:

$$R_{s+1}\omega_{j,s+1} = R_s\omega_{js} + R_s\tau_s + R_{s+1}P_s y_s - R_s P_s c_{js} \quad (15)$$

Household  $j$  maximizes the expected value of his utility (1) subject to the sequence of budget constraints (15). This yields the first order condition:

$$\frac{1}{R_s P_s c_{j_s}} = \beta E_s \left( \frac{1}{R_{s+1} P_{s+1} c_{j_{s+1}}} \right) \quad (16)$$

We can approximate (16) to the first order as:

$$E_s (R_{s+1} P_{s+1} c_{j_{s+1}}) = \beta R_s P_s c_{j_s} \quad (17)$$

and take the expectation as of time  $t$  of both sides:

$$E_t (R_{s+1} P_{s+1} c_{j_{s+1}}) = \beta E_t (R_s P_s c_{j_s}) \quad (18)$$

Now let us take the expectation of (15) as of time  $t$ , and aggregate all these constraints from time  $t$  to infinity. Assuming that  $R_s \omega_{j_s}$  goes to zero as  $s$  goes to infinity (this is the usual transversality condition), we obtain the intertemporal budget constraint of the household (in expected terms):

$$E_t \sum_{s=t}^{\infty} R_s P_s c_{j_s} = R_t \omega_{j_t} + E_t \sum_{s=t}^{\infty} (R_{s+1} P_s y_s + R_s \tau_s) \quad (19)$$

Applying repeatedly formula (18) and inserting it into (19) we find the consumption function for agents born in period  $j$ :

$$R_t P_t c_{j_t} = (1 - \beta) \left[ R_t \omega_{j_t} + E_t \sum_{s=t}^{\infty} (R_{s+1} P_s y_s + R_s \tau_s) \right] \quad (20)$$

Summing over all  $N_t$  households alive in  $t$  we obtain the aggregate consumption function:

$$R_t P_t C_t = (1 - \beta) \left[ R_t \Omega_t + N_t E_t \sum_{s=t}^{\infty} (R_{s+1} P_s y_s + R_s \tau_s) \right] \quad (21)$$

Now using the definition of “total taxes”  $\Gamma_s$  (equation 12), this can be rewritten:

$$R_t P_t C_t = (1 - \beta) \left[ N_t E_t \sum_{s=t}^{\infty} R_s P_s y_s + R_t \Omega_t - N_t E_t \sum_{s=t}^{\infty} \frac{R_s \Gamma_s}{N_s} \right] \quad (22)$$

We see that generations alive in  $t$  will pay at time  $s > t$  only a fraction  $N_t/N_s$  of total taxes. From this the wealth effect will arise.



### 3.3 The wealth effect

Combining the two equations (14) and (22), we rewrite the consumption function as:

$$R_t P_t C_t = (1 - \beta) \left[ N_t E_t \sum_{s=t}^{\infty} R_s P_s y_s + E_t \sum_{s=t}^{\infty} \frac{N_s - N_t}{N_s} R_s \Gamma_s \right] \quad (23)$$

Note that  $N_s - N_t$  is the number of agents alive in period  $s$ , but yet unborn at period  $t$ . So the wealth of agents currently alive consists of two things: (a) the discounted sum of their incomes (b) the part of taxes that will be paid by future generations in order to reimburse the current financial wealth.

Now we may wonder what is the part of financial wealth that will be considered as “real wealth” by the currently alive generations. For that let us insert (13) into (22). We obtain:

$$R_t P_t C_t = (1 - \beta) \left[ N_t E_t \sum_{s=t}^{\infty} R_s P_s y_s + R_t \Omega_t + N_t E_t \sum_{s=t}^{\infty} \frac{R_{s+1} \Omega_{s+1} - R_s \Omega_s}{N_s} \right] \quad (24)$$

Rearranging we find:

$$R_t P_t C_t = (1 - \beta) N_t E_t \left[ \sum_{s=t}^{\infty} R_s P_s y_s + \sum_{s=t}^{\infty} R_{s+1} \Omega_{s+1} \left( \frac{1}{N_s} - \frac{1}{N_{s+1}} \right) \right] \quad (25)$$

Consider the case where, by an adequate fiscal policy,  $\Omega_t$  remains constant in time and equal to  $\Omega_0$ . In such a case the “supplementary wealth” beyond discounted incomes is, from (25), equal to:

$$\left[ N_t \sum_{s=t}^{\infty} R_{s+1} \left( \frac{1}{N_s} - \frac{1}{N_{s+1}} \right) \right] \Omega_0 \quad (26)$$

It is easy to see that the coefficient into brackets is between 0 and 1, and other things equal, larger when the rate of increase of the population is larger.

## 4 Dynamic equilibrium

In equilibrium we have  $C_t = Y_t$ , so equation (25) becomes, after dividing by  $N_t$ :

$$R_t P_t y_t = (1 - \beta) E_t \left[ \sum_{s=t}^{\infty} R_s P_s y_s + \sum_{s=t}^{\infty} R_{s+1} \Omega_{s+1} \left( \frac{1}{N_s} - \frac{1}{N_{s+1}} \right) \right] \quad (27)$$

Let us rewrite it for  $t + 1$ , take the expectation as of time  $t$ , and subtract from (27). We find:

$$E_t (R_{t+1} P_{t+1} y_{t+1}) = \beta R_t P_t y_t - (1 - \beta) \left( \frac{1}{N_t} - \frac{1}{N_{t+1}} \right) R_{t+1} \Omega_{t+1} \quad (28)$$

Multiplying by  $N_{t+1}/R_{t+1}$ , and taking  $N_{t+1}/N_t = 1 + n$  we obtain:

$$E_t (P_{t+1} Y_{t+1}) = \beta (1 + n) (1 + i_t) P_t Y_t - (1 - \beta) n \Omega_{t+1} \quad (29)$$

This will be our central dynamic equation. Note that the “non-Ricardian” character appears through the last term, which shows that, unlike in the Ricardian model where  $n = 0$ , financial wealth does matter.

Now since the model is non Ricardian, the dynamics will depend on the actual fiscal transfer policy. To simplify the dynamics below, and since our focus is on monetary policy, we shall choose the simplest fiscal policy and assume that the government balances its budget period by period. Fiscal transfers will thus exactly compensate interest payments on bonds:

$$T_t = -i_t B_t \quad (30)$$

Combining (7), (8) and (30) we find that under the balanced budget policy (30) total financial wealth will remain constant:

$$\Omega_t = \Omega_0 \quad \text{for all } t \quad (31)$$

The dynamic equation (29) then becomes:

$$E_t (P_{t+1} Y_{t+1}) = \beta (1 + n) (1 + i_t) P_t Y_t - (1 - \beta) n \Omega_0 \quad (32)$$

and since  $M_t = P_t C_t = P_t Y_t$ , this can be rewritten:

$$E_t M_{t+1} = \beta (1 + n) (1 + i_t) M_t - (1 - \beta) n \Omega_0 \quad (33)$$

## 5 Liquidity effects

We shall now see that the non-Ricardian character of the economy, i.e. the fact that  $n > 0$ , will produce a liquidity effect<sup>3</sup>.

### 5.1 The nominal interest rate

We can actually solve explicitly equation (33) for the nominal interest rate:

$$1 + i_t = \frac{1}{\beta(1+n)} E_t \left( \frac{M_{t+1}}{M_t} \right) + \frac{(1-\beta)n\Omega_0}{\beta(1+n)M_t} \quad (34)$$

We see that the first term, which is present even if  $n = 0$ , displays the “inflationary expectations effect”: indeed, the nominal interest rate will rise if a positive monetary shock announces future money growth, i.e. if:

$$\frac{\partial}{\partial M_t} \left[ E_t \left( \frac{M_{t+1}}{M_t} \right) \right] > 0 \quad (35)$$

which is what is generally found empirically. In the example above (equation 9) this will occur if  $\rho > 0$ . We shall assume in what follows that the money process satisfies condition (35).

Now the second term, which appears only if  $n > 0$ , i.e. if we are in a non-Ricardian framework, clearly introduces a liquidity effect, since an increase in money directly decreases the nominal interest rate. The higher  $n$ , the stronger this effect.

We can give an even simpler expression. Assume that money  $M_t$  is stationary around the value  $M_0$ . From (33), the corresponding stationary value of the interest rate,  $i_0$ , is related to  $M_0$  and  $\Omega_0$  by:

$$M_0 = \beta(1+n)(1+i_0)M_0 - (1-\beta)n\Omega_0 \quad (36)$$

Let us define:

$$\theta = \beta(1+n)(1+i_0) \quad (37)$$

If we want to have a “wealth effect”, net financial assets must be positive, i.e.  $\Omega_0 > 0$ . As a consequence, from (36) the parameter  $\theta$  must satisfy<sup>4</sup>:

<sup>3</sup>The appendix shows in an intuitive manner in an IS-LM framework how the liquidity effect arises from the wealth effect.

<sup>4</sup>We may note that condition (38), which is similar to Wallace’s (1980) condition for the viability of money in an OLG structure, appears in relation to other issues concerning money and monetary policy. For example it is shown in Bénassy (2002) that (38) is a condition for price determinacy when the central bank uses interest rate rules à la Taylor (1993).

$$\theta > 1 \quad (38)$$

Now combining (34), (36) and (37), we obtain:

$$\frac{1 + i_t}{1 + i_0} = \frac{1}{\theta} E_t \left( \frac{M_{t+1}}{M_t} \right) + \left( 1 - \frac{1}{\theta} \right) \frac{M_0}{M_t} \quad (39)$$

We see that formula (39) gives a balanced view between the new non-Ricardian liquidity effect and the traditional inflationary expectations effect. We can note that the higher  $\theta$  (and thus notably the higher  $n$ ), the stronger the liquidity effect will be.

## 5.2 The real interest rate

The real interest rate  $r_t$  is defined as:

$$1 + r_t = (1 + i_t) \frac{P_t}{P_{t+1}} \quad (40)$$

Now let us assume that endowments per head are constant in time, so that  $Y_{t+1}/Y_t = 1 + n$ . Combining this with  $M_t = P_t Y_t$ , and equations (36), (37) and (39) we obtain:

$$\frac{1}{1 + r_t} = \beta \frac{M_{t+1}/M_t}{E_t (M_{t+1}/M_t) + (\theta - 1) (M_0/M_t)} \quad (41)$$

and:

$$E_t \left( \frac{1}{1 + r_t} \right) = \beta \frac{E_t (M_{t+1}/M_t)}{E_t (M_{t+1}/M_t) + (\theta - 1) (M_0/M_t)} \quad (42)$$

In view of assumption (35) we see that the real interest rate will react negatively to a positive money shock, in the sense that:

$$\frac{\partial}{\partial M_t} \left[ E_t \left( \frac{1}{1 + r_t} \right) \right] > 0 \quad (43)$$

## 6 A generalization

We shall now consider the more general case where households' resources diminish at the rate  $\xi < 1$ . The income and transfer of an agent of generation  $j$  are  $\xi^{t-j}$  times those of an agent of generation  $t > j$ . To be more precise:

$$y_{jt} = \xi^{t-j} y_t \quad \tau_{jt} = \xi^{t-j} \tau_t \quad (44)$$

where  $y_t$  and  $\tau_t$  are the income and transfer of a newborn agent in period  $t$ .

In such a case it can be shown<sup>5</sup> that the dynamic equation (33) is replaced by the more general one:

$$\xi E_t M_{t+1} = \beta (1+n) (1+i_t) M_t - (1-\beta) (1+n-\xi) \Omega_0 \quad (45)$$

We can again solve directly for the interest rate:

$$1+i_t = \frac{\xi}{\beta(1+n)} E_t \left( \frac{M_{t+1}}{M_t} \right) + \frac{(1-\beta)(1+n-\xi)\Omega_0}{\beta(1+n)M_t} \quad (46)$$

in which we see that the second term does indeed produce a liquidity effect. Now from (45) the stationary value of money  $M_0$  and the stationary interest rate  $i_0$  are related by:

$$\xi M_0 = \beta (1+n) (1+i_0) M_0 - (1-\beta) (1+n-\xi) \Omega_0 \quad (47)$$

We can now give a more general definition of the parameter  $\theta$ :

$$\theta = \frac{\beta(1+n)(1+i_0)}{\xi} \quad (48)$$

Combining (46), (47) and (48) we find that the interest rate is given by:

$$\frac{1+i_t}{1+i_0} = \frac{1}{\theta} E_t \left( \frac{M_{t+1}}{M_t} \right) + \left( 1 - \frac{1}{\theta} \right) \frac{M_0}{M_t} \quad (49)$$

This is exactly the same expression as (39), but the expression of  $\theta$  has been generalized from (37) to (48). We see that a lower value of  $\xi$  increases the value of  $\theta$  and therefore enhances the non-Ricardian liquidity effect. In the extreme case where  $\xi = 0$  (i.e. when agents have all their income in the first period of their life),  $\theta$  is infinite and the liquidity effect totally dominates.

## 7 The persistence of the liquidity effect

We shall now develop an example showing that our liquidity effect can be quite persistent. Let us loglinearize equations (39) or (49), which yields:

$$\frac{i_t - i_0}{1+i_0} = \frac{1}{\theta} (E_t m_{t+1} - m_t) - \left( 1 - \frac{1}{\theta} \right) (m_t - m_0) \quad (50)$$

where the Ricardian particular case is obtained by taking  $\theta = 1$ . Let us consider the following stationary money process:

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<sup>5</sup>The proof is similar to that for  $\xi = 1$  in sections 3 and 4. It is available upon request from the author.

$$m_t - m_0 = \frac{\varepsilon_t}{(1 - \rho L)(1 - \mu L)} \quad 0 < \rho < 1 \quad 0 < \mu < 1 \quad (51)$$

where  $\varepsilon_t$  is i.i.d.. Then:

$$E_t m_{t+1} - m_t = \frac{(\mu + \rho - 1 - \mu \rho L) \varepsilon_t}{(1 - \rho L)(1 - \mu L)} \quad (52)$$

If  $\mu + \rho > 1$ , then a positive monetary innovation  $\varepsilon_t > 0$  creates the expectation of a monetary increase next period, which is the assumption traditionally associated with the “inflationary expectations effect”. So we shall assume  $\mu + \rho > 1$  so as to have this effect.

Now combining (50), (51) and (52) we can compute the full effect of monetary shocks on the interest rate:

$$\frac{i_t - i_0}{1 + i_0} = \frac{(\mu + \rho - \theta - \mu \rho L) \varepsilon_t}{\theta (1 - \rho L)(1 - \mu L)} \quad (53)$$

We first see that if  $\mu + \rho > 1$ , the Ricardian version of the model ( $\theta = 1$ ) always delivers an increase in interest rates on impact in response to monetary injections. We thus obtain the traditional “inflationary expectations effect”.

Let us now move to the non-Ricardian case  $\theta > 1$ . Looking at formula (53), we see that the first period impact  $\mu + \rho - \theta$  is negative as soon as:

$$\theta > \mu + \rho \quad (54)$$

We shall see that this liquidity effect is persistent, and that condition (54) is actually sufficient for a monetary injection to have a negative effect on the interest rate in all subsequent periods. Formula (53) can indeed be rewritten as:

$$\frac{i_t - i_0}{1 + i_0} = \frac{1}{\theta(\mu - \rho)} \left[ \frac{\rho(\theta - \rho) \varepsilon_t}{1 - \rho L} - \frac{\mu(\theta - \mu) \varepsilon_t}{1 - \mu L} \right] \quad (55)$$

This can be written as a distributed lag of all past innovations in money  $\varepsilon_{t-j}$ ,  $j \geq 0$ :

$$\frac{i_t - i_0}{1 + i_0} = \sum_{j=0}^{\infty} \omega_j \varepsilon_{t-j} \quad (56)$$

with:

$$\omega_j = \frac{\rho^{j+1}(\theta - \rho) - \mu^{j+1}(\theta - \mu)}{\theta(\mu - \rho)} \quad (57)$$

We want to show now that condition (54) is a sufficient condition for  $\omega_j < 0$  for all  $j$ . This is done simply by rewriting (57) as:

$$\omega_j = \frac{\mu + \rho - \theta}{\theta} \left( \frac{\mu^{j+1} - \rho^{j+1}}{\mu - \rho} \right) - \frac{\mu\rho}{\theta} \left( \frac{\mu^j - \rho^j}{\mu - \rho} \right) \quad (58)$$

The second term is always negative or zero. The first term is negative if  $\theta > \mu + \rho$ . So condition (54) is sufficient for the non-Ricardian liquidity effect to dominate the usual inflationary expectations effect, not only on impact, but for all subsequent periods as well.

## 8 Conclusions

We developed in this article a new mechanism through which liquidity effects are introduced into dynamic monetary models.

The basic channel is the following: (a) in a non Ricardian economy, accumulated financial assets represent, at least partly, real wealth to the generations alive, simply because part of the future taxes that are a counterpart to this financial wealth will be paid by the next generations. This is not the case in the ‘‘Ricardian’’ framework since there is no such thing as the ‘‘next generations’’, and: (b) this wealth effect gives rise to a liquidity effect as follows: An increase in money raises prices, which decreases the real value of financial wealth. Because of the wealth effect this reduces aggregate demand. In order to maintain aggregate demand at the market clearing level the real interest rate goes down. This creates, *ceteris paribus*, the liquidity effect<sup>6</sup>.

## 9 Appendix

To guide our intuition as to why the wealth effect leads to a liquidity effect, let us consider a simple traditional IS-LM model augmented with a ‘‘wealth effect’’ à la Pigou or Patinkin. To make the exposition particularly simple we write this model in loglinear form:

$$y = -a(i - \pi^e) + b(\omega - p) + cy \quad IS \quad (59)$$

$$m - p = -di + ey \quad LM \quad (60)$$

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<sup>6</sup>See the appendix for a more detailed elaboration of this reasoning in a very simple model.

$$y = y_0 \quad (61)$$

where  $\pi^e$  is the expected rate of inflation,  $\omega = \text{Log } \Omega$ ,  $p = \text{Log } P$  and:

$$a > 0 \quad b > 0 \quad c > 0 \quad d > 0 \quad e > 0 \quad (62)$$

The third equation expresses market clearing (which we assumed throughout the article). The IS equation says that output is equal to demand, which itself depends negatively on the real interest rate  $i - \pi^e$ , and positively on real wealth  $\omega - p$ . Note that the presence of this “real wealth” term, i.e.  $b > 0$ , is specific of the non Ricardian framework, as was shown rigorously in this article. Now we can solve for the nominal interest rate  $i$ , the real interest rate  $r = i - \pi^e$ , and price level  $p$ . Omitting irrelevant constants this yields:

$$i = \frac{a\pi^e - bm}{a + bd} \quad r = -b\frac{m + d\pi^e}{a + bd} \quad p = \frac{ad\pi^e + am}{a + bd} \quad (63)$$

We first see, differentiating the expression of  $i$  in (63), that:

$$\frac{\partial i}{\partial m} = \frac{1}{a + bd} \left( a \frac{\partial \pi^e}{\partial m} - b \right) \quad (64)$$

We recognize the two effects identified in the article. First the “inflationary expectations effect”, which is positive if a positive money shock raises inflationary expectations ( $\partial \pi^e / \partial m > 0$ ). Secondly the negative “liquidity effect”, itself due to the wealth effect ( $b > 0$ ).

Now the underlying mechanism for the liquidity effect is the following: an increase in money creates a price increase (63). This price increase decreases demand because of the real wealth effect (the second term in equation 59). To maintain total demand at the market clearing level, the first term in (59) must increase, i.e. the real rate of interest must decrease. This decrease in the real interest rate creates the liquidity effect.

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