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Public Markets Tailored for the Cartel

- Favoritism in Procurement Auctions -[*]

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Abstract

In this paper, we investigate interaction between two firms, which are engaged in a repeated procurement relationship modelled as a multiple criteria auction, and an auctioneer (a government employee) who has discretion in devising the selection criteria.

A first result is that, in a one-shot context, favoritism turns the asymmetric information (private cost) procurement auction into a symmetric information auction (in bribes) for a common value prize. In a repeated setting we show that favoritism substantially facilitates collusion. It increases the gains from collusion and contributes to solving basic implementation problems for a cartel of bidders that operates in a stochastically changing environment. A most simple allocation rule where firms take turn in winning independently of stochastic government preferences and firms' costs achieves full cartel efficiency including price, production and design efficiency. In each period the selection criteria is fine-tailored to the in-turn winner: the "environment" adapts to the cartel. This result holds true when the expected punishment is a fixed cost. When the cost varies with the magnitude of the distortion of the selection criteria (compared with the true government's preferences), favoritism only partially shades the cartel from the environment. We thus find that favoritism generally facilitates collusion at a high cost for society. Our analysis suggests some anti-corruption measures that can be effective to curb favoritism and collusion in public markets. It also shows that the rotation of officials is not one of them.

Keywords: auction, collusion, favoritism, procurement

JEL: D44, D73, H57

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1   Introduction

Many cartels operate in a stochastically changing environment. In particular, this is the case of firms involved in public procurement. The public demand for e.g., construction works typically depends on a number of factors that are difficult to predict. They include social needs, elected representatives' political agenda, internal budget concerns etc... In addition, firms' technology changes with time. Altogether this implies a significant uncertainty about the profitability of future contracts. In face of such an uncertain environment, a cartel of firms must devise a mechanism that while being responsive to changes does not open up for gaming opportunities. In this paper we claim that favoritism can contribute to solving key problems for a cartel of bidders that operate in a stochastically changing environment. A main motivation for the paper is the mounting body of evidence that collusion and corruption often go hand in hand in public procurement.

In France, practitioners and investigators in courts of accounts, competition authorities, and in the judiciary have long been aware of the close links between collusion and corruption in public procurement.[1] According to one of the leading Parisian anti-corruption judges, there exists in France, almost not a single case of large stake collusion in public procurement without corruption.[2] Beside empirical motivations, there are theoretical motivations for investigating the links between favoritism and collusion. In particular, a cartel typically faces a tension between the efficiency goal and the need to provide firms with incentives to reveal private information. A fair amount of attention has been given to the theoretical problems facing a cartel that operates within an imperfectly or privately observable environment. The general results of most relevance to our issue are due to Fudenberg, Levine and Maskin (1994). But their central efficiency result has not always been applicable when approaching more concrete issues. Recently, Athey and al. (2004) show that it can simply be too costly

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[1] The testimony of J. C. Mery provides suggestive evidence of those links (*Le Monde*, September 22 and 23, 2000). J. C. Mery, a City Hall official, admitted that for ten years (1985-94) he organized and arbitrated collusion in the allocation of most construction and maintenance contracts for the Paris City Hall. In exchange, firms were paying bribes used to finance political parties.

A recent judgment in 'Les Yvelines' (Cour d'Appel de Versaille, January 2002) provides a vivid illustration as well.

[2] This judge from the Pole Financier was among other things involved in the investigation of corruption allegations in the procurement of a 4.3 billion euros market for the recontruction of Paris's lycees (see *Le Monde* April 23 2005). The case resulted in a series of convictions.

for a price cartel to provide the right incentives for firms to reveal private information about shocks to costs so the optimal mechanism entails price rigidity.[3] Our analysis is concerned with a cartel of bidders that face both incomplete information about demand i.e., government preferences and asymmetric information about shocks to firms' costs. Most closely related to our paper is Athey and Bagwell (2001). They study a price cartel that operates on a market with a given demand but under asymmetric information about costs. They show that cartel efficiency (including production and price efficiency) is achievable in a scheme where firms are rewarded for truthfully reporting high costs by future market shares. Their efficiency result relies on assumptions that secure the existence (with sufficiently high probability) of states where firms have an identical cost structure.[4] In those states utility can be transferred at no cost for the cartel. As a consequence future market shares can be used to provide incentives for revealing private information without relinquishing production or price efficiency. Our main contribution is to show that, in an auction context, corruption can solve the cartel's information revelation problem in a situation characterized both by asymmetric information and stochastic government demand. Full cartel efficiency including, production, price and design efficiency (the contract is fine-tailored to the cartel) is achievable in a very simple scheme relying on a non-contingent allocation rule so that firms take turn to win in a pre-determined manner. Favoritism effectively shades the cartel from hazards in the environment. The expected cost of corruption determines the extent of favoritism. This result is established for the case the expected punishment cost is independent of the magnitude of the distortion of government preferences. When the expected punishment varies with that magnitude, favoritism only partially shades the cartel from hazards in the environment. We find that favoritism generally exacerbates the social costs of collusion: the selected specification is socially inefficiency and the price paid by the government is higher than in the absence of favoritism.

We model the procurement procedure as a "first score auction". Two firms characterized by a vector of cost parameters compete in scores with offers that include a specification of the project and a price. Public preferences are stochastic. The procedure is administered by an auctioneer who is a government employee. At the beginning of the period the auctioneer, privately observes a signal of public preferences. His duty is to devise and announce a scoring

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[3]See also Green and Porter (1984) for the analysis of a price cartel on a market with a demand subject to shocks.

[4]Even then cartel efficiency is only achievable under some configuration of the parameters.

rule that reflects the (current) public preferences. In the absence of favoritism, the procedure selects the socially efficient specification of the project.

The presence of asymmetric information between the government and its auctioneer implies that the auctioneer has some discretion when deciding over the scoring rule. We call favoritism the act of biasing the scoring rule in favor of one of the firms. Corruption is modelled as an auction-like procedure that takes place before the official auction. Firms compete in (menus of) corrupt "deals" including a bribe and a demanded scoring rule. We find that with favoritism the procedure selects a non-standard specification of the project. The intuition is that the associated scoring rule induces minimal competition and thus maximal profit-if-win in the official auction. In the one-shot setting favoritism turns the asymmetric information private cost procurement auction, into a symmetric information auction (in bribes) for a common value prize corresponding to influence over the design of the contract. A main intuition here is that corruption works as a revelation mechanism: firms truthfully reveal their private information to the corrupt agent who uses that information to maximize their rents because that maximizes the bribe he receives.

We then consider a situation where firms meet repeatedly, each period on a new market (the auctioneers are short-run players). We show that favoritism fully solves the cartel's problems related to stochastic government preferences and privately observable costs. Provided each firm is efficient at producing some specification of the project, the cartel can earn the maximal income in a scheme that selects the winner independently of the true preferences and of firms' costs. The intuition is that a corrupt auctioneer has own incentives to fine-tailor the scoring rule to the in-turn winner. Firms' main concern is to contain competition in bribes. That is achieved by opting for a fixed in-turn allocation rule which makes any defection from the equilibrium strategies immediately observable.

In an extension we investigate a case where the expected punishment for favoritism is a function of the magnitude of the distortion between the announced scoring rule and the true preferences. We find that the central insights from the fixed punishment case carry over. In the stage game competition in bribes does not dissipate all the firms' rents however. And in the repeated setting the cartel may face a problem due to imperfect public information. For high cost of punishment, the optimal scheme is contingent on the true government preferences which are never observed. The official auction outcome is bounded away from full cartel efficiency. For low cost of punishment the pre-determined in-turn allocation rule is optimal

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and full cartel efficiency obtains.

The equilibrium allocation patterns emerging from the analysis is consistent with empirical findings. There exists ample evidence e.g., in developing countries of problems of maintenance of construction objects due to the non-standard design that was selected in the international procurement procedure (see Rose-Ackerman 1999). Evidence from corruption scandals in France also show that the tender winner is the most efficient firm and that its profits often are larger than the average in the branch (30% contra 5%) as in the case with the court case concerned with the series of constructions contracts in Paris.

A central policy implication is that since collusion and corruption are linked they must investigated conjointly. A second implication of our analysis is that increasing the severeness of punishment can have a real impact on the extent of favoritism. On the other hand the much advocated anti-corruption policy aiming at reducing the time in any particular office i.e., making procurement agents short-run players finds no support in our analysis.

This paper contributes to a growing literature on corruption in auction.[5] The auctioneer's abuse of discretion to devise the selection rule has been studied in Che and Burget (2004) in the context of a single auction. The present article is most closely related to Compte, Lambert-Mogiliansky and Verdier (2005) and Lambert-Mogiliansky and Sonin (2006). Both articles are concerned with links between corruption and collusion. They address a cartel's enforcement problem in a one-shot setting and focus on the impact of the auctioneer's abuse of discretion to let firms readjust their bid. In Compte et al. the auctioneer sells an illegal opportunity to resubmit, which is shown to permit sustaining collusion in a single object auction. In Lambert-Mogiliansky and Sonin, the auctioneer abuses a legal right to let all firms simultaneously readjust their offer in the context of a multiple-object auction. As a consequence collusive market-sharing becomes sustainable. The contribution of the present paper is to demonstrate corruption's role with respect to another central problem of a cartel: how to achieve (cartel) efficiency in a stochastically changing environment.

The paper is organized as follows. The model is described in section 2. Section 3 offers an analysis of the one-stage game. In Section 4 we derive our central results. Section 5 proposes an extension to the case with varying punishment cost. Central assumptions are discussed in section 6 where we also suggest policy implications for procurement and control agencies.

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[5]See for instance Laffont J-J. and J. Tirole (1991), Celentani, M. and J. Ganunza, (2002), Che and Burget (2004), Compte, Lambert-Mogiliansky and Verdier (2005).

2 The model

In each time period a project is allocated. A project allows for a multiplicity of specifications. A specification is a vector $\mathbf{q} = (q_1, ...q_k)$ where q_j represents the level of the j (quality) component. There are two firms indexed i, $i = 1, 2$, which are characterized by their cost function

$$c_i \left(\mathbf{q}; \boldsymbol{\theta}_i^t \right) = \sum_{j=1}^{k} \frac{\theta_{ij}^t q_j^2}{2}$$

where $\theta_{ij}^t \in \{\underline{\theta}, \overline{\theta}\}$, $j = 1, ..., k$ is firm $i's$ cost parameter associated with quality component q_j in period t. The vector of cost parameters $\boldsymbol{\theta}_i^t = \left(\theta_{i1}^t, ...\theta_{ik}^t \right)$ is firm i's private information. In each period there is a new draw of $(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2)$. For the sake of convenience we remove the realizations $\boldsymbol{\theta}_i^t = (\overline{\theta}, \overline{\theta}, ..., \overline{\theta})$, $i = 1, 2$ from the support and we assume that each firm has a comparative advantage in at least one component.[6] The probability of the realizations left are proportional to the probabilities which we would have if parameters θ_{ij}^t are i.i.d. with $prob(\theta_{ij}^t = \underline{\theta}) = \rho$ across all i, j and t.[7]

The government derives utility from the realization of a project in period t:

$$W \left(\mathbf{q}^t, p^t; \boldsymbol{\alpha}^t \right) = \alpha_1^t q_1^t + ... + \alpha_k^t q_k^t - p^t; \tag{1}$$

$$\text{with } \alpha_j^t \geq 0, \ \forall j = 1...k, \ \sum_{j=1}^{k} \alpha_j^t = 1, \tag{2}$$

where p^t is the price paid to the firm that delivers the project and $\boldsymbol{\alpha}^t = \left(\alpha_1^t, ..., \alpha_k^t \right)$ is a vector of parameters representing the true social preference in period t. A zero value for a component j, $\alpha_j^t = 0$ is understood as no social value of q_j above a *minimal level* that defines a "basic good". The vector $\boldsymbol{\alpha}^t$ is random with full support Δ^{k-1}. The government does not know the true $\boldsymbol{\alpha}^t$. It hires an auctioneer who privately observes a signal of the true $\boldsymbol{\alpha}_t$ at the beginning of each period. For simplicity we assume that the signal is fully informative.[8]

The auction rule

At the beginning of each period the auctioneer announces a selection criteria which is a function of both price p and quality $\mathbf{q} = (q_1, ..., q_k)$. We consider a class of selection criteria

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[6] These restrictions basically imply that we disregard some special cases and focus on the main results. Where that is relevant we comment on some implications of relaxing the restrictions.

[7] The precise characterization of the probability is rather complex but its details play no role for our results.

[8] This is a not an assumption crucial to our results.

similar to the government's utility function:

$$S\left(\mathbf{q},\widehat{\boldsymbol{\alpha}}\right) = s\left(\mathbf{q},\widehat{\boldsymbol{\alpha}}\right) - p = \sum_{j=1}^{k}\widehat{\alpha}_j q_j - p, \ \sum_{j=1}^{k}\widehat{\alpha}_j = 1,$$

where $\widehat{\boldsymbol{\alpha}}$ is the vector of parameters *announced* by the auctioneer (see *Timing* below). Throughout the paper we refer to $\widehat{\boldsymbol{\alpha}}$ as the "scoring rule" which is a slight abuse of language since the score of an offer is determined by its price also according to the selection criterion. The firms simultaneously submit in a sealed envelop an offer including a project specification $\mathbf{q}_i^t$ and a price p_i, $i = 1, 2$. The contract is awarded to firm i^{*t} whose offer maximizes (among submitted offers) the announced selection criteria subject to a "reserve score" normalized to zero:

$$
\begin{aligned}
i^{*t} &\in \ \arg\max_{i=1,2} S\left(\mathbf{q}_i^t, p_i^t, \widehat{\boldsymbol{\alpha}}\right) \\
s.t &: \ S\left(\mathbf{q}_i^t, p_i^t, \widehat{\boldsymbol{\alpha}}\right) \geq 0.
\end{aligned}
\tag{3}
$$

The winner is due to deliver the specification $\mathbf{q}_{i*}^t$ at price p_{i*}^t. In case of tie in scores the project is awarded to the firm whose "quality score"(i.e., $s\left(\mathbf{q},\widehat{\boldsymbol{\alpha}}\right)$) is highest. In case of tie in both price and quality the auctioneer randomizes. We refer to this procedure as a First Score Auction (FSA).

The firm i's per-period profit-if-win is

$$\pi_i^t = p_i^t - c_i\left(\mathbf{q}_i^t; \boldsymbol{\theta}_i^t\right).\tag{4}$$

Profit-if-lose is zero. We assume that when a firm is indifferent between winning with zero profit and losing, it chooses (so as) to win. The game is infinitely repeated with the same two firms but with a different auctioneer in each period. The firms discount future gains with a common factor δ. Their payoff for the whole game is the discounted sum of the per period payoffs.

Corruption

The auctioneer is opportunistic. He accepts bribes in exchange for announcing a scoring rule i.e., some $\widehat{\boldsymbol{\alpha}}$. The auctioneer's utility is

$$U = w + b - m,$$

where w is a wage that we normalize to 0, b is a bribe and $m \geq 0$ is a term that captures the expected punishment cost including moral and other costs associated with distorting

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government preferences.[9] In the basic model expected punishment is a fixed cost. This is consistent with e.g., the French legislation (Code Penal 432-14, 432-11).[10] In an extension we consider the special case the expected punishment depends on the magnitude of the distortion of social preferences and where $k = 2$, so $U = b - m\left(\widehat{\alpha}_1 - \alpha_1\right)^2$. Such a model can be relevant when the magnitude of the distortion significantly affects the probability of detection. We discuss these assumptions in section 6.

Corruption is modelled as a procedure whereby the firms compete in corrupt "deals" where a deal is an offer to pay a bribe in exchange for a specific scoring rule. The two firms simultaneously and secretly submit a menu of deals $M_i = \{(\boldsymbol{\alpha}_{il}, b_{il}),\ l = 1, ..., n_i\}$, where n_i is (finite and) freely chosen by firm i.

The bribe is only paid by the official auction's winner if the announced scoring rule corresponds to one he demanded. The assumption here is that the agent has some discretion to intervene after the submission of the official offers. He may invalidate some bids appealing to a formal default, or ask for a resubmission appealing to default in the tendering documents.[11] As a result the auction procedure does not result in any allocation under the current period. Although this might in practice occur before the official opening, for simplicity we refer to this action as "rejecting the auction's outcome". The key feature is that the agent can take an action that results in zero payoffs for all. This makes the payment of bribes enforceable which is accounted for as an assumption. Another important consequence is that it also prevents free-riding in corruption which we will show in the analysis.

3   The stage game

The stage game is defined by the following *Timing*:

step 0: Firms privately learn their cost parameters $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$;

step 1: The auctioneer learns $\boldsymbol{\alpha}$, the firms submit each its menu of deals $M_1 = \{(b_{1l}, \boldsymbol{\alpha}_{1l}),\ l = 1, ..., n_1\}$ and $M_2 = \{(b_{2l}, \boldsymbol{\alpha}_{2l}),\ l = 1, ..., n_2\})$ respectively;

step 2: The auctioneer makes an announcement $\widehat{\boldsymbol{\alpha}}$, $\widehat{\boldsymbol{\alpha}} \in \Delta^{k-1}$;

step 3: The firms simultaneously submit their offer $(\mathbf{q}_i, p_i)$, $i = 1, 2$;

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[9] The government can engage a procedure to find out its true preferences and punish the auctioneer if he distorted them in his announcement.

[10] The punishment for favoritism is independent of the social-economic loss it induced.

[11] As argued in Lambert-Mogiliansky and Sonin (2006) empirical evidence shows that there is ample opportunities for the auctioneer to intervene in the procedure so as to effectively relaunch it and delay its outcome.

step 4: The auctioneer publicly opens the envelops and he selects the firm whose offer maximizes the selection criteria corresponding to the announced scoring rule;

step 5: The auctioneer decides to confirm or reject the outcome of the auction. If he confirms the outcome, the winner i^* pays a bribe b if and only if $(\widehat{\boldsymbol{\alpha}}, b) \in M_{i*}$. Otherwise no bribe is paid. If he rejects the outcome no transaction takes place.

We first establish a result applying to the First Score Auction described by the *Timing* above when deleting step 1 from consideration and as we show later applying to any subgame starting from step 2. Throughout the paper we consider equilibria which are subgame perfect.

Lemma 1 *The subgame perfect equilibrium offers of the FSA are characterized by specification efficiency:* $\mathbf{q}_i^* = \arg \max s(\mathbf{q}, \widehat{\boldsymbol{\alpha}}) - c(\mathbf{q}; \boldsymbol{\theta}_i)$.

All proofs are gathered in the appendix.

The result exploits separability between quality and price in the selection criteria. It can be shown that for any offer not including the efficient values for the components, we can find another offer with the same score but that yields a higher expected profit.

The result in Lemma 1 greatly simplifies the forthcoming analysis. Lemma 1 allows us to, *at step 3*, separate between firms' offer of project specification and their price bid.[12] The equilibrium values of the components are the efficient ones corresponding to the announcement

$$\mathbf{q}_i^* = \arg \max s(\mathbf{q}, \widehat{\boldsymbol{\alpha}}) - c(\mathbf{q}; \boldsymbol{\theta}_i)$$

$$q_{ij}^*\left(\theta_{ij}, \widehat{\alpha_j}\right) = \frac{\widehat{\alpha_j}}{\theta_{ij}}, \ i = 1, 2, \ j = 1..., k.$$

When the announcement corresponds to the true government preferences, Lemma 1 implies social efficiency in the specification of the project.

3.1 Favoritism

We now proceed to investigate the one-shot game described in *Timing* above.

Proposition 1 *There exists a Perfect Bayes-Nash equilibrium such that*

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[12] A similar result can be found in Che (1993).

i. For $m \leq \frac{(\bar{\theta}-\underline{\theta})}{2\theta\bar{\theta}}$ the equilibrium scoring rule is $\widehat{\boldsymbol{\alpha}}^* = (0,...,1_j,...0)$ for some $\theta_j; \theta_{1j} \neq \theta_{2j}$ and $b_{1j}^* = b_{2j}^* = \frac{(\bar{\theta}-\underline{\theta})}{2\underline{\theta}\bar{\theta}}$ for all j. When $m > \frac{(\bar{\theta}-\underline{\theta})}{2\underline{\theta}\bar{\theta}}$ there is no favoritism.

ii. The equilibrium offers are the competitive offers relative to the announced scoring rule.

A first result is that whatever the true government preferences, favoritism always entails an extreme (single-peaked) scoring rule $\widehat{\boldsymbol{\alpha}}^* = \widehat{\boldsymbol{\alpha}}^j = (0,...,1_j,...0)$ for some j.[13] The intuition for the single-peakness result is that the winner's profit is maximal when the scoring rule emphasizes a single component for which he has a comparative advantage. Alternatively, a selection rule including a single-peaked scoring rule induces the "weakest possible competitive pressure" among all FSA generated by any selection rule from the relevant class.

The interpretation of this result is that with favoritism the scoring rule tends to drive to a minimum the weight given to most components while emphasizing quite exclusively a component characterized by weak competition in production.[14] This means that the winning project has a specification that tends to be "non-standard" in the sense of being unusual. We note that the true government preferences have minimal impact on the announced scoring rule. In case of ties in the corruption game, the auctioneer may choose the deal that is most congruent with the true preferences.

Quite remarkably we find that firms' asymmetric information is a minor concern in our context. The intuition is that a corrupt auctioneer has incentives to use that information to devise a scoring rule that *maximizes* the winning firm's rents as it also maximizes his bribe.[15] Therefore firms have an incentive to reveal their cost. Yet, the agent's favor is costly and there may exist incentives for firms to free-ride. Indeed with some probability the two firms have low cost on the same component. Therefore there is an incentive to let the other firm demand a scoring rule in exchange for a bribe and undercut its official bid to win. Such deviation deprives the auctioneer from the bribe. He has therefore incentives to prevent it which he can do by credibly threatening to reject the auction's result. In the appendix we

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[13] We show in section 5 below (Extensions) that this result is robust to other specification of the punishment costs.

[14] Strictly speaking the interpretation of zero weight as a minimal level is equivalent to assuming that firms are (more or less identical) in the production of a "basic good" while they differ in the production of specifications of the project in excess of the requirement defining the "basic good".

[15] An honest auctioneer uses information about costs to minimize firms' rents. If he knew the firms' costs, he would simply give the contract to the most efficient firm and pay it its actual cost i.e., he would leave no rents to the winner.

show that in equilibrium there is no free-riding so firms' cost structure is fully revealed to the auctioneer in the submitted menus of deals. The firms infer all relevant information about each other's cost structure from the announced scoring rule. The equilibrium offers are the (unique) competitive Nash equilibrium offers of the symmetric information FSA defined by the announced scoring rule (for details see the appendix). The offered specifications are efficient relative to $\widehat{\alpha}^*$ and the equilibrium price is determined by the second score.

Competition for favors drives up the bribe to $b^* = \frac{(\bar{\theta}-\underline{\theta})}{2\underline{\theta}\bar{\theta}}(\geq m)$ which is the profit that yields with a scoring rule that is most favorable to the winner. Since this profit is the same for the two firms, the auctioneer captures the totality of the winning firm's rents.[16] In the remaining of the paper we assume that $m < \frac{(\bar{\theta}-\underline{\theta})}{2\underline{\theta}\bar{\theta}}$ so the stage game is characterized by favoritism.[17] Note that this feature i.e., that the equilibrium yields zero payoff to both firms makes it a particularly good candidate for serving as a threat equilibrium in the repeated game that we investigate next.

We see that, in effect, favoritism turns the asymmetric information (private cost) auction, into a symmetric information common value auction (in bribes) for a prize. The prize is influence over the design of the selection rule which has a common value corresponding to the gain when winning the official auction with a maximally favorable selection criteria. This gain is common knowledge and identical for both firms.[18] The social cost of favoritism is twofold. First, a socially inefficient project specification is selected. Second, the price paid by the government is higher than in the absence of favoritism. The bias in project specification due to favoritism minimizes competition between firms. The equilibrium depicted in Proposition 1 will serve a threat point in the collusive schemes we study next.

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[16] We wish to remark that if firms have an identical cost structure, favoritism has no value. If one of the firms is inefficient on all components but not the other, we would have favoritism but no competition so the bribe would just cover the punishment cost m.

[17] When reviewing court cases, it appears quite clear that the cost of favoritism is very low. The only instances of conviction for favoritism in France pertain to cases where the auctioneer explicitly required a firm specific technology. (Cour des Graces 2002).

[18] We consider a symmetric case but the logic would be the same if we allowed for some asymmetry in the cost structure. All that a firm needs to know is the other firm's value of winning the contract under the most favorable circumstances i.e., with a selection rule that gives full weight to a component such that the firm has the largest comparative advantage in its production.

4   Collusion and Favoritism: A Strategic Complementarity

We now proceed to investigate a situation when the two firms interact repeatedly. In each period they meet on a public market administered by a new auctioneer, e.g., different local governments. In each period there is a new draw of $\left(\boldsymbol{\theta}_1^t, \boldsymbol{\theta}_2^t, \boldsymbol{\alpha}^t\right)$. We are interested in collusion between the two firms under the assumption that transfers between them are precluded.

Information assumptions: At the end of each period the submitted contract offers are publicly observed by the two firms and the active auctioneer. The corrupt deal offers remain private information to the involved parties. The true value of $\boldsymbol{\alpha}$ is never revealed. Each auctioneer is appointed for one period only and there is no communication between auctioneers from different periods.

We consider a repetition of the game described in *Timing* (Section 3). Proposition 2 constitutes the central result of this paper.

Proposition 2 *i. There exists $\underline{\delta_1} < 1$ and $\overline{m}$ such that for $\delta \geq \underline{\delta_1}$ and $m \leq \overline{m}$ full cartel efficiency is achievable in a Perfect Bayes-Nash equilibrium of the repeated game.*

ii. In the official auction firms take turn in winning independently of government preferences and firms' costs.

iii. The equilibrium scoring rule is single-peaked (i.e., $\widehat{\alpha} = (0, ..., 1, ...0)$) and the winning firm i^* pays a bribe $b_{i^*} = m$.

Full cartel efficiency is defined for the official auction as follows. i. In each period the winner is (one of) the most efficient firms relative to the announced selection criteria (productive efficiency); ii. The price paid to the winner is the highest price the government is willing to pay (price efficiency); iii. The selection criteria that applies yields the highest gains to the winning firm from among all possible selection criteria (design efficiency). Note that the third part of our criteria goes beyond the standard definition of cartel efficiency. Proposition 2 establishes that with favoritism full cartel efficiency is achievable in spite of incomplete and asymmetric information.[19],[20] The cartel needs not adapt to the "environment" i.e., to the current cost structure or to the current government preferences. Instead the environment

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[19] Notice that the firm that wins first has a higher expected discounted payoff. We can equate the two firms' discounted payoffs by randomly designing the first winner.

[20] We here wish to remark that the results in proposition 2 hold true even when firms have an identical cost structure (a case that we excluded in the model). However full cartel efficiency can obviously not obtain if one of the firm has high cost on all components (but the other has not).

adapts to the cartel: in each period the auctioneer fine-tailors the scoring rule to the in-turn winner. The optimal allocation rule is extremely simple: firms take turn for winning in a non-contingent manner. A main concern for the cartel is to contain competition in bribes which can be very costly as we learned from Proposition 1. Proposition 2 shows that competition for favors is eliminated when opting for a non-contingent in-turn allocation rule. At the corruption stage both firms offer a menu of deals each with a single-peaked scoring rule as in Proposition 1. The out-of-turn firm offers a zero bribe while the in-turn winner offer a bribe that just covers the expected punishment cost m. The out-of-turn firm may deviate and (unobserved) bribe the auctioneer to announce a scoring rule favorable to itself. This is immediately detected however - the pre-determined in-turn rule is violated - and punished by reverting to the equilibrium of proposition 1 which yields zero payoff to both firms from next period on. This explains why the bribe can be kept to a minimum of m. In the official auction the out-of-turn firm submits an offer that scores at most zero. Since contract offers become public information any defection at that stage is detected after the official opening and punished similarly.

We thus see that favoritism facilitates collusion in several ways. The gains from collusion are higher than with an honest auctioneer: the scoring rule is fine-tailored to maximize the winner's profit. While the threat payoffs are lower than in the absence of corruption because competition in bribes dissipates the rents. Most importantly we find that favoritism solves key problems for a repeated cartel in a stochastic environment. The auctioneer's self-interested determination of the scoring rule effectively shades the cartel from fluctuations in the profitability of projects due to stochastic government preferences and changing costs. The environment "adapts" to the cartel and ex-post efficiency i.e., efficiency relative to the *announced* scoring rule, is secured. But this comes at a cost, the bribe.

Finally we note that in the equilibrium of the one-shot game the agent is limited to choosing a scoring rule that minimizes competition i.e., a scoring rule emphasizing a (single) component such that firms have differentiated technologies (costs). In the repeated setting with collusion, the agent needs not bother about cost differentials. So, in a sense, collusion makes favoritism "easier": the agent's equilibrium choice set is larger.

4.1 The impact of the expected punishment cost

In this section we discuss the impact of an increase in the expected cost of punishment on players' incentives, payoffs and strategies. In the appendix we show that defection from the equilibrium strategies supporting our result in Proposition 2 can be deterred iff $\frac{\delta}{(1-\delta^2)}\left(\frac{1}{2\underline{\theta}}-m\right) \geq \frac{1}{2\underline{\theta}}$. Recalling that all our analysis is performed for $m < \frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}} = \pi^{NE}$ we know that $\frac{1}{2\underline{\theta}}-m > 0$ and so $\underline{\delta_1} < 1$. A first interesting point is that $\frac{\partial\underline{\delta_1}}{\partial m} > 0$ which means that the larger the cost of punishment, the more patient firms must be to be able to sustain collusion with favoritism. This suggests that anti-corruption policy may have a real impact on competition in procurement. But as favoritism becomes more expensive (m grows), there may exist equilibria without favoritism but with collusion that Pareto dominate the equilibrium of proposition 2. In order to investigate that issue we must spend some time on "pure collusion".

Pure collusion in our context means that firms do not compete with each other but there is no manipulation of the selection criteria which always reflects the true government preferences. We do not go into the details of the pure collusion case. Instead, we shall rely heavily on Athey and Bagwell (2001) who provide a thorough analysis of similar case. We only focus on a few features of relevance to our discussion.

Under pure collusion there is no manipulation of the selection criteria. As a consequence the selection of the winner must be contingent on $\left(\boldsymbol{\theta}_1^t, \boldsymbol{\theta}_2^t, \boldsymbol{\alpha}^t\right)$ i.e., on firms' cost parameters and on current government preferences in order to secure the cartel's production efficiency goal. Clearly, for any realization of $\left(\boldsymbol{\theta}_1^t, \boldsymbol{\theta}_2^t, \boldsymbol{\alpha}^t\right)$, the gain from winning the contract is largest for the firm whose cost structure is most congruent with current government preferences.

Assume first that the agent is honest. Under symmetric information about costs First Best can be achieved in a simple mechanism that designates the winner as the firm with the cost structure most congruent with current government preferences. The designated winner submits a bid that scores exactly 0 and the other firm a bid that scores less than zero. Any deviation from those strategies is observable and triggers a reversal to a competitive Nash equilibrium.[21] Let $E\pi^{FBc}$ denote firms expected First Best collusive payoff. Appealing to our result in Proposition 1(i) (that establishes the optimality of the single-peaked scoring

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[21] We do not characterize the equilibrium of the stage game without favoritism. By lemma 1, we know that the equilibrium specification is socialy efficient. The firms compete in price given their beliefs about the other's cost structure.

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rule) we know that $\pi^{FBc} < \frac{1}{2\bar{\theta}}$ where the left-hand-side is the pure collusion profit-if-win and the right-hand-side of the inequality is the profit-if-win (gross of the bribe) under favoritism (see Proposition 2).

Under asymmetric information things are much more tricky, production efficiency requires a mechanism that induces firms to truthfully reveal their private information. Athey and Bagwell (2001) show that for some combination of parameters, the cartel's first-best is achievable provided there exist states where utility can be transferred at no cost for the cartel. A mechanism similar to theirs could be devised. A full characterization of such a mechanism is beyond the scope of this section. We just note that a key feature of the mechanism is to exploit states where firms are equally efficient to secure incentives for truth-telling.[22] In our context such states do also exist i.e., when firms are equally efficient relative to government preferences.[23] Just as in Athey and Bagwell, cartel efficiency is demanding on parameters so it is likely that the optimal pure collusion mechanism is plagued by some inefficiency so $E\pi^c < E\pi^{FBc}$ where $E\pi^c$ is the expected profit from pure collusion.[24] But that is not crucial to our argument.

We now return to the case with a corruptible agent. Suppose that $m = m_t$ so m changes with time. Assume that in period t, $\frac{1}{2\bar{\theta}} - m_t < E\pi^c$ where $E\pi^c (\leq E\pi^{FBc})$ is the expected payoff of pure collusion. Then both (the in-turn and the out-of-turn) firms would prefer to refrain from bribery if they could collect the expected pure collusion payoff. In the mechanism of proposition 2 that would not happen however. A violation of the in-turn rule leads to a reversal to the zero payoff Nash equilibrium of the stage game. But firms could make their strategies contingent on m. Namely for $m_t \leq \frac{1}{2\bar{\theta}} - E\pi^c$, they play the equilibrium of proposition 2 but for $m_t > \frac{1}{2\bar{\theta}} - E\pi^c$ they play an optimal pure collusion equilibrium. Note importantly that the presence of a corruptible agent relaxes the (off-schedule) incentive constraint compared to the case when the agent is honest.[25] This is because the threat payoffs

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[22] In our context truth-revelation could be achieved by conditioning firms's probability to be selected as the winner on their cost announcement.

[23] Assume that government only values the two first components and that firms have the same cost for those, then they are equally efficient.

[24] When truth revelation is not achievable costlessly, it may be achieved at some cost in productive efficiency or price efficiency.

[25] This terminology is borrowed from Athey and Bagwell (2001). The off-schedule incentive constraint reflects firms' incentives to take observable defection steps. As opposed to the in-schedule incentive constraint that are related to incentives to take actions designed for their type. Defection in that respect is not observable.

are the ones corresponding to the Nash equilibrium of the one-stage game *with favoritism* which yields zero payoffs.

The discussion above shows that the cost of favoritism to society depends critically on the expected cost of punishment m. For $m < \frac{1}{2\underline{\theta}} - E\pi^c$ firms achieve the highest payoff in an equilibrium with favoritism. The social cost of favoritism includes a distortion of the selection criteria. The winning project is not the social efficient one and it is paid at the high collusive price. In that equilibrium the higher the cost of punishment the more patient firms need to be to be able to sustain collusion in equilibrium. For $m \geq \frac{1}{2\underline{\theta}} - E\pi^c$ the social cost due to the presence of corruptible agent may be limited to facilitating collusion by making it sustainable for lower level of the discount factor (because the "threat payoff" is then zero). There is no social cost due to design inefficiency (but we may have production inefficiency i.e., when the First Best is not feasible) and the government pays a lower (collusive) price than in the case of favoritism. Finally we note that further increase in m (up to $\frac{(\overline{\theta} - \underline{\theta})}{2\underline{\theta}\overline{\theta}}$) has no impact on players' incentives.

5   Extensions

In this section we extend the analysis by considering the case when the expected punishment for favoritism depends on the magnitude of the distortion of social preferences. We do that in a simpler setting with $k = 2$ so $\alpha_1 = \alpha$ and $\alpha_2 = (1 - \alpha)$ and α is uniformly distributed on $[0, 1]$. The auctioneer's utility is $U = b - m(\widehat{\alpha} - \alpha)^2$. Note that since, by assumption, each firm has a comparative advantage in at least one component (see the model description), the $k = 2$ case boils down to symmetric information.[26]

The time-line of events in the stage game is as follows:

step 0: Firms learn privately their cost parameters $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$.

step 1: The auctioneer learns α, the firms submit their corruption deals $\{(b_{1l}, \alpha_{1l})\}$ and $\{(b_{2l}, \alpha_{2l}), \}$, $l = 1, ..n_i$, $I = 1, 2$;

step 2: The auctioneer makes an announcement $\widehat{\alpha}$, $\widehat{\alpha} \in [0, 1]$;

step 3: The firms submit their contract offers $(\mathbf{q}_i, p_i)$;

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[26] We learned from proposition 1 that asymmetric information is not a big concern in our setting. Therefore, the $k = 2$ provides a satisfying model to focus on the new features due to a variable expected punishment cost.

step 4: The auctioneer publicly opens the envelops and he selects the firm whose offer maximizes the selection criteria corresponding to the announced scoring rule;

step 5: The auctioneer decides to confirm or reject the outcome. If he confirms the winner i^* pays a bribe b if and only if $(\widehat{\boldsymbol{\alpha}}, b) \in M_{i*}$. Otherwise no bribe is paid. If he rejects no transaction takes place.

Proposition 3 characterizes symmetric Perfect Bayes-Nash equilibria of the stage game described above. We show that for $m \leq \frac{4}{5} \left(\frac{(\overline{\theta} - \underline{\theta})}{2\underline{\theta}\overline{\theta}} \right)$

Proposition 3 *Any symmetric Perfect Bayes-Nash equilibrium is characterized by*

i. The equilibrium scoring rule is $\widehat{\alpha}^* (m, \alpha) = \begin{cases} 1 \text{ for } \alpha \geq 1/2 \\ 0 \text{ for } \alpha < 1/2 \end{cases}$;

ii. The equilibrium bribe offer is $b_1^* = b_2^* = b^* (m) = \frac{(\overline{\theta} - \underline{\theta})}{2\underline{\theta}\overline{\theta}} - m$;

iii. The contract offers are the competitive equilibrium offers relative to the announced scoring rule.

A first important result is that the equilibrium scoring rule is single-peaked as in the fixed punishment case. In the appendix we prove this as a lemma. The intuition is that in the corruption game firms compete in the auctioneer's utility levels. This utility is separable in bribe and expected punishment cost. We show that for any deal with $\alpha \notin \{0, 1\}$ that achieves a given utility to the auctioneer there exists a deal with $\alpha \in \{0, 1\}$ that achieves the same utility level but yields a higher expected profit for the firms. We also note that, as in Proposition 1, the official auction offers are the (efficient) competitive equilibrium offers relative to the announced scoring rule.

In contrast with earlier results competition for favors does not dissipate all firms' rents. The intuition is that contingent punishment costs introduces an asymmetry between firms: the firm whose demanded scoring rule is closer to the true preferences has more bribing power than the other firm. Firms' incomplete information about the true preferences therefore induces a continuity of the probability to win in the submitted bribe. As a result competition for favor is mitigated.

The results in proposition 3 apply for $m \leq \frac{4}{5} \left(\frac{(\overline{\theta} - \underline{\theta})}{2\underline{\theta}\overline{\theta}} \right)$. Since $\frac{(\overline{\theta} - \underline{\theta})}{2\underline{\theta}\overline{\theta}}$ is the competitive profit-if-win associated with the most favorable scoring rule, this range covers most interesting

cases.[27] Over that range of value for m the profit-if-win is simply $\pi_{i^*} = m$.

Summing up, with an expected punishment cost that is a function of distortion, the scoring rule always induces a "non-standard" project. What government expenditure concerns there is no advantage in the more sophisticated punishment rule. Finally, because it mitigates competition in favors, some of the rents stays with the firms. We conclude that in the stage game, the sophisticated punishment scheme offers no advantage from the point of view of social efficiency.

We now consider a repeated version of the game described above. As in the case with k-components, the two firms meet in each period with a new (short-run) auctioneer. At the end of each period, the submitted contracts offers are publicly observed by the two firms and the active auctioneer. The corrupt deal offers remain private information to the involved parties. The true value of α is never revealed. There is no communication between auctioneers from different periods.

Proposition 4 *i. For $\delta \geq \underline{\delta_2} \in (0, 1)$, there exists a Public Perfect Equilibrium equilibrium of the repeated game with collusion in contract offers and in corruption deals.*

ii. For m small a simple pre-determined in-turn allocation rule is optimal while for m large any optimal collusive scheme entails a contingent allocation rule.

A first important remark is that collusion in contract offers and in bribes is achievable in a simple pre-determined in-turn scheme at $b^* = m$. The reasoning is similar to that in proposition 2. However, for m relatively large, the simple scheme implies a significant loss in revenue for the cartel. This is because in such a scheme the bribe always covers the punishment cost associated with the maximal distortion of the scoring rule relative to the true one. The bribe cost can be reduced in a contingent scheme but that may not always be worthwhile because of imperfect public information which induces new inefficiencies.

We first note that once the winner has been designated, collusion in the official auction is sustainable relying on a standard folk theorem argument. This is because offers become public information with the official opening of the envelops. As in earlier Propositions, in the scheme

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[27] In an earlier version we investigated the whole solution. For $m > \frac{4}{5} \left(\frac{D}{2\theta\overline{\theta}} \right)$ the equilibrium bribe is equal to the expected punishment cost and favoritism occurs less often. As the cost grows larger, it becomes fully prohibitive.

of Proposition 4, the announcement of the agent results from competition in corruption deals and determines the winner (as the firm whose cost structure is most congruent with the announced scoring rule). A main issue for the cartel is therefore to sustain collusion in corrupt deals in order to contain competition in bribes and make an efficient use of stochastic government preferences as an allocation rule. The problem is that firms do not know the true scoring rule and do not observe the submitted bribe deals. They only observe the announced scoring rule which is an imperfect public signal of firms' action in the corruption game. Therefore firms must sometime be "punished" even when complying (this result is a similar to results in Green and Porter (1986) and Radner, Myerson and Maskin (1986)). In the appendix we provide an example showing that collusion is sustainable in a Public Perfect Equilibrium (PPE) with $b^* = \frac{1}{4}m$. A PPE is a profile of public strategies that, beginning any date t and given any public history till time t, forms a Nash equilibrium.[28] Deterrence from defection at the bribing stage is achieved by the threat of competition in the official auction. In case a firm wins twice in a row, it is "punished" by the other firm which then submits an offer that scores more than zero. This reduces the cartel's revenue (we have price inefficiency). In our example, we have an equilibrium with a contingent scheme that is bounded away from full cartel efficiency but that dominates the fixed in-turn rule scheme for m not too small.

We did not aim at characterizing an optimal contingent mechanism but it can be shown that any contingent mechanism is bounded away from full cartel efficiency. This may seem to conflict with the results in Fundenberg, Levine and Maskin (1994). A main reason is that our model does not satisfy their property of pairwise identifiability. But most importantly efficiency fails because the cartel only has a limited set of instruments to achieve conflicting goals. When applying the general theory to real interaction situations Athey and Bagwell (2001) hit upon that problem. In their model efficiency is achievable but only under some configuration of the parameters (and with direct communication between firms). The situation is similar here with a tension between information revelation, design efficiency and production or price efficiency. In the present context with only two components and with government preferences uniformly distributed over $[0, 1]$, there is no state where firms are identically efficient (and thus utility cannot be transferred at no cost for the cartel). As a consequence to achieve information revelation (or collusion in bribes) one must relinquish

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[28] A strategy for player i is public if, at each time t, the strategy depends only on the public history and not on i's private history.

price efficiency (see appendix). In an earlier version we considered a case where there existed states where firms were identical. Even then full cartel efficiency including, price, production and design efficiency could not obtained because we have a trade-off between design efficiency and bribe minimization.

The main insight from Proposition 4 is that even with a variable expected punishment cost favoritism facilitates collusion. First, for m not too large, favoritism increases the collusive gain and it always reduces the threat payoffs. Second, for m not too large the simple fixed in-turn rule is optimal. For larger m and unlike in the case with a fixed punishment cost, favoritism here does not fully shade firms from future hazards in preferences and costs. Yet, matters are simplified for firms. With favoritism the profit-if-win is fully known by force of single-peakness and depends minimally on the environment. As a result favoritism allows for a reasonably simple contingent collusive scheme to sustain collusion.

Our conclusion is that the central insight from proposition 2 i.e., that favoritism facilitates collusion in face of demand uncertainty (incomplete information about the true government preferences) and privately observed shocks to costs, partly carries over to the case when the punishment cost that varies with the magnitude of the distortion. Favoritism relaxes the incentive constraints by increasing the collusive gain and decreasing the threat payoff. With a variable punishment cost, favoritism may not always bring about larger gains in terms of a simplified scheme compared with no corruption collusion.

6   Discussion

The main insights of the analysis can be summarized as follows:

- Favoritism facilitates collusion because

  - It induces the revelation of firms' private information as that information is used by the corrupt auctioneer to *maximize* the winner's rent;

  - It shades firms from fluctuations in government preferences. The selected contract specification reflects the cartel's interests instead of social preferences;

- Favoritism exacerbates the cost of collusion for society. The contract specification is socially inefficient and the price is higher than with collusion alone.

- Anti-corruption policy can have a real impact on firms' behavior.

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The analysis thus reveals that favoritism fundamentally perverts the auction mechanism both what concerns the use of firms' private information (about their costs) and that of the agent's private information about government preferences.

A central intermediary result is that the equilibrium scoring rule is extreme i.e., "single-peaked". In the one-shot setting this allows to minimize competition between firms. As a result the selected project tends to be non-standard in the sense that the winning firm is alone to be efficient at its production. In the repeated setting competition is less of an issue because of collusion. As a result the winning firm's rents can be maximize for a larger range of project specifications each of which responding to a single-peaked scoring rule. Most procurement codes include provisions that preclude the use of non-standard (a fortiori firm specific) specifications and that encourage generic technical specification. Interestingly, even for the simplest objects such as print paper one may not be able to define a unique standard (see Compte and Lambert-Mogiliansky (2000)). When dealing with complex procurement projects, it is simply not realistic to expect being able to define a unique generic specification. Choices have to be made either by settling for a technical solution or in a scoring rule. Often it is mistakenly believed that a first price auction of a technically specified object precludes favoritism. In Compte and Lambert-Mogiliansky (2000), it is demonstrated that such procedure can be even more vulnerable to favoritism. A technical specification can bias competition at a larger cost for the government than a scoring rule. Generally, the use of a scoring rule (that weights technical components or performance measures) increases competition and thereby reduces the stake of favoritism. Our analysis applies within the spectrum of discretion consistent with typical anti-favoritism provisions. It says that within that spectrum, favoritism results in the selection of a project specification that maximizes the winner's rent. It also says that collusion relaxes a constraint on equilibrium scoring rule (i.e., it needs not minimize competition), which presumably makes favoritism more difficult to detect.

Single-peakness as the solution to rent maximization, obtains from the conjunction of a series of assumptions most of them are standard or reasonable. Two assumptions deserve some comments: separability in costs between components and separability in bribes and punishment cost. There is a natural way to reinterpret the single-peakness result when relaxing the assumption of separability in costs. If we have complementarities in costs, one should group components that are complementary in production into a composite component that

is given full weight in a proper manner. Clearly, a more involved cost structure would entail more complex computation of the demanded scoring rule(s) and a more involved operation to compute the scoring rule that maximizes the winner's rent (used in the stage game). A conjecture is that the menu of deal offers is sufficiently rich a message language to allow for quite sophisticated information to be revealed so the auctioneer can minimize competition as in the basic model. With (ex-ante) symmetric firms the prize i.e., winning the contract with minimal competition is the same for both firms in which case most of the results carry over. Some additional analysis may be required if we want to relax the assumption about separability in bribes and expected punishment. In particular to investigate the case when the auctioneer is not willing to take a bribe so high that it covers the expected cost of all distortions. However, evidence suggests (see footnote 8 and policy implications below) that the expected cost is rather low in which case the problems related to bribe cap would not arise.

Our conjecture is thus that the main insights of the analysis do not depends on the fine details of the model but capture central features of the reality of favoritism in procurement as revealed by empirical evidence. First there exists numerous anecdotal evidence e.g., from developing countries. In one case an Africa country set its telephone specification to require equipment that could survive in frigid climate. Only one telephone company from Scandinavia could satisfy this obviously worthless specification (Rose-Ackerman (1999), p.64). Similarly problems of maintenance of construction objects are often due to the non-standard project specification that was selected by the international procurement procedure. Second, the allocation pattern emerging from the analysis: a pre-determined in-turn rule that allocates the contract to the most efficient firm while generating large profits is very close to the patterns observed in Paris Hall case mentioned in the Introduction. Interestingly, people have argued that the fact that the contract were allocated to the most efficient firm was an indication that there was no collusion. The present analysis shows that it is sufficient that each firm has a comparative advantage in some component for this outcome to obtain in a collusive equilibrium with favoritism.

7 Policy implications

A central message of the analysis is that the risks of collusion and favoritism are linked and must be addressed simultaneously. Yet, the investigation of collusion is often the jurisdiction of Competition Authorities while that of corruption is the jurisdiction of criminal courts. A first recommendation is to develop cooperation to overcome this institutional separation so as to improve efficiency in the prosecution of cases that involve both favoritism (corruption) and collusion.

The analysis confirms earlier results (see e.g., Laffont and Tirole (1993) and Che and Burget (2004)) that discretion to devise the scoring rule is subject to capture by firms. This seems to suggest that one should eliminate the agent's discretion i.e., let the agent administer a first price auction. But that would be highly naive a conclusion. Indeed scoring rules are used to give flexibility in design which generally increases competitive pressure. In a pure first price auction the object has to be fully defined by the technical specifications. Compte and Lambert-Mogiliansky (2000) show that the decisions related to the technical specification is even more sensitive to capture than those related to the scoring rule because they are linked with higher rents. On the other hand if a first price auction is associated with a standardization of the technical specification, the agent's discretion can be truly reduced. Our result gives support to a policy that reduces the agent's discretion to devise the scoring rule but only when the technical specification can be standardized. When standardization is too costly (or not feasible), the auctioneer's decision should be subjected to close scrutiny. This recommendation is in line with Steven Kelman (1994)[29] who argues in favor of flexibility in association with increased accountability of procurement officials. Concretely this means for instance an obligation to motivate their decisions in writing. Another type of measures recognizes that firms often have a superior information about each other than the government has. They can be in a position to recognize when a scoring rule is fine-tailored to some other firm. A recommendation would be to consider devising a mechanism to reveal this information e.g., by performing an anonymous consultation prior the official submission.

Our results suggest that there is a real role to play for anti-corruption policy (controls and punishments) to play. From proposition 2 we know that the minimal discount factor$\underline{\delta_1}$ increases with m from $\underline{\delta_1}(m = 0) \approx .62$. An anti-corruption policy aiming at induc-

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[29]Harvard professor Steven Kelman was the director of the Office of Federal Procuremnt Policy under 1993-1997.

ing a positive m has the immediate effect of precluding the most impatient firms. As we noted in section 4.1, further increasing in m over some threshold may even induce firms to refrain from favoritism altogether so the cost to society is then limited to a high collusive price. The risk of favoritism disappears when the expected punishment cost is so high that it exceeds the stage game competitive payoff.

The potential efficiency of the repressive tools contrasts with the current legislation in the European Union that makes it very difficult to convict for favoritism. A central reason for this is that favoritism is difficult to prove. Indeed, generally any selection criteria would favor some firm(s) at the expense of others. "Deciding to build a swimming pool rather than a stadium is good for firms that have a comparative advantage in building swimming pools." The problem is thus to compare between selection criteria that favor different firms. The honest auctioneer picks up the one that is congruent with public preferences while the corrupt selects another one. But public preferences are seldom so well-defined that congruence can be measured in a way that is non-controversial (which also suggests that a fixed punishment cost model maybe the relevant one). Generally, detecting and proving the occurrence of favoritism is difficult. An implication of the analysis is that attention should be paid to a careful study of allocation patterns over time. Unfortunately courts tend to focus on bribery and few cases of favoritism are brought to court. We thus suggest that sophisticated economic expertise be given more power in cases where there is a suspicion of favoritism. Indeed, while this is the rule in cases of standard collusion, economist expertise appears to be seldom requested in cases involving corruption and favoritism.

Finally, in our analysis we have assumed that the agents were short-run players. The idea is that the firms are rather specialized and meet on public markets that are set up in different jurisdictions and therefore administrated by different agents. The demand for e.g., public sport facilities is not recurrent in any single jurisdiction. But we could also interpret our results in the context of firms who meets on public markets organized by the same administration but with officials who are often moved from one position to another. Such a policy is often advocated to prevent corruption which is presumed to be easier to sustain within the frame of long-running relationships. Our result shows that this presumption is not warranted here. On the contrary the short-run character of the agents permits firms to earn all the rents from collusion. So our results suggest that a high turnover of officials certainly does not make favoritism more difficult.

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A Proof of Lemma 1

For any announcement $\widehat{\boldsymbol{\alpha}}$, the efficient specification for firm i is defined: $\mathbf{q}_i^* = \arg\max s\left(\mathbf{q}, \widehat{\boldsymbol{\alpha}}\right) - c_i\left(\mathbf{q}, \boldsymbol{\theta}_i\right)$. We claim that in the equilibrium of the FSA both firms offer the efficient specification corresponding to their cost structure. Assume that this was not the case i.e., that, in equilibrium, firm i offers $(\widehat{\mathbf{q}}, \widehat{p})$ with $\widehat{\mathbf{q}} \neq \mathbf{q}_i^*$. We first note that under asymmetric information $prob\left\{\left.\text{win}\right| ((\widehat{\mathbf{q}}, \widehat{p}))\right\} > 0$. To show this we order the firms' type according to the number of components for which they have high cost (so each such type corresponds to a group of "elementary types") $\overline{\boldsymbol{\theta}}^0 <, ..., < \overline{\boldsymbol{\theta}}^{k-1}$, where $\overline{\theta}^{k-1}$ is the highest cost type (recall the fully inefficient types have been deleted from the distribution). Let $S^*\left(\boldsymbol{\theta}\right) = \max_q s\left(\mathbf{q}, \widehat{\boldsymbol{\alpha}}\right) - c_i\left(\mathbf{q}, \boldsymbol{\theta}\right)$, in our model we have that $prob\left\{\left.\text{win}\right| S^*\left(\overline{\boldsymbol{\theta}}^{k-1}\right)\right\} > 0$. If the offer $(\mathbf{q}, p)$ for some $\overline{\boldsymbol{\theta}}^j < \overline{\boldsymbol{\theta}}^{k-1}$ was such that $prob\left\{\left.\text{win}\right| (\mathbf{q}, p)\right\} < prob\left\{\left.\text{win}\right| S^*\left(\overline{\boldsymbol{\theta}}^{k-1}\right)\right\}$, it could not be an equilibrium offer since the score in decreasing in cost i.e., there would exist another offer that would yield higher expected profit.

We now show that offer $(\mathbf{q}_i^*, p')$ with $p' = \widehat{p} + s\left(\mathbf{q}_i^*, \widehat{\boldsymbol{\alpha}}\right) - s\left(\widehat{\mathbf{q}}, \widehat{\boldsymbol{\alpha}}\right)$ dominates $(\widehat{\mathbf{q}}, \widehat{p})$. Note that $S\left(\mathbf{q}_i^*, p'\right) = S\left(\widehat{\mathbf{q}}, \widehat{p}\right)$ so in particular $prob\left\{\left.\text{win}\right| (\widehat{\mathbf{q}}, \widehat{p})\right\} = prob\left\{\left.\text{win}\right| (\mathbf{q}_i^*, p')\right\}$. Now the expected profit from submitting $(\mathbf{q}_i^*, p')$ is

$$
\begin{aligned}
\pi_i\left(\mathbf{q}_i^*, p'; \boldsymbol{\theta}_i\right) &= \left[\, p' - c_i\left(\mathbf{q}_i^*, \boldsymbol{\theta}_i\right)\right] prob\left\{\left.\text{win}\right| \left(\mathbf{q}_i^*, p'\right)\right\} \\
&= \left[\widehat{p} - c_i\left(\widehat{\mathbf{q}}, \boldsymbol{\theta}_i\right) + s\left(\mathbf{q}^*, \widehat{\boldsymbol{\alpha}}\right) - c_i\left(\mathbf{q}_i^*, \boldsymbol{\theta}_i\right) - \left(s\left(\widehat{\mathbf{q}}, \widehat{\boldsymbol{\alpha}}\right) - c_i\left(\widehat{\mathbf{q}}, \boldsymbol{\theta}_i\right)\right)\right] prob\left\{\left.\text{win}\right| (\widehat{\mathbf{q}}, \widehat{p})\right\} \\
&> \left[\widehat{p} - c_i\left(\widehat{\mathbf{q}}, \boldsymbol{\theta}_i\right)\right] prob\left\{\left.\text{win}\right| (\widehat{\mathbf{q}}, \widehat{p})\right\} = \pi_i\left(\widehat{\mathbf{q}}, \widehat{p}; \boldsymbol{\theta}_i\right).
\end{aligned}
$$

The last inequality holds because $s\left(\mathbf{q}^*, \widehat{\boldsymbol{\alpha}}\right) - c_i\left(\mathbf{q}_i^*, \boldsymbol{\theta}_i\right) > s\left(\widehat{\mathbf{q}}, \widehat{\boldsymbol{\alpha}}\right) - c_i\left(\widehat{\mathbf{q}}, \boldsymbol{\theta}_i\right)$.

The argument applying to the symmetric information case which we also use below is even simpler. Consider the case when firm 1 has a cost structure that is more congruent with the announced scoring rule than firm 2. Firm 1 is sure to win when submitting the second highest score (corresponding to firm 2's efficient specification associated with a price bid equal to its cost) because the tie breaking rule favors quality. Suppose firm 2 submits an offer that does not include the efficient specification and firm 1 matches that score. Then firm 2 could switch to an offer that includes the efficient specification to achieve a higher score and win. Suppose now that firm 1 matches firm 2's score with an offer that does not include the efficient specification. Appealing to the argument above (setting the winning probability equal to 1), we see that it cannot be optimal. Firm 1 could earn a higher profit with an

offer that scores the same but includes the efficient specification. Similar reasoning applies when firms are identically efficient. Hence, in equilibrium firms submit offers that include the efficient specification. *QED*

B   Proof of proposition 1

We consider the following strategies for the players:

Firms:

At *step 1* submit a menu of deal offer $\left\{\left(\boldsymbol{\alpha}^j, b^*\right)\right\}$ with a deal for *each* component j; $\theta_{ji} = \underline{\theta}$ with a single-peaked scoring rule i.e., $\boldsymbol{\alpha}^j = (0, ..., 1_j, ..., 0)$. The same bribe is offered in each one of the deals belonging to the offered menu. Both firms offer $b^* = \frac{(\overline{\theta} - \underline{\theta})}{2\underline{\theta}\overline{\theta}}$.

At *step 3*, when the announced scoring rule is single-peaked, firms submit the competitive equilibrium offers under the assumption that they are anti-symmetric in cost (for $\widehat{\boldsymbol{\alpha}} = \boldsymbol{\alpha}^j$ if $\theta_{ji} = \underline{\theta}$ then $\theta_{j-i} = \overline{\theta}$). When the scoring rule is not single-peaked, the (non-deviating) firm believes the other has a cost structure fully congruent with the announced scoring rule (i.e., that the firm has low cost on all emphasized components) and submits the corresponding competitive offer.

The auctioneer:

At *step 2,* the auctioneer selects from among the submitted corrupt deals a deal that includes the highest bribe provided the bribe covers the cost m. From among the highest bribe deals he selects a one associated with a scoring rule only demanded by one firm if there is any. If there are none, he randomizes among the highest bribe deals. If there are several deals demanded by one firm only he randomizes among those. He announces the associated scoring rule.

At *step 5,* the auctioneer maintains the outcome of the auction if the winner is one who demanded the favor (and by assumption will pay for it) or if no favors were demanded. Otherwise he rejects the outcome.

Note that the players's action at *Step 4* are not specified. This is because they are fully determined by the rules and assumptions of the game.

We below show that the strategies described above form a Perfect Bayes-Nash equilibrium of the game.

For that purpose it is useful to first derive the competitive offers that form the Nash equilibrium of the First Score Auction described by step 3 and 4 with no bribes with the following out-of-equilibrium beliefs. The non-deviating firm (2) plays competitively as if the other firm has the cost structure which is most congruent with the announced scoring rule i.e., low cost on all emphasized components). The deviating firm (1) plays as if the other firm had the least congruent cost structure i.e., high cost on all emphasized components. Although these beliefs are not consistent they generate the largest possible gain from deviation with respect to the demanded scoring rule at the corruption deal submission stage. Those beliefs are designed so as to make the deviation most attractive.

We know from lemma 1 that firms choose $\mathbf{q}_i^* = \arg\max_\mathbf{q} s\left(\mathbf{q}, \widehat{\boldsymbol{\alpha}}\right) - c\left(\mathbf{q}, \boldsymbol{\theta}_i\right)$. Suppose $\widehat{\boldsymbol{\alpha}}$ emphasizes n component $n < k$, firm 1 expects firm 2 to make an offer including $\mathbf{q}_2^* = q_{ij}^*\left(\theta_i\right)$ so its quality score is $\sum_j \alpha_j \left(\frac{\alpha_j}{\bar{\theta}}\right) = \sum_j \frac{\alpha_j^2}{\bar{\theta}}$, for $j = 1, ...n$. By a standard argument, firm 2 bids the lowest price that just secures non-negative profit

$$p_2^* = c\left(\mathbf{q}_2^*, \boldsymbol{\theta}_2\right) = \sum_j^n \frac{\widehat{\alpha}_j^2}{2\bar{\theta}}. \tag{5}$$

Firm 1's best response is to bid the highest price that secures win:

$$p_1^* = c\left(q_2^*, \theta_2\right) + s\left(q_1^*\right) - s\left(q_2^*\right) = p_2^* + \frac{\sum_j^n \widehat{\alpha}_j^2 \left(\bar{\theta} - \underline{\theta}\right)}{\underline{\theta}\bar{\theta}}. \tag{6}$$

The deviation profit (gross of bribe) is

$$\pi_1 = \frac{\sum_j^n \widehat{\alpha}_j^2 \left(\bar{\theta} - \underline{\theta}\right)}{2\underline{\theta}\bar{\theta}}. \tag{7}$$

We call the bids $\left(\mathbf{q}_1^*, p_1^*\right)$, $\left(\mathbf{q}_2^*, p_2^*\right)$ defined in Lemma 1 and the formulas in (5) and (6) the out-of-equilibrium competitive bids.

We now proceed to investigate the whole game by backward induction.

At *step 5*, the auctioneer chooses whether to reject or maintain the outcome. If the winner is not a firm that demanded the scoring rule the auctioneer expects no bribe. Rejecting the auction outcome earns him zero as well so it is optimal to do so. When the winner is a firm that demanded the favor, if he rejects he earns 0 while if he maintains the outcome he collects the bribe. So maintaining the outcome is optimal. At *Step 3* firms make their offers. By lemma 1 we know that any offer includes the cost efficient specification. Consider first the subgame which corresponds to single-peaked $\widehat{\boldsymbol{\alpha}} = \alpha^j$ for some $j = 1, ..., k$ and say it favors

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firm 1. In equilibrium firms submit deal on all their low cost components with the same bribe. By assumption each firm has a comparative advantage on at least one component and we know that the auctioneer chooses to announce a scoring rule demanded by one firm only. Firms therefore correctly infer from $\widehat{\boldsymbol{\alpha}} = \alpha^j$ that they are anti-symmetric in cost with respect to θ_j so in particular when a firm has high cost (firm 2), it infers that its opponent has low cost. We note that these are precisely the beliefs that we assumed when deriving the competitive equilibrium above. The only distinction is that they apply to $\widehat{\boldsymbol{\alpha}} = \alpha^j$. Firm 2's best response is to to bid p_2^* defined in (5) for $\widehat{\boldsymbol{\alpha}} = \alpha^j$. If 2 wins, it pays no bribe. Firm 1 bids p_1^* as defined in (6) which secures win and precisely covers the bribe cost $b^* = \frac{(\overline{\theta} - \underline{\theta})}{2\theta\overline{\theta}}$. In an (out-of-equilibrium) subgame where $\hat{\alpha} \neq \alpha^j$ the non-deviating firm believes that the opponent has low cost on all emphasized components, so it is optimal to submit the Nash equilibrium offers with the price bid given by (5). By construction this is optimal.

At *step 2,* the auctioneer chooses a deal among the submitted menus (M_1, M_2) with $M_i = \{(\boldsymbol{\alpha}_{il}, b_{il})\}_{l=1}^{n_i \leq k}$. The auctioneer expects firms to ask for scoring rules that emphasize components in which they have low cost. Since $U = b - m$, he selects a deal $(\hat{\alpha}, b_k)$ such that $b_k \in max\{b_{1l}, b_{2l}\}_{l=1}^{n_i \leq k}$ and $b_k \geq m$. If no such deal has been submitted the auctioneer announces the true α. From among acceptable highest bribe deals $(b_k \geq m)$ he is indifferent between those with a scoring rule demanded by one or by the two firms. So it is optimal to choose a deal that emphasizes a component demanded by one firm only. By assumption there are at least two such deals at equilibrium.

At *step 1,* we know from *step 2* that the auctioneer selects a deal associated with the highest bribe and among those deal with a scoring rule demanded by one firm only. We show that (i) firms demand single-peaked scoring rules, (ii) they submit deals for each low cost component, (iii) they bid the same bribe b^* in each deal.

i. The scoring rule only determines the profit-if-win which for arbitrary $\widehat{\boldsymbol{\alpha}}$ is $\pi_1 = \frac{\sum_j^n \widehat{\alpha_j}^2 (\overline{\theta} - \underline{\theta})}{2\theta\overline{\theta}}$, since $\sum_j^n \widehat{\alpha_j} = 1$, π_1 is maximized with any $\boldsymbol{\alpha}_1^j = (0, ..., 1_j, ..., 0)$; $\theta_{1j} = \underline{\theta}$ and $\theta_{2j} = \overline{\theta}$. Hence, firms demand a single peaked-scoring rule.

ii. The firms don't know each other's cost, but they assume the other firm acts according to the equilibrium strategy i.e., submits deals on each of its low cost component. Consider firm 1's incentives to submit a deal with α_1^j and some b and $\theta_{1j} = \overline{\theta}$. It knows that the agent only chooses a deal with a highest bribe so the bribe must be b^*. But we show below that b^* corresponds to the rents of a low cost firm winning over a high cost firm so if the firm was to

win with such a deal on a high cost component it would earn a negative payoff. Assume that $\theta_{1j} = \underline{\theta}$. When firm 1 submits a deal, it wins if firm 2 has high cost (and thus 1 is alone to submit on that component). If firm 1 does not submit a deal with $\boldsymbol{\alpha}_1^j$, it can still win if firm 2 has low cost (and 2 is alone to submit a deal on that component) by slightly undercutting 2's offer in the official auction. But firm 1 knows that in that case the auctioneer will reject the auction's outcome because the winner will not pay the bribe which results in zero payoffs for the firm so free-riding does not pay. Now for any given $M_{2(1)}$ the probability that the auctioneer finds a deal with a scoring rule demanded by only firm 1 only increases with the number of submitted deals by firm 1. So it is optimal to submit a deal on each low cost component.

iii. A firm's profit-if-win with any of the α^j it demands is equal to $\frac{(\bar{\theta}-\underline{\theta})}{2\underline{\theta}\bar{\theta}}$. The corruption game boils down to a symmetric information common value auction. By a standard argument, firms submit the common value $b_1^* = b_2^* = \frac{(\bar{\theta}-\underline{\theta})}{2\underline{\theta}\bar{\theta}}$ for all component with low cost.QED

C   Proof of Proposition 2

We show that full cartel efficiency can be achieved in an equilibrium supported by a Trigger strategy with a punishment phase corresponding to the play of the equilibrium of proposition 1. The cooperative phase is characterized by the following:

Firms' strategy

At *step 1* the in-turn-firm (referred to with subscript *in)* submits a menu $M_{in} = \{(b^*, \boldsymbol{\alpha}^j)\}$ with $\theta_{in,j} = \underline{\theta}$, $b^* = m$. The out-of-turn firm (referred to with subscript *out)* submits $M_{out} = \{(0, \boldsymbol{\alpha}^j)\}$ for some $\theta_{out,j}$, $b^* = 0$.

At *step 3* for any $\widehat{\boldsymbol{\alpha}}^t$ the in-turn firm submits an offer that scores less than zero. The out-of-turn firm bids to score strictly less than zero.

The auctioneer's strategy:

At *step 2* the auctioneer selects from among the submitted corrupt deals a one associated with the highest bribe. If that bribe covers the costs, he announces the associated scoring rule.

At *step 5* the auctioneer maintains the outcome if the winner is the one who demanded the favor (and by assumption will pay for it) otherwise he rejects the outcome.

Let $H_{t-1} = H^*$ denote a public history of the game when it is in a cooperative phase i.e., in all $t' = 1, ..., t-1$ the outcome is characterized by the firm wining in alternation i.e., every second period.

The trigger strategy entails that in any subgame following $H_{t-1} \neq H^*$, the firms move to (stay in) the punishment phase. Since it is a Nash equilibrium, conforming is by construction a best response for all players.

We now consider a subgame following $H_{t-1} = H^*$ to show that cooperating according to the strategies defined above is optimal. We proceed by backward induction.

At *step 5* the short-run auctioneer has the same incentives as in the stage game, see proof of proposition 1.

At *step 3* whatever $\widehat{\boldsymbol{\alpha}}^t$, the in-turn firm expects the out-of-turn firm to bid less that zero. The maximal payoff $\pi^c = \frac{1}{2\underline{\theta}}$ yields when the in-turn firm offers the efficient specification and a price so its offer scores just zero. So the proposed strategy is optimal. The out-of-turn firm may deviate. The most profitable deviation occurs when the announced scoring rule is single-peaked and the out-of-turn firm also has low cost on the emphasized component and submits $p = \frac{1}{\underline{\theta}} - \varepsilon$, $\varepsilon > 0$. Its gain is $\pi^d = \frac{1}{2\underline{\theta}} - \varepsilon$. However the in-turn rule is violated and from the next period on the firms revert to the zero payoff competitive equilibrium of proposition 1. So the out-of-turn firm complies with the collusive strategy whenever

$$IC : \frac{\delta}{(1-\delta^2)} \left(\frac{1}{2\underline{\theta}} - m \right) \geq \frac{1}{2\underline{\theta}} - \varepsilon \tag{8}$$

which is satisfied for $\delta \geq \underline{\delta_1}$, $\underline{\delta_1} \in (0,1)$ with $\frac{\partial \underline{\delta_1}}{\partial m} > 0$.

At *step 2* since the auctioneer is a short-run player, the argument developed in the proof of proposition 1 carries over. A distinction is that the auctioneer may announce a scoring demanded by both. This is because there is no competition in the official auction, the out-of-turn firm submits an offer that scores less than zero.

At *step 1* the firms submit their menu of deals. Since the auctioneer only cares about the bribe the argument of proposition 1 carry over and firms always propose deals with single-peaked scoring rules. The in-turn firm expects the out-of-turn firm to offer $b = 0$. It is sufficient to offer $b = m$ to cover the auctioneer's cost so he announces one of the in-turn firm's preferred scoring rule. The out-of-turn firm can defect and offer $b = m + \varepsilon$ associated with a menu including a most preferred scoring rule $\boldsymbol{\alpha}_{out}$. It knows that the auctioneer would respond by announcing that $\widehat{\boldsymbol{\alpha}} = \boldsymbol{\alpha}_{out}$. But such a deviation brings at most a profit

of $\frac{1}{2\theta} - m - \varepsilon$. Since we know that such a win triggers a punishment phase defection is not profitable under (8) since there the gain from defection is larger. Hence for δ satisfying (8) the proposed strategies do form a Bayes-Nash equilibrium of the repeated game. The cartel's gain is maximized. In each period, the scoring rule is the most favorable to the winner, the price is given by the reserve score and the bribe is the lowest possible.*QED*

D Proof of Proposition 3

Firms' strategy:

At *step 1* submit a menu of deal offers $\left\{ \left(\boldsymbol{\alpha}^j, b^* \right) \right\}$ with a deal for *each* component j; $\theta_{ji} = \underline{\theta}$, $j = 1, 2$, with $\boldsymbol{\alpha}^1 = (1, 0)$ and $\boldsymbol{\alpha}^2 = (0, 1)$. If the menu contains two deals, the same bribe $b^* (m)$ defined below is offered in both.

At *step 3* firms submit the competitive equilibrium offers under the assumption that they are anti-symmetric in cost.

The auctioneer:

At *step 2* the auctioneer selects from among the submitted corrupt deals a deal that includes the highest bribe provided the bribe covers the cost $m (\alpha - \widehat{\alpha})^2$. From among the highest bribe deals he selects a one associated with a scoring rule only demanded by one firm if any. If there are several such deals he randomizes. He announces the associated scoring rule.

At *step 5* the auctioneer maintains the outcome of the auction if the winner is a firm who demanded the favor (and by assumption will pay for it). Otherwise he rejects the outcome.

We below show that the strategies described above form a symmetric Perfect Bayes-Nash equilibrium with favoritism. We develop the proof in terms of firm 1 which has its advantage in the production of q_1. Firm 2 is symmetric with advantage in component 2. We proceed by backward induction.

The reasoning for *step 5* is the same as in proposition 1 and 2. At *Step 3,* the reasoning here is identical to the one in the proof of proposition 1 for $k = 2$, $\alpha_1 = \alpha$ and $\alpha_2 = (1 - \alpha)$.

A step 2 the auctioneer's utility function is $b_i - m (\alpha_i - \alpha)^2$. So it is optimal to choose a deal among the submitted ones as follows $\widehat{i} = \arg\max_{\{(b_1, \alpha_1)(b_2, \alpha_2)\}} b_i - m (\alpha_i - \alpha)^2$ s.t. $b_i \geq m (\alpha_i - \alpha)^2$. In case of ties, he is indifferent and may just as well select a deal demanded

by one firm only. By assumption and given the firms' equilibrium strategies, we know that there is at least one such. The auctioneer announces $\alpha_{\widehat{i}}$ if $i^*\left(\alpha_{\widehat{i}}\right) = \widehat{i}$. If no bribe deal can secure win in the official auction or if $b_{\widehat{i}} < m\left(\alpha_{\widehat{i}} - \alpha\right)^2$, the auctioneer announces the true alpha.

A step 1 we start with a Lemma

Lemma 2 *In a symmetric equilibrium firms always demand the "cartel efficient" scoring rule contingent on their cost structure i.e., $\alpha_1^* = 1$ and correspondingly $\alpha_2^* = 0$.*

For firm 1, the "cartel efficient" scoring rule is defined $\alpha_1^* = \arg\max_{\widehat{\alpha}_1} \left\{ \frac{(\overline{\theta} - \underline{\theta})}{2\underline{\theta}\overline{\theta}} (2\widehat{\alpha}_1 - 1) - m\left(\widehat{\alpha}_1 - \alpha\right)^2 \right\}$. It is the scoring rule that maximizes the cartel's payoff given that there is a cost associated with deviations from the true scoring rule. We know that the auctioneer selects the firm whose deal maximizes $U(b_i, \alpha_i) = b_i - m\left(\alpha_i - \alpha\right)^2$. Suppose by contradiction that an equilibrium offer is (b_1, α_1) with $\alpha_1 \neq \alpha_1^*$. We now construct offer (b_1', α_1^*) with $b_1' = b_1 - m(1/2 - \alpha_1)^2 + m(1/2 - \alpha_1^*)^2$. By construction

$$prob\left(U\left(b_1', \alpha_1^*\right) > U\left(b_2, \alpha_2\right)\right) = prob\left(U\left(b_1, \alpha_1\right) > U\left(b_2, \alpha_2\right)\right) = 1/2.$$

Now

$$
\begin{aligned}
E\pi\left(b_1', \alpha_1^*\right) &= \left[\frac{(\overline{\theta} - \underline{\theta})}{2\underline{\theta}\overline{\theta}} - b_1'\right] prob\left(U\left(b_1', \alpha_1^*\right) > U\left(b_2, \alpha_2\right)\right) \\
&= \left[\frac{(\overline{\theta} - \underline{\theta})}{2\underline{\theta}\overline{\theta}} - b_1 + m(1/2 - \alpha_1)^2 - m(1/2 - \alpha_1^*)^2\right]\frac{1}{2} \\
&= \left[\left(\frac{(\overline{\theta} - \underline{\theta})}{2\underline{\theta}\overline{\theta}}(2\alpha_1 - 1) - b_1\right) + \left(2\frac{(\overline{\theta} - \underline{\theta})}{2\underline{\theta}\overline{\theta}} - m\alpha_1\right)(1 - \alpha_1)\right]\frac{1}{2} \\
&> \left[\frac{(\overline{\theta} - \underline{\theta})}{2\underline{\theta}\overline{\theta}}(2\alpha_1 - 1) - b_1\right] prob\left(U\left(b_1, \alpha_1\right) > U\left(b_2, \alpha_2\right)\right)
\end{aligned}
$$

where the inequality holds because $m < \frac{(\overline{\theta} - \underline{\theta})}{2\underline{\theta}\overline{\theta}}$.

Hence $\alpha_1^* = 1$ and similarly for firm 2: $\alpha_2^* = 0$.

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We now consider the determination of b_1 and b_2. The expected profit of firm 1

$$
\begin{aligned}
E\pi_1\left(b_1; b_2\right) &= \left(\frac{\left(\overline{\theta} - \underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - b_1\right) prob\left\{U\left(b_1, 1\right) > U\left(b_2, 0\right)\right\} \\
&= \left(\frac{\left(\overline{\theta} - \underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - b_1\right) prob\left\{b_1 - m\left(1 - \alpha\right)^2 > b_2 - m\left(\alpha\right)^2\right\} \\
&= \left(\frac{\left(\overline{\theta} - \underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - b_1\right) prob\left\{\alpha > \frac{1}{2} + \frac{\left(b_2 - b_1\right)}{2m}\right\} \\
&= \left(\frac{\left(\overline{\theta} - \underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - b_1\right)\left(\frac{1}{2} - \frac{\left(b_2 - b_1\right)}{2m}\right).
\end{aligned}
$$

Taking the derivative with respect to b_1 an interior solution satisfies

$$
b_1^* = \frac{1}{2}b_2 + \frac{1}{2}\left(\frac{\left(\overline{\theta} - \underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - m\right).
$$

In a symmetric equilibrium we obtain

$$
b^* = b_1^* = b_2^* = \frac{\left(\overline{\theta} - \underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - m, \tag{9}
$$

In a symmetric equilibrium the auctioneer never distorts more than by .5 so the highest cost for distortion is $\frac{1}{4}m$. Firms bid the bribe in (9) which is a best response to the other firm. Hence, for $\frac{\left(\overline{\theta} - \underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - m > \frac{1}{4}m \Leftrightarrow m < \frac{4}{5}\frac{\left(\overline{\theta} - \underline{\theta}\right)}{2\underline{\theta}\overline{\theta}}$, the investigated strategies described above including the deal offers $(b^*, 1)$ and $(b^*, 0)$ form a Bayes-Nash equilibrium of the FSA with favoritism. QED.

E   Proof of proposition 4

In this proof we consider two types of collusion, characterize the condition for their sustainability and compare them in terms of cartel efficiency.

"In-turn rule" collusion:

This type of collusion is similar to the one in Proposition 2. The strategies are the same as the ones described in the proof of proposition 2 when putting $k = 2$. Any deviation from those strategies triggers the play of the Nash equilibrium of proposition 3 from the next period on.

As usual we investigate the game by backward induction and focus on incentives to comply in a period following a history of compliance play.

At *step 5* the same argument as in the proof of proposition 3 applies since our agent is short-run.

At *Step 3* the designated winner, say firm 1, has no incentive to deviate while the other firm might undercut the offer of firm 1 and get at most $\frac{1}{2\bar{\theta}}$ in the current period and a continuation payoff of $\frac{\delta}{1-\delta^2)}\left\{\frac{(\bar{\theta}-\underline{\theta})}{2\underline{\theta}\bar{\theta}} - b^*(m)\right\}$ afterwards. Given that $b^*(m) = \frac{(\bar{\theta}-\underline{\theta})}{2\underline{\theta}\bar{\theta}} - m$, the incentive constraint writes

$$\frac{\delta}{1-\delta^2}\left\{\frac{1}{2\underline{\theta}} - m\right\} \geq \frac{1}{2\bar{\theta}} + \frac{\delta}{1-\delta^2}m. \tag{10}$$

where the rhs is the compliance payoff and the lhs is the deviation payoff. We show below that deterring deviation at *step 1* is more demanding so we postpone the derivation of the limit on the discount factor.

At *step 2* there is no incentive to deviate for the agent for reason similar to those in proposition 3. At *step 1* firm 2 might make a secret bribe bid of $b \in (0, 2m]$ and demand $\hat{\alpha} = 0$. The agent will grant firm 2 the favor of choosing its demanded scoring rule when $\alpha \in [0, b/(2m)]$. Firm 2 then gets a payoff of $\frac{1}{2\underline{\theta}} - b$ and a continuation payoff $\frac{\delta}{1-\delta^2}\left\{\frac{(\bar{\theta}-\underline{\theta})}{2\underline{\theta}\bar{\theta}} - b^*(m)\right\}$. The expected payoff from this deviation is

$$E\pi = \frac{b}{2m}\left\{\frac{1}{2\underline{\theta}} - b\right\} + \frac{b}{2m}\frac{\delta}{1-\delta^2}m + \left\{1 - \frac{b}{2m}\right\}\frac{\delta}{1-\delta^2}\left\{\frac{1}{2\underline{\theta}} - m\right\}.$$

In order for $b = 0$ to be an equilibrium strategy, the above expression needs to be maximized at $b = 0$. Since the expression is strictly concave in b we get the following restriction for the discount factor

$$\frac{\partial E}{\partial b}(b = 0) \leq 0 \text{ or } \frac{1}{2\underline{\theta}} + \frac{\delta}{1-\delta^2}\left\{m - \frac{1}{2\underline{\theta}}\right\} \leq 0 \tag{11}$$

The expression in the bracket is negative and increasing in m. By assumption we have $m < \frac{4}{5}\frac{(\bar{\theta}-\underline{\theta})}{2\underline{\theta}\bar{\theta}}$ but to simplify the calculation, we check for $m = \frac{(\bar{\theta}-\underline{\theta})}{2\underline{\theta}\bar{\theta}}$ the constraint is more restrictive then. Thus a conservative formulation of the constraint on the discount factor yields $\delta \geq \underline{\delta}$ where

$$\underline{\delta} : \frac{\underline{\delta}}{(1-\underline{\delta}^2)} = \frac{\bar{\theta}}{\underline{\theta}}$$

For $\frac{\bar{\theta}}{\underline{\theta}} = 2.2$, $\underline{\delta} = 0.8$.

Contingent rule

We use the notation: $w_{1(2)}(YES)$ and $w_{1(2)}(NO)$ to denote firm 1(2) continuation after a period when it won respectively when it lost. We focus on a smaller set of strategies

including $b \in \{\frac{1}{4}m, \frac{5}{4}m\}$ so in particular we only consider a defection that secures win. To out-compete firm 1 when the true scoring rule is most favorable to 1 e.g. $\alpha = 1$, firm 2 must offer $\frac{5}{4}m$. Restricting the set of possible deviations is not crucial to the result but it simplifies the presentation.

Let $H_{t-1} = H^*$ denote a history of the game where in all periods t', $t' = 0, ..t - 1$, we have $\widehat{\alpha}^{t'} \in \{0, 1\}$ and $S\left(p^{*t'}, q^{*t'}\right) = 0$.

We propose the following strategies for the players:

i. If $H_{t-1} \neq H^*$, the firms and the auctioneer play the equilibrium strategies depicted in proposition 3.

ii. If $H_{t-1} = H^*$, the firms' strategy is

At *step 0* firm 1 (2) learn its cost structure.

At *step 1* firm 1(2) submits a menu of deals one for each component where cost is low demanding a single-peaked scoring rule and offering the same bribe $b = \frac{1}{4}m$.

At *step 3*

- If $\widehat{\alpha}^t$ is single-peaked, the firm with the congruent cost structure submits an offer including the corresponding efficient specification such that the offer scores zero. Firm 2 with the non congruent cost structure submits an offer that scores at most zero.

- If the announced scoring rule is not single peaked, the winner is designated by a random rule characterized below. The designated winner submits and offer that scores zero and the other firm submits an offer that scores slightly less than zero.

The auctioneer's strategy

At *step 2*, the auctioneer selects the corruption deal among that maximize his utility provided the bribe covers expected costs and he announces the corresponding scoring rule. In case of ties he randomizes.

At *step 5* if the scoring rule is single peaked but the winner is not a firm that demanded that scoring rule he rejects the auction's result. Otherwise he maintains it.

We below show that these strategies form a Perfect Public Equilibrium of the repeated game with the stage game as described in section 5. Collusive bidding at *step 3* is sustainable relying on an argument similar to the one developed in Proposition 2 when setting $k = 2$. The non-favored (say 2) firm's incentives to comply with the collusive strategy is satisfied for $\delta > \underline{\delta_2}$ where $\underline{\delta_2}$ is defined by the following equality $\frac{\delta}{(1-\delta)}\frac{1}{2}w_2\left(NO\right) = \frac{1}{2\overline{\theta}} - \varepsilon + \frac{\delta}{(1-\delta)}\frac{1}{2}\pi^{ne}$ the lowest compliance payoff is set equal to the defection payoff.

At *step 2* the proposed strategy is optimal for the auctioneer appealing to the same argument as in proposition 3. At *step 1* the firms may consider defection and offer a deal with a bribe equal to $\frac{5}{4}m$. The defection payoff is at most $\pi^d = \pi^c - \frac{5}{4}m$ while the expected compliance payoff is $\frac{1}{2}\left[\pi^c - \frac{1}{4}m\right]$. We first note that for $m \geq \frac{4}{9}\pi^c$ there is no incentive to defect. But for $m < \frac{4}{9}\pi^c$ (recall that we only consider the interval $m < \frac{4}{5}\frac{(\overline{\theta}-\underline{\theta})}{\theta\underline{\theta}}$ implying $m < \frac{4}{9}\pi^c$). The following incentive constraint, applies in any period t preceded by $\alpha_{t-1} = 0$:

$$\frac{1}{2}\left[\pi^c - \frac{1}{4}m\right] + \delta\left[\frac{1}{2}w_1\left(YES\right) + \frac{1}{2}w_1\left(NO\right)\right] > \left(\pi^c - \frac{5}{4}m\right) + \delta w_1\left(YES\right)$$

$$\delta\left[\frac{1}{2}w_1\left(NO\right) - \frac{1}{2}w_1\left(YES\right)\right] > \left(\frac{1}{2}\pi^c - \frac{9}{8}m\right)$$

$$\delta\left[w_1\left(NO\right) - w_1\left(YES\right)\right] \geq \left(\pi^c - \frac{9}{4}m\right) \tag{12}$$

So as $\delta \longrightarrow 1$, $w_1\left(NO\right) - w_1\left(YES\right) \to \left(\pi^c - \frac{9}{4}m\right)$ the continuation payoff of firm 1 following an announcement of $\widehat{\alpha} = 1$ must be lower that the one following $\widehat{\alpha} = 0$. This payoff is achieved by letting firm 2 submit an offer in the official auction that induces a lower profit to firm 1. For $m = \frac{4}{9}\pi^c$ (the highest m for which there is an incentive to deviate), the rhs is equal to 0. Consider the following strategy that satisfies the constraint. The full punishment is taken in the next-following period and "the clock is reset" i.e., the next following payoffs in $t + 2$ are determined as if $\widehat{\alpha}_t = 0$. Note that when (12) holds incentives to comply in a period following an announcement of $\alpha = 1$ also are satisfied. This is because the gain from defection are lower then. Hence, for $\delta \geq \underline{\delta}_2$ and $w_1\left(1\right) = w_2\left(0\right)$ satisfying (12) the proposed strategies form a Perfect Public Equilibrium of the repeated game.

As $m \to \frac{4}{9}\pi^c$ the equilibrium average expected payoff of the contingent scheme tends toward $\frac{1}{2}\left(\pi^c - m/4\right) = \frac{4}{9}\pi^c$ while the average payoff of the fixed in-turn rule is $\frac{\frac{5}{9}\pi^c}{(1+\delta)}$. So for any $\delta > \frac{5}{4} - 1$, the contingent scheme yields a higher expected payoff. We note that the condition $m \geq \frac{4}{9}\pi^c$ is consistent with the condition in proposition 4 $m \leq \frac{4}{5}\pi_{ne}$ when $\frac{4}{9}\frac{1}{2\underline{\theta}} \leq \frac{4}{5}\frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}}$ which requires $\underline{\theta} \leq \frac{4}{9}\overline{\theta}$.

On the other side when $m \to 0$ the rhs of (12) tends to π^c implying $\pi_1^{t+1}\left(1\right) \to 0$ corresponding to an average equilibrium expected payoff in the contingent scheme of $\frac{1}{4}\pi^c$ which is strictly smaller than the average payoff in the in-turn scheme $\frac{\pi^c}{(1+\delta)}$. *QED*