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Nonlinear and chaotic patterns in Japanese video game console sales and consequences for management control

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Abstract

This paper investigates the behaviour of weekly hardware sales in the Japanese video game sector. It is found that weekly hardware sales exhibit significant linear and non-linear behaviours during the product cycle. We are going to analyse the implications of our findings for management control in the video game sector.

Keywords: forecasting, marketing, time series, systems dynamics, chaos theory

1.Introduction

The existence of non-linear dependance in financial asset returns, in cardiology and in physics have received considerable attention in the literature. Advances in the theorical literature on chaos and nonlinear dynamics have resulted in a number of statistical tests that can be used to detect nonlinearity in time series data. The application of these tests reveals that there is a significant nonlinear dependence in organization (Thiétart and Forgues, 1997), in innovation process (Cheng and Van de Ven, 1996), in financial returns of many of the world's major equity markets (see, for exemple, Scheinkman and Lebaron, 1989; Guégan, 2003; Harris and Küçüközmen, 2000; Takala and Virén, 1996). some sectors exist in management and market studies where nonlinearity is plausible theorically but has never been tested: fashion sales (Nakayama and Nakamura, 2004; Granovetter and Soong, 1986), addictive good sales (Feichtinger et al. , 1995), innovative good sales (Granovetter and Soong, 1986).

In theory, nonlinear or even chaotic behaviour is highly plausible for fashion good sales. Indeed, Granovetter and Soong (1986) show that nonlinearity is analytically possible for such goods in threshold models of interpersonal effects. Moreover, from another perspective, Nakayama and Nakamura (2004) demonstrate analytically this possibility . The model backgrounds are consumer

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microeconomics and statistical physics. So, theorically, the plausibility of a nonlinear behaviour is higher than for addictive good sales (Feichtinger et al., 1995), innovative goods (Granovetter and Soong, 1986), because two papers from different perspectives have got the same conclusions.
These conclusions are questionned because some marketing researchers have modelised the fashion good sales as a linear dynamic system (Miller et al., 1993). Miller et al. (1993) have formalized the fashion theory as a linear dynamic for explaining bandwagon and counter-bandwagon effects in consumption. Their article can be seen as an alternative theory or hypothesis.
These rival hypotheses seem to rest untested in management or economics literature. A reason could be the ephemeral aspect of some fashion goods. In order to test nonlinearity, the tests impose to have long time series with more than 500 points. Nevertheless, the existence of nonlinear behaviours in sales could be interesting for purposes of management control. Notably, this possibility and the presence of chaotic patterns could call into question the previsibility of sales and the budgeting process in some sectors.
The rest of the paper is organised as follows. Section 2 shows some theorical implications of nonlinear and possible chaotic patterns for management control. Section 3 describes the data and the methodology used. Section 4 presents the empirical results. Section 5 concludes.

2.The implications of nonlinear behaviour for management control

The implications of nonlinear patterns in sales are numerous for management control: impredictibility, controlability, fraud auditability, among other things.
In this paper, we only focus on the implications of chaotic patterns. The presence of chaos is a possibility for some nonlinear system parameters. These patterns can invite the management control to change the time horizon of predictability and thus the budget. It can suggest to drop traditional tools useful for linear dynamics to optimize profit and to detect fraud.
The possibility of chaotic behaviour could call into question the idea of predictability and the budget process. In fact, because of high sensitivity to initial conditions, if the reporting process has some mesurement errors and a bias, the forecast error could exponentially grow. A mesure of this exponential growth is the caracteristic Lyapunov exponent. By estimating this quantity, one can give the time horizon of predictability, if the exponent is signicantly positive. Thus, the presence of chaos limits budget time horizon.
Another interesting implication is the way of controlling a nonlinear chaotic system. Shinbrot, Ott, Grebogi et Yorke (1990; 1992; 1992; 1993) have shown that it is possible to control a chaotic system. The idea is to maintain a cyclical pattern with small pertubations. These perturbations aim to bring the system in the desired trajectory. However, for management purposes, the idea of

maintaining the dynamic at a cyclical trajectory must be impossible because one cannot know the equations governing the dynamics (Kopel et al., 2008) and because chaotic behaviour could be the solution of a profit optimization problem (Kopel et al., 1998). Thus, Kopel et al.(2008) introduce a tool based on short term trajectories to predict catastrophic outcomes. It can be interesting to see the tool effectiveness on real data.

Last but not least, the nonlinear chaotic behaviour could be of interest for fraud audit. In fact, Tolle et al. (2000) show on simulation that chaotic experimental time series could not be significantly in adequation with the Benford's law. During the last few decades, fiscal administrations in western countries have used increasingly Benford's law to detect fraud. They are conforted by numerous papers in accounting fields (see, for exemple, Carslaw, 1998, Van Caneghem, 2004 ; Niskanen and Keloharju, 2000; Skousen et al., 2004; Thomas, 1989; Kinnunen and Koskela, 2002). Thus, it could be interesting to show if fashion sales with nonlinear chaotic behaviour follow significantly the Benford's law. It could limit the application of this tool for fiscal audit purpose.

Section 3 justifies the data and methods used.

3.Data and methodology

3.1. Data

The raw data used in this paper is drawn from the VG-Chartz data base of weekly video game console sales in Japan for the period 15th april 1989 to 15th april 2009. According to the table 1 and following Varian (1972), the data base seems to be without significative fraud given its adequacy with Benford's law.

Table 1
Goodness-of-fit of data base and japanese data with Benford's Law

Statistics	Data base	Japanese data
Chi-square statistics, χ^2	1,53	6,33
P-value (χ^2)	0,99	0,61
Kolmogorov-Smirnov statitics, D	0,11	0,22
P-value (KS)	1	0,99
Cramer Smirnov Von Mises statistics C	0,03	0,05
P-value (C)	1	0,95

The KPSS tests have indicated that a good way to stationarize time series is the first-difference filter. This yields a total of 20 consoles sales time series. In order to compute the nonlinearity tests, we have only taken the series with more than 400 points (weeks). According to this condition, we have used only Game Boy-GB, Playstation- PS-, Super Nintendo -SNES- and Playstation 2 -PS2-

weekly sales. Table 2 reports some summary statistics for each stationarized series, including the first moments, the minimum and the maximum, the skewness and the excess kurtosis coefficients, and the jarque-Bera statistics for normality. Normality is rejected at the one-percent level using the Jarque-Bera test and the first-differenced series are not significantly non-stationary according to KPSS tests.

Table 2
Preliminary statistics[a]

Hardware	GB	PS	PS2	SNES
Mean	-119,12	0,30517	-1582,5	-37,370
Standard deviation	25842,	19223	35067	22364
Skewness	-0,072682	-0,34252	-9,7951	-0,051933
Excess kurtosis	56,487	22,140	150,49	22,846
Minimum	-2,9285e+005	-1,7706e+005	-5,7157e+005	-1,7903e+005
Maximum	3,03E+005	1,2383e+005	1,1181e+005	1,9096e+005
Jarque-Bera statistic	105697***	11857,4***	455840***	10525,8***
KPSS statistics (original series)	0,815409***	4,01059***	3,0671***	5,11893***
KPSS statistics (first differenced series)	0,00957838	0,0242597	0,265552	0,02
Number of observations	795	580	475	484

[a] The table reports the preliminary statistics for the first differences weekly sales in volume for the period april 1989 to april 2009. For the Jarque-Bera and the KPSS statistics, *** denotes significance at the 1% level.

3.2. BDS test, nonlinear prediction error and time reversibility test

In order to detect significant nonlinear dependence in hardware sales, we can use the test developed by Brock, Dechert and Scheinkman-BDS test- or, with Monte Carlo simulation, the reversibility test or the nonlinear prediction error test.

This three tests are useful because the BDS test is the most common one for nonlinear detection, despite some critics about its robustness and its discrimination power with noisy data. Indeed, Schreiber and Schmitz (1997) show that the BDS test has a good discrimination power and noise robustness for some nonlinear dynamics but not for all the nonlinear systems, like the time reversibility test (Van der Heyden et al., 1996). Thus, they are sufficient and powerful indicators of nonlinearity, but not necessary conditions. On the contrary, the nonlinear prediction error test seems to have a good overall performance. Finally, it could be rational to triangulate these tests to improve results validity.

3.2.1.The BDS test

Based on the correlation integral of a time series (Grassberger and Procaccia, 1983), the BDS statistic tests the assumption of independent and identical distribution- i.i.d.. With this test, one can test the presence of nonlinearity in time series if and only if applied on the residual of an ARMA model (Guégan, 2003).

For a time series $\{x_1, \ldots, x_T\}$, the correlation integral estimates the probability that any two-dimensional vectors of observations, $x_t(m)=\{x_t, \ldots, x_{t+m-1}\}$, are within a certain distance, ε, of each other in phase space. Formally, one can define the correlation integral as:

$$C_m(\varepsilon, m) = \lim_{T \to \infty} \frac{2}{(T-m+1)(T-m)} \sum_{t=1}^{T-m} \sum_{s=1}^{T-m} I(x_t(m) - x_t(m)), \tag{1}$$

where I(.) is an Heaviside's function which is equal to one if the euclidean distance between the two-dimensional vectors of observations, $x_t(m)$ and $x_s(m)$, is less than ε. m is the embedding dimension. Under the i.i.d. assumption, Brock et al.(1987) show that $C_m(\varepsilon,T)= C_1(\varepsilon, T)^m$.

Thus, they propose a statistic, which follows a gaussian distribution if the serie is i.i.d.:

$$W_m(\varepsilon, T) = \frac{\sqrt{T} \cdot (C_m(\varepsilon, T) - C_1(\varepsilon, T)^m}{\sigma_m(\varepsilon, T)} \tag{2}$$

where $\sigma_m(.)$ is the asymptotic standard deviation of $C_m(.)$ estimated from the data (Brock et al., 1987).

Following Brock et al.(1987) and Hsieh (1991), the maximum distance, ε, is set to 0.5σ, 0.75σ, σ, 1,25σ, 1.5σ, 1,75σ and 2σ, where σ is the standard deviation of the series. Following Lin (1997), the number of observations divided by the embedding dimension must be greater than or equal to 200. This last condition imposes to test the i.i.d. assumption on AR filtered PS2 and SNES stationarized sales for m=2, AR filtered PS first differenced series for m=2 and m=3, and m=2,...,4 for AR filtered GB stationarized sales.

According to Theiler and Prichard (1996), the ARMA model residuals must be used because the BDS statistic is « pivotal ».

The two other statistics (time reversibility and nonlinear prediction errors ones) are not « pivotal ». Thus, following Theiler and Prichard (1996), the surrogates and the stationarized time series have the same power spectrum. Since the probability distributions of such nonlinear statitics are generally not known analytically, they must be estimated by Monte Carlo resampling of the data. The method of surrogate data provides a rigorous statistical test for the hypothesis that a series has been generated by a linear stochastic process (Theiler et al, 1992; Schreiber and Schmitz, 1997). Schreiber and Schmitz (1996) method is used to generated surrogate data. Such surrogates are obtained by taking the Fourier transform of the stationarized series, randomizing the phases and inverting the transform. Then, Schreiber et al.'s two steps (Schreiber and Schmitz, 1996; Schreiber and Schmitz, 1997) are iterated to obtain surrogates with the same distribution and the same power spectrum as the data. 19 surrogates have been generated for each series to have a 90% level of significance for these two-sided tests (B=2/α-1= 2/0,1-1=19). The size of the tests, 10%, has been fixed to limit the computational burden.

3.2.2. The nonlinear prediction error test

In the literature, many quantities have been used in order to test the nonlinearity dependence (Kaplan and Glass, 1992; Kennel et al., 1992; Schreiber and Schmitz, 1997). We use a stable representative of the class of predictability measure (Schreiber and Schmitz, 1997). Consequently, a nonlinear prediction error with respect to a locally constant predictor F can be defined by:

$$t^{PE}(m,\tau,\varepsilon)=\sqrt{\sum_{i=1}^{T}\left(x_{i+1}-F(x_i)\right)} \qquad (3)$$

The prediction about one period, i+1, is performed by averaging over the future values of all neighboring delay vectors closer than ε in m dimensions. In order to make these predictions for the stationarized series and the 19 surrogates, we use the TISEAN software lfo-test algorithm with default parameters.

3.2.3. The time reversibility test

Following Schreiber and Schmitz (1997), a simple statistic is used to detect deviations from time-reversibility:

$$t^{PE}(\tau)=\left(x_i-x_{i-\tau}\right)^3 \qquad (4)$$

The parameter τ has been fixed equal to 1 because it is the one fixed by Schreiber and Schmitz (1997) for Henon equation. The Henon equation equals the one by Granovetter and Soong (1986).

3.3. Chaos detection with noisy data

A classic test for chaos detection in management is the one by Wolf et al. (1985). However, this test is not noise robust (see, for exemple, Rosenstein et al., 1993; Liu et al., 2005) and it is not a statistical test (Vialar, 2005). Thus, the modified Rosenstein et al.'s maximum Lyapunov exponent estimation has been used because it is more robust to noise than other methods and gives a significance level for the maximum Lyapunov exponent estimation (Liu et al., 2005).
The TISEAN software « lyapr- » algorithm has been used. Then, Liu et al.'s nonlinear adjustement with R software has been computed for each « lyapr- » output in order to estimate the maximum Lyapunov exponent and its significance level.

4.Results

4.1. Evidences of non-linearity

4.1.1. Linear dependance

In order to identify any linear dependance in stationarized sales, table 3 reports the first 9 autocorrelation coefficients for the series. The table also reports Ljung-Box portemanteau statistics for 3 and 7 lags. For exemple, for stationarized GB sales, the 3th- and the 7th-order autocorrelation coefficients are respectively -0,1908 and -0,0735. The remaining autocorrelations are insignificant at the 1% level. The Ljung-Box statistics are significant at the one-percent level. Thus, there is evidences of statistically very significant linear dependence in GB stationarized hardware sales. Although the magnitude of this dependence is not large. On the basis of the estimated autocorrelation coefficients, the Schwartz Bayesian criterion (SBC) and the diagnostics tests for the residual, we estimated an AR model for the data with a lag length of 3 and 7. In order to remove the linear dependence in stationarized sales, we filtered the series using the estimated AR(7) model. In order to verify the appropriateness of the estimated model, table 3 reports the autocorrelation coefficients and the Ljung-Box statistic for the residuals from the AR model. For the AR-filtered and stationarized GB sales, all of them are insignificant at the one-percent level. For the PS ones, all of them are insignificant at the ten-percent level. For the PS2 and SNES ones, AR-filtering usefulness seems to be questionable.

Table 3
Linear dependence in stationarized sales [a]

Lag	GB	AR(7)-filtered GB	PS	AR(2)-filtered PS	PS2	AR(2)-filtered PS2	SNES	AR(2)-filtered SNES
1	-0,1929***	0,002	-0,1078***	0,0007	0,0566	0,0019	-0,1669***	-0,0236
2	-0,02	0,016	-0,03	-0,0136	-0,0651	0,03	-0,0720	0,0422
3	-0,1908***	-0,020	-0,195***	-0,05	-0,0701	0,0115	-0,1258***	-0,0631
4	-0,05	-0,015	0,03	-0,0297	0,0284	-0,1014**	-0,0191	-0,0897**
5	-0,004	-0,007	-0,0723*	-0,0915**	0,0441	-0,0368	0,0421	-0,1250***
6	-0,1198***	-0,026	-0,03	-0,0968**	-0,0557	-0,1621***	-0,0260	-0,0162
7	-0,0735**	-0,023	-0,05	-0,0700*	-0,0200	-0,1001**	-0,1216***	-0,1815***
8	0,1286***	0,007	0,06	0,0035	0,0307	-0,0204	-0,0474	-0,0459
9	0,0751**	-0,02650	0,03	0,0209	-0,0159	-0,0646	-0,0425	-0,1059**
Q(3)	36,3769***	0,5272	29,5079***	1,5030	5,9244	0,3763	23,8426***	3,0684
Q(7)	54,0842***	1,7208	35,4815***	15,2590**	8,9383	23,3720***	32,5142***	30,8813***
conclusion	AR-filtering useful		AR-filtering useful		AR-filtering useless		AR-filtering useless	

[a] *,**,*** denote significance at the 10%, 5%, 1% level, respectively.

4.1.2 The nonlinearity tests

Table 4 reports the outputs of the BDS test applied to the AR-filtered and stationarized sales. Results are reported for m-histories 2 for SNES and PS2 series, from 2 to 3 for PS sales and from 2 to 4 for GB series, and for values of the maximum distance parameter ε from 0,5 to 2 times the standard deviation of the AR filtered stationarized sales. For all these values of both m and ε, one can strongly reject the I.i.d. assumption, suggesting that nonlinear dependence is significantly detected in addition to the linear dependence identified above. The positive values of BDS statistics for all m and ε for each series implies that there are behaviours which appear more frequently than in random series.

Table 4

BDS statistics of AR filtered stationarized series [a]

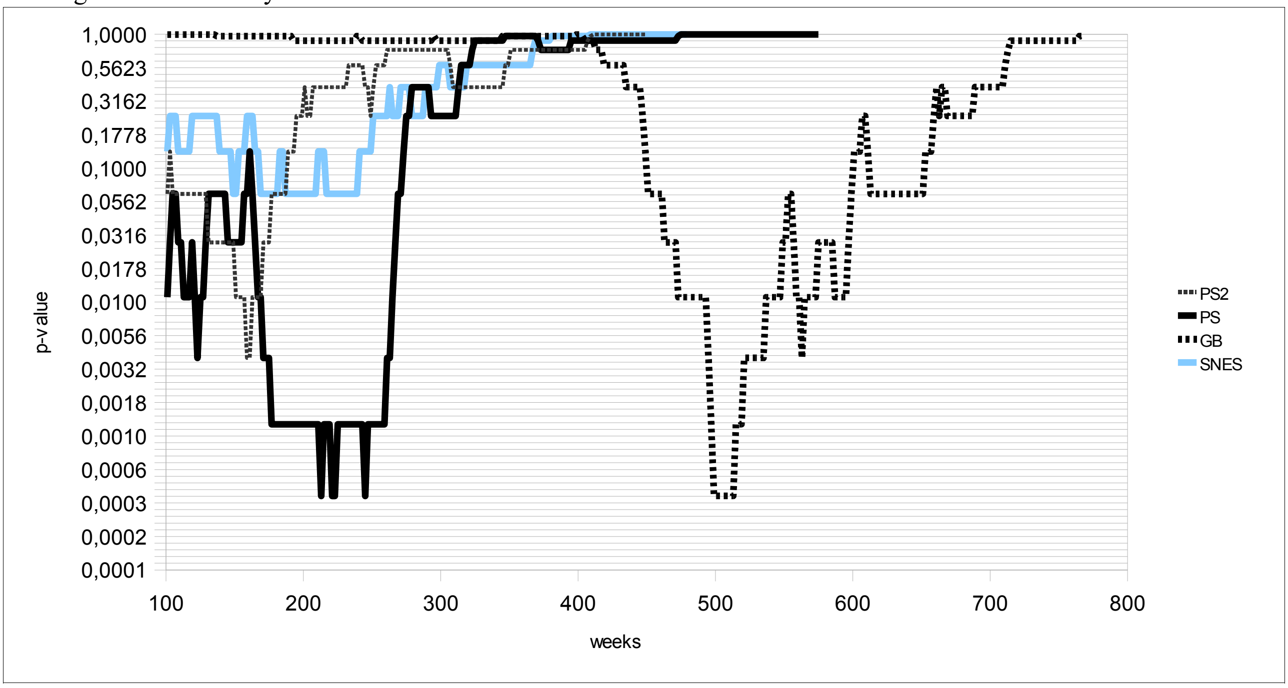
console	m	ε/σ	0,5	0,75	1	1,25	1,5	1,75	2
GB	2		13,96***	13,27***	12.63***	12,01***	12,68***	12,61***	12,80***
	3		17,81***	16,17***	15,15***	14,52***	15,08***	14,69***	14,52***
	4		21,03***	17,5***	15,95***	15,14***	15,41***	14,91***	14,6***
PS	2		15,33***	13,79***	13,12***	11,88***	11,08***	10,74***	9,86***
	3		19,62***	16,46***	15,17***	13,54***	12,35***	12,03***	11***
PS2	*2*		*11,87****	*11,13****	*12,22****	*11,45****	*10,48****	*9,17****	*8,9****
SNES	*2*		*12,62****	*8,36****	*9,17****	*8,48****	*8,07****	*7,3****	*7,02****

[a] *** denotes significance at the 1% level. Items in italics denote that AR-filtering is questionable.

In order to understand the simultaneous presence of linear and nonlinear dependences, time reversibility and nonlinear error prevision tests could be helpful because the BDS statistics is sufficient and powerful indicator of nonlinearity, but not necessary condition. Moreover, for PS2 and SNES stationarized sales, AR-filtering usefulness seems to be questionable.

Figure 1 reports the significance level of time reversibility tests of stationarized sales during each product life cycle. Outputs indicate a period during the products cycle with significant time irreversibly at the 10% level of significance for SNES and even 1% for other hardwares. It seems difficult to infer a law from these outputs about a significant nonlinear period and a linear period in the products cycle. Nevertheless, it seems possible to infer that at the end of the product cycle the sales patterns are less nonlinear.

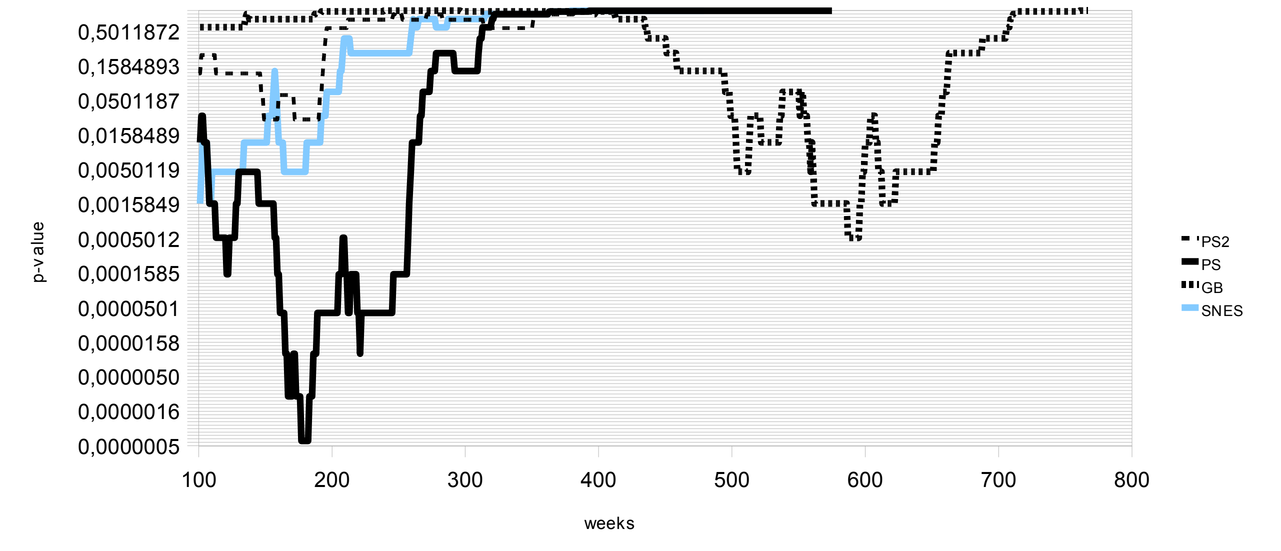
Figure 1
Moving time reversibility test of stationarized sales [a]



[a] The size of the test, computed for the last 100 weekly sales, was taken to be 0.1.

Figure 2 reports the significance level of the nonlinear prediction error test during each console life cycle. Results indicate the existence of periods in each hardware life cycle with significant nonlinearity at the 5% level of significance for PS2 and even 1% for other video game console sales. One can observe the same nonlinearity loss at the end of the product cycle.

Figure 2
Moving nonlinear prediction error test [a]



[a] The size of the test, computed for the last 100 weekly sales, was taken to be 0.1.

Thus, we will use the chaos detection test for the four stationarized sales series because there are all significant evidences of nonlinearity. The nonlinearity lost at the end of the product life-cycle could be used in management control and marketing in order to predict changes in consumers' behaviour.

4.2. Chaos detection and implications for the management control area

Table 5 reports the Liu et al.'s estimation of the maximum Lyapunov exponent and the significance level (Liu et al., 2005). The maximum Lyapunov exponent is significantly positive for PS2, GB and PS stationarized sales AR-filtered or not. The *predictability* is limited to a time horizon of 3-14 weeks. Nevertheless, one cannot infer a general predictability time horizon for all video game hardware sales because of contingencies. Following these results, one can infer that the sales budget is not always possible for a time horizon of one year in management control.

Table 5
Maximum Lyapunov exponent estimation, predictability in weeks, sum of square errors, Fisher statistics and p-value [a]

series	λ_1	a	k	95% max/ predictability	$\sum e^2$	F	p-value
PS2 AR filtered stationarized sales	0,03335	-2,53263	0,04039	14 weeks	0,00075	125,39	0,00794
PS2 stationarized sales	0,03175	-3,19518	0,03938	14 weeks	0,00027	333,63	0,00299
GB AR filtered stationarized sales	0,06545	-4,17401	0,09402	10 weeks	0,00202	705,38	0,0000007
GB stationarized sales	0,15328	-4,85851	0,23311	6 weeks	0,01089	71,86	0,01373

PS AR filtered stationarized sales	0,21707	-4,29578	0,34673	8 weeks	0,01394	114,17	0,00865
PS stationarized sales	0,07727	-3,54559	0,09993	9 weeks	0,00324	75,97	0,01302
SNES AR filtered stationarized sales	0,40031	-4,51433	0,73464	4 weeks	0,00618	*	*
SNES stationarized sales	0,39624	-5,30642	0,72264	3 weeks	0,00032	*	*

[a] λ_1 is the maximum Lyapunov exponent estimation with Rosenstein et al.'s modified algorithm, a and k are other estimated parameters, 95% max/ predictability indicate how many weeks predictability is limited, $\sum e^2$ is the sum of quare errors, F is the Fisher statistics of the nonlinear adjustment, *indicate that it is not technically possible to compute this information.

The significant presence of chaos can have an implication concerning firm *controllability*. Indeed, one cannot control a nonlinear system like a linear system with linear commands far from equilibrium. For a firm who wants to control its sales in video game sector, it would be useful to apply Kopel et al.'s tool (Kopel et al., 2008). This tool is simple to apply because one does not have to know the actual equations of the dynamics. Nevertheless, for Japanese video games hardware sales we have tried to apply this method without success. One can justify this non-applicability by the high noisy aspect of the data. Another argument could be that the dynamics of sales is not stable: the parameters change but also the structure of the system (Dosi and Metcalfe, 1991). This could be a major difference between physical systems and social systems.

Following Tolle et al. (2000), a chaotic system could not follow the Benford's law. Table 6 reports the adequation with Benford's law test for our original series. For all the consoles, the null hypothesis, which states that series follow Benford's law, is strongly rejected for the first number, suggesting that the hardware sales do not follow the Benford law. For all number, the original GB and PS weekly sales in volume do not significantly follow the Benford's law. For *fiscal auditors*, this result seems to show that Benford's law could be ineffective in this sector in order to detect fraud.

Table 6
Goodness-of-fit Khi-square statistics with Benford's law [a]

Console/ number	First number	Second number	Third number	Fourth number	Fifth number
GB	206,31***	156,24***	159,01***	115,4***	70,23***
PS2	32,11***	5,27	9,4	3,91	5,57
PS	44,3***	34,34***	35,61***	97,36***	143,62***
SNES	51,01***	11,02	138,04***	5,83	10,78

[a] *,**,*** denote significance at the 10%, 5%, 1% level, respectively.

5.Summary and conclusion

The existence of nonlinear dependence and chaotic patterns in fashion sales has important implications for management control. In this paper, we investigate the existence and the nature of nonlinear dependence in one type of fashion sales (video games hardware sales) and its implications for management control. Firstly, there is significant linear dependence combining with nonlinear dependence in the life-cycle of video game hardware sales. Secondly, for each series, one can find a significantly positive maximum Lyapunov exponent and thus chaotic patterns. This second result induces some implications in management control for budgeting, controllability and fiscal audit purposes.

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