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Supply Chain Control: A Theory of Vertical Integration*

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Abstract

Improving a company's bargaining position is often cited as a chief motivation to vertically integrate with suppliers. This paper expands on that view in building a new theory of vertical integration. In my model firms integrate to gain bargaining power against other suppliers in the production process. The cost of integration is a loss of flexibility in choosing the most suitable suppliers for a particular final product. I show that the firms who make the most specific investments in the production process have the greatest incentive to integrate. The theory provides novel insights to the understanding of numerous stylized facts such as the effect of financial development on the vertical structure of firms, the observed pattern from FDI to outsourcing in international trade, the effect of technological obsolescence on organizations, etc.

Keywords: vertical integration, supply chain, bargaining, outside options

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1 Introduction

I consider how vertical integration affects the bargaining power of the integrating firms against non-integrated firms in the supply chain. Integration has costs because it limits an assembler's flexibility in choosing the most suitable suppliers for a particular end product. However, by gaining bargaining power the assembler can appropriate a larger share of the total revenue which can make integration a profitable strategy for the assembler.

I use two examples from the PC and the cell phone industry to motivate my analysis. In the PC industry, IBM and Apple Inc. followed very different strategies. IBM only controlled the hardware of the original PC and had Microsoft provide the operating system. In contrast, Apple controlled both the operating system and the hardware of the Macintosh PC from the start. Within a few short years, Microsoft became the dominant player in the PC industry and in 2005 IBM exited the PC business by selling its remaining factories. Apple, on the other hand, was able to keep its PC business highly profitable and thriving. Apple's decision did not come without costs because the company was often slow in updating its operating system.¹ Ultimately, however, Apple's decision to integrate software and hardware and sacrifice flexibility proved profitable. In the cell phone industry, there has been substantial disagreement about the optimal level of vertical integration and the boundary between in-house and outside procurement has shifted a number of times during the past 15 years. In the 1990s, large handset manufacturers such as Motorola, Nokia and Ericsson outsourced a lot of the design and software development to suppliers in Taiwan, Singapore and India. These suppliers gained crucial knowledge and expertise as a result which allowed some of them to become fierce competitors on their own right.² The business press has acknowledged the importance of vertical integration for bargaining with suppliers. For example, the *Financial Times* stated in a special report on vertical integration that "An-

¹For example, Windows 95 was considered a more stable system than the Apple OS 7. In fact, the main reason behind IBM's decision not to develop the operating system in-house was a desire to bring the PC to market as quickly as possible.

²For example, HTC now produces own brand smart phones as well as those of its clients. The same is true for Compal and the goal of Flextronics' CEO Michael E. Marks -as reported by Business Week (March 21, 2005)- is to make Flextronics a low-cost, soup-to-nuts developer of consumer-electronics and tech gear.

other reason to integrate vertically is to affect bargaining power with suppliers” (November 29, 1999).

In my model, each final product requires a continuum of inputs which are each produced by a specialized supplier. Inputs in my model are complements and each supplier has ex-ante equal ability to hold up assembly of the final product. I assume that inputs differ in their *specificity*, defined by the extent to which the revenue produced by each supplier is subject to hold up. The assembler of the final product has the opportunity to purchase suppliers and integrate them into a single company. The integrated company obtains bargaining power that is disproportionately larger than the share of the production process that is being integrated in the single company. In return, the assembler loses the ability to choose the most suitable companies for production of the final product.

As long as the inefficiency that is caused by integration is not too severe a well defined integrated equilibrium exists. I show that the vertically integrated company will incorporate those suppliers who are required to make the most specific investments. The intuition behind this result is that those suppliers are most vulnerable to hold-up and therefore benefit the most from an increase in bargaining power. In my basic model, integration always has negative welfare consequences because integration affects the distribution of revenue between integrated and non-integrated firms but actually decreases total available revenue (due to the decrease in flexibility). In an extension of the model I allow firms to vary the level of investment. I show that, under certain conditions, integration can improve welfare because firms with specific investments will invest more after integration since their investments are better shielded from expropriation.

My model predicts that a greater incidence of incremental innovation, the increased human to physical capital ratio and the rise of modern financial markets (Rajan and Zingales (2001); Acemoglu, Johnson, and Mitton (2005)) lead to less vertical integration which is consistent with recent trends in developed economies. On the other hand, I also show that industries with a high level of basic research give rise to vertically integrated firms (Acemoglu, Aghion, Griffith, and Zilibotti (2004)). The prediction that the vertically integrated company incorporates those suppliers who are required to make the most specific investments explains why Japanese auto makers have historically been unwilling to import US auto parts

with high technological content ([Spencer and Qiu \(2001\)](#); [Qiu and Spencer \(2002\)](#)). My model also helps to explain why vertically integrated firms are mostly found in developed countries while developing countries host predominantly small-sized firms. Using a similar approach as in [Antràs \(2005\)](#) my model can also explain the recent shift from FDI to outsourcing in international trade (see also [Vernon \(1966\)](#)).

My work shares several characteristics with the Property Rights theory of the firm as developed in [Grossman and Hart \(1986\)](#) and [Hart and Moore \(1990\)](#). Just as in the property-rights view firms' investments can be appropriated by a partner. However, my model analyzes the bargaining power of integrated (inside) firms versus non-integrated (outside) firms. My model also contributes to our understanding of the optimal boundary of the firm. This question was famously posed by [Coase \(1937\)](#) and became the cornerstone of Transaction Cost Economics ([Williamson \(1985\)](#), [Williamson \(2002\)](#)). Although my model resembles models found in the literature on vertical foreclosure ([Salinger \(1988\)](#), [Hart and Tirole \(1990\)](#), [Kranton and Minehart \(2002\)](#)) the mechanism is quite different. In the foreclosure literature different assemblers produce an homogeneous final good and compete with each other - integration serves as a means to exclude the competing assembler from access to crucial supplier. In contrast, each assembler in my model is a monopolist in the final product market. Finally, my model shares some features of [Acemoglu, Antràs, and Helpman \(2007\)](#), most notably the way contractual incompleteness is modeled -more on this later.

In the next section I introduce and analyze the basic model. Section [3](#) endogenizes the investment decision of firms. In section [4](#) I apply the model to explain a series of stylized facts. Section [5](#) concludes.

2 Theory

This section introduces a simple model that exhibits the tradeoff between the bargaining advantage that vertical integration provides and the loss in flexibility.

2.1 Model Setup

I model an industry that produces L variants of a final product (such as the automobile industry or the cell phone industry). Each final product variant is assembled from a set of individual *essential* components³ which I index by $i \in [0, 1]$. Each type i component is produced by a type i firm. Since there are different variants there are also multiple firms of each type, one per variant. Components are put together by an *assembler* - there is one assembler for each final product variant.

Each component can be either “perfect” or imperfect for the final product. The share of perfect components in a product, x , determines the value of the final product variant in the market. For the sake of simplicity I assume a simple linear specification and define the final revenue, S , as:

$$S = \pi x \tag{1}$$

In order to produce each firm has to make an investment of 1. Therefore, total profits for a final product variant with a share x of perfect components are $\pi x - 1$.

Firms differ in their ability to appropriate the available revenue S . I assume that each firm type i can appropriate a revenue of $r(i)S$ by investing where $r(i)$ is an increasing function that lies between 0 and 1. The remaining revenue $\tilde{S} = \int_0^1 [1 - r(i)] S$ is subject to negotiation between firms which I describe in greater detail below. The function $r(i)$ is the *specificity function* and captures the extent to which a firm’s claim to the revenue is subject to negotiation between firms. For example, the revenue of a firm with specificity $r(i) = 0$ fully depends on its ability to negotiate with other component suppliers. On the other hand, a firm with specificity $r(i) = 1$ does not have to rely on negotiation at all. I consider the specificity function as determined by production technology⁴. In section

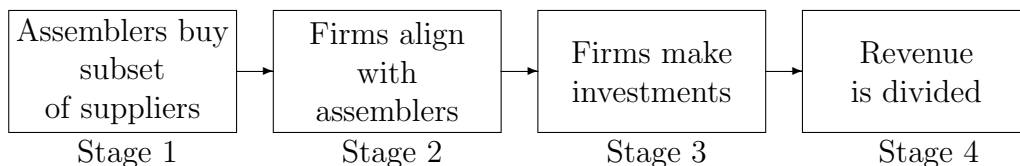
³A final product cannot be produced and/or sold without each and every component, hence components are essential. In other words, the production function is Leontief.

⁴Modeling investment specificity without reference to the existence of a second best -outside option- market is not novel. For instance [Acemoglu, Antràs, and Helpman \(2007\)](#) model contractual incompleteness by allowing suppliers’ activities to be only *partially* verifiable: the degree of verifiability of a supplier’s activities is a primitive in their model and determines her bargaining outcome.

Moreover, a large literature dating back at least to [Macaulay \(1963\)](#) attests that, when writing

4 I analyze how vertical integration decisions differ across industries with different specificity functions.

Figure 1: Timing for vertical integration game and negotiation over revenue



The remainder of the model describes precisely how firms divide the appropriate revenue \tilde{S} among themselves and how they can affect the distribution of revenue through vertical integration. Figure 1 lists the four main stages of the model.

Stage 1: Vertical Integration Game

In the first period each assembler can buy a subset $I \subset [0, 1]$ of component firms and form a *vertically integrated firm*. To simplify notation I will focus on the cases where assemblers use symmetric strategies such that each assembler buys the same portfolio of firms. I will later show that the assemblers will prefer to buy a contiguous set of firms $I = [0, N]$ where $0 \leq N \leq 1$ and $N = \int_I di$. However, the model does not restrict assemblers to such strategies

In equilibrium the assembler has to offer each firm the profit the firm would derive from remaining non-integrated.

Stage 2: Alignment Stage

In the second period non-integrated firms choose assemblers. I assume that each non-integrated firm of type i produces a “perfect” component for exactly one final product variant. For all other variants the firm’s component is non-perfect. This gives rise to a simple assignment of non-integrated firms to assemblers: each type i firm will choose its optimal variant.

breach of contract provisions, businessmen don’t actually believe they have a way out the contract other than the payment of damage fees. In other words, when dealing with complex transactions, managers normally don’t take it seriously the existence of an outside market to sell their specific components in case the original contract is breached; rather, in such cases they rely on the payment of damage fees at best.

I assume that each component produced by an integrated firm is perfect with probability $\gamma \leq 1$. The parameter γ is determined by technology and captures the loss of flexibility that vertical integration entails. The lower γ the more costly is vertical integration.

Integration by itself therefore induces a welfare loss because it bounds a portfolio of component suppliers together *before* uncertainty about a supplier's suitability for a particular final product variant is resolved. For example, a vertically integrated car manufacturer might discover that it needs to increase the share of fuel efficient cars in its lineup. However, because the company is integrated it might be forced to make use of gas-guzzling internally produced engines rather than source engines from suppliers of fuel-efficient engines⁵.

Stage 3: Investment Stage

In this stage all firms decide whether to invest one unit of capital. Production of a final product variant is Leontief - therefore it cannot take place unless all suppliers make the required investment.

Stage 4: Bargaining Stage

In the bargaining stage both integrated and non-integrated firms divide the appropriable revenue \tilde{S} :

$$\tilde{S} = \pi(1 - N + N\gamma) \int_0^1 [1 - r(i)] di \quad (2)$$

I assume that firms engage in Nash bargaining⁶. One problem with Nash bargaining in the presence of a vertically integrated firm is that integration tends to reduce bargaining power. To see the problem, consider simple bargaining over a pie of size 1 with three firms with equal bargaining power and outside option 0. Each firm will receive $\frac{1}{3}$ of the pie. Now assume that two firms integrate and bargain as a single entity with the third firm. Now the integrated firm will receive $\frac{1}{2}$ of the pie - therefore integration hurts a firm's bargaining power.

⁵Of course an assembler might well procure *both* internally and from outside at the same time, possibly at a higher cost. Here I focus on a more drastic in or out decision.

⁶Another widely used concept is the Shapley Value. I use Nash Bargaining because *a*) being a non cooperative solution concept it fits better the zero-sum opportunistic nature of the game studied here; and *b*) given the Leontief production function, the marginal contribution of each supplier in Shapley Value would be equal to the whole value of production and the resulting share would be trivially equal across suppliers.

I follow [Kalai \(1977\)](#) and assume that the integrated firm has Nash weight N^7 . This implies that integration *per se* does neither decrease nor increase a firm's bargaining power. Instead, in my model the division of the pie will be affected by the outside option of integrated and non-integrated firms.

I assume that a non-integrated firm has outside option 0⁸. The integrated firm, however, can replace any non-integrated firm with a *fringe supplier*. Fringe suppliers always produce imperfect components⁹. Therefore, if bargaining in the last period breaks down the integrated firm will bargain with fringe suppliers in an auxiliary bargaining round where the appropriable revenue is now¹⁰:

$$\tilde{S}^{\text{aux}} = \pi N \gamma \int_0^1 [1 - r(i)] di \quad (3)$$

In this auxiliary round the outside option of both integrated firm and fringe suppliers is zero.

2.2 Discussion

Intuitively, in my model vertical integration improves the bargaining power of integrating firms in the bargaining stage but prevents them from optimally mixing and matching component suppliers to maximize the share of perfect components when producing the final product variant.

One immediate implication of the model is that integration always decreases welfare even though integration can be privately optimal for the assembler. This is no longer necessarily true when investment is endogenous: if firms can vary the *amount* of their investment integration can improve investment incentives for the

⁷[Kalai \(1977\)](#) deals with group aggregation of players in multi-players Nash bargaining problems and, loosely speaking, suggests that a group be weighted by the number of its components. This implies that a group-player enjoys a share of the pie proportional to its size.

⁸One could find intuitive to set the outside option of a component supplier proportional to $r(i)$. However this contrasts with the discussion about breach of contract provisions and specificity modeling. See footnote 4 for more on this.

⁹One can think of such components as of refurbished components, such that they can be employed as substitutes for some missing parts but it would not be possible to assemble a final good out refurbished components only.

¹⁰The difference between \tilde{S} and \tilde{S}^{aux} is that $(1 - N + N\gamma)$ is replaced by $N\gamma$ because now, given the inferior quality of fringe suppliers' components, the only perfect components are the $N\gamma$ components produced by the integrated .

integrating firms because it provides a shield against expropriation at the bargaining stage. Under certain conditions the welfare gain through higher investment can offset the welfare loss from lower flexibility. I discuss this extension in section 3.

2.3 Analysis

I begin the analysis with the benchmark case of *linear specificity*, such that $r(i) = i$. This case is analytically particularly tractable. I will later generalize the specificity function.

2.3.1 Linear specificity

Under linear specificity the appropriable revenue becomes:

$$\tilde{S} = \pi(1 - N + N\gamma) \int_0^1 (1 - i) di \quad (4)$$

where N is the number of component firms bought by the assembler at stage 1. The appropriable revenue is maximized when there is no integration ($N = 0$) because integration always reduces productivity since integrated firms always produce a share of non-perfect components ($\gamma < 1$).

Without integration a firm of type i which invests one unit of capital obtains πi units of “private” revenue plus an equal¹¹ share of the appropriable revenue through Nash bargaining. Its profit can be written as

$$\pi \left[i + \int_0^1 (1 - j) dj \right] - 1 \quad (5)$$

which is positive for any firm as long as π is large enough. I assume that π is large enough to make production individually rational. It follows from (5) that firms with the least specific investments (high i) will be more profitable than firms with more specific investments because they are less exposed to hold-up.

I solve the model backwards by analyzing the bargaining stage where the assembler has integrated N firms. Assembler and non-integrated suppliers bargain

¹¹All the firms have zero outside option in this case. The share is 1 because there is a mass 1 of firms.

over the appropriable revenue \tilde{S} . Each non integrated supplier has an outside option of zero. The outside option of the assembler, on the other hand, is whatever can be obtained in the auxiliary bargaining round which occurs if assembler and suppliers cannot reach agreement in the first round. In this case the appropriable revenue is (3) and the integrated assembler is entitled to a share N of it, as prescribed by Nash bargaining *à la* Kalai. Thus, the outside option of the integrated assembler in the first round is $\pi N^2 \gamma \int_0^1 (1-i) di$.

Since the integrated assembler has a better outside option than the non integrated component firms, the assembler can secure a share of revenue in the first round, $F(N)$, which exceeds its share of production N . Formally, the assembler's share of appropriable revenue, $F(N)$, solves the following Nash maximization problem:

$$F(N) = \arg \max_s \left\{ \ln \left(s\pi(1-N+N\gamma) \int_0^1 (1-i) di - \pi N^2 \gamma \int_0^1 (1-i) di \right)^N + (1-N) \ln \left(\frac{1-s}{1-N} \pi(1-N+N\gamma) \int_0^1 (1-i) di \right) \right\} \quad (6)$$

The second term in this expression describes the division of revenue among the $1-N$ non-integrated firms, each one receiving an equal share of the revenue not assigned to the assembler, $1-s$. The revenue share of the assembler can be derived as:

$$F(N) = N + \frac{\gamma N^2 (1-N)}{1-N+\gamma N} \quad (7)$$

Clearly $F(N) > N$ and $F'(N) > 0$ for any $N \in [0, 1]$: there is a bargaining premium which comes with size. Also, $F(N) = 0$ for $N = 0$ and $F(N) = 1$ for $N = 1$. Finally, the share of revenue of the assembler does not depend at all on which suppliers he buys nor on the appropriable revenue generated by production, but solely on its size and on the inefficiency parameter γ : this is because the amount of investment is the same for all firm types.

To illustrate the power balances resulting from vertical integration, notice that the shares of revenue enjoyed by different firms can always be represented by means of a cumulative distribution function. Figure 2 plots the cumulative share distribution as a function of N , with the corresponding shares of two representative firms:

an ideal (or average) component of the integrated firm, j , and a non integrated firm, k . The s-shape curve indicates, for each N , the share of the corresponding vertical integration while the 45 degrees line is the share cumulative distribution under non integration. Under non integration each firm has a share of one. When there is a vertically integrated assembler, its share is $F(N) > N$: thus, the average share of an ideal component of the vertical integration, $F(N)/N$, is larger than one, while the share of a single supplier is lower than one.

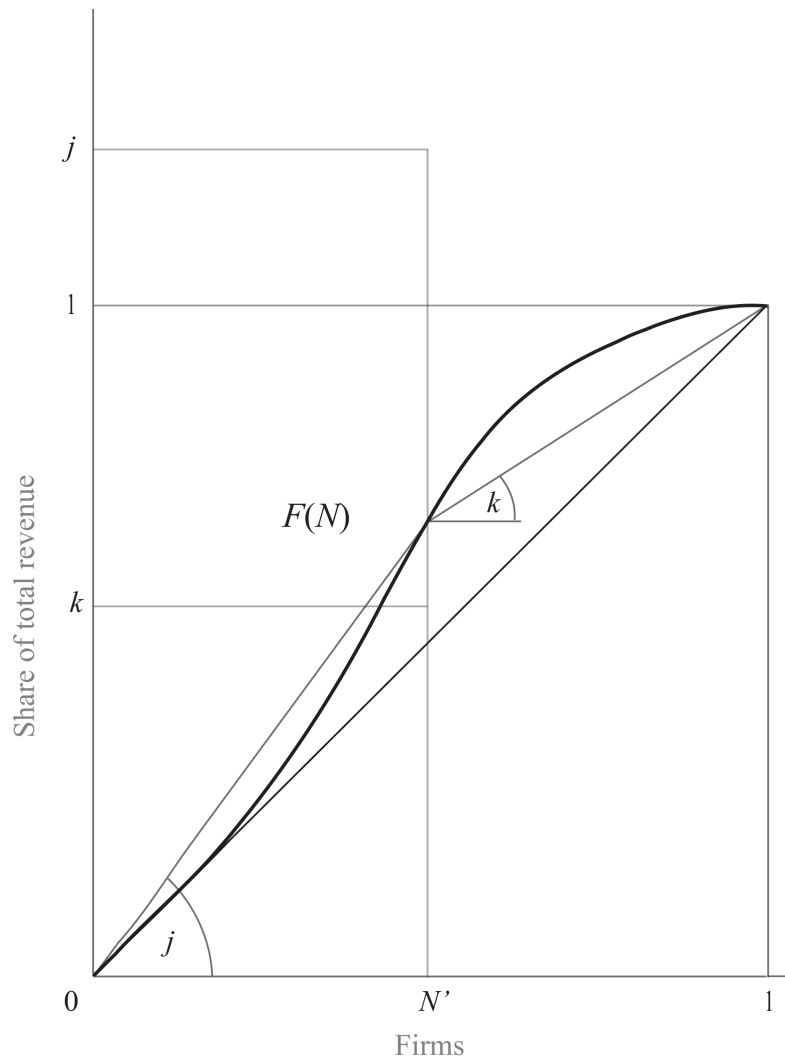


Figure 2: $F(N)$, Bargaining Power and Revenue Shares

I can now turn to the analysis of the assembler's problem at stage 1 (Vertical

Integration Game). If an assembler buys a subset $I \subset [0, 1]$ of firms with Lebesgue measure N and merges them into an integrated company, the assembler's profits are:

$$\pi(1 - N + \gamma N) \left[\int_{i \in I} i di + F(N) \int_0^1 (1 - j) dj \right] - N \quad (8)$$

The assembler's problem is twofold: he must choose how many firms to buy and which ones. The answer to the first question is simple: the assembler buys component firms until the marginal profit from buying an extra firm is equal to its cost. The cost of a firm is the profit the firm would derive from remaining non integrated, which is what it would obtain bargaining *ex-post* if it refused to be bought *ex-ante*. The condition is then:

$$\frac{\partial(8)}{\partial N} = \pi(1 - N + \gamma N) \left[i + \frac{1 - F(N)}{1 - N} \int_0^1 (1 - j) dj \right] - 1 \quad (9)$$

where the left hand side is self explaining while the right hand side is the cost of the marginal firm i bought by the assembler, which is (5) taking into account that the share of a single supplier decreases from 1 to $(1 - F(N))/(1 - N) < 1$ when there is an N -size integrated assembler.

The answer to the second question is intuitive: an assembler should start buying firms from the one with the highest specificity, that is the one with the least "private" revenue. In fact, buying a firm has two consequences. First, it improves the outside option of the vertical integration. This effect is independent of the type of firm bought. Second, it reduces productivity (which is $\pi(1 - N + N\gamma)$ with $\gamma < 1$): this effect has a different impact on the assembler's revenue depending on the firms he has already bought. By first buying firms with the least "private" investment the assembler minimizes the expected efficiency loss from buying further firms. In terms of condition (9), buying first high specificity firms maximizes the difference between right and left hand sides, *i.e.* it maximizes the marginal profit of the assembler net of the marginal cost.

Hence, if there is an optimal degree of vertical integration N^* , then the assembler optimally buys a portfolio $I = [0, N^*]$ of firms and produces internally the N^* most specific components.

The problem of the assembler is then:

$$\begin{aligned} \frac{\partial}{\partial N} \left(\pi(1 - N + \gamma N) \left[\int_0^N idi + F(N) \int_0^1 (1 - j) dj \right] - N \right) = \\ = \pi(1 - N + \gamma N) \left[i + \frac{1 - F(N)}{1 - N} \int_0^1 (1 - j) dj \right] - 1 \end{aligned} \quad (10)$$

where the left hand side is the derivative of (8) with respect to N with the first integral taken over the set $I = [0, N]$ and $F(N)$ is as in (7). The right hand side is (5) for the marginal firm $i=N$ taking into account the decrease in the share of appropriable revenue.

The following proposition holds:

Proposition 1 *For γ sufficiently large, there exists an optimal number of firms, N^* , which solves the problem of the assembler. N^* is a well defined positive number in the interval $(0,1]$.*

I have established that there exists an optimal degree of integration as long as integration does not cause too much inefficiency. In particular, it is optimal for firms with a highly specific investment to become part of an integrated company. In fact, such firms are the ones which suffer the most from expropriation in the bargaining game. By the same token these firms are less affected by efficiency losses, because, by contributing more to the appropriable revenue, they also split most of the loss with the firms they bargain with. Therefore, the assembler has a stronger incentive to integrate high specificity firms than low specificity ones. In this way he realizes the gains from power at the minimum efficiency cost.

In this base version of the model the optimal degree of integration solely depends on the inefficiency of the assembler, γ : this is because I have constrained the specificity function to a very simple specification. In the next section I introduce a two parameters specificity function which allows for different specificity patterns among the firms of a production process. This allows to study how the technological characteristics of an industry affect its integration structure.

For now, however, it is useful to notice the following:

Proposition 2 *An increase in the inefficiency of large organizations (a decrease of γ) implies that the degree of vertical integration decreases.*

2.3.2 General specificity functions

I next consider the following two-parameter family of specificity functions:

$$r(i) = \alpha + (1 - \alpha)i^\beta, \quad 0 \leq \alpha < 1, \beta > 0 \quad (11)$$

Figure 3 offers a graphical representation of this more general case.

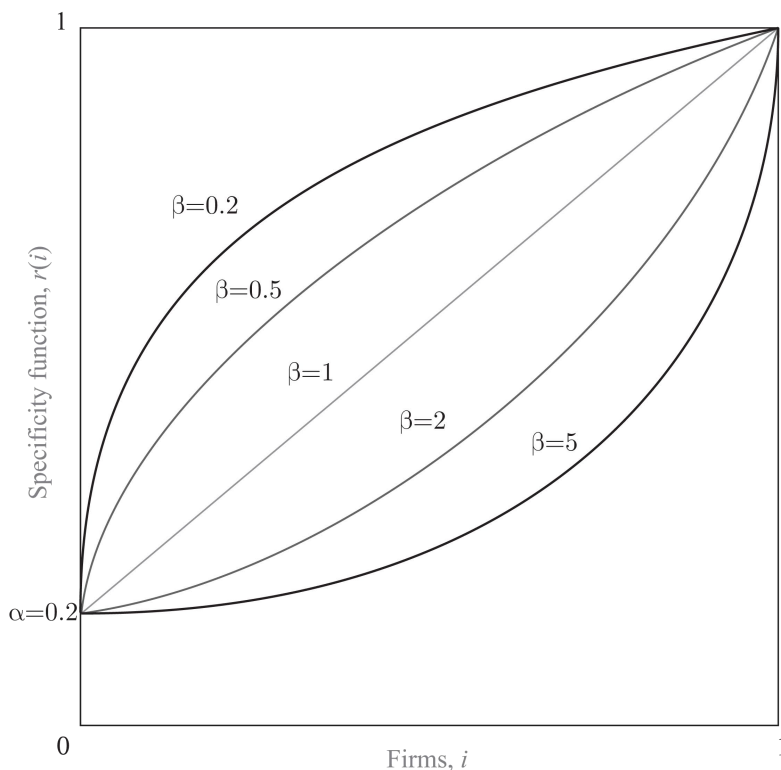


Figure 3: General specificity functions: $\alpha = 0.2$ and $\beta = 0.2; 0.5; 1; 2; 5$

The two parameters α and β summarize the state of technology in an industry and have intuitive interpretations. The parameter α captures the *average* specificity of an industry while β captures differences in the *distribution* of types.

An industry with larger α uses a production technology that is, on average, less specific. One would expect that as an industry matures, α increases. Intuitively, we would expect that lower average specificity makes integration less profitable because the hold-up problem is reduced. The next proposition confirms this intuition:

Proposition 3 *Industries with lower average specificity (higher α) are less integrated.*

The parameter β governs the shape of the distribution of specificity across firm types. When $\beta = 1$ the specificity function is uniformly distributed between α and 1. An increase in β corresponds to relatively more firms having high versus low specificity. Intuitively, this correlates with a more complex production process. Conversely, a decrease in β implies that fewer firms make highly specific investments. Intuitively, we expect that sophisticated industries with complex production processes are most prone to holdup. The next proposition confirms this conjecture:

Proposition 4 *If production becomes more complex (β increases) vertical integration increases.*

Finally, it can be easily shown that Proposition 2 holds in the extended model.¹²

3 Endogenous Investment

In the model introduced in the previous section integration is privately optimal but socially wasteful. The total industry profit of both integrated and non-integrated firms is greater under non-integration compared to integration, $\pi - 1 > \pi(1 - N^* + \gamma N^*) - 1$. This is because integrated firms are less flexible and produce more imperfect components. Integration *per se* only allows the assembler to capture a disproportionate share of the appropriable revenue and hence is equivalent to a welfare-neutral redistribution.

However, by fixing the level of investment at 1 for all firms my model shuts down one potentially important channel through which integration might actually increase welfare. Intuitively, integrated firms can shield their investment better from expropriation. Since integrated firms make the most specific investments

¹²It suffices to show that the derivative of (16) with respect to γ is positive. Indeed it is:

$$\frac{\partial(16)}{\partial\gamma} = \frac{N\left(N^\beta(1-\alpha) + \alpha + \beta(1+2(1-\alpha)(1-2N+N^2))\right)}{(1-\alpha)\beta(1-(1-\gamma)N)^2} > 0 \quad \forall N > 0$$

within an industry this might increase the willingness of the integrated company to invest in production.

The social planner might therefore allow integration because it may give rise to a socially preferable second-best equilibrium.

I endogenize the level of investment using a highly simplified version of the basic model:

1. There are two types of firms only: half have completely specific investment ($r = 0$), half have not at all specific investment ($r = 1$)
2. Firms can invest either 1 or 2 units of capital.
3. The productivity of investment is π_1 (or $\pi_1(1 - N + \gamma N)$) if a firm invests 1, while it is π_2 (or $\pi_2(1 - N + \gamma N)$) if it invests 2, with $\pi_1 > \pi_2 > 1$
4. The productivities are sufficiently large but not too close one another (in particular $\frac{2}{\gamma} < \pi_1 < \frac{7}{3}$ and $\frac{\pi_1}{2} + \frac{1}{2\gamma} < \pi_2 < \frac{\pi_1}{2} + 1$)

Under the above assumptions the following holds:

Proposition 5 *If γ is close enough to 1, then the following statements are true:*

1. *Under non integration high specificity firms invest 1 and low specificity firms invest 2*
2. *The equilibrium is characterized by full integration ($N^* = 1$)*
3. *In equilibrium the assembler invests 2 in all the divisions of the vertically integrated firm*
4. *The total profit of the industry is greater in the integrated equilibrium than under non integration*

The proposition above demonstrates that integration is not necessarily detrimental to welfare. In fact, by providing a protection against revenue expropriation, integration can provide greater incentives to invest. In some cases, as shown in the proposition, this is sufficient to overcome the efficiency loss caused by integration and, consequently, integration can enhance welfare.

4 Applications and predictions of the model

Vertical integration has generally diminished over the last decades (Rajan and Wulf (2006), Brynjolfsson, Malone, Gurbaxani, and Kambil (1994)). Companies have increasingly outsourced activities that were previously carried out inside the firm (Spencer (2005), Hummels, Ishii, and Yi (2001)). Firms have gained flexibility by purchasing intermediate components on the market rather than producing them inside the firm. These trends were accompanied by a deepening of financial markets and rapid growth in some developing countries.

In the following I use the model developed in section 2.3.2 to provide a unified interpretation of a variety of inter-related phenomena.

Applied research and process innovation. For a given industry, advances in applied research and process innovation improve the efficiency and efficacy of *existing* technologies. This kind of innovation tend to give physical capital generally more flexible production capabilities: technological advances have made machinery both more responsive to production timing needs and market demand rhythms as well as normally able to produce less standardized goods with a comparable amount of invested capital. As an example, consider the diffusion of robotics and just-in-time plant management techniques: Nemetz and Fry (1988) point out that such flexible manufacturing technologies favor organizational forms with a narrow span of control, a lower number of vertical layers and a more decentralized decision making process as compared to mass production technology organizations. In the context of my model these changes are equivalent to an increase of α , the general state of technology. In fact, such advancements reduce the costs associated to a certain technology and slowly help it spread throughout the economy, making it increasingly standard. This, in turn, implies that investment in such technology becomes less specific. In addition, it is likely that, if any, applied research decreases β : in fact, those production stages which are less complex and tend to be less specific are more likely to become standard first. Both these effects imply a decrease of the optimal degree of vertical integration. This explains some important aspects of the general trend in industries like automobiles: in such industries the introduction of more flexible production techniques has made it convenient the outsourcing of many activities to specialized firms. These in fact are now able to

provide different products for different customers with a comparable amount of investment.

Human capital. It is well known that today's economies are characterized by a high and increasing level of human capital¹³. As modern economies move toward services and knowledge intensive sectors, human capital has gained importance as arguably the major factor of production. Now, human capital is by nature much more flexible than physical capital and, to a great extent, it is non relation-specific. In fact, if it is true that it is probably difficult for a nuclear physicist to become a financial broker overnight, the personal histories of many businessmen, professionals and scientists demonstrate how easily human capital transfers across and within single firms and sectors of the economy. Therefore investments in human capital intensive technologies tend to be less specific to the relationship and to generate less quasi rents than a investments in physical capital. In terms of the model a generalized increase of the ratio of human to physical capital is equivalent to a rise in α as it touches, at least to some extent, most industries and most production stages of a given industry. This leads to a decrease of the optimal degree of vertical integration. However, it is not clear how the rise in relative importance of human vs. physical capital affects β making it hard to say what the final effect is. [MacDonald \(1985\)](#) finds that the use of vertical integration is more prevalent in capital intensive industries while [Hortaçsu and Syverson \(2007\)](#) document that, within an industry, vertically integrated firms have a higher capital-to-labor ratio than non integrated firms. These findings, which appear to be robust, suggest that more human capital as compared to physical capital leads to less integration.

Financial markets. As pointed out by [Rajan and Zingales \(2001\)](#), another reason for why physical capital is today less crucial than in the past is the huge development of financial markets in the last decades which has made it much less of a constraint the acquisition of machineries, the building of new plants and the investment in equipment in general. In fact, various authors ([Rajan and Zingales \(2001\)](#); [Acemoglu, Johnson, and Mitton \(2005\)](#)) have studied the relationship be-

¹³As Gary S. Becker puts it, "Human capital is increasingly important in modern economies. Skills and knowledge are highly valuable in more high-tech economies" (from a public conference in Milan, the 22nd of June 1998). More concretely [Berman, Bound, and Machin \(1998\)](#) provide evidence on the rise of skilled labor demand and wages worldwide in the past decades.

tween vertical integration and the development of financial markets. The argument behind such studies is that more efficient financial markets tend to reduce the hold up problem and, *a fortiori*, the degree of vertical integration. The mere existence of efficient credit markets -the argument goes- makes hold up threats less credible because they provide entrepreneurs with more easily accessible outside options. For instance, if a partner threatens to withdraw from production, an efficient credit market might well mean that the threatened partner is able -or at least have more chances- to buy and/or build the machineries to internally produce the missing component. In other words, “with capital easy to come by, alienable assets such as plant and equipment have become less unique” (Rajan and Zingales (2001)). This means that physical investment is less relation-specific as a consequence of more efficient financial markets. Ideally, with perfect financial markets, no hold up may be based on any asset which could be possibly borrowed. In terms of the model the development of financial markets can be formalized as an increase of α . Moreover, it is likely that efficient credit markets are worth most to firms producing complex products which have very specific investments. Thus, if any, β should decrease as financial markets become more efficient. Which means that the degree of vertical integration tends to diminish as a consequence of financial development.

Industries comparison. The model predicts that complex products requiring high-tech and sophisticated machinery are produced in industries whose structure tends to be more vertically integrated. In fact, one can interpret the parameter β as a proxy for technology intensity or complexity. A sophisticated or complex final product involves a relatively large number of sophisticated intermediate products which require investment in highly specific machinery: this corresponds to a high β (see Figure 3). A relatively standard product, on the contrary, involves relatively fewer complex stages of production, with less specific investments: this corresponds to a small β . Thus, technology intensive products will be produced by more integrated industries than standard products. Evidence of the above is very neatly provided by Novak and Eppinger (2001) who explicitly test the hypothesis that product complexity and vertical integration are complements. More evidence has been provided by Wilson (1977) who finds that licensing is more attractive the less complex the good involved is, and by Kogut and Zander (1993) whose results show that the probability of internalization is lower the more codifiable, teachable,

and the less complex the technology is.

North-South differences. As a consequence of the previous point, poor countries, producing less technology intensive products than rich countries, tend to have a lower number of vertical conglomerates. This is consistent with the observation that the economies of poor countries are generally characterized by household-style, non integrated firms.

USA vs. Japanese *keiretsu*. Various authors ([Spencer and Qiu \(2001\)](#); [Qiu and Spencer \(2002\)](#)) have studied the case of the Japanese *keiretsu* and the reluctance of such conglomerates to import auto parts from abroad. They stress the fact that the Japanese import only parts of limited technological content, such as seat covers. The model presented in this paper predicts this phenomenon. In fact, the model gives a rationale for why the most complex and technology intensive parts of a production process should be produced internally. Those parts, in fact, are the ones which require the most specific investments and which therefore will be carried out by the vertical integration. Which, in the context of the Japanese automobile industry, is the *keiretsu* itself. Only the less sophisticated parts are produced by non integrated firms and therefore can be imported.

Trade patterns. Some authors ([Vernon \(1966\)](#); [Antràs \(2005\)](#)) have explained the pattern which leads firms to produce certain parts first internally, then in regime of FDI and finally outsourcing them to less developed countries. In particular, [Antràs \(2005\)](#) gives a Property Rights interpretation of such pattern. The model presented here is able to explain the pattern quite naturally by assuming that *a*) there is a (labor) cost advantage of poor countries (P); that *b*) rich countries (R) have a technological supremacy which reduces over time as poor countries develop; and that *c*) there is technological progress¹⁴ and product standardization over time. Suppose there are four periods.

$t=0$ The product is invented in R, P has no skills and cannot produce. All production is in R and, given technology, the optimal degree of vertical integration is *VI*. The rest is produced in regime of domestic outsourcing

¹⁴In particular applied research and process innovation may reduce the importance of high tech, critical inputs with the age or maturity of the final good as in [Antràs \(2005\)](#).

t=1 P develops, it becomes able to produce some parts: the firm in R buys some standard pieces from P in foreign outsourcing regime

t=2 P develops further, it can produce more complex parts which are critical to the firm: these are produced in regime of FDI with the firm operating directly in P

t=3 Technology evolves: (some of) the parts produced in FDI are sufficiently standard -no longer critical- to be bough outside the firm, hence the move from FDI to outsourcing

The pattern just described is only a plausible example but many other patterns could arise. It is depicted in Figure 4 where the horizontal lines represent the spectrum of firms aligned from the most specific to the least, with the vertical integration in bold.

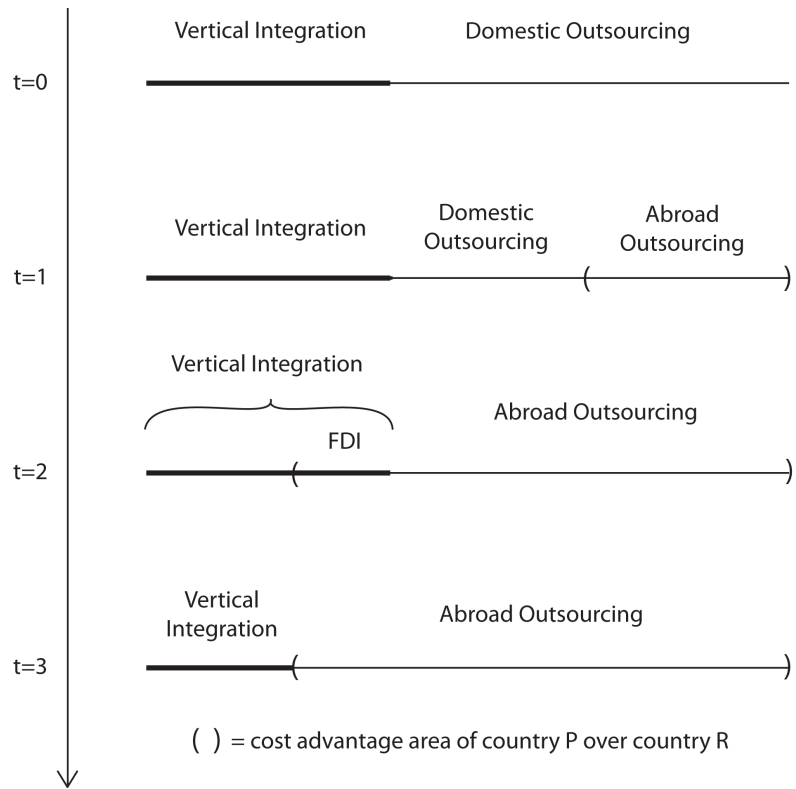


Figure 4: FDI and outsourcing

Basic research. From the previous points it may seem that technological progress will eventually undo vertical integration. Even though such perspective is intriguing, vertically integrated firms are likely to endure. In fact, scientific progress and basic research keep “creating” new products which, at least at the beginning of their life cycle, are usually complex and non standard (robotic machines, hydrogen cars, new pharmaceuticals, etc.). Such products normally require very specific investments before the technologies involved become standard and the industry mature. This means that, as long as new discoveries are made, there will still be a “demand for” vertical integration.

5 Conclusion

I have presented a theory of vertical integration which explains why and to what extent firms in an industry become vertically integrated. The perspective of this paper emphasizes the bargaining problem associated to the vertical integration decision rather than the incentives to invest. As such, it is not mutually exclusive with the prevalent Grossman-Hart-Moore perspective but rather complementary.

My model predicts that integration is the privately optimal response of a subset of firms to their investments being appropriated by other firms with less specific investments. Integration is viewed here as a means to gain bargaining power with respect to non integrated firms. The context is a relationship between an assembler and several suppliers which concur to the production of a final good and which expropriate each other’s revenue because their investments are specific to the relationship. Integration is the *ex-ante* optimal response of an assembler who has to decide how much production to do in-house. He optimally integrates the firms more exposed to expropriation in the *ex-post* bargaining problem: by merging them into a larger organization he gains bargaining power and enjoys a disproportionately larger share of the total appropriable revenue. This happens at the cost of foregone flexibility: the assembler is no longer able to choose the best suppliers for its final good.

This paper views vertical integration as an optimal economizing strategy in the presence of asymmetric exposure to expropriation: firms which are more exposed to expropriation benefit the most from the increased power provided by integration.

As Williamson pointed out, there is, in the real world, a continuum of specificity degrees: as the degree of specificity increases, the market becomes less and less feasible for some firms and integration becomes a better response to the hold-up problem. Coherent with a view which regards the market as the optimum, the model displays a welfare loss in any equilibrium which involves integration.

However, the paper also considers the effect of integration on incentives and demonstrates that there are conditions under which integration is indeed welfare improving. In fact, by providing a shield against expropriation, integration improves the incentive to invest of the assembler. In an extension of the model where firms are allowed to vary the size of their investment it is shown that, under certain conditions, the shield provided by integration leads to more investment and to a higher welfare.

The paper also illustrates in which sense the technological development of the several past decades might have impacted the organizational structure of firms causing a wave of externalizations and a surge in outsourcing strategies. I argue that process innovation has made capital less specific over time reducing the average degree of specificity of a given industry. This has made integration less valuable over time determining the observed pattern of organizational structures.

Finally, the model offers a key to interpret the different propensity to vertically integrate in different industries. I argue that when a majority of the stages of a production process involve highly specific investments then it will be observed a more vertically integrated organization of production and *vice versa*.

The model has a number of testable implications and provides an interesting opportunity for the empirical researcher to further investigate the relationship between specific investment, technology and vertical integration.

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APPENDIX

Proof of Proposition 1: rearranging terms, (10) becomes:

$$F'(N) - \frac{1 - F(N)}{1 - N} - \frac{1 - \gamma}{1 - N + \gamma N} \left(F(N) + \frac{\int_0^N i di}{\int_0^1 (1 - j) dj} \right) = 0$$

In studying the above expression I will always assume $\gamma > 0$: technically, this makes all the involved functions continuous for each N between 0 and 1; economically, it means that efficiency is not completely lost when the assembler integrates firms (if $\gamma = 0$ a vertical integration cannot produce any perfect component).

The condition above, once all terms are substituted, simplifies to:

$$N(-1 + \gamma(3 - N) - N) = 0 \tag{12}$$

The LHS equals zero at $N = 0$ and $-2(1 - \gamma) \leq 0$ at $N = 1$. Its first derivative is:

$$-1 + \gamma(3 - 2N) - 2N \tag{13}$$

which equals $3\gamma - 1$ at $N = 0$. Therefore LHS(12) is increasing at the origin as long as $\gamma > 1/3$. Finally, the second derivative of LHS(12) is:

$$-2(1 + \gamma) < 0 \tag{14}$$

The considerations above imply that, for $\gamma > 1/3$, it exists $N^* \in (0, 1]$ such that, for $N < N^*$ LHS(12) is positive, while for $N > N^*$ it is negative. Therefore N^* is a solution to the problem of the assembler. In fact, it is the only equilibrium because at the corner solution $N = 0$ the marginal revenue of buying a firm is positive. Therefore in this benchmark case the problem of the assembler (12) has a simple closed form solution:

$$N^* = \frac{3\gamma - 1}{1 + \gamma} \in (0, 1] \quad \text{iff} \quad \gamma \in \left(\frac{1}{3}, 1 \right] \tag{15}$$

Q.E.D.

Proof of Proposition 2: follows immediately from the expression for N^* . In fact, its derivative with respect to γ is $4/(1 + \gamma)^2$.

Proof of Proposition 3:

Part 1: we first have to prove Proposition 1 in the new context. I'll follow the same steps as in the proof of 1. Remember that $\alpha \in [0, 1)$, $\beta > 0$ and, again, $\gamma > 0$. The condition for N^* is now:

$$N(-1 + \gamma(3 - N) - N) + \frac{(1 - \gamma)N(\beta N - N^\beta - \alpha(1 + \beta + \beta N - N^\beta))}{(1 - \alpha)\beta} = 0 \quad (16)$$

The LHS of (16) equals 0 at $N = 0$ and $-(1 + \beta)(1 - \gamma)/(1 - \alpha)\beta \leq 0$ at $N = 1$. Its first derivative with respect to N is:

$$\frac{-1}{(1 - \alpha)\beta} \left[N^\beta(1 - \alpha)(1 + \beta)(1 - \gamma) + 2\beta\gamma(1 - \alpha)(2N - 1) + (\alpha + \beta)(1 - \gamma) \right] \quad (17)$$

The values of (17) at $N = 0$ and $N = 1$ are:

$$(17)|_{N=0} = \frac{-1}{(1 - \alpha)\beta} \left[\beta + \alpha(1 - \gamma) + \beta\gamma(2\alpha - 3) \right] > 0 \text{ iff } \gamma > \frac{\alpha + \beta}{\alpha + \beta(3 - 2\alpha)}$$

$$(17)|_{N=1} = \frac{-1}{(1 - \alpha)\beta} \left[1 - \gamma + \beta(2 - \alpha(1 + \gamma)) \right] < 0$$

Therefore the LHS of (16) is increasing at the origin as long as γ is sufficiently large. Finally, the second derivative of LHS(16) is:

$$- \frac{N^\beta(1 + \beta)(1 - \gamma) + 4\gamma}{N} < 0 \quad (18)$$

The considerations above prove that LHS(16), as LHS(12) in Proposition 1, is a concave function which starts out at zero and increases above zero if γ is sufficiently large; then it decreases below zero as N approaches one. At some point, it takes the value of zero which proves the existence of an equilibrium. As in Proposition 1 the corner solution $N = 0$ is not an equilibrium as the marginal revenue of buying a firm is positive.

Part 2: we now have to show that an increase of α implies a decrease of N^* . To this end, it suffices to show that the LHS of (16) is decreasing in $\alpha \forall N \in [0, 1]$, provided that the condition on γ , $\gamma > (\alpha + \beta)/(\beta(3 - 2\alpha) + \alpha)$, remains satisfied. In fact, the derivative of LHS(16) with respect to α is:

$$\frac{\partial \text{LHS}(16)}{\partial \alpha} = -\frac{(1 + \beta)(1 - \gamma)N}{(1 - \alpha)^2 \beta (1 - (1 - \gamma)N)} < 0$$

which proves Proposition 3. *Q.E.D.*

Proof of Proposition 4: we have to show that an increase of β implies an increase of N^* . To this end, it suffices to show that the LHS of (16) is increasing in β , provided that the condition on γ , $\gamma > (\alpha + \beta)/(\beta(3 - 2\alpha) + \alpha)$, remains satisfied. In fact, the derivative of LHS(16) with respect to β is:

$$\frac{\partial (16)}{\partial \beta} = \frac{(1 - \gamma)N \left(\alpha + N^\beta (1 - \alpha) (1 - \beta \ln(N)) \right)}{(1 - \alpha) \beta^2 (1 - (1 - \gamma)N)} > 0$$

which proves Proposition 4. *Q.E.D.*

Proof of Proposition 5: first, it is easy to check that condition 4 guarantees result 1. It also guarantees that, if an assembler purchases any number of suppliers and integrates them, single firms supplying the assembler invest 1 (2) if they are of the high (low) specificity type -*i.e.* they stick with their non integration investment decisions. For the rest, the proof follows the same lines as the proof of previous propositions, with some *caveats*.

As before, the assembler will buy firms as long as the marginal revenue from buying one more firm is equal to the cost of the next firm to buy. The cost of a firm is unchanged and, in particular, because of conditions 1 and 2, it does not vary within types as the assembler integrates suppliers -other than, of course, for the efficiency loss γ . The marginal revenue of the assembler, however, varies depending on the types of firms he buys as well as on how much he invests in each new integrated division. Notice that, because the production function is linear, the assembler will invest the same in all of its integrated divisions of a given type.

It is not difficult to show that, for γ sufficiently close to 1, the marginal profit

from integrating any firm is positive irrespective of how many firms the assembler has already bought, of their type and of how much he invests in them after integration. In particular, comparing the marginal profits, it can be shown that: the assembler first integrates high specificity firms and invests 1 in each integrated division; then he keeps buying high specificity firms but invests 2 in all his high specificity integrated divisions; after having bought all high specificity firms he starts buying low specificity firms and invests 2 in both types of integrated divisions until he has bought all the firms.

Finally, it is straightforward to check that the total profit is higher under full integration with the assembler investing 2 in all the integrated divisions than under non integration. In fact, under integration the total profit is $2\pi_2\gamma - 2$, while under non integration it is $\frac{1}{2}(\frac{\pi_1}{2} - 1) + \frac{1}{2}(2\pi_2 - 2 + \frac{\pi_1}{2}) = \pi_2 + \frac{\pi_1}{2} - \frac{3}{2}$. The difference is $(2\gamma - 1)\pi_2 - \frac{\pi_1}{2} - \frac{1}{2}$, which is clearly positive under the above conditions. *Q.E.D.*