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Abstract

With quarterly data of a sample period starting from 1973, the conventional unit root tests reject the null of nonstationarity in favor of the alternative of linear stationarity for short-term real interest rates (RIRs) of non-European industrial countries. There is evidence of nonlinearities in many European countries' RIRs, most of which appear to be stationary exponential smooth transition autoregressive processes.

Keywords: Real interest rates; Unit root; Nonlinear stationarity

JEL classifications: C22; E4; G1

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1. Introduction

Real interest rate plays a crucial role in investment, savings and almost all intertemporal decisions. Its statistical characterization has become an important issue in macroeconomic studies. The well-known Fisher hypothesis predicts that the long-run equilibrium value of real interest rate stays around a constant and thus for the rate to behave stationarily. The Fisher hypothesis has been widely accepted and real interest rate is usually assumed to be stationary in the studies using the capital asset pricing model or econometric methodologies such as the generalized moments methods. However, empirical investigations on the time series properties of real interest rates display mixed findings (see Rapach and Weber, 2004, for a further discussion). Empirical studies using conventional unit root tests and/or cointegration tests based on linear models often found a nonstationary real interest rate and/or the rejection of the Fisher hypothesis (see, for examples, Rose, 1988, Mishkin, 1992, and Koustas and Serletis, 1999).

In a few recent studies (e.g., Kapetanios et al., 2003; Haug and Siklos, 2006), it has been argued that some short-term interest rates or real interest rates may exhibit nonlinear mean reverting tendency. If short-term real interest rates follow nonlinear stationary processes, the alternative hypothesis of linear stationarity in the conventional unit root tests would not be suitable. There are a number of explanations for the possible existence of non-linearity in real interest rates (RIRs). RIR is measured by the difference between nominal interest rate, a financial market variable, and inflation rate, a goods market variable. Transaction costs existing in these markets may differ in one another and may change over time. In addition, international interest rate parity conditions and the growing integration of capital markets may closer link between interest rates across countries. There are also several countries which have adopted inflation targeting policy. All these may generate friction in the relation between a country's short-term interest rate and inflation rate, resulting in nonlinearities in adjustment toward the equilibrium relation between the two rates. As pointed out by Kapetanios, Shin, and Snell (2003,

hereafter, KSS) “owing to transaction costs and other frictions, it is quite plausible that the more these variables...”, such as inflation and interest rates, “...deviate from their equilibrium values, the larger will be the investment/arbitrage adjustment flows that drive them back again (p. 369).”

KSS (2003) have developed a testing procedure for the null hypothesis of nonstationarity against an alternative of nonlinear stationary smooth transition autoregressive (STAR) process. They have applied their tests to RIRs of 11 industrial countries and illustrated that their tests are more powerful than the conventional unit root tests in revealing the time series behavior of variables for series that may revert to their mean nonlinearly.

The present paper intends to contribute to research in this area in three aspects. First, we extend the work of KSS (2003) to the real interest rates of 22 industrial countries for a more comprehensive study of time series behavior of industrial countries' RIRs. Second, in addition to examining the stationarity of RIRs, an effort is made on detecting the presence of nonlinearity in RIRs as well as on specifying which model would be more appropriate in presenting the nonlinearity of RIRs. Third, the study also shows the difference in time series properties of RIRs of European countries from those of non-European industrial countries. Because of faster growing integration of goods and capital markets among European countries, there are closer links among interest rates and also among inflation rates of these countries than those of other industrial areas. More friction may exist in adjustment toward equilibrium relation between inflation and short-term interest rates within a European country when these rates are closely connected to those of other European countries. It is thus expected that RIRs of European countries are more likely to behave nonlinearly.

2. Methodological issues

Consider a smooth transition autoregressive (STAR) model of order L (see Granger and Terasvirta, 1993):

$$x_t = \alpha_{10} + \alpha_1' z_t + (\alpha_{20} + \alpha_2' z_t) \Phi(x_{t-d}) + v_t \quad (1)$$

where $z_t = (x_{t-1}, \dots, x_{t-L})'$, $\alpha_j' = (\alpha_{j1}, \dots, \alpha_{jL})'$, $j = 1, 2$, $d \geq 1$ is the delay parameter, v_t is an i.i.d. error with zero mean and constant variance, and $\Phi(x_{t-d})$ is a transition function. The exponential transition (ESTAR) function may have the following form:

$$\Phi(x_{t-d}) = 1 - \exp[-\theta(x_{t-d} - c)^2] \quad (2)$$

where $\theta > 0$ and $\Phi(x_{t-d})$ is symmetrically U-shaped around c . The alternative logistic transition (LSTAR) function has the form below:

$$\Phi(x_{t-d}) = \{1 + \exp[-\theta(x_{t-d} - c)]\}^{-1} \quad (3)$$

which is asymmetrically S-shaped around c . Both ESTAR and LSTAR models assume that the speed of adjustment of x_t varies with the extent of deviation from equilibrium. The main difference between the two is that ESTAR assumes a symmetric response of x_t to either positive or negative shocks, while LSTAR implies an asymmetric response of x_t to shocks of different signs.

If a real interest rate appears to revert to its mean nonlinearly, the issue of whether it responds to shocks of opposite signs symmetrically or asymmetrically would be worthy of empirical examination. The issue will be addressed in the present study by testing for an appropriate model, either ESTAR or LSTAR, to describe the behavior of real interest rates.

3. Data and empirical testing procedures

The nominal short-term interest rates adopted for the study are 3-month Treasury bill rates or money market rates. For many industrial countries, these interest rates were not available until the late 1960s or 1970s. In order to include more countries in the study and to have test results comparable across countries for a similar period, the sample period of the present study starts from 1973. Quarterly data used in the study are obtained from the International Financial

Statistics (IFS) online for 22 industrial countries, namely Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the U.K., and the U.S. The 3-month Treasury bill rates (*IFS* line 60c) are collected for Belgium, Canada, France, Greece, Japan, New Zealand, Sweden, the U.K., and the U.S. For the other 13 countries, the money market rates (*IFS* line 60b) are employed when their short-term T-bill rates are either not available or only available for a shorter sample period than that for the money market rates. In line with earlier studies in existing literature, the annualized quarterly interest rate is transformed into a compounded rate. The quarterly inflation rate is the annual growth rate of the consumer price index (*IFS* line 64). The short-term real interest rate at time t is calculated by subtracting the inflation rate ahead of period t from the compounded interest rate at period t . The sample period of the data for each country is reported in Table 1. The period runs from the first quarter of 1973 to the first quarter of 2007, or is shorter due to lack of data availability.

The study is conducted first by applying the augmented Dickey–Fuller (ADF) test, a conventional unit root test, to the real interest rates of 22 industrial countries. The ADF test is carried out with the number of augmentations k to be selected using a sequential testing procedure based on the significance of augmentation terms (i.e., insignificant terms are excluded). The maximum number of k is set to be 8 for using quarterly data. The test statistics are denoted as ADF(Sig). To check the sensitivity of the test results to the number of augmentations, the ADF test is also conducted with k to be selected on the basis of the Schwarz criterion (SC) and the Ljung-Box statistic for serial correlation among the residuals.¹ The corresponding test statistics are denoted as ADF(SC).

¹ The Schwarz criterion (SC) tends to select a model with shorter lag length. If the Ljung-Box statistic indicates serial correlation in the residuals of a model selected by SC, the model would be augmented with additional lag terms until the Ljung-Box statistic shows no significant serial correlation in the residuals.

For the RIRs in which ADF tests show no evidence of linear stationarity, the presence of nonlinearity in RIR is tested by a three-step procedure following the suggestion of Terasvirta (1994). First, a linear autoregressive (AR) model for RIR is specified. The lag length L of the model is selected based on the Schwarz criterion and the Ljung-Box statistic for serial correlation. The residuals of the AR model are saved from the chosen model and denoted as u . Second, we test for the presence of nonlinearity with different values of d , the delay parameter, through the estimation of the following model:

$$u_t = \beta_0 + \sum_{j=1}^L \beta_{1j} R_{t-j} + \sum_{j=1}^L \beta_{2j} R_{t-j} R_{t-d} + \sum_{j=1}^L \beta_{3j} R_{t-j} R_{t-d}^2 + \sum_{j=1}^L \beta_{4j} R_{t-j} R_{t-d}^3 + v_t \quad (3)$$

where R_t is the real interest rate. Define $\gamma_2 = (\beta_{21}, \beta_{22}, \dots, \beta_{2L})$, $\gamma_3 = (\beta_{31}, \beta_{32}, \dots, \beta_{3L})$, and $\gamma_4 = (\beta_{41}, \beta_{42}, \dots, \beta_{4L})$. The linearity of RIR is tested on the null $H_0: \gamma_2 = \gamma_3 = \gamma_4 = 0$ using the estimation of equation (3), with the delay parameter d corresponding to the lowest p -value in testing H_0 among a set of estimations of (3) having different values of d from 1 to 8. The last step is to choose which STAR model, LSTAR or ESTAR, would be appropriate in presenting the time series behavior of the real interest rate. This is carried out through a set of F tests on testing a sequence of hypotheses with equation (3):

$$H_{01}: \gamma_4 = 0 \quad \text{versus} \quad H_{11}: \gamma_4 \neq 0$$

$$H_{02}: \gamma_3 = 0 | \gamma_4 = 0 \quad \text{versus} \quad H_{12}: \gamma_3 \neq 0 | \gamma_4 = 0$$

$$H_{03}: \gamma_2 = 0 | \gamma_4 = \gamma_3 = 0 \quad \text{versus} \quad H_{13}: \gamma_2 \neq 0 | \gamma_4 = \gamma_3 = 0$$

Following the suggestion of Granger and Terasvirta (1993) and Terasvirta (1994), the choice of STAR model is made on the basis of the lowest p -value of these F tests. If the lowest p -value is associated with the rejection of H_{01} or H_{03} , it implies the selection of LSTAR as an appropriate model. If the rejection of H_{02} is accompanied by the lowest p -value, then the ESTAR model would be more appropriate than the LSTAR model.

The results of the above tests show that most of the RIRs in the study appear to be well described by the ESTAR model. The stationarity of these rates is then examined by the non-linear unit root tests of KSS (2003) in the ESTAR framework. For y_t being the de-meaned series of interest, the KSS tests are based on the following auxiliary regression:

$$\Delta y_t = \delta y_{t-1}^3 + \sum_{j=1}^k \rho_j \Delta y_{t-j} + \text{error} \quad (4)$$

which is obtained from a first-order Taylor series approximation of an ESTAR model specified in KSS (2003).² The null hypothesis of nonstationarity to be tested with (4) is $H_0: \delta = 0$ against

the alternative of (nonlinear) stationarity $H_1: \delta < 0$. The augmentations $\sum_{j=1}^k \rho_j \Delta y_{t-j}$ are included

to correct for serially correlated errors. Kapetanios et al. (2003) use the t -statistic for $\delta = 0$ against $\delta < 0$, referred to as the KSS statistic, but show that it does not have an asymptotic standard normal distribution. They tabulated the asymptotic critical values of the KSS statistics via stochastic simulations. In this study, the KSS tests are applied to the de-meaned data of real interest rates for which the conventional unit root tests provide no evidence of linear stationarity and the tests of Terasvirta (1994) reject the null of linearity. The test statistics, denoted as KSS(Sig) and KSS(SC), are obtained using the augmented model (4) with the lag length k to be selected based on significance testing procedure for KSS(Sig), and on the Schwarz criterion and the Ljung-Box statistic for serial correlation for KSS(SC).

4. Empirical test results

Table 1 reports the test statistics of the ADF test with a constant term in the model for the real interest rates of 22 countries. For 8 out of 22 rates, the null of nonstationarity is rejected by both ADF(Sig) and ADF(SC) statistics and thus could be considered as linear stationary series. For the

² See Kapetanios et al. (2003) for more detailed derivation.

RIR of Norway, while the ADF(SC) statistic rejects the null at the 5% significance level, the ADF(Sig) is not able to do so. This indicates that the test results are sensitive to the lag length of the model, and therefore they do not provide solid evidence for the linear stationarity of the rate. It is interesting that RIRs of all five non-European industrial countries appear to be linearly stationary. Yet, for 14 of 17 European countries, the ADF tests fail to reject the nonstationarity of their RIRs. The presence of nonlinearity in these 14 rates is then tested by the procedures of Terasvirta (1994).

The results reported in the first 6 columns of Table 2 are obtained using the estimation of equation (3) with lag length L in AR model being determined by the Schwarz criterion and the Ljung-Box statistic for serial correlation and the delay parameter d corresponding to the lowest p -value in testing the null. The test statistics indicate that the null hypothesis of linearity ($H_0: \gamma_2 = \gamma_3 = \gamma_4 = 0$) is rejected for all 14 RIRs at the 5% level of significance, except for the rate of France where the null is rejected at the 10% significance level. For the specification of the nonlinear model, among the three F tests on the null $H_{01}: \gamma_4 = 0$, $H_{02}: \gamma_3 = 0 | \gamma_4 = 0$, and $H_{03}: \gamma_2 = 0 | \gamma_4 = \gamma_3 = 0$, the lowest p -value is found on testing H_{02} for 13 out of 14 RIRs. The results suggest that the nonlinearity in RIRs can be well presented by the ESTAR model for most of the European countries. Therefore, it seems to be appropriate to apply the KSS test procedure to detect the presence of nonstationarity against nonlinear but globally stationary ESTAR process (KSS 2003, p. 359).³

The KSS test statistics are reported in the last two columns of Table 2. The null of nonstationarity is rejected by both KSS(Sig) and KSS(SC) in favor of the alternative of nonlinear stationarity at the 5% significance level for 9 out of 14 European RIRs under testing. The results

³ Although the ESTAR specification in KSS (2003) may not fit the ESTAR model with a large delay parameter, the rejection of the null of nonstationarity by the KSS tests and not by the ADF tests may still indicate that the series under testing tends to revert to its mean nonlinearly.

indicate that most of the RIRs of European countries have a nonlinear mean-reverting tendency and have time series properties of a stationary ESTAR process.

5. Conclusions

This study provides stronger evidence than earlier studies for the argument that most of industrial countries' RIRs are stationary.⁴ With the quarterly data of a sample period starting from 1973, evidence based on conventional unit root tests shows that the short-term real interest rates of non-European industrial countries are linearly stationary. The tests based on Terasvirta (1994) reveal the presence of nonlinearity in the RIRs of many European countries, and most of them appear to have properties of the ESTAR process: that is, they tend to respond to positive and negative shocks symmetrically. The results obtained using the KSS (2003) tests, which allow for nonlinearity in the series under testing, further show that most of European countries' RIRs have a tendency to revert nonlinearly toward their equilibrium positions. These results seem to imply that growing economic integration among European countries in the past three decades may have played an important role in driving the RIRs of these countries to move nonlinearly around their equilibrium positions. The policies of economic integration, such as the Maastricht Treaty's convergence criteria, have generated convergence in inflation rates as well as reduction in interest rate spreads among these countries.⁵ Close links of a country's inflation and interest rates to those of other countries may have resulted in deviations from equilibrium relation between own country's inflation and short-term interest rates, along with adding more friction and thus nonlinearities in their adjustment process to restore the equilibrium relation within the country.

⁴ In KSS (2003), the ADF test rejects the null of nonstationarity for only 2 out of 11 RIRs at the 5% significance level. While the KSS test gives "...stronger overall support to the long run Fisher hypothesis" (pp. 371-372), it rejects the null of nonstationarity for 7 out of 11 RIRs at the 10% but only for 5 of them at the 5% significance level.

⁵ Although some European countries are not directly involved in the Maastricht Treaty, they may still be affected due to traditionally close economic connections among almost all European countries.

References

- Granger, C. and T. Terasvirta, 1993. Modeling nonlinear economic relationships. (Oxford University Press, New York).
- Haug, A.A. and P.L. Siklos, 2006. The behavior of short-term interest rates: international evidence of non-linear adjustment, *Studies in Nonlinear Dynamics and Econometrics*, 10(4), 1-32.
- Kapetanios, G., Y. Shin, and A. Snell, 2003. Testing for a unit root in the nonlinear STAR framework. *Journal of Econometrics* 112(2), 359-379.
- Koustaş, Z. and A. Serletis, 1999. On the Fisher effect, *Journal of Monetary Economics* 44(1), 105–130.
- Mishkin, F.S., 1992. Is the Fisher effect for real? A reexamination of the relationship between inflation and interest rates. *Journal of Monetary Economics* 30(2), 195-215.
- Rapach, D.E. and C.E. Weber, 2004. Are real interest rates really nonstationary? New evidence from tests with good size and power. *Journal of Macroeconomics* 26, 409-430.
- Rose, A.K., 1988. Is the real interest rate stable? *Journal of Finance* 43, 1095-1112.
- Terasvirta, T., 1994. Specification, estimation, and evaluation of smooth transition autoregressive models. *Journal of the American Statistical Association* 89(425), 208-218.

Table 1
 ADF unit root test results for real interest rates

Country	Sample period	ADF(Sig)	ADF(SC)
Australia	1973:1 - 2007:1	-3.01 ^a	-3.31 ^a
Austria	1973:1 - 1998:4	-2.50	-2.50
Belgium	1973:1 - 2007:1	-3.51 ^b	-3.35 ^a
Canada	1973:1 - 2007:1	-3.00 ^a	-3.32 ^a
Denmark	1973:1 - 2007:1	-2.28	-2.28
Finland	1978:1 - 2007:1	-2.01	-1.64
France	1973:1 - 2004:3	-2.10	-2.10
Germany	1973:1 - 2007:1	-2.57	-2.58
Greece	1983:1 - 2007:1	-1.81	-1.81
Iceland	1987:1 - 2007:1	-4.65 ^b	-5.53 ^b
Ireland	1973:1 - 2006:2	-2.36	-2.66
Italy	1973:1 - 2007:1	-2.50	-2.50
Japan	1973:1 - 2007:1	-3.90 ^b	-3.90 ^b
Netherlands	1973:1 - 1998:4	-2.38	-2.38
New Zealand	1978:1 - 2007:1	-2.95 ^a	-2.95 ^a
Norway	1973:1 - 2006:4	-2.77	-3.17 ^a
Portugal	1981:1 - 2000:1	-2.18	-2.18
Spain	1974:1 - 2007:1	-2.58	-2.58
Sweden	1973:1 - 2006:2	-1.88	-2.01
Switzerland	1975:4 - 2007:1	-2.36	-2.36
UK	1973:1 - 2007:1	-3.41 ^a	-3.41 ^a
US	1973:1 - 2007:1	-3.22 ^a	-2.93 ^a

Notes: These are the standard ADF test statistics with a constant term in the model for testing. ADF(Sig) and ADF(SC) are obtained using the augmented model with the number of augmentations being selected based on the significance testing procedure for ADF(Sig), and on the Schwarz criterion and the Ljung-Box statistic for serial correlation for ADF(SC). The 5% and 1% asymptotic critical values for the tests are -2.86 and -3.43, respectively. ^a and ^b denote rejection of the null hypothesis of nonstationarity at the 5% and 1% significance levels, respectively.

Table 2
Testing for nonlinearities and nonstationarity in real interest rates

Country	L	d	Nonlinearity	H_{01}	H_{02}	H_{03}	KSS(Sig)	KSS(SC)
Austria	7	2	0.015	0.108	0.030	0.011*	-2.03	-2.03
Denmark	7	1	0.002	0.005	0.002*	0.207	-3.29 ^a	-3.06 ^a
Finland	8	2	0.012	0.017	0.004*	0.344	-3.86 ^b	-3.60 ^b
France	1	3	0.068	0.745	0.003*	0.837	-2.56	-2.56
Germany	4	1	0.034	0.092	0.004*	0.479	-3.08 ^a	-3.34 ^a
Greece	4	4	0.014	0.043	0.003*	0.454	-4.06 ^b	-3.27 ^a
Ireland	7	1	0.000	0.000	0.000*	0.000	-4.27 ^b	-4.26 ^b
Italy	4	2	0.000	0.093	0.000*	0.001	-4.40 ^b	-4.40 ^b
Netherlands	3	4	0.002	0.176	0.000*	0.190	-2.14	-2.14
Norway	2	1	0.000	0.002	0.000*	0.212	-5.49 ^b	-5.49 ^b
Portugal	7	1	0.000	0.000	0.000*	0.036	-3.19 ^a	-3.19 ^a
Spain	6	2	0.000	0.000	0.000*	0.000	-4.59 ^b	-4.59 ^b
Sweden	7	1	0.033	0.606	0.001*	0.131	-2.27	-2.34
Switzerland	3	2	0.026	0.128	0.003*	0.353	-2.43	-2.80

Notes: The lag length L in the AR model is determined by the Schwarz criterion and the Ljung-Box statistic for serial correlation. The delay parameter d is selected corresponding to the lowest p -value of the linearity test using the residual series of the AR process. Numbers in the column below 'Nonlinearity' are p -values corresponding to the test with the null of linearity. Numbers in the columns below H_{01} , H_{02} , and H_{03} are p -values of the F tests on testing the null H_{01} : $\gamma_4 = 0$, H_{02} : $\gamma_3 = 0 | \gamma_4 = 0$, and H_{03} : $\gamma_2 = 0 | \gamma_4 = \gamma_3 = 0$, respectively. * denotes the lowest p -value among the three tests. A * with the p -value below H_{01} or H_{03} implies the appropriateness of selecting a LSTAR model, while a * with the p -value below H_{02} supports the choice of an ESTAR model. KSS(Sig) and KSS(SC) are the KSS test statistics obtained using the methods similar to those used in attaining ADF(Sig) and ADF(SC), respectively. The 5% and 1% asymptotic critical values for the KSS tests are -2.93 and -3.48, respectively, taken from Kapetanios et al. (2003, p. 364). Also see notes to Table 1.