HTHE UNIVERSITY OF TEXAS AT SAN ANTONIO, COLLEGE OF BUSINESS

Working Paper SERIES

Date June 24, 2009

WP # 0096ECO-566-2009

Liquidity Risk and Banks' Asset Composition: Implications for Monetary Policy

Edgar A. Ghossoub
Department of Economics
University of Texas at San Antonio

UTSA COLLEGE OF BUSINESS KNOWLEDGE FOR A NEW WORLD
ONE UTSA CIRCLE
SAN ANTONIO, TEXAS 78249-0631
210 458-4317 | BUSINESS.UTSA.EDU

Liquidity Risk and Banks' Asset Composition: Implications for Monetary Policy

Edgar A. Ghossoub*
The University of Texas at San Antonio

April 17, 2009

Abstract

Monetary growth models in which the government is a net debtor demonstrate that inflation adversely affects capital formation through the crowding out effect. Interestingly, the results are at odds with empirical evidence. In particular, recent studies point out to an asymmetric relationship between inflation and the real economy across countries. Specifically, inflation and output are negatively correlated in poor countries. In contrast, inflation is associated with higher levels of economic activity in advanced economies. I present a monetary growth model where the exposure to risk is inversely related to the level of income. In this setting, I demonstrate that the effects of monetary policy depend on the level of economic activity and the portfolio composition of financial institutions. In poor countries, banks' portfolios consist primarily of government liabilities. Therefore, a higher rate of money creation inhibits capital formation in these economies. In contrast, banks devote more resources towards productive uses in advanced countries. Consequently, monetary policy generates a Tobin effect.

JEL Codes: E31, E41, E44, O42

Keywords: Economic Development, Banks, Monetary Policy

1 Introduction

Empirical evidence finds that the long run relationship between inflation and the real level of economic activity varies across countries. If a significant correlation between inflation and economic activity is found, it is generally positive for

*Department of Economics, One UTSA Circle, University of Texas at San Antonio, San Antonio, TX 78249; Email: Edgar.Ghossoub@UTSA.edu; Phone: (210) 458-6322.

advanced economies and negative for poor countries.[1,2]

Interestingly, monetary growth models with public debt show that inflation has adverse effects on capital formation. For instance, in Schreft and Smith (1997), when the government is a net borrower, a higher rate of money creation enables it to issue more bonds. The higher amount of debt in the economy crowds out capital formation in private markets.[3] While the channels of operation of monetary policy are interesting, the results are at odds with the empirical evidence.

In this paper, I demonstrate that the effects of monetary policy depend on the level of income (development) and the distribution of assets in banks' portfolios. In particular, if banks' holdings of government debt and cash reserves exceed some threshold level, a higher rate of monetary growth adversely affects capital formation in poor countries. In contrast, monetary policy generates a Tobin effect in high income economies when the amount of government liabilities occupies a small fraction of banks' assets.

I proceed by outlining the details of my modeling framework. I consider a two-period overlapping generations economy with production similar to Diamond (1965). There are three types of assets: physical capital, government bonds, and fiat money. Following Townsend (1987) and Schreft and Smith (1997), agents are born on one of two geographically separated locations. If an individual moves to another location, she cannot establish and trade claims to assets due to limited communication. Therefore, spatial separation and limited communication create a role for money. Furthermore, there is a government that issues illiquid bonds and currency in order to satisfy its budget constraint. In addition, the monetary authority adopts a constant money growth rule.

In this economy, young individuals are subject to random relocation shocks. As money is the only asset that can cross locations, relocated agents must liquidate all their asset holdings into currency. Thus, random relocation is analogous to the liquidity preference shocks in Diamond and Dybvig (1983). As a result, the model illustrates the risk pooling role of financial intermediaries (or banks).

In contrast to standard random relocation models, and following Ghossoub and Reed (2009), I assume that the probability of a liquidity shock is inversely related to the aggregate capital stock. Since income is higher under higher levels of capital, this assumption reflects the linkages between the level of income and

[1] Studies that find a positive relationship between inflation and the real economy in advanced countries include Bullard and Keating (1995), Ahmed and Rogers (2000), and Crosby and Otto (2000). In a recent study, Rapach (2003) finds a positive long-run correlation between inflation and the real level of output in 14 industrialized countries. While Gillman and Nakov (2003) find evidence that inflation is negatively correlated with growth in the U.S. and the U.K., inflation and the level of output are positively correlated.

[2] In the work by Bullard and Keating (1995), the authors find no relationship between inflation and the level of output for 11 out of 16 countries in group A. For four countries, Austria, Germany, Finland, and the U.K., they find evidence of a Tobin effect. In contrast, a reverse Tobin-effect is observed for Peru. Additionally, a recent study by Bae and Ralti (2000) finds evidence of a reverse Tobin effect for Argentina and Brazil.

[3] Schreft and Smith (1998) find a similar relationship between inflation and capital formation.

liquidity risk observed in many studies. In particular, previous work indicates that individuals are more exposed to risk at low levels of income. As a result, they are more likely to liquidate their asset holdings.[4]

Under conditions provided in the text, the model predicts there should be two different classes of steady-state equilibria. In one class of steady-states, the government is a net borrower. In the other class, it is a net lender in financial markets. Since governments primarily incur budget deficits, I focus most of my attention on steady-states in which the government issues debt. In this class, I provide conditions under which two steady-state equilibria exist. In the steady-state with low level of economic activity, banks allocate a large fraction of their deposits into government liabilities. Despite that agents are highly exposed to liquidity risk, they are poorly insured against it. Thus, financial markets are highly distorted. The other steady-state has a higher level of capital formation and agents are less exposed to liquidity risk. Further, it is accompanied by lower interest rates.

I proceed to discuss the impact of monetary policy on each steady-state. In this environment, monetary policy affects the economy through two channels: The price channel (direct effect) and the balance sheet channel (indirect effect). Each channel has an opposing effect on the economy. Thus, the net impact of monetary policy depends on which effect dominates.

The price channel of monetary policy reflects the direct impact of a change in the rate of money growth on relative rates of return. In particular, a higher rate of money growth lowers the return to money and therefore, the benefit from holding it.

In the steady-state with low levels of financial market activity, the degree of liquidity risk is significant. As a result, banks hold highly liquid portfolios to insure their depositors against relocation shocks. Thus, a higher inflation rate significantly reduces the marginal benefit from holding cash. In addition, changes in the stock of capital will generate substantial gains from reducing the degree of liquidity risk. Rather than holding less cash, banks collectively choose to reduce their capital investment. By doing so, they raise the value of money since agents are more likely to relocate.

In contrast to poor countries, the degree of liquidity risk is not significant in advanced economies. Because the need for cash is not significant, the benefit from holding money is not significantly affected by a higher rate of money growth. Therefore, banks respond to a lower value of money by holding less of it and by raising their investment in physical capital.

The balance sheet channel reflects the indirect impact of a change in the rate of money growth on banks' balance sheets through the government's budget. For a given rate of interest, a higher rate of money creation enables the government to increase its indebtedness. The higher amount of debt in the economy reduces the amount of resources in banks' portfolios that can be allocated

[4] Deaton (1991) concludes that households accumulate assets to insure themselves against income uncertainties. That is, they rely on assets as buffer stocks. Further, at low levels of income, people might face borrowing constraints. This suggests that low income households are more likely to liquidate assets in the event of adverse shocks.

towards physical capital and cash reserves. In addition, a higher amount of debt implies a higher consumption to agents who do not relocate. Therefore, a lower cost of holding money.

At the low capital steady-state, banks hold a lot of cash reserves relative to other assets. Further, financial markets are highly distorted by high interest rates. As a result, the cost of holding money (marginal utility of non–relocated agents) is significant and thus declines sharply when the amount of bonds in banks' portfolios rises. Consequently, banks respond to a lower cost of holding money by collectively choosing to hold more capital to reduce their depositors' exposure to risk. By doing so, banks are able to substantially reduce the amount of money holding in their portfolios. Thus, they can acquire the additional debt issued by the government without lowering their capital investments.

By comparison, the need to liquidate assets is relatively low in the steady-state with high levels of capital formation. Consequently, banks allocate a significant fraction of their deposits towards long term investments. Moreover, at high levels of income, the exposure to liquidity risk is relatively unaffected by investment activity. Therefore, banks respond to a higher amount of debt by reducing capital investment and holding more cash. In these economies, the balance sheet channel of monetary policy calls for a negative relationship between the rate of money growth and capital investment.

In sum, agents at different levels of income face different degrees of exposure to risk. Hence, they respond differently to price changes and monetary policy. As I demonstrate in the text, the net impact of monetary policy at each stage of development depends on the portfolio composition of financial intermediaries. Specifically, if the amount of government liabilities is above some threshold level, monetary policy adversely affects capital formation. Notably, the threshold amount of government liabilities is much higher in poor countries.

The analysis above suggests that the asymmetric relationship between inflation and the real economy observed across countries is explained by the fact that the price effect dominates at all levels of income. As I demonstrate in the text, this occurs when the rate of return to money is relatively low. Intuitively, when the rate of return to money balances is low, agents receive little consumption in the event they relocate. The lower consumption by relocated depositors renders banks' balance sheets highly sensitive to changes in rates of return. Thus, the price effect dominates the balance sheet effects under a higher rate of money growth.

This paper complements recent work by Ghossoub and Reed (2008) that demonstrates how the effects of monetary policy differ across the stages of development. In their work, Ghossoub and Reed (2008) show that the degree of exposure to liquidity risk at different stages of development plays an important role in the determination of monetary policy. Since government debt plays a primary factor in the transmission of monetary policy, I view that it is important to study how the interaction between different markets affect the conduct of monetary policy. Interestingly, I show that the distribution of assets in banks portfolios is also an important determinant of monetary policy at all stages of development. Monetary policy generates a reverse-Tobin effect in poor countries because banks allocate a large fraction of their deposits towards cash reserves and government debt. Conversely, a higher rate of money creation is likely to generate a Tobin effect in rich countries as long as the amount of government liabilities in banks portfolios is relatively small.

Finally, the results in this study shed light on current policies in advanced countries such as Europe and the United States. Specifically, governments in these economies have been incurring large budget deficits in recent years. Combined with very low real interest rates, an inflationary monetary policy could hinder economic activity in the long run if the amount of government liabilities in banks' portfolios are high enough.

The paper is organized as follows. In Section 2, I describe the model and study the impact of monetary policy. I offer concluding remarks in Section 3. Most of the technical details are presented in the Appendix.

2 Environment

Consider a discrete-time economy with two geographically separated locations or islands. Each location is populated by an infinite sequence of two-period lived overlapping generations. Let $t = 1, 2, ..\infty$, index time. At the beginning of each time period, a new generation of individuals is born on each island with a unit mass.

Each young agent is endowed with one unit of labor effort which she supplies inelastically. In contrast, agents are retired when old. Further, agents derive utility from consuming the economy's single consumption good, c when old. The preferences of a typical agent are expressed by $U(c) = \frac{c^{1-\theta}}{1-\theta}$.

The consumption good is produced by a representative firm which rents capital and hires labor from young agents. The production function is denoted by $Y_t = F(K_t, L_t)$, where K_t is the aggregate capital stock and L_t denotes the amount of labor hired. Equivalently, output per worker is expressed by $y_t = f(k_t)$ and satisfies standard Inada conditions. Further, the capital stock completely depreciates in the production process.

There are three types of assets in this economy: money (fiat currency), capital, and government bonds. Denote the per worker nominal monetary base, capital stock, and real government debt, by $\bar{m}_t$, k_t, and b_t respectively. At the initial date 0, the generation of old agents at each location is endowed with the aggregate capital, K_0 and money supply, M_0. Since the total population size is equal to one, these variables also represent aggregate values. Moreover, one unit of investment by a young agent in period t becomes one unit of capital next period. Equivalently, i_t units of goods invested become k_{t+1} units of capital in the subsequent period.

Assuming that the price level is common across locations, I refer to P_t as the number of units of currency per unit of goods at time t. Thus, in real terms, the supply of money per worker is, $m_t = \bar{m}_t/P_t$.

Moreover, individuals in the economy are subject to relocation shocks. Each period, a fraction of young agents must move to the other island. These agents are called "movers." Limited communication and spatial separation make trade difficult between different locations. As in standard random relocation models, fiat money is the only asset that can be carried across islands.[5] Furthermore, currency is universally recognized and cannot be counterfeited — therefore, it is accepted in both locations.

Since money is the only asset that can cross locations, depositors who learn they will be relocated will liquidate all their asset holdings into currency. Random relocation thus plays the same role that liquidity preference shocks perform in Diamond and Dybvig (1983). As banks provide insurance against the shocks, each young depositor will put all of her income in the bank rather than holding assets directly.

Following recent work by Ghossoub and Reed (2008), the probability of a liquidity shock, π_t, is inversely related to the aggregate capital stock. Since aggregate income is higher under high levels of capital, this assumption serves as a proxy for the connections between the level of income and liquidity risk. Specifically, previous studies such as Rosenzweig and Wolpin (1993) point out that individuals are more likely to liquidate assets at low levels of income. Hence, in any time period I assume that:

$$\pi_t = \pi\left(K_t\right) = \frac{\pi_0}{K_t} \tag{1}$$

where $\pi_0 \in (0, K_t)$ is a parameter reflecting the degree of liquidity risk independent of the economy. From this expression, the probability of relocation is significant at low levels of capital formation (income). Moreover, a change in the capital stock has a significant impact on the exposure to risk at low levels of capital.

In addition to depositors, there is a central bank that follows a constant money growth rule. The aggregate nominal stock of cash in period t is expressed by $M_t = \sigma M_{t-1}$, where $\sigma > 1$ is the gross rate of money creation. In real per capita real terms:

$$m_t = \sigma \frac{P_{t-1}}{P_t} m_{t-1} \tag{2}$$

where $\frac{P_{t-1}}{P_t}$ is the gross rate of return on money balances between period $t-1$ and t. In addition to seigniorage revenue, the government adjusts the amount of new liabilities in order to finance interest payments on previously issued debt. The expenditures and revenues make up the government budget constraint:

$$b_t = \frac{M_t - M_{t-1}}{P_t} - R_{t-1} b_{t-1} \tag{3}$$

where R_{t-1} is the gross real interest rate on government bonds issued in period $t-1$. At $t = 0$, I assume that the government commits to any amount of bonds (loans) demanded by young agents at the market interest rate. In this manner,

[5] Because government bonds are issued in large denominations, they are not sufficiently liquid to be used as a medium of exchange.

the new amount of government bonds and money created at the initial period are endogenous.

2.1 Trade

2.1.1 Factor markets:

Under perfect competition in factor markets, factor inputs are paid their marginal product. The rental rate and wage rates in period t are respectively:

$$r_t = f'(k_t) \tag{4}$$

$$w_t = w(k_t) = f(k_t) - k_t f'(k_t) \tag{5}$$

where r_t is the gross rental rate on capital in period t.

2.1.2 A typical bank's problem

In this economy, financial intermediaries arise to insure agents against liquidity risk. Therefore, agents choose to deposit all of their income at banks. Due to perfect competition in the banking sector, banks choose portfolios to maximize the expected utility of each depositor. A bank promises a gross real return r_t^m if the young individual will be relocated and a gross real return r_t^n if not. Moreover, banks take the aggregate capital stock as given. Since the market for deposits is perfectly competitive, financial intermediaries take the return on deposits as given.

The bank's portfolio choice in period t, involves determining the amounts of real money balances, m_t, capital investment, k_{t+1}, and government bonds, b_t. The bank's balance sheet is expressed by:

$$m_t + k_{t+1} + b_t \leq w_t \ ; \ t \geq 0 \tag{6}$$

Announced deposit returns must satisfy the following constraints. First, since currency is the only asset that can be transported across locations, relocated agents will choose to liquidate their asset holdings into currency. Depending on the bank's money holdings and the inflation rate, the return to movers satisfies:

$$\pi(K_t)\, r_t^m w_t \leq m_t \frac{P_t}{P_{t+1}} \tag{7}$$

In addition, I choose to study equilibria in which money is dominated in rate of return. Therefore, banks will not carry money balances between periods t and $t+1$. The bank's total payments to non-movers are therefore paid out of its return on capital and government bonds in $t+1$:

$$(1 - \pi(K_t)) r_t^n w_t \leq r_{t+1} k_{t+1} + R_t b_t \tag{8}$$

The bank's problem is summarized by:

$$\underset{r_t^m, r_t^n, m_t, k_{t+1}, b_t}{Max} \pi(K_t) \frac{(r_t^m w_t)^{1-\theta}}{1-\theta} + (1-\pi(K_t)) \frac{(r_t^n w_t)^{1-\theta}}{1-\theta} \tag{9}$$

subject to (6)-(8).

In equilibrium, a bank is willing to hold government bonds and capital simultaneously as long as they yield the same rate of return. In particular, the no-arbitrage condition between capital and government bonds must hold:

$$r_{t+1} = R_t \tag{10}$$

The solution to the bank's problem generates the demand for real money balances:

$$m_t = \frac{1}{1 + \frac{K_t - \pi_0}{\pi_0} \left(R_t \frac{P_{t+1}}{P_t} \right)^{\frac{1-\theta}{\theta}}} w(k_t) \tag{11}$$

Alternatively,

$$\gamma \left(K_t, R_t, \frac{P_{t+1}}{P_t} \right) = \frac{m_t}{w(k_t)} = \frac{1}{1 + \frac{K_t - \pi_0}{\pi_0} \left(R_t \frac{P_{t+1}}{P_t} \right)^{\frac{1-\theta}{\theta}}} \tag{12}$$

where γ is the reserves to deposits ratio.

As pointed out earlier, banks make their portfolio choices taking the aggregate level of economic activity as given. If the economy is at high levels of capital formation (income), agents are less exposed to liquidity risk. Thus, for a given interest rate, banks allocate a small fraction of their deposits into cash reserves. Furthermore, as in standard monetary models, for a given level of capital, banks demand less cash reserves if the return to money falls relative to other assets.

Finally, using (7), (8), (10), and (12), the relative return to depositors can be expressed by:

$$\frac{r_t^n}{r_t^m} = (R_t \sigma)^{\frac{1}{\theta}} \tag{13}$$

which indicates that banks provide little insurance against relocation shocks when interest rates are high.

2.2 General Equilibrium

I now combine the results of the preceding section and characterize the equilibrium for the economy. In equilibrium labor receives its marginal product, (5), and the labor market clears:

$$L_t = 1 \tag{14}$$

Furthermore, by substituting the money demand equation into the bank's balance sheet, (6), we get the optimal demand for government bonds in period t:

$$b_t = \left(1 - \gamma\left(k_t, R_t, \frac{P_{t+1}}{P_t}\right)\right) w_t - k_{t+1} \tag{15}$$

where $k_t = K_t$ in equilibrium. That is, an individual bank's choice of capital investment is equal to the average level in the banking sector.

On the supply side, using the constant money growth rule, (2), the government's budget constraint, (3), and the demand for real money balances, (12), the supply of bonds by the government must satisfy:

$$b_t = R_{t-1} b_{t-1} - \frac{\sigma - 1}{\sigma} \gamma\left(k_t, R_t, \frac{P_{t+1}}{P_t}\right) w\left(k_t\right) \tag{16}$$

Finally, in equilibrium, the capital market clears when the supply of capital by banks, (15), is equal to its demand by firms, (4).

Conditions (2), (4), (10), (15), and (16), characterize the behavior of the economy at each point in time. In this paper, I focus on stationary equilibria. Thus, I turn to study the behavior of the economy in the steady-state.

2.3 Steady-State Analysis

From the constant money growth rule, (2), the rate of money creation, σ, is equal to the rate of inflation, $\frac{P_{t+1}}{P_t}$ in the long run. By imposing steady-state on (16), we obtain the stationary supply of government bonds:

$$b = \frac{1 - \frac{1}{\sigma}}{R - 1} \gamma\left(k, R, \sigma\right) w\left(k\right) \tag{17}$$

Denote the amount of government bonds supplied and demanded per unit of deposits by, β^S and β^D respectively. From (17):

$$\beta^S = \frac{1 - \frac{1}{\sigma}}{R - 1} \gamma\left(k, R, \sigma\right) \tag{18}$$

Imposing steady-state on banks' demand for bonds, (15):

$$\beta^D = 1 - \gamma\left(k, R, \sigma\right) - \Omega\left(k\right) \tag{19}$$

where $\Omega\left(k\right) = \frac{k}{w(k)}$, is the fraction of deposits allocated towards capital investment. In addition, the fraction of deposits allocated to cash reserves in the steady-state is:

$$\gamma\left(k, R, \sigma\right) = \frac{1}{1 + \frac{k - \pi_0}{\pi_0} \left(R\sigma\right)^{\frac{1-\theta}{\theta}}} \tag{20}$$

In this environment, the government is either a net borrower or a net lender. In particular, for a given inflation tax base, the government is a net borrower when the real return to government bonds is positive. That is, $\beta^S > 0$ when $R > 1$. Moreover, for a given seigniorage tax base, the government's obligations are higher as the interest rate on government debt rises. Consequently, the government lowers its indebtedness. Conversely, the government is a net lender when the real return to bonds is negative. That is, $\beta^S < 0$ if $R < 1$. In contrast to the previous case, the government's subsidy to the private sector increases with the return on bonds.

On the demand side, the demand for cash reserves is decreasing in its opportunity cost. Therefore, for a given level of capital investment, banks demand more bonds under higher interest rates.

By imposing equilibrium on the bonds market and using the expression for wages, (5), we obtain the economy's supply of capital:

$$\Omega(k) = 1 - \frac{R - \frac{1}{\sigma}}{(R-1)}\gamma(k, R, \sigma) \tag{21}$$

From (4) and (10) in the steady state, the inverse demand for capital by firms can be expressed by:

$$R = f'(k) \tag{22}$$

The steady-state behavior of the economy is characterized by the supply and demand for capital in the steady state, (21) and (22), respectively.

It is clear that the demand for capital, from (22), is strictly decreasing in R as shown in Figure 2 below.[6] In addition, Proposition 1 describes the behavior of the supply of capital, (21), in the steady-state:

Lemma 1. *The locus defined by* (21) *satisfies the following:*

a. If $R > 1$*:*

i. $\frac{dR}{dk} < (\geq) 0$ *for all* $k < (\geq) \widetilde{k}$*, where* $\tilde{k} : -\frac{R-\frac{1}{\sigma}}{(R-1)}\frac{\partial \gamma(k,R,\sigma)}{\partial k}|_{k=\tilde{k}} = \Omega'\left(\tilde{k}\right)$.

ii. $\lim\limits_{k \to \pi_0} R \to \infty$ *and* $\lim\limits_{k \to \Omega^{-1}(1)} R \to \infty$.

b. if $R < 1$*:*

i. $\frac{dR}{dk} > 0$ *and* $\lim\limits_{R \to 1} k \to \infty$.

ii. $\left(\Omega^{-1}(1), \frac{1}{\sigma}\right)$ *satisfies* (21).

Lemma 1 demonstrates that the effects of interest rates on the supply of capital depend on the net lending position of the government and the level of economic activity. In particular, suppose the government is a net borrower, with $R > 1$. At low levels of output, banks supply less capital under higher interest

[6] Equation (22), can also be interpreted as a no-arbitrage condition between capital and government bonds. In this case, the relationship between R and k is explained as follows. A higher return on government bonds requires a higher return to capital for both assets to be held simultaneously by banks. This implies that the capital stock must decline.

rate. In contrast, at high levels of income, equity returns are associated with higher investment activity. By comparison, if the government is a creditor in capital markets, interest rates and the level of capital stock are strictly positively related.

Unlike standard random relocation models, the level of investment by one financial institution (agent) depends positively on the level of investment by other financial institutions. This happens because the higher investment by one agent reduces the exposure of other agents to liquidity risk. This in turn reduces the demand for cash reserves and raises capital formation. Therefore, strategic complementarities from investment in physical capital occur.

Moreover, the portfolio choice by one financial institution positively affects the payoffs by other banks. In particular, a higher level of investment activity by one bank raises the aggregate level of capital, which reduces the exposure of agents to risk. Since the population size is unity, a lower probability of relocation also implies a smaller number agents who relocate. Thus, other banks make smaller payments to their depositors (as a group) who relocate. As a result, there are positive spillovers from capital accumulation in this framework.[7]

Due to the presence of strategic complementarities, there are two levels of capital that clear the bonds market for a given positive interest rate, as illustrated in Figure 1 below. Therefore, a given return to equity can be associated with two different levels of capital supplied by banks. The intuition is as follows.

The supply of capital, (21), reflects combinations of R and k that clear the bonds market. As a benchmark, consider an environment such as Schreft and Smith (1997), where the degree of liquidity risk does not vary with the level of economic activity. In their setting, the inflation tax base is independent from economic activity. Hence, the supply of bonds per unit of deposits does not vary with the level of income. Moreover, the fraction of deposits allocated into government bonds is inversely related with the level of investment. That is, at higher levels of capital formation, banks allocate a smaller fraction of their deposits into bonds to satisfy their balance sheets. Consequently, there exists a unique value of k for a given R that clears the bonds market.

By comparison, for a given return on bonds, $R = R_0 > 1$, the supply of government debt is strictly declining in the level of economic activity in this setting. Specifically, at high levels of capital formation, say k_A, the probability of liquidity shock is low. Therefore, the demand for cash in the economy is relatively small. The low amount of seigniorage revenue results in a low supply of government debt. Moreover, banks allocate a small fraction of their deposits into government bonds. Thus, the bonds market clears at a point like A in Figure 1 below, where $\beta^D_{k_A}$ and $\beta^S_{k_A}$ denote the demand and supply of bonds, respectively at k_A.

In contrast, the degree of liquidity risk is significant at low levels of capital formation such as $k_B < k_A$. As a result, banks hold a lot of cash reserves in their portfolios. Since seigniorage revenue is high, the supply of bonds is high

[7]Please refer to Cooper and John (1988) for additional information on spillovers and strategic complementarities.

as well. Consequently, the bond market also clears at point B as shown in the Figure.

Interestingly, the above analysis suggests that for a given interest rate, banks' portfolios at low levels of income consist mainly of cash reserves and government bonds $(\beta + \gamma)$. In contrast, banks at higher levels of economic activity devote more resources towards capital formation and less towards government liabilities.

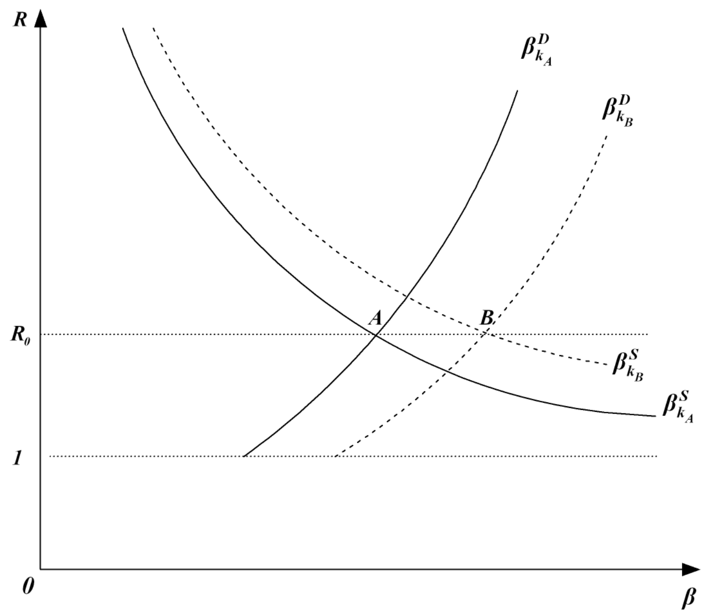

Figure 1: Bonds Market Equilibrium

In addition, the level of economic activity plays a significant role in determining the relationship between the return on capital and the supply of capital by banks. To better understand the effects of inflation on banks' portfolios, it is useful to refer to the condition for optimal money holding, (23). A bank chooses to hold money such that:

$$\left(\frac{1}{\sigma}\right) u'\left(c^m\right) = Ru'\left(c^n\right) \tag{23}$$

where c^m and c^n are the consumption of a mover and a non-mover, respectively. In addition, the term on the left-hand side of (23) reflects the benefit from holding one additional unit of money. Additionally, the term on the right hand side is marginal cost of holding cash. The cost of holding one unit of money is equal to the return to alternative investments times the loss in utility to non-movers from holding one extra unit of currency (one less unit of other assets). Equivalently, under CRRA preferences:

$$\left(\frac{1}{\sigma}\right)^{1-\theta}\left(\frac{\frac{\pi_0}{k}}{\gamma w\left(k\right)}\right)^{\theta}=R^{1-\theta}\left(\frac{1-\frac{\pi_0}{k}}{\left(1-\gamma\right)w\left(k\right)}\right)^{\theta} \tag{24}$$

For a given portfolio choice, a change in the return to capital has two effects on asset allocation. First, a higher return to capital raises the cost of holding money directly. Further, the supply of capital incorporates the supply of bonds by the government. Specifically, it takes into account the reaction by the government to changes in interest rates. Thus, banks adjust their portfolios to conditions in the bonds market. For instance, suppose the government is a net borrower. Under a higher real interest rate, the government lowers its indebtedness. The lower amount of bonds in the economy translates into a lower consumption to non-movers $(1-\gamma=\beta+\Omega)$. Due to a higher marginal utility from consumption to non-movers, the cost of holding money increases. In this manner, higher interest rates indirectly raise the cost of holding money through the government's budget.

If banks are unable to influence the exposure to risk, banks respond to a higher cost of holding money by holding less of it. By doing so, movers receive a lower consumption and utility is maximized.

Interestingly, the above mechanism does not necessarily hold if the degree of liquidity risk depends on the aggregate level of investment. As shown earlier, there are two levels of capital that clear the bonds market for a given interest rate. At low levels of capital formation, $k<\widetilde{k}$, the degree of liquidity risk is significant. Therefore, banks hold a lot of cash reserves to insure their depositors against risk. From (1), the probability of relocation is highly sensitive to changes in the amount of capital accumulation. In response to a lower consumption to non-movers, banks collectively choose to raise the value of money by raising the probability of relocation. That is, banks invest less in capital. By raising the probability of relocation, the marginal utility of movers is higher and the expected utility of depositors is maximized. Thus, the capital supply curve is downward sloping for all $k<\widetilde{k}$ as illustrated in Figure 2 below. In addition, from (19), this also implies that the amount of government liabilities in banks' portfolio is increasing with the interest rate in poor economies.

In contrast, at high levels of capital, $k>\widetilde{k}$, individuals are less susceptible to liquidity risk. Further, changes in the stock of capital have little impact on the value of money. In response to a higher cost of holding money, banks raise their capital investment. In turn, the supply curve of capital slopes upward in economies with large amount of resources. Further, financial intermediaries in these economies hold less government liabilities in their portfolios under higher interest rates.

By comparison, when the government is a net lender, the amount of subsidy provided by the government to the private sector increases with interest rates. Moreover, the degree of liquidity risk is not significant. Consequently,the amount of capital supplied by banks increases under a higher return to bonds.

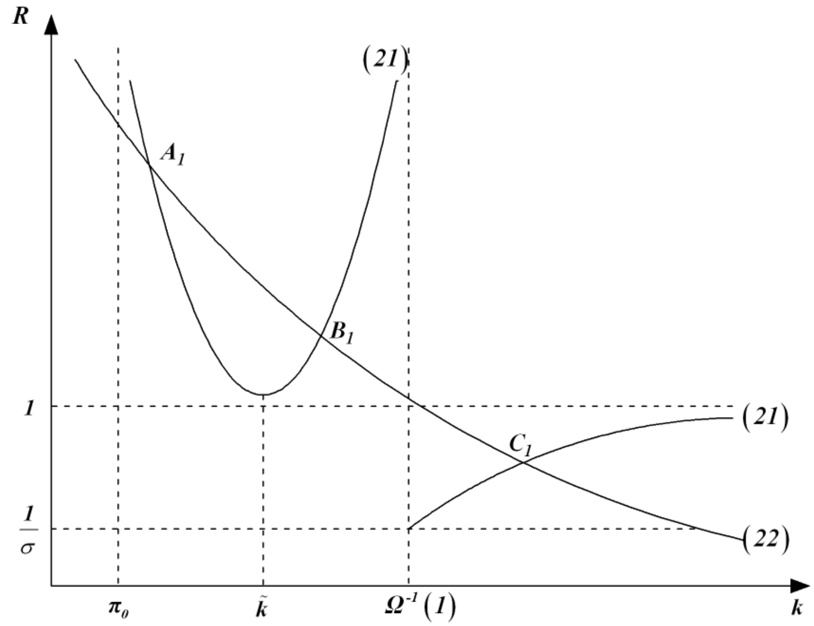

Figure 2: Steady-State Equilibria

I proceed by studying existence of steady-state equilibria where money is dominated in rate of return, $R > \frac{1}{\sigma}$.

Proposition 1. *Suppose* $\pi_0 < \overline{\pi_0}$, where $\overline{\pi_0} = \frac{\tilde{k} - \frac{(1-\Omega(\tilde{k}))}{\Omega'(\tilde{k})}}{1-(\sigma f\prime(\tilde{k}))^{-\frac{1-\theta}{\theta}}}$. *Under this condition, there are exactly three steady-states if* $\sigma > \underline{\sigma}_0$, where $\underline{\sigma}_0 : \Omega^{-1}(1) = \left(f\prime\left(\frac{1}{\underline{\sigma}_0}\right)\right)^{-1}$. *For two steady states, the government is a net borrower* $(R > 1)$ *and for the other steady state, the government is a net lender* $(R < 1)$. *In contrast, there are exactly two steady-states with* $R > 1$ *if* $\sigma < \underline{\sigma}_0$.

An equilibrium in which the government is a net borrower requires that (21) and (22) intersect at $R > 1$ as illustrated in Figure 2 above. This happens if the degree of liquidity risk is not too significant. Thus, the lowest real interest rate associated with banks' supply of capital is no too high. Under this condition, there are always two steady states in which the government is a net borrower. In particular, the first steady-state, A_1, is characterized with a low capital stock, high real interest rates, and a high degree of liquidity risk. Alternatively, the other steady-state, B_1, has a higher capital stock, a lower real interest rate, and a lower exposure to liquidity risk.

Moreover, the supply and demand for capital also intersect at $R < 1$, but the steady steady state may not be feasible since money may not be dominated in rate of return. As indicated in the proposition, a steady-state where the government is a net lender exists if the rate of return to money (inflation) is significantly low (high). If this condition does not hold, equilibrium C_1 is not feasible and there are exactly two steady-states where the government is a net borrower.

Since governments primarily incur budget deficits, I focus my attention on equilibria with positive interest rates. I proceed to discuss the impact of a higher rate of money growth on the supply of capital, (21).

Lemma 2. *Suppose $k < \tilde{k}$. Define $\widehat{R}_A$ and $\widehat{k}_A : \frac{1}{\left(\widehat{R}_A\sigma-1\right)} = \frac{1-\theta}{\theta}\left(1-\gamma\left(\widehat{k}_A, \widehat{R}_A, \sigma\right)\right)$. If $R \geq (<)\, \widehat{R}_A$, $\frac{\partial k}{\partial \sigma} \leq (>)\, 0$. In contrast, suppose $k > \tilde{k}$. Define $\widehat{R}_B$ and $\widehat{k}_B : \frac{1}{\left(\widehat{R}_B\sigma-1\right)} = \frac{1-\theta}{\theta}\left(1-\gamma\left(\widehat{k}_B, \widehat{R}_B, \sigma\right)\right)$, with $\widehat{R}_A > \widehat{R}_B$ and $\widehat{k}_A < \widehat{k}_B$. If $R \geq (<)\, \widehat{R}_B$, $\frac{\partial k}{\partial \sigma} \geq (<)\, 0$.*

The effects of inflation on banks' portfolios may be better understood by referring to a bank's optimal choice of money balances, (23). In this setting, monetary policy affects banks' portfolios through two primary channels. First, for a given real interest rate, a higher rate of money growth directly lowers the return to money relative to other assets. This results in lower payments to relocated agents and therefore a lower benefit from holding cash. I refer to this effect as the price channel.

Additionally, monetary policy influences banks' balance sheets indirectly through the government's budget. Specifically, a higher rate of money creation raises the ability of the government to issue debt (or loans when $R < 1$). The higher amount of debt reduces resources in banks' portfolios that can be devoted towards alternative assets. In addition, a higher amount of debt implies a higher consumption to agents who do not relocate. Therefore, a lower cost of holding money. I refer to this effect as the balance sheet channel.

Interestingly, the impact of each channel on capital investment depends on the degree of liquidity risk and the level of economic activity. I first examine the price channel of monetary policy at different levels of income.

At low levels of capital formation (income), agents are highly exposed to liquidity risk. Thus, banks hold highly liquid portfolios. Consequently, a higher inflation rate significantly reduces the benefits from holding cash reserves. Further, the probability of relocation is highly sensitive to changes in the level of investment. Rather than holding less cash to raise the value money, banks collectively choose to reduce their capital investment. By doing so, they raise their depositors' exposure to risk and the benefit from holding money.

In contrast to poor countries, the degree of liquidity risk is not significant in advanced economies. Because the need for cash is not significant, the benefit from holding it is slightly affected by a higher rate of money growth. Therefore, banks respond to a lower value of money by holding less of it and by raising their investment in physical capital.

I proceed to examine the balance sheet channel of monetary policy for a given interest rate. At low levels of income, agents receive little consumption in the event they do not relocate. Therefore, the cost of holding money is significant and falls drastically when the amount of bonds in banks portfolios increases. As the probability of relocation is highly sensitive to changes in the capital stock, banks respond to a lower cost of holding money by jointly choosing to hold more capital to reduce their depositors' exposure to risk. By doing so, banks are able to substantially reduce the amount of money holding in their portfolios. Thus, they can acquire the additional debt issued by the government without lowering their capital investments.

By comparison, banks hold little amounts of cash reserves in their portfolios at high levels of development. Due to high levels of capital investment, non-movers also receive high levels of consumption. Consequently, the cost of holding money slightly falls due to the higher amount of debt in the economy. Banks respond to a lower cost of holding money by holding more of it. In these economies, government debt crowds out capital formation.

In sum, for a given R, the net impact of monetary policy on the supply of capital depends on which effect dominates at different levels of income. Specifically, if the price effect dominates, banks supply less (more) capital under a higher rate of money growth in poor (rich) countries. In contrast, banks supply more (less) capital under a higher σ if the balance sheet effect dominates in poor (rich) economies. Interestingly, the price effect dominates if the real interest rate is above some threshold level as illustrated in Figure 3 below.

Intuitively, from (13), non-movers receive a relatively higher rate of return under a higher R. The higher consumption by non-movers renders banks' balance sheets less sensitive to changes in the amount of government debt. Thus, when government debt increases under a higher rate of money growth, its marginal impact through the balance sheet channel weakens. There exists a real interest rate, above which the price effect dominates. As I demonstrate in the appendix, the threshold level of interest for poor economies, $\widehat{R}_A$, is much higher than that in advanced countries, $\widehat{R}_B$.

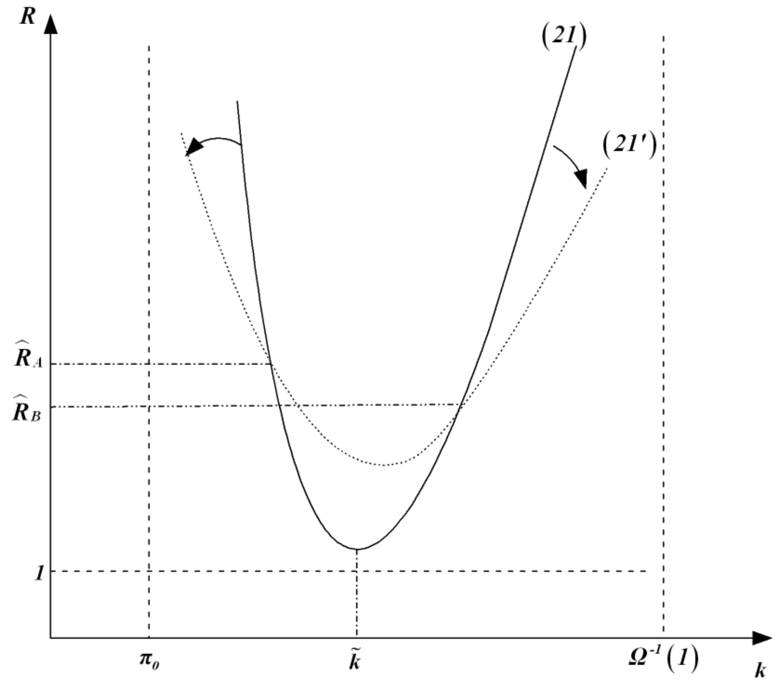

Figure 3: Effects of Monetary Policy on the Supply of Capital

As emphasized in the discussion of Lemma 1, the amount of government liabilities in banks portfolios is relatively large in economies with low levels of income, $\left(k < \widetilde{k}\right)$. Moreover, banks supply less capital under higher interest rates. From (19), this also implies that the amount of government liabilities in banks' portfolio is increasing with the interest rate in poor economies. Consequently, Lemma 2 suggests that an inflationary monetary policy adversely affects capital formation in poor countries when the amount of government liabilities in banks' portfolios is significantly large. In contrast, monetary policy generates a Tobin effect if the amount of government liabilities is relatively small.

By comparison, banks supply more capital under higher interest rate in high income countries, $\left(k > \widetilde{k}\right)$. Hence, financial intermediaries allocate less resources towards government liabilities. The condition in Lemma 2 indicates that banks supply more capital under a higher rate of money growth if the amount of government liabilities in their portfolios is relatively small. In contrast, banks reduce their investment in physical capital under higher inflation rates if their portfolios contain a substantial amount of government liabilities.

The work above suggests that the effects of monetary policy should vary at each steady-state. In the following proposition, I present a condition under which the results in the data are obtained.

Proposition 2. *Suppose* $\sigma > \underline{\sigma}$, where $\underline{\sigma}$:$\overline{k}\left(\widehat{R}_B\right) = \left(f\prime\left(\widehat{R}_B\right)\right)^{-1}$. *Under this condition, inflation adversely affects capital formation at the low-capital steady-state. In contrast, inflation generates a Tobin effect at the economy with a high level of economic activity.*

Figure 4 depicts the effects of a higher rate of money growth on each steady-state. As demonstrated in Lemma 2, a higher rate of money growth promotes capital formation in advanced economies, $k > \widetilde{k}$, if the return to capital exceeds some threshold, $\widehat{R}_B$. Since interest rates are higher at the low capital economy, it is sufficient that the price effect dominates at the high-capital economy, B_1. This happens if at $\widehat{R}_B$, there is an excess demand for capital. The rate of money growth, $\underline{\sigma}$, is such that $\widehat{R}_B$ equals the real interest rate at the high capital economy (capital market is in equilibrium). As I demonstrate in the appendix, $\widehat{R}_B$ is falling in σ. Therefore, for all $\sigma > \underline{\sigma}$ there is an excess demand for capital. This guarantees that the price effect dominates at both steady-states.

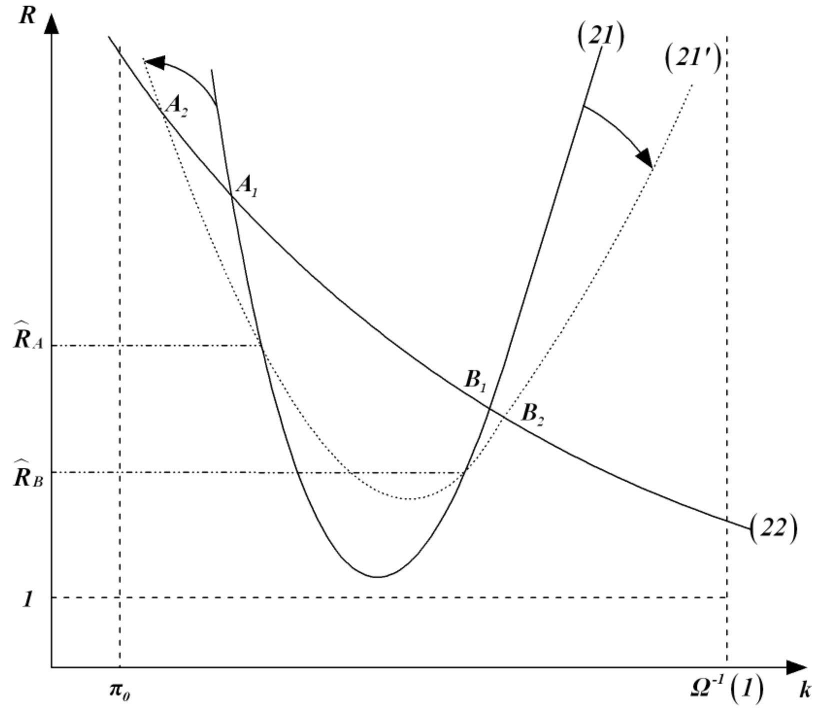

Figure 4: Effects of Monetary Policy on Steady-State Equilibria

Intuitively, when the rate of return to money balances is low, movers receive little consumption in the event they relocate. The low consumption by movers renders banks' balance sheets highly sensitive to changes in rates of return. Thus, the price effect dominates the balance sheet effects under a higher rate of money growth.

At low levels of income, individuals are significantly exposed to liquidity risk. However, financial markets are highly distorted as banks provide inadequate amount of insurance to their depositors. Moreover, banks allocate a significant fraction of their depositors towards government liabilities. Inflation exacerbates these problems by adversely affecting capital formation.

In contrast to poor countries, resources are allocated more efficiently in advanced economies. Due to higher levels of income, people are less exposed to liquidity risk. Consequently, banks allocate a small fraction of their deposits towards less productive assets. In these economies, a higher rate of money creation will most likely promote capital formation as long as the amount of government liabilities is relatively small.

3 Conclusions

Recent evidence points out to an asymmetric relationship between inflation and the level of economic activity across countries. In poor economies, inflation is associated with low levels of output. In contrast, inflation and output are positively correlated in advanced countries. Recent work by Ghossoub and Reed (2009) attribute these asymmetries to the large differences in the degree of exposure liquidity risk across countries. In this paper, I demonstrate that the portfolio composition of financial institutions also has significant implications for monetary policy at all levels of development. For a given level of development a higher rate of money growth adversely affects capital formation if the amount of government liabilities in banks portfolios is above some threshold level. In contrast, investment activity increases under a higher inflation rate if the amount of government liabilities in banks portfolios is relatively low.

In this manuscript, I provide a condition under which the results in the data hold. In less developed economies, financial intermediation is highly distorted by high interest rates. Because agents are highly exposed to liquidity risk, government liabilities crowd out capital formation in banks' portfolios. Consequently, a higher rate of money creation inhibits capital formation in these countries. In contrast, agents are less exposed to liquidity risk in advanced countries and financial markets are less distorted. Further, banks invest a large fraction of their deposits in capital formation. As a result, monetary policy generates a Tobin effect.

References

Ahmed, S. and J.H., Rogers, 2000. Inflation and the Great Ratios: Long Term Evidence From the U.S. Journal of Monetary Economics 45, 3-35.

Bae, S.K. and R.A. Ratti, 2000. Long-run Neutrality, High Inflation, and Bank Insolvencies in Argentina and Brazil. Journal of Monetary Economics 46, 581-604.

Bullard, J. and J.W., Keating, 1995. The Long-Run Relationship Between Inflation and Output in Postwar Economies. Journal of Monetary Economics 36, 477-496.

Cooper, R. and A. John, 1988. Coordinating Coordination Failures in Keynesian Models. Quarterly Journal of Economics 103, 441-463.

Crosby, M. and G. Otto, 2000. Inflation and the Capital Stock. Journal of Money, Credit and Banking 32, 236-253.

Deaton, D., 1991. Saving and Liquidity Constraints. Econometrica 59, 1221-1248.

Diamond, D. and P. Dybvig, 1983. Bank Runs, Deposit Insurance, and Liquidity. Journal of Political Economy 91, 401-19.

Diamond, P.A., 1965. National Debt in a Neoclassical Growth Model. American Economic Review 55, 1126-1150.

Ghossoub, E.A., and R.R. Reed, 2008. Liquidity Risk, Economic Development, and the Effects of Monetary Policy. Mimeo, University of Alabama.

Gillman, M. and A. Nakov, 2003. A Revised Tobin Effect from Inflation: Relative Input Price and Capital Ratio Realignments, USA and UK, 1959–1999. Economica 70, 439–450.

Rapach, D.E., 2003. International Evidence on the Long-Run Impact of Inflation. Journal of Money, Credit and Banking 35, 23-48.

Rosenzweig, M.R. and K. Wolpin, 1993. Credit Market Constraints, Consumption Smoothing and the Accumulation of Durable Production Assets in Low-Income Countries: Investments in Bullocks in India. Journal of Political Economy 101, 223-244.

Schreft, S.L. and B.D. Smith, 1997. Money, Banking, and Capital Formation. journal of economic theory 73, 157-182.

Schreft, S.L. and B.D. Smith, 1998. The Effects of Open Market Operations in a Model of Intermediation and Growth. The Review of Economic Studies 65, 519-550.

Townsend, R., 1987. Economic Organization with Limited Communication. American Economic Review 77, 954-971.

4 Technical Appendix

1. **Proof of Lemma 1**. Define $S(R,\sigma) \equiv \frac{R-\frac{1}{\sigma}}{(R-1)}$. The supply of capital, (21) is expressed as:

$$S(R,\sigma)\gamma(k,R,\sigma) = 1-\Omega(k) \tag{25}$$

Next, I take the derivative with respect to k. With some algebra, we get:

$$\frac{dR}{dk} = \frac{-\Omega'(k) - S(R,\sigma)\frac{\partial\gamma(k,R,\sigma)}{\partial k}}{\left[\gamma(k,R,\sigma)\frac{\partial S(R,\sigma)}{\partial R} + S(R,\sigma)\frac{\partial\gamma(k,R,\sigma)}{\partial R}\right]} \tag{26}$$

It is easy to verify that $\frac{\partial S(R,\sigma)}{\partial R} < 0$, therefore, the denominator is negative. Thus, the sign of $\frac{dR}{dk}$, depends on the sign of the term in the numerator. In particular, $\frac{dR}{dk} \geq (<) 0$ if $-\Omega'(k) - S(R,\sigma)\frac{\partial\gamma(k,R,\sigma)}{\partial k} \leq (>) 0$. Thus, $\frac{dR}{dk} = 0$ if $-\Omega'(k) - S(R,\sigma)\frac{\partial\gamma(k,R,\sigma)}{\partial k} = 0$. By definition of γ, $\frac{\partial\gamma(k,R,\sigma)}{\partial k} = -\frac{(R\sigma)^{\frac{1-\theta}{\theta}}}{\pi}\gamma^2$. Further, using (21), $\frac{dR}{dk} = 0$ if

$$\pi_0\Omega'(k) = \frac{1-\Omega(k)}{(R\sigma)^{-\frac{1-\theta}{\theta}} + \frac{k-\pi_0}{\pi_0}} \tag{27}$$

It is easy to verify from (27) that for a given $R = \widetilde{R} > 1$, there exists a unique value of $k, \widetilde{k} \in \left(\pi_0, \Omega^{-1}(1)\right)$ such that (27) holds and $\frac{dR}{dk} = 0$. In addition for all $k < (>) \widetilde{k}$, the numerator is positive (negative) and $\frac{dR}{dk} < (>) 0$. Finally, from (26), $\frac{dR}{dk} > 0$ for all $R < 1$ as $S(R,\sigma) < 0$. This completes the proof of Lemma 1.

2. **Proof of Proposition 1.** As pointed out in the next, two conditions for existence must be satisfied. First, when the government is a net borrower, the supply and demand curves must intersect. This happens if the inflection point of (21) lies above (22). Thus, at $\widetilde{k}$, the real interest rate that satisfies the demand for capital, $f\prime\left(\tilde{k}\right)$, must exceed the return that satisfies the supply of capital by banks, $\widetilde{R} = R\left(\tilde{k}\right)$. By definition of $\tilde{k}$, (27) and some algebra,

$$\widetilde{R} = R\left(\tilde{k}\right) = \frac{1}{\sigma}\left(\frac{1-\Omega\left(\tilde{k}\right)}{\pi_0\Omega'\left(\tilde{k}\right)} - \frac{\tilde{k}-\pi_0}{\pi_0}\right)^{-\frac{\theta}{1-\theta}}$$

$f\prime\left(\tilde{k}\right) > R\left(\tilde{k}\right)$ implies that $\pi_0 < \overline{\pi_0} = \frac{\tilde{k} - \frac{\left(1-\Omega\left(\tilde{k}\right)\right)}{\Omega'\left(\tilde{k}\right)}}{\left(1-\left(\sigma f\prime\left(\tilde{k}\right)\right)^{-\frac{1-\theta}{\theta}}\right)}$.

In equilibrium, money must be dominated in rate of return. Therefore, all equilibria must occur above the $R = \frac{1}{\sigma}$ line. When the government is a net borrower, $R > 1$, this condition is automatically satisfied. Thus, there are always two steady-states in which the government is a net borrower, as long as the condition derived above holds. Next, we need to find conditions such that the steady-state with $R < 1$ exists. This happens if (21) and (22) intersect above the $R = \frac{1}{\sigma}$ line as in Figure 2. This happens if there is an excess demand for capital at $R = \frac{1}{\sigma}$.

At $R = \frac{1}{\sigma}$, the amount of capital supplied and demanded is $\Omega^{-1}(1)$ and $\left(f\prime\left(\frac{1}{\sigma}\right)\right)^{-1}$. Money is dominated in rate of return when $R < 1$ if $\Omega^{-1}(1) < \left(f\prime\left(\frac{1}{\sigma}\right)\right)^{-1}$. Define $\underline{\sigma} : \Omega^{-1}(1) = \left(f\prime\left(\frac{1}{\underline{\sigma}}\right)\right)^{-1}$. Therefore, for all $\sigma > \underline{\sigma}$, money is dominated in rate of return when the government is a net lender. This completes the proof of Proposition 1.

3. **Proof of Lemma 2.** Taking the derivative of (21) with respect to σ, holding R constant generates:

$$\frac{\partial k}{\partial \sigma} = \frac{\frac{\partial S(R,\sigma)}{\partial \sigma}\gamma(k,R,\sigma) + S(R,\sigma)\frac{\partial \gamma(k,R,\sigma)}{\partial \sigma}}{-\Omega'(k) - S(R,\sigma)\frac{\partial \gamma(k,R,\sigma)}{\partial k}} \tag{28}$$

From Lemma 1, we know that for all $k < \tilde{k}$, $-S(R,\sigma)\frac{\partial \gamma(k,R,\sigma)}{\partial k} - \Omega'(k) > 0$ and for all $k \geq \tilde{k}$, $-S(R,\sigma)\frac{\partial \gamma(k,R,\sigma)}{\partial k} - \Omega'(k) \leq 0$. Next, we need to sign the term in the numerator of (28). Specifically, $\frac{\partial k}{\partial \sigma} = 0$ if $\frac{\partial S(R,\sigma)}{\partial \sigma}\gamma(k,R,\sigma) + S(R,\sigma)\frac{\partial \gamma(k,R,\sigma)}{\partial \sigma} = 0$. Using the expression for $S(R,\sigma)$ and γ, where $\frac{\partial S(R,\sigma)}{\partial \sigma} = \frac{\frac{1}{\sigma^2}}{(R-1)}$, $\frac{\partial \gamma(k,R,\sigma)}{\partial \sigma} = -\frac{1-\theta}{\theta}\frac{k-\pi_0}{\pi_0}(R\sigma)^{\frac{1-\theta}{\theta}}\sigma^{-1}\gamma^2$, and $\frac{k-\pi_0}{\pi_0}(R\sigma)^{\frac{1-\theta}{\theta}} = \frac{1-\gamma}{\gamma}$, $\frac{\partial k}{\partial \sigma} = 0$ if $\frac{1}{(R\sigma-1)} = \frac{1-\theta}{\theta}(1-\gamma)$. This polynomial can be expressed as:

$$\psi(k,R) \equiv \frac{\theta}{1-\theta} - \mu(k,R) = 0 \tag{29}$$

where $\mu(k,R) \equiv (1-\gamma)(R\sigma - 1)$.

It trivial to show that the locus defined by (29) is such that $\frac{dR}{dk} < 0$, $\underset{R\to\infty}{lim}\, k \to \pi_0$, and $\underset{R\to\frac{1}{\sigma}\left(1+\frac{\theta}{1-\theta}\right)}{lim} k \to \infty$. Below (above) the $\psi(k,R) = 0$ locus, $\psi(k,R) > (<)\, 0$. Equivalently, from (29), there exists an $R = \widehat{R} : \frac{\theta}{1-\theta} = \left(1-\gamma\left(k,\widehat{R}\right)\right)\left(\widehat{R}\sigma - 1\right) \equiv \mu\left(k,\widehat{R}\right)$. For all $R < \widehat{R}$, $\frac{\theta}{1-\theta} - (1-\gamma)(R\sigma-1) > 0$ and the numerator of (28) is positive. In contrast, for all $R \geq \widehat{R}$, the numerator of (28) is negative. Finally, it is clear that $\widehat{R}$ is strictly increasing in k. Therefore, I denote $\widehat{R}_A$ and $\widehat{R}_B$ to be the values of R satisfying (21) such that $\frac{\partial k}{\partial \sigma} = 0$ at $k < \tilde{k}$ and $k > \tilde{k}$, respectively. Clearly, at $R = \widehat{R}_B > \widetilde{R}$, there are two values of k solving (21). Denote the roots of (21) at $R = \widehat{R}_B$ by $\underline{k}$ and $\widehat{k}_B$, with $\underline{k} < \widehat{k}_B$. This completes the proof of Lemma 2.

4. **Proof of Proposition 2**. As emphasized in the text, the price effect dominates if a steady-state equilibrium occurs above the $R = \widehat{R}$ line. This

holds at the steady-state with high economic activity, B if at $\widehat{R}_B$, there is an excess demand for capital. As explained above, $\widehat{k}_B$ is the maximum amount of capital supplied when $R = \widehat{R}_B$. On the other hand, the demand for capital at $\widehat{R}_B$ is: $\left(f\prime\left(\widehat{R}_B\right)\right)^{-1}$. Consequently, we need $\widehat{k}_B < \left(f\prime\left(\widehat{R}_B\right)\right)^{-1}$. From (29), it is easy to verify that for a given stock of capital, $\widehat{R}$ is decreasing in σ. Specifically, the $\psi(k, R) = 0$ locus shifts downwards. Thus, we can define $\underline{\sigma}$: $\widehat{k}_B = \left(f\prime\left(\widehat{R}_B\right)\right)^{-1}$. An excess demand for capital at $R = \widehat{R}_B$ occurs if $\sigma > \underline{\sigma}$. This completes the proof of Proposition 2.