

THE UNIVERSITY OF TEXAS AT SAN ANTONIO, COLLEGE OF BUSINESS

# Working Paper SERIES

August 19<sup>th</sup>, 2009

Wp# 0070ECO-566-2009

## LIQUIDITY RISK, ECONOMIC DEVELOPMENT, AND THE EFFECTS OF MONETARY POLICY

Edgar A. Ghossoub  
Department of Economics  
University of Texas at San Antonio  
6900 North Loop 1604 West  
San Antonio, TX 78249-0634

Robert R. Reed  
University of Alabama  
Tuscaloosa  
AL, 35487  
Tel: (205) 348-8667

*Department of Economics,  
University of Texas at San Antonio,  
San Antonio, TX 78249, U.S.A*

*Copyright © 2008, by the author(s). Please do not quote, cite, or reproduce without permission from the author(s).*

# Liquidity Risk, Economic Development, and the Effects of Monetary Policy\*

Edgar Ghossoub  
The University of Texas at San Antonio

Robert R. Reed III<sup>†</sup>  
The University of Alabama

August 19, 2008

## Abstract

Empirical evidence indicates that monetary policy is not super-neutral in many countries. In particular, in high inflation economies, inflation is negatively related to economic activity. By comparison, inflation may be positively correlated with output in low inflation countries. We present a neoclassical growth model with money in which the incidence of liquidity risk is inversely related to aggregate capital formation. Interestingly, there may be multiple monetary steady-states where the effects of monetary policy vary. In poor economies, the financial system is highly distorted and higher rates of money growth are associated with less capital formation. In contrast, in advanced economies, a Tobin effect is observed. Since inflation exacerbates distortions from a coordination failure in the low capital steady-state, individuals become much more exposed to liquidity risk. Consequently, optimal monetary policy depends on the level of development.

*JEL Codes:* E41, E52, E31, O42

*Keywords:* Economic Development, Banks, Monetary Policy

## 1 Introduction

There is a growing awareness that monetary policy is not super-neutral in many countries. In particular, in high inflation economies, a significant amount of

---

\*We thank Joe Haslag, Jenny Minier, Harris Schlesinger, and two anonymous referees for their insightful comments. We have also received valuable feedback from seminar participants at the University of Alabama, the 2007 Southern Economic Association Meetings (New Orleans), and the Spring 2008 Midwest Economic Theory Meetings at the University of Illinois.

<sup>†</sup>For correspondence: Robert R. Reed, Department of Economics, Finance, and Legal Studies, 200 Alston Hall, University of Alabama, Tuscaloosa, AL 35487; Email: rreed@cba.ua.edu; Phone: (205) 348-8667.

evidence indicates that inflation is negatively related to economic activity.<sup>1</sup> For example, in their study of Argentina and Brazil, Bae and Ratti (2000) find that higher rates of money growth are associated with lower levels of output. By comparison, inflation may be positively correlated with output in low inflation economies. Notably, Bullard and Keating (1995) demonstrate that inflation is positively correlated with output in some low inflation countries while in others there is no relationship. Ahmed and Rogers (2000) focus on the U.S. economy. In their analysis, inflation and output are positively correlated. It has also been observed that inflation is generally higher in developing countries than industrialized economies.

*Why* do the effects of monetary policy vary across countries? In this paper, we propose an interesting explanation based on the degree of liquidity risk at different stages of economic development. In particular, in poor countries, individuals are more susceptible to events which cause them to liquidate their holdings of assets. This behavior is well documented in a number of studies of developing countries.<sup>2,3</sup> Since the exposure to liquidity risk varies across countries, individuals respond differently to rates of return in low income countries than in advanced economies. As a result, the effects of monetary policy will also vary between developing and advanced countries.

We proceed by outlining the details of our modeling framework. We study an overlapping generations economy with production similar to Diamond (1965). Following Townsend (1987) and Schreft and Smith (1997), there are two different geographically separated locations. There are also two types of assets: fiat money and physical capital. Within each location, agents have complete information regarding others' asset holdings. However, across locations, there is incomplete information such that individuals do not have the ability to establish and trade claims to assets. If an individual is forced to trade outside of his location of residence, he must acquire money balances. In this manner, private information leads to a transactions role for money.

---

<sup>1</sup>A number of studies investigate the relationship between inflation and the *growth rate* of output across countries. Barro (1995) concludes that inflation is negatively related to growth – regardless of the inflation rate. In contrast, Bruno and Easterly (1998) find that high inflation crises are associated with lower rates of growth.

<sup>2</sup>For example, Rosenzweig and Wolpin (1993) point out that households in poor countries are more likely to liquidate holdings of physical capital in response to adverse productivity shocks. In addition, Jacoby and Skoufias (1997) contend that poor, agrarian households withdraw children from school in the face of low realizations of income – thus, in developing countries, families are far more likely to liquidate investments in human capital. Moreover, the provision of social insurance (or lack of it) also plays a significant role. In particular, Chetty and Looney (2005) calculate that expenditures on social insurance programs in developed countries are nearly three times the amount of developing economies. As a result, individuals in developing countries are more likely to sell holdings if adverse shocks occur. Gertler and Gruber (2002) stress that health shocks would lead to less disruption of consumption smoothing if countries had more generous social insurance programs. The same arguments apply to labor market outcomes – publicly provided unemployment assistance would mitigate the economic costs of bad shocks.

<sup>3</sup>McPeak (2006) emphasizes that it is easier for wealthy households in poor countries to enter into risk-sharing arrangements. As a result, wealthy households do not need to liquidate investments as often as poor households.

Furthermore, individuals are subject to random relocation shocks. As money is the only asset that can cross locations, relocated agents must liquidate all their asset holdings into currency. Thus, random relocation is analogous to liquidity preference shocks in Diamond and Dybvig (1983). As a result, the model illustrates the risk pooling role of financial intermediaries.

In contrast to previous work, we assume that the probability of a liquidity shock is inversely related to the aggregate capital stock. We view that this relationship serves as a proxy for the linkages between economic development and liquidity risk observed in many studies. As in a standard random relocation model such as Schreft and Smith, an individual's return from bank deposits is stochastic. Expected income depends on the probability distribution of an individual's location status and the return in each state. However, in the standard random relocation model, the probability of a liquidity shock is independent of real variables. *By comparison, in our framework, the probability distribution depends on the amount of capital accumulation.* Since money is dominated in rate of return, income will be lower if an individual is forced to liquidate assets early. Moreover, relocation is less likely if capital accumulation is higher. From this perspective, *the probability distribution of income in advanced economies first-order stochastically dominates the probability distribution in developing countries.*<sup>4</sup>

As the distribution of income in an economy with a high capital stock dominates the probability distribution in an economy with a low capital stock, there are *positive spillovers* from capital accumulation in our model. Moreover, the economy-wide stock of capital influences the returns of a bank – if the probability of a relocation shock is low, individuals are more likely to derive earnings from physical capital. As a result, each financial institution will devote more resources to capital if the economy-wide stock of capital is high. In this manner, *strategic complementarities from investment in physical capital* are an important aspect of our modeling framework.<sup>5</sup>

Due to the presence of strategic complementarities, multiple monetary steady-states can occur. In the economy with a low amount of capital accumulation, an individual is highly likely to need to liquidate her asset holdings. Consequently, an individual's expected utility will be low. Moreover, her expected income from investment in capital will be low. In turn, banks acquire large amounts

---

<sup>4</sup>This definition follows Hadar and Russell (1969). Foster and Shorrocks (1988) propose that income distributions between countries can be compared using various degrees of stochastic dominance. Bishop et. al. (1991) construct data on the income distributions of 26 different countries. Based on their evidence, international comparisons of income distributions can often be ranked according to first-order stochastic dominance. Moreover, their results indicate that the stochastic dominance of one income distribution over another generally depends on each country's level of economic development. That is, the income distributions of developed economies tend to first-degree dominate the income distributions of developing countries. This is consistent with the primary assumption of our modeling framework – the probability distribution of income in an economy with a high amount of capital accumulation dominates an economy with a low stock of capital.

<sup>5</sup>As discussed in Drazen (1987) and Cooper and John (1988), strategic complementarities may be observed in situations where an individual's payoff depends on economy-wide aggregates.

of money balances and devote little resources to productive activity.<sup>6</sup> The reduced state of economic development exacerbates the problem of liquidity risk, increasing the need for banks to hold money. In this manner, a coordination failure occurs – the level of income is inefficiently low since no individual agent realizes any gains from deviating from equilibrium behavior.<sup>7</sup> In addition, as in Schreft and Smith, the low amount of capital formation leads to high nominal interest rates – another sign of the degree of inefficiency in the financial system. In contrast, in the economy with a high capital stock, there is little incentive for banks to acquire liquid assets. This allows banks to channel more resources to investment in physical capital which further stimulates the amount of income in the economy.

Interestingly, the model generates important insights regarding the impact of monetary policy between developing and advanced countries. In the steady-state with a low amount of capital accumulation, the banking system is highly distorted and financial institutions will hold highly liquid portfolios.<sup>8</sup> *This implies that the marginal benefit from holding money will be much lower if inflation is higher.*

If the probability of relocation is independent of the economy’s stage of development, banks respond to inflation by reallocating asset holdings so that the costs of holding money fall. This is accomplished by reducing their holdings of money and acquiring more physical capital. As a result, the marginal utility of money balances is partially restored.

However, in our framework, individuals’ need to liquidate assets depends on the level of economic development. In particular, in poor countries, the degree of liquidity risk is highly sensitive to changes in the stock of capital. Rather than lowering money balances in order to increase the marginal utility of an individual who experiences a positive realization of the location shock, *banks in a highly distorted financial system increase the value of money by taking actions that collectively increase the degree of liquidity risk* – that is, they lower investment in productive assets. Due to the increased level of poverty, individuals are more likely to liquidate their deposits.<sup>9</sup> Therefore, in a poor country, higher inflation

---

<sup>6</sup>Kochar (2004) concludes that the portfolio choice of assets in developing countries depends on the likelihood that a household will suffer from an adverse shock. In his work, households anticipating a higher likelihood of poor health outcomes will devote less income to illiquid assets. Since life expectancies in developed countries are longer than in poor countries, it is reasonable to infer that individuals in developing countries are more likely to experience adverse health shocks than in advanced economies. Therefore, portfolios of assets in developing countries will be relatively more liquid than in advanced economies.

<sup>7</sup>Diamond (1982) develops a model with trading externalities where a coordination failure can arise as an equilibrium outcome. In addition, Laing, Palivos, and Wang (1997) construct a search-theoretic model of the labor market with endogenous human capital accumulation. In their work, multiple balanced growth paths may exist due to positive feedback between the likelihood of successful job matching and the returns to investment in human capital. Murphy, Shleifer, and Vishny (1989) show that pecuniary externalities and fixed costs may be responsible for multiple equilibria with different levels of industrialization.

<sup>8</sup>Haslag and Koo (1999) and Rousseau and Wachtel (2001) discuss that financial institutions in high inflation countries hold a relatively large amount of liquid assets.

<sup>9</sup>Bencivenga and Smith (1991) demonstrate that financial intermediaries, through provision of risk pooling among depositors, reduce socially unnecessary holdings of liquid assets. This

rates are associated with a reverse-Tobin effect.

In contrast, financial institutions in advanced countries devote more resources to productive activity. Since these countries are more developed, individuals are less susceptible to liquidity risk. Furthermore, changes in the stock of capital will have little impact on the marginal utility of money. As a result, the high costs of holding money become the dominant factor in the portfolio choice of intermediaries in advanced economies. Consequently, in developed nations, banks will acquire additional amounts of physical capital at higher inflation rates. Therefore, inflation will be associated with a Tobin effect in advanced countries.

The analysis concludes by investigating the behavior of dynamical equilibria. To begin, we derive a phase diagram to examine the global information on the stability properties of the steady-states. The results demonstrate that strategic complementarities lead to meaningful insights into the stability of economies with different levels of initial resources. Notably, economies must have sufficient resources in order to stabilize over time. For example, the low capital steady-state is a source. Investigation of local dynamics also demonstrates that undamped oscillations are possible – especially at high inflation rates. That is, in relatively poor economies, high rates of money growth can lead to endogenous fluctuations that never disappear. Therefore, the effects of high inflation policies may be particularly unpredictable. By comparison, the high capital steady-state is saddle-path stable. At sufficiently high amounts of initial resources and low nominal interest rates, the economy can converge to the steady-state. Since there is a unique path to the steady-state for advanced economies, it is also possible to determine the impact of a change in monetary policy along the transition to the new long-run equilibrium.

Our work contributes to a growing literature that examines the interactions between economic development, financial market activity, and monetary policy such as Schreft and Smith (1997) and Antinolfi, Landeo, and Nikitin (2007). In order to understand how monetary policy affects real activity, it is important to identify systematic differences in the characteristics of low and high income countries. For example, Schreft and Smith point out that banks allocate a large fraction of deposits to government bonds in developing countries. Antinolfi et. al. highlight that individuals in developing countries hold a large amount of foreign currency. However, there is another important factor – as emphasized, individuals in poor countries are more susceptible to liquidity risk.

As all three papers address different aspects of the development process, they also lead to different monetary transmission mechanisms. In Schreft and Smith, inflation affects the ability of the government to intervene in private financial markets. In developing economies, higher rates of inflation allow the government to issue more bonds which crowds out capital formation. Advanced economies are associated with government budget surpluses. As a result, the

---

suggests that the development of the banking system reduces an economy's reliance on money. In addition, Boyd, Levine, and Smith (2001) find that higher rates of inflation are associated with a deterioration in financial sector activity. Thus, it is reasonable to infer that inflation distorts risk sharing and induces individuals to hold more cash balances.

actions of the monetary authority have an impact on the amount of lending to the banking system. Antinolfi et. al. demonstrate that at significantly high inflation rates, individuals will hold more foreign currency if the rate of return to domestic currency falls. In our framework, there are only two assets available – fiat money and capital. In developing countries, as the exposure to liquidity risk is high, inflation has a significant impact on agents’ expected utility since there is a high probability of a liquidity shock. In advanced countries, exposure to liquidity risk is relatively unaffected by investment activity. Higher inflation rates raise the cost of holding money and induce individuals to switch to the productive asset. This motivation is quite similar to standard Tobin-effect logic – yet, individuals’ need for cash is random; it also depends on the extent of capital accumulation.

The paper is organized as follows. In Section 2, we describe the model and study the impact of monetary policy. Section 3 studies the design of optimal monetary policy. Section 4 discusses the stability properties of our framework. Finally, we offer concluding remarks in Section 5. Most of the technical details are presented in the Appendix.

## 2 Environment

We consider a discrete-time economy populated by an infinite sequence of two-period lived overlapping generations, plus an initial old generation. In particular, the economy consists of two spatially separated locations. At the beginning of each time period, a new generation of individuals is born on each island with a population measure equal to one. Although the population resides in two separate locations, there is a single consumption good available on both islands. Individuals derive utility from consuming the economy’s consumption good ( $c$ ) when old. The utility function is expressed by  $U(c) = \frac{c^{1-\theta}}{1-\theta}$ , with  $\theta \in [0, 1)$ .

When young, agents are endowed with one unit of labor which they supply inelastically. In contrast, agents are retired when old. As a result, the total labor supply at each date is equal to the total population mass of young individuals. Private information serves as the primary trade friction in the economy. Although each island is characterized by complete information, communication across islands is not possible. Consequently, private liabilities do not circulate.

There are two types of assets in this economy: money (fiat currency) and capital. Define the per worker monetary base and capital stock by  $\bar{m}_t$  and  $k_t$  respectively. At the initial date 0, the generation of old agents at each location is endowed with the aggregate money and capital stocks. Since the total population is equal to one, these variables also represent aggregate values. Moreover, one unit of investment by a young agent in period  $t$  becomes one unit of capital next period. Equivalently,  $i_t$  units of goods invested become  $k_{t+1}$  units of capital in the subsequent period.

Assuming that the price level is common across locations, we refer to  $P_t$  as the number of units of currency per unit of goods at time  $t$ . Thus, in real terms, the supply of money per worker is  $m_t = \bar{m}_t/P_t$ .

The consumption good is produced by a representative firm which rents capital and hires labor from young agents. The production function is given by  $Y_t = F(K_t, L_t)$ , where  $K_t$  denotes the aggregate capital stock and  $L_t$  denotes the amount of labor hired. Equivalently, output per worker is expressed by  $y_t = f(k_t)$  and satisfies standard Inada conditions. To simplify the algebra, we assume that capital depreciates completely during the production process. Due to perfect competition, factor inputs are paid their marginal products. The rental rate and wage rates in period  $t$  are respectively:

$$R_{t-1} = f_{k_t}(k_t) \quad (1)$$

$$w_t = w(k_t) = f(k_t) - k_t f_{k_t}(k_t) \quad (2)$$

Moreover, individuals in the economy are subject to relocation shocks. Each period, a fraction of young agents must move to the other island. These agents are called “movers.” Limited communication and spatial separation make trade difficult between different locations. As in standard random relocation models, fiat money is the only asset that can be carried across islands. Furthermore, currency is universally recognized and cannot be counterfeited — therefore, it is accepted in both locations.

Since money is the only asset that can cross locations, depositors who learn they will be relocated will liquidate all their asset holdings into currency. Random relocation thus plays the same role that liquidity preference shocks perform in Diamond and Dybvig (1983). As banks provide insurance against the shocks, each young depositor will put all of her income in the bank rather than holding assets directly.

In contrast to previous work such as Schreft and Smith (1997), the probability of a liquidity shock is inversely related to the aggregate capital stock. This assumption reflects the linkages between economic development and liquidity risk observed across countries. In particular, in poor countries, individuals are more susceptible to events which force them to liquidate their asset holdings. Therefore, we posit that in any time period  $\pi_t = \pi(K_t)$ , where  $0 < \pi(K_t) < 1$  and  $\pi'(K_t) < 0$ . Alternatively, one might interpret that an economy’s reliance on cash balances depends on the level of economic development. For tractability,  $\pi(K_t) = \frac{\pi_0}{K_t}$ , where  $0 < \pi_0 < K_t$ . At a given amount of capital formation, a higher value of  $\pi_0$  indicates that individuals are more exposed to liquidity risk. It also reflects the magnitude of the external impact from the aggregate capital stock to the probability of relocation since individuals take the aggregate capital stock as given.<sup>10</sup>

In addition to depositors, there is a central bank that follows a constant money growth rule. The aggregate nominal stock of cash in period  $t + 1$  can be

---

<sup>10</sup>Recent work by Bhattacharya, Haslag, and Martin (2007) considers that the probability of relocation depends on the amount of effort exerted by agents. In particular, agents can reduce the probability of moving by exerting costly effort. However, in their framework, individuals internalize that their choices affect the likelihood of a liquidity shock.



expressed by  $M_{t+1} = \sigma M_t$ , where  $\sigma$  is the gross rate of money creation. In per capita real terms:

$$m_{t+1} = \sigma \frac{P_t}{P_{t+1}} m_t \quad (3)$$

where  $\frac{P_t}{P_{t+1}}$  is the gross rate of return on money balances between period  $t$  and  $t + 1$ .

## 2.1 Trade

### 2.1.1 The bank's problem

Due to perfect competition in the banking sector, banks choose portfolios to maximize the expected utility of each depositor. Since financial intermediaries reduce depositors' consumption variability, each of them chooses to deposit all of their income. The bank promises a gross real return  $r_t^m$  if the young individual will be relocated and a gross real return  $r_t^n$  if not. Since the market for deposits is perfectly competitive, financial intermediaries take the return on deposits as given. As previously mentioned, banks take the aggregate capital stock as given.

As of period  $t$ , a bank determines the amount of real money balances to hold,  $m_t$ , and chooses how much to invest in capital,  $i_t$ . The bank's balance sheet constraint is expressed by:

$$m_t + k_{t+1} \leq w_t ; t \geq 0 \quad (4)$$

Announced deposit returns must satisfy the following constraints. First, since currency is the only asset that can be transported across locations, relocated agents will choose to liquidate their asset holdings into currency. Depending on the bank's money holdings and the inflation rate, the return to movers satisfies:

$$\pi(K_t) r_t^m w_t \leq m_t \frac{P_t}{P_{t+1}} \quad (5)$$

In addition, we choose to study equilibria in which money is dominated in rate of return. Therefore, banks will not carry money balances between periods  $t$  and  $t + 1$ . The bank's total payments to non-movers are therefore paid out of its return on capital in  $t + 1$ :

$$(1 - \pi(K_t)) r_t^n w_t \leq R_t k_{t+1} \quad (6)$$

Perfectly competitive banks choose return schedules and portfolio allocations to maximize a typical depositor's expected utility subject to the constraints described above. Alternatively, one might view that each bank makes choices to maximize the *total* level of welfare among its depositors. In either case, a bank will choose values of  $r_t^m, r_t^n, m_t$ , and  $k_{t+1}$  in order to solve the problem:

$$\underset{r_t^m, r_t^n, m_t, k_{t+1}}{Max} \quad \pi(K_t) \frac{(r_t^m w_t)^{1-\theta}}{1-\theta} + (1-\pi(K_t)) \frac{(r_t^n w_t)^{1-\theta}}{1-\theta} \quad (7)$$

subject to (4), (5), and (6).

In our framework, it is particularly important to understand the bank's decision to acquire money balances. The bank's choice of money holdings is such that:

$$\left(\frac{1}{\sigma}\right) U' \left(\frac{m_t}{\sigma} * \frac{1}{\frac{\pi_0}{K_t}}\right) = RU' \left(\frac{R}{1 - \frac{\pi_0}{K_t}} * (w_t - m_t)\right) \quad (8)$$

From (8), banks acquire money balances so that the marginal benefit of money among movers is equal to the marginal cost incurred by non-movers who derive income from earnings on capital. Given depositors' utility functions, the bank's money demand function is:

$$m_t = \frac{w(k_t)}{\left[1 + \frac{(1-\pi(K_t))}{\pi(K_t)} \left(R_t \frac{P_{t+1}}{P_t}\right)^{\frac{1-\theta}{\theta}}\right]} \quad (9)$$

Alternatively,

$$\gamma \left(R_t, K_t, \frac{P_{t+1}}{P_t}\right) = \frac{m_t}{w(k_t)} = \frac{1}{\left[1 + \frac{(1-\pi(K_t))}{\pi(K_t)} \left(R_t \frac{P_{t+1}}{P_t}\right)^{\frac{1-\theta}{\theta}}\right]} \quad (10)$$

where  $\gamma$  is the fraction of deposits allocated to money balances.

As in standard random relocation models in which the probability of relocation is independent of real variables, the demand for money balances depends on the likelihood that individuals will need to liquidate their portfolios early. However, our framework incorporates the idea that the degree of liquidity of risk varies across different stages of development. In poor countries, individuals are more susceptible to events that lead them to liquidate their holdings of assets. For a variety of reasons, these problems are not as severe in advanced economies. Consequently, in economies with a larger capital stock, banks demand less money.

Finally, in equilibrium, the rate of return to movers and non-movers can be expressed as:

$$r_t^m = \frac{\gamma \left(R_t, K_t, \frac{P_{t+1}}{P_t}\right) P_t}{\pi(K_t) P_{t+1}} \quad (11)$$

and

$$r_t^n = \frac{1 - \gamma \left(R_t, K_t, \frac{P_{t+1}}{P_t}\right)}{1 - \pi(K_t)} R_t \quad (12)$$

## 2.2 General Equilibrium

We now combine the results of the preceding section and characterize the equilibrium for the economy. In equilibrium labor receives its marginal product, (2), and the labor market clears:

$$L_t = 1 \quad (13)$$

From the bank's balance sheet, (4), and its demand for cash reserves, (10), we can obtain the total supply of capital in period  $t + 1$  :

$$k_{t+1} = \left[ 1 - \gamma \left( R_t, k_t, \frac{P_{t+1}}{P_t} \right) \right] w(k_t) \quad (14)$$

where  $k_t = K_t$  in equilibrium. That is, in equilibrium, an individual bank's choice of capital investment is equal to the average level in the banking sector.

Moreover, the capital market clears when the supply of capital, (14), is equal to its demand by firms, (1). Finally, using the central bank's money growth rule, (3), money market clearing requires that:

$$\gamma \left( R_{t+1}, k_{t+1}, \frac{P_{t+2}}{P_{t+1}} \right) = \gamma \left( R_t, k_t, \frac{P_{t+1}}{P_t} \right) \sigma \frac{P_t}{P_{t+1}} \frac{w(k_t)}{w(k_{t+1})} \quad (15)$$

Conditions (1), (3), (14), and (15) characterize the behavior of the economy at each point in time.

## 2.3 Steady-State Analysis

Define the nominal return to capital by  $I_t = \frac{P_{t+1}}{P_t} R_t$ . Imposing steady-state on (1), (3), (14), and (15), the following two loci characterize the behavior of the economy in the long-run:

$$\Omega(k) = \frac{k}{w(k)} = 1 - \gamma(k, I) \quad (16)$$

$$I = \sigma f_k(k) \quad (17)$$

where  $\Omega(k)$  is the fraction of deposits allocated towards capital investment. Moreover,  $\Omega'(k) > 0$ ,  $\Omega''(k) < 0$ , and  $\Omega(0) = 0$ . In addition, the fraction of deposits allocated to cash reserves in the steady-state is:

$$\gamma(k, I) = \frac{1}{\left[ 1 + \frac{k - \pi_0}{\pi_0} I^{\frac{1-\theta}{\theta}} \right]} \quad (18)$$

Equation (16) represents the relationship between the supply of capital by banks and the nominal interest rate. In addition, (17) reflects the profit-maximizing amount of capital by firms. The demand for capital, from (17),

is negatively associated with  $I$  as shown in Figure 1 below. Lemma 1 describes the behavior from (16):

**Lemma 1.** *The locus defined by (16) behaves as follows:*

- (a)  $\lim_{k \rightarrow \pi_0} I(k) \rightarrow \infty$  and  $\lim_{k \rightarrow \Omega^{-1}(1)} I(k) \rightarrow \infty$
- (b) Define  $\hat{k}$  such that  $-\frac{\partial \gamma(k, I)}{\partial k} \Big|_{k=\hat{k}} = \Omega'(\hat{k})$ . If  $k \leq (>)\hat{k}$ ,  $\frac{\partial I}{\partial k} \leq (>)0$ .

The first result from part (a) of Lemma 1 indicates that the probability of a liquidity shock will be bounded below 1. At very low levels of capital formation,  $k \rightarrow \pi_0$ , the probability of relocation is very high. Consequently, banks would want to acquire large amounts of money. In turn, the nominal return to equity must be very high so that banks hold *both* types of assets. Therefore, the nominal interest rate will adjust so that the probability of a liquidity shock is bounded. The second result from part (a) of Lemma 1 shows that investment in capital cannot exceed bank deposits. That is, in advanced economies, the return to capital (which may be viewed as the cost of holding money) must be extremely high in order for banks to devote nearly all of their deposits to capital holdings.

The remainder of Lemma 1 demonstrates that the relationship between equity returns and the supply of capital is different than standard monetary growth models. For low levels of capital, equity returns and capital accumulation are negatively related. By comparison, in economies with more capital, equity returns are associated with higher investment activity.

In order to better understand the determinants of the supply of capital, it is useful to review the incentives of financial institutions across different levels of capital accumulation. For example, it is possible that two different economies (capital stocks) can be associated with the same rate of return.

In contrast to the standard random relocation model, the probability distribution of deposit returns depends on the amount of capital accumulation. Notably, as the return to capital is higher than the return to money balances, the probability distribution of income in advanced economies first-order stochastically dominates the income distribution in developing countries. In this manner, there are *positive spillovers from capital accumulation* in our framework.

In an advanced economy where the probability of a liquidity shock is low, individuals are more likely to derive earnings from capital. In turn, each financial institution will seek to devote more funds to investment in physical capital. Alternatively, at lower amounts of capital formation, an individual's expected utility will be low since it is unlikely that she will be able to gain income from capital. As a result, in economies with little resources, banks allocate less funds to productive activity. That is, they will hold a large amount of liquid assets and acquire less capital to rent to firms. Since an economy-wide aggregate, the stock of capital, affects the choices of each financial institution, *strategic complementarities from investment in physical capital* occur.

Consequently, a given return to capital can be associated with two different levels of the supply of capital by banks. In poor economies, individuals are more susceptible to liquidity shocks. In order for banks to be willing to hold both money and capital, the return to capital must be high. In advanced economies, liquidity risk is less significant – a high return to capital is associated with a large cost of holding money. In turn, financial institutions choose to invest a relatively large amount of funds in physical capital.

Furthermore, the economy’s level of development plays a strong role in determining the relationship between equity returns and investment in physical capital. To begin, we discuss the impact of equity returns on the portfolio choice of financial intermediaries in developing countries (economies in which  $k < \hat{k}$ ). If the return to capital increases, *the cost of holding money (from the foregone earnings on capital in the good state) is higher.*

If banks were unable to influence the probability of individuals to experience each state, as in a model where the probability of relocation is independent of real variables, then they would not be able to affect depositors’ need for cash balances relative to capital. Accordingly, intermediaries would restore the marginal benefit and cost from holding money by adjusting portfolios to provide more income to individuals who experience the good state. By lowering the amount of money balances, non-movers would obtain higher earnings from capital. As a result, the marginal benefit of holding money (for movers) would also be higher.

*However, the incentives of the bank are much different when aggregate portfolio choices affect the need for cash (the probability of the bad state).* This is particularly important in developing countries. In our framework, at low levels of capital formation, the probability of the bad state ( $\frac{\pi_0}{k}$ ) is highly sensitive to changes in capital accumulation. If investment in capital changes, the probability distribution of income will be significantly affected. Accordingly, if the return to capital is higher, financial institutions do not need to provide more income to individuals who enter the good state. Instead, banks can re-optimize and increase the value of money by accumulating less capital so that more individuals will need to liquidate their deposits. *That is, they can increase the value of money by taking actions that collectively increase the degree of liquidity risk.*

As mentioned, if the capital stock is initially low, this is easy to accomplish since the probability of the bad shock is highly sensitive to the amount of capital accumulation. To be specific, it is relatively easy to increase the value of money balances if  $k < \hat{k}$ . Therefore, the capital supply curve is downward-sloping as long as  $-\frac{\partial \gamma(k, I)}{\partial k} > \Omega'(k)$ .

In contrast, in advanced economies, individuals are less susceptible to liquidity risk. Moreover, changes in the stock of capital will have little impact on the value of money;  $-\frac{\partial \gamma(k, I)}{\partial k}$  is relatively low if  $k > \hat{k}$ . This indicates that portfolio decisions by financial institutions in advanced countries will be dominated by the costs of holding money at higher rates of return to capital. In turn, the supply curve of capital is upward-sloping in economies with large amounts of

resources.

We proceed by studying existence of steady-state equilibria with positive nominal interest rates. As listed in Proposition 1 below, there may be multiple monetary steady-state equilibria in which  $I > 1$ .

**Proposition 1. Existence of Monetary Steady-States.** *Suppose that  $\sigma > \underline{\sigma}_0$ , where  $\underline{\sigma}_0$  satisfies  $\underline{\sigma}_0 f_k(\hat{k}) = \left( \pi_0 \frac{\Omega'(\hat{k})}{(1-\Omega(\hat{k}))^2} \right)^{\frac{\theta}{1-\theta}}$ . If this condition holds, multiple monetary steady-states may exist. Consider the following:*

(a) *Let  $\pi_0 > \underline{\pi}$  where  $\underline{\pi}$  is such that  $1 = \underline{\pi} \frac{\Omega'(\hat{k})}{(1-\Omega(\hat{k}))^2}$ . Under these conditions, there are two monetary steady-states that exist.*

(b) *Suppose that  $\pi_0 < \underline{\pi}$ . Define  $\underline{\sigma}_1$  and  $\underline{\sigma}_2$  so that  $\underline{\sigma}_1 = \frac{1}{f_k(\hat{k})}$  and  $\underline{\sigma}_2 = \frac{1}{f_k(\hat{k})}$ , with  $\underline{\sigma}_1 < \underline{\sigma}_2$ . If  $\sigma > \underline{\sigma}_2$ , there are two monetary steady-states that exist. However, if  $\sigma \in (\underline{\sigma}_1, \underline{\sigma}_2]$ , a unique steady-state occurs.*

The first condition in Proposition 1,  $\sigma > \underline{\sigma}_0$ , reflects behavior between financial institutions and firms which rent the services of capital. In particular, it guarantees that the demand curve for capital will always lie above the capital supply curve at the inflection point of the supply curve. Suppose that the nominal interest rate associated with the inflection point is listed as  $\hat{I}$ . Alternatively, it could be stated that  $\hat{I}$  represents the lowest nominal interest rate that satisfies a bank's balance sheet condition. For example, consider an economy in which banks choose to acquire  $\hat{k}$  units of physical capital. If the nominal return to capital is higher than  $\hat{I}$ , the demand for money would not be high enough to exhaust available deposits. Yet, at  $k = \hat{k}$  and  $\sigma > \underline{\sigma}_0$ , the marginal revenue of capital is relatively high – consequently, at  $\hat{I}$ , there would be an excess demand for capital. As a result, there are two combinations of nominal interest rates and capital formation where the two curves intersect. Otherwise, a steady-state will not exist.

Case (a) demonstrates that multiple steady-states will occur if there is significant exposure to liquidity risk. Please refer to Figure 1 below for an example – in this situation, there is a fairly high degree of risk regardless of the amount

of capital accumulation.

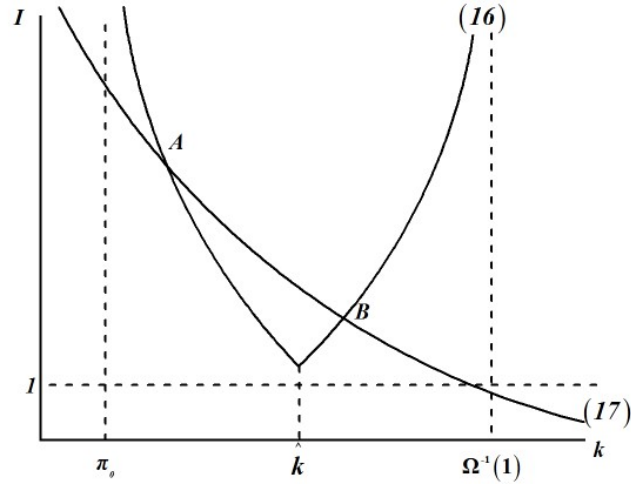


Figure 1: Risk and Multiple Steady-States

As a result, the lowest nominal interest rate associated with banks' demand for money balances can also be relatively high. Consequently, at moderate rates of money growth and a relatively significant need to liquidate assets, multiple steady-states with positive nominal interest rates can exist.

By comparison, case (b) considers economies in which individuals are not as likely to experience liquidity shocks. Nevertheless, if the inflation rate is sufficiently high, multiple nontrivial steady-states may still exist. This possibility is illustrated in Figure 2.

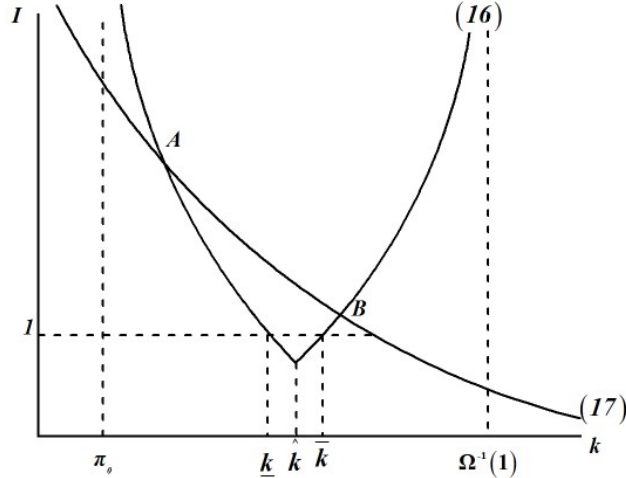


Figure 2: Inflation and Multiple Steady-States

However, if there is only a moderate degree of liquidity risk and inflation is not that high, a valid monetary steady-state for the high capital economy (represented by point  $B$ ) will not occur. Since the money growth rate is not high enough, the rate of return to capital in economy  $B$  will not be high enough to dominate the return to money.

Why are multiple steady-state equilibria possible in our framework? The externality from the aggregate capital stock is a source of inefficiency in the economy. Moreover, from the discussion of Lemma 1, strategic complementarities from investment in physical capital take place in our model. As a result, in the economy represented by point  $A$  in Figure 1, a coordination failure is a distinct possibility – the level of income is inefficiently low since no individual bank realizes any gains from increasing investment in physical capital. The economy has a low level of development where depositors are highly likely to need to liquidate their asset holdings. This lowers expected utility and the returns to capital. The reduced state of economic development exacerbates the problem of liquidity risk, increasing the need for banks to hold money. By comparison, in the economy with a high capital stock, there is little incentive for banks to acquire liquid assets. This allows banks to channel more resources to investment in physical capital which further stimulates the amount of income in the economy. Case (a) demonstrates that multiple steady-state equilibria will occur as long as the externality from the aggregate capital stock is sufficiently strong. Alternatively, there will be multiple steady-states if there is significant exposure to liquidity risk, regardless of the economy’s level of development – even in the high capital economy (represented by point  $B$ ), the demand for money will be fairly high. Consequently, there is a positive return to equity and money will be dominated in rate of return.



Case (a) may be considered as a statement on conditions regarding the supply of capital. If  $\pi_0$  is high enough, multiple steady-states with positive nominal interest rates will exist because the supply of capital is sufficiently low. In comparison, case (b) demonstrates that multiple steady-state equilibria will exist if the inflation rate is significantly high. This case could be considered to be a statement on conditions regarding the demand for capital – if the demand for capital goods is sufficiently high, there will be multiple steady-states with a positive return to equity. Nevertheless, as is clear from Lemma 1, the rationale for multiple steady-states relies on the idea that the degree of liquidity risk depends on a country’s stage of development. As a result, the effect of equity returns depends on the extent of capital formation.

*The Effects of Monetary Policy*

Our framework provides a useful interpretation for the asymmetric effects of monetary policy between developing and advanced countries. If there are multiple valid steady-state equilibria, the effects of monetary policy will vary across the equilibrium level of capital accumulation. If the rate of money growth increases, the demand for capital by firms shifts up as illustrated in Figure 3:

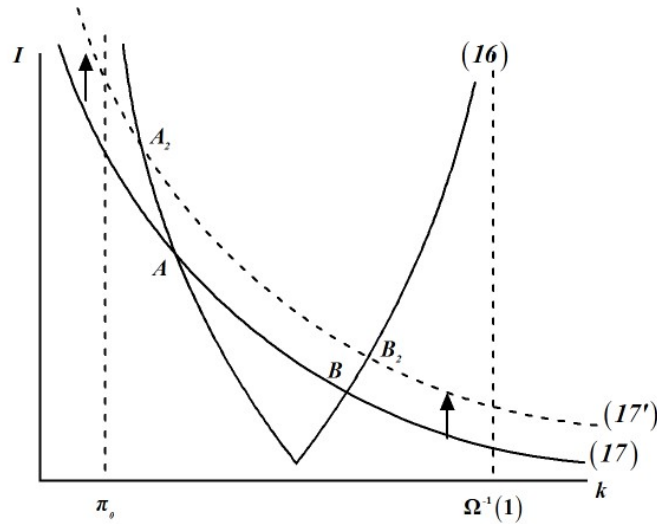


Figure 3: Effects of Monetary Policy

As shown in the figure, the effects of monetary policy depend on the level of economic development. In poor countries, higher rates of inflation adversely affect capital accumulation. However, in advanced countries, the model predicts that a Tobin effect will be observed.

In order to gain insight into the effects of monetary policy, it is useful to review a bank’s condition for its portfolio choice. To begin, consider the low capital steady-state,  $k = k_A$  :

$$\left(\frac{1}{\sigma}\right) U'\left(\frac{m}{\sigma} * \frac{1}{\frac{\pi_0}{k_A}}\right) = RU'\left(\frac{R}{1 - \frac{\pi_0}{k_A}} * (w - m)\right)$$

At a higher inflation rate, the marginal benefit from holding money will be lower. Since the economy is relatively poor, portfolios are highly liquid and sensitive to changes in capital accumulation. Rather than reducing holdings of money to increase the marginal utility of an individual in the bad state, banks collectively choose allocations to increase the value of money. Banks in a highly distorted financial system increase the value of money by taking actions that increase the probability of a liquidity shock.

In advanced economies, financial institutions devote more resources to productive activity. Since the economy is more developed, individuals are less susceptible to liquidity risk. Changes in the stock of capital will have little impact on the value of money. Therefore, the high costs of holding money represent the dominant factor in the portfolio choice of financial institutions in developed countries. This leads to a Tobin-effect.

In addition to the asymmetric effects of monetary policy on capital accumulation across countries, our model suggests that the welfare costs of inflation are also likely to depend on the level of development. In poor countries, inflation will be associated with a significant increase in the degree of exposure to liquidity risk. In advanced countries, there is little impact. This indicates that the design of monetary policy *should* vary across countries. We turn to this issue in the following Section.

### 3 Monetary Policy and Welfare

We now turn to examine the effects of monetary policy on welfare at different steady-states. As a benchmark, we follow Antinolfi and Keister (2006) by studying a world with complete information. In a centralized environment, communication between locations is possible. Therefore, money will not be held in equilibrium and the optimal level of capital formation is the golden rule:  $f_k(k) = 1$ . As all income is invested in capital, the consumption of each individual is equal to  $R_{GR}w(k_{GR})$ , where the subscript  $GR$  refers to the allocation under the golden rule. Since each agent consumes the same amount regardless of her type and location, the level of utility of each depositor is:  $\frac{(w(k_{GR}))^{1-\theta}}{1-\theta}$ .

While the first best allocation may not be attained in a decentralized world with private information and limited communication, it serves as a benchmark to examine the degree of distortions across steady-states. In addition, we are able to compare the allocation under the optimal monetary policy to the first best allocation. From (7), (2), (4) - (6), (16) and (17), the expected utility of a representative individual in the steady-state is expressed as:

$$\Lambda(\sigma) = \frac{\left(\frac{\pi}{k(\sigma)}\right)^\theta \left(\frac{\gamma(k(\sigma), I(k(\sigma)))}{\sigma}\right)^{1-\theta} + \left(1 - \frac{\pi}{k(\sigma)}\right)^\theta \{f_k(k(\sigma)) [1 - \gamma(k(\sigma), I(k(\sigma)))]\}^{1-\theta}}{(1-\theta)} w(k(\sigma))^{1-\theta} \quad (19)$$

While these comparisons are non-trivial to prove analytically, numerical calculations provide important insights. We assume that the production function is given by  $f(k) = \Psi k^\alpha$ , where  $\Psi$  is a technology parameter and  $\alpha$  is the capital share of total output. The baseline set of parameters is:  $\pi_0 = 1.2$ ,  $\theta = 0.5$ ,  $\alpha = 0.5$ , and  $\Psi = 5.36$ .<sup>11</sup> The results are illustrated in the following figures. Exact numerical values are provided in tables in the Appendix. Under these parameters, we first note that a valid monetary steady-state will not exist if  $\sigma < 0.8$ . This takes place because the demand for capital is not sufficient to exhaust the supply of capital (i.e., the curves from (16) and (17) curves do not intersect).

The relationship between inflation and capital formation at each steady-state confirms our work from section 2 above. Moreover, for a given rate of money growth, the market choice of investment is sub-optimal. Figure 4 illustrates the impact of inflation on capital formation along with a comparison to the golden rule level of capital accumulation:

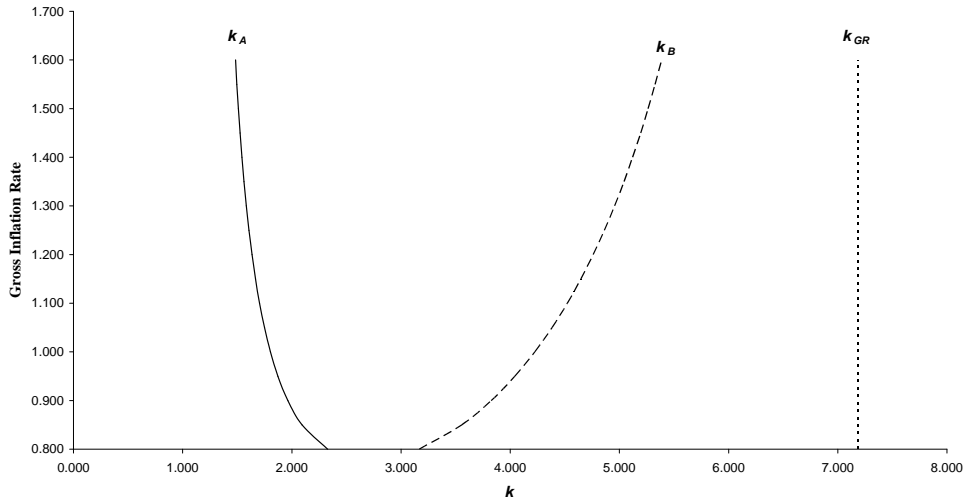


Figure 4: The Effects of Inflation on Capital Formation

<sup>11</sup>The parameter space under which our results in this section hold is significantly large.

The locus  $k_A$  shows the relationship between inflation and capital accumulation for low capital steady-states while  $k_B$  reflects behavior in the high capital steady-state. Since the level of investment activity affects the degree of liquidity risk, it results in a positive externality that leads to under-investment in capital. However, the degree of inefficiency is much higher at low levels of development. Inflation exacerbates distortions associated with the coordination failure in the low capital steady-state, leaving individuals much more exposed to liquidity risk. This occurs because the probability of relocation is highly sensitive to changes in capital accumulation at low levels of capital formation. Please refer to Figure 5 to examine the impact of inflation on the degree of liquidity risk:

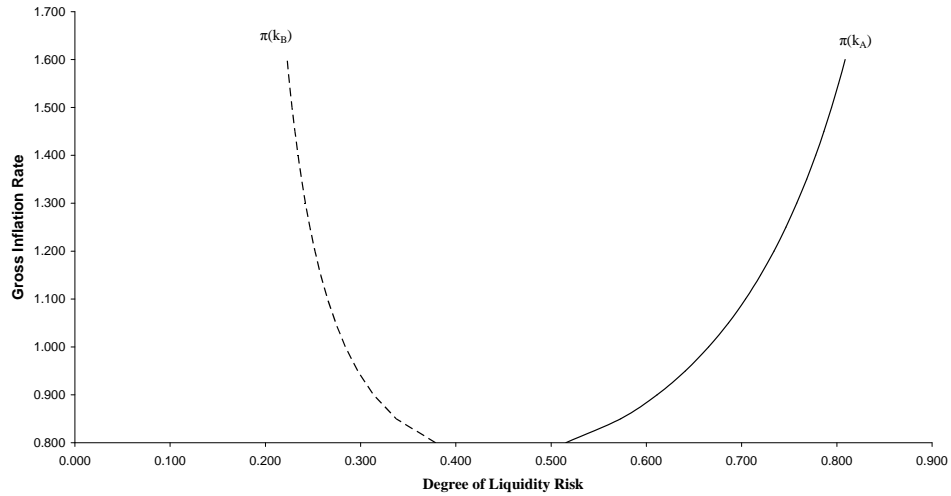


Figure 5: The Impact of Inflation on the Degree of Liquidity Risk

The relationship between inflation and welfare across steady-states is shown

in Figure 6:

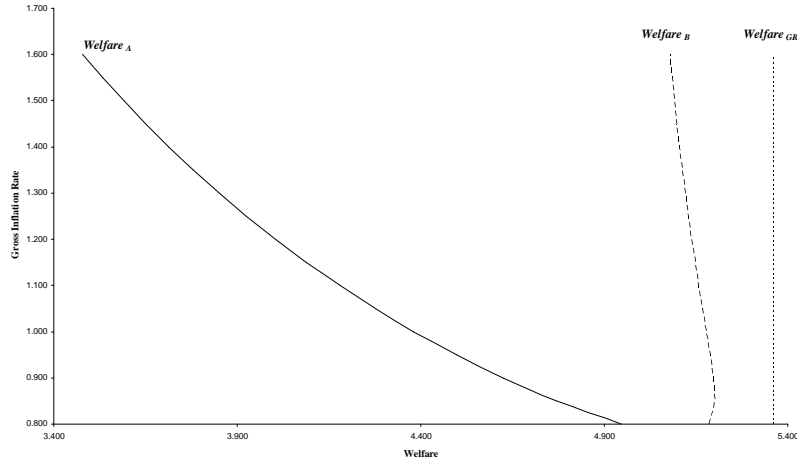


Figure 6: The Impact of Inflation on Welfare

As observed in the Figure, the welfare loss from higher rates of money growth is particularly significant in the low capital economy. In turn, optimal monetary policy depends on the level of economic development. Under the baseline set of parameters, the pair  $(k, \sigma)$  that maximizes (19) for the high capital economy is  $(3.6, 0.857)$ . By comparison, the level of welfare for the low capital economy is maximized at  $(2.3, 0.8)$ . This is associated with the lowest money growth rate in which multiple steady-states exist.

## 4 Dynamical Equilibria

In section 2, we demonstrate that two steady-states may exist. In this section, we study dynamical equilibria. We first derive a phase diagram to examine the global information on the stability properties of the steady-states. Subsequently, we study the local stability properties of the system in the neighborhood of the steady-state equilibria.

### 4.1 A Phase Diagram

In constructing the phase diagram, it is important to simplify the structure of the economy somewhat to make the analysis more tractable. To begin, we consider a production function of the Cobb-Douglas form:  $y_t = f(k_t) = \Psi k_t^\alpha$ . In addition, we assume that  $\theta = \frac{1}{2}$ .

The nominal return to capital between period  $t$  and  $t + 1$  is given by  $I_t = \sigma f'(k_{t+1})$ . From (1),  $R_t = f'(k_{t+1})$ . By (14), the law of motion of capital is:

$$\Delta k_t = k_{t+1} - k_t = [1 - \gamma(k_t, I_t)] w(k_t) - k_t \quad (20)$$

Using the definition of  $\gamma(I_t, k_t)$ , (10), and the evolution equation for cash reserves, (15), the law of motion of  $I_t$  can be written as:

$$\Delta I_t = I_{t+1} - I_t = \left( \frac{(1-\alpha)}{\alpha} \frac{1}{\sigma} \frac{k_{t+1}(k_t, I_t)}{w(k_t)} \frac{I_t}{\gamma(k_t, I_t)} - 1 \right) \frac{\pi_0}{k_{t+1}(k_t, I_t) - \pi_0} - I_t \quad (21)$$

In the Appendix we show that the  $\Delta k_t = 0$  locus is convex as illustrated in Figure 7 below. Further, the capital stock is rising over time if  $[1 - \gamma(k_t, I_t)] - \delta(k_t) > 0$ , where  $\delta(k_t) = \frac{k_t}{w(k_t)}$ . This condition holds for all points above the  $\Delta k_t = 0$  locus since  $\frac{\partial \gamma(k_t, I_t)}{\partial I_t} < 0$ .

Substituting  $k_{t+1}$  from (20) into (21), we obtain the  $\Delta I_t = 0$  locus:

$$\left( \frac{(1-\alpha)}{\sigma\alpha} \frac{1}{\gamma(k, I)} - \frac{w(k)}{\pi_0} \right) (1 - \gamma(k, I)) + \left( 1 - \frac{1}{I} \right) = 0 \quad (22)$$

Define  $\left( \frac{(1-\alpha)}{\sigma\alpha} \frac{1}{\gamma(k, I)} - \frac{w(k)}{\pi_0} \right) (1 - \gamma(k, I)) + \left( 1 - \frac{1}{I} \right) = \psi(k, I)$ . It is easy to show  $(\pi_0, 1)$  and  $\left( \delta^{-1}\left(\frac{\sigma\alpha}{(1-\alpha)}\right), 1 \right)$  are two points on (22). In addition, taking the derivative of (22) with respect to  $k$  yields:

$$\frac{dI}{dk} = \frac{\left( \frac{(1-\alpha)}{\sigma\alpha} - \frac{1}{\pi_0} \gamma^2(k, I) w(k) \right) \frac{1}{\pi_0} I - \frac{1}{\pi_0} (1 - \gamma(k, I)) \alpha \frac{w(k)}{k}}{- \left( \frac{(1-\alpha)}{\sigma\alpha} - \frac{1}{\pi_0} \gamma^2(k, I) w(k) \right) \frac{k - \pi_0}{\pi_0} - \frac{1}{I^2}} \quad (23)$$

In general,  $\frac{dI}{dk}$  has an ambiguous sign. However, in the Appendix, we prove the following lemma:

**Lemma 2.** *The denominator in (23) is negative for all  $I \geq 1$  and  $k > \pi_0$ . In addition,  $\frac{dI}{dk} \leq (>) 0$  for all  $k \geq (<) \tilde{k}$ , where  $\tilde{k} \in (\pi_0, \delta^{-1}(\frac{\alpha\sigma}{1-\alpha}))$  satisfies  $\frac{(1-\alpha)}{\sigma\alpha} \delta(\tilde{k}) - \gamma^2(\tilde{k}, I) = \alpha \frac{[1-\gamma(\tilde{k}, I)]}{I}$ .*

Lemma 2 implies that the sign of the slope from (22) depends on the sign of the term in the numerator of (23). In particular, for all  $k \geq (<) \tilde{k}$ , the slope is negative (positive). In addition,  $\tilde{k}$  is not significantly larger than  $\pi_0$ . Therefore, the  $\Delta I_t = 0$  locus is as illustrated in Figure 7:

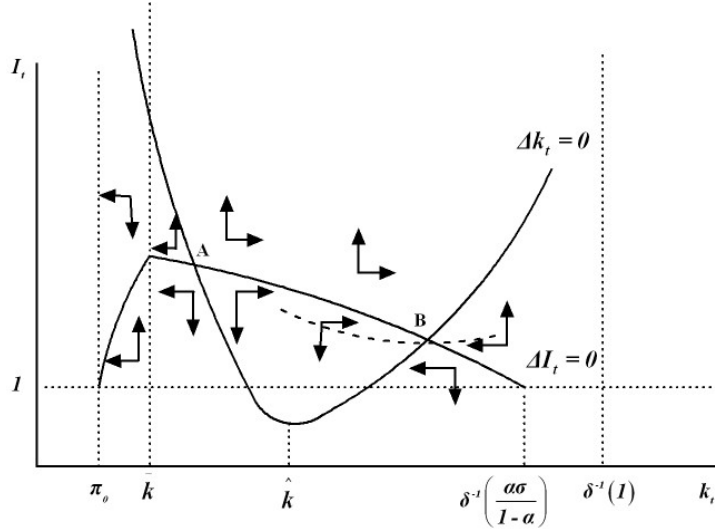


Figure 7: A Phase Diagram

Interestingly, the dynamics for the nominal interest rate depend on the size of the capital stock. In particular, above the  $\Delta I_t = 0$  locus,  $I_{t+1} < I_t$  if  $k_t < \tilde{k}$ . However,  $I_{t+1} > I_t$  if  $k_t > \tilde{k}$ . In this manner, the law of motion of  $I_t$  is as illustrated in the Figure.

The phase diagram in Figure 7 indicates that the low capital steady-state is a source. For example, the behavior of the nominal interest rate changes around  $\tilde{k}$  which reflects that there are initial conditions which lead to non-monotonic behavior away from the low capital steady-state. In contrast, there is a unique trajectory that leads to the steady-state with high economic activity. This suggests that economies must have sufficient initial resources in order to be able to stabilize over time.

The stability properties of the steady-states are consistent with previous models with strategic complementarities such as Diamond (1982). In contrast, in Schreft and Smith (1997), the low capital steady-state is a saddle while the high capital steady-state is a sink. In our framework, since there is a unique path to the steady-state for advanced economies, it is possible to determine the impact of a change in monetary policy along the transition to the new long-run equilibrium.<sup>12</sup>

<sup>12</sup>Under higher rates of money growth, the  $\Delta I_t = 0$  locus shifts upward. The high-capital steady-state converges to a steady-state with higher levels of capital formation.

## 4.2 Local Dynamics

We proceed by studying the local stability properties of the system in the neighborhood of the steady-states. As in the previous sub-section, the dynamic behavior of the economy is summarized by (20) and (21). The stability properties of a steady-state depend on the eigenvalues of the Jacobian matrix:

$$J = \begin{bmatrix} \frac{\partial k_{t+1}}{\partial k_t} & \frac{\partial k_{t+1}}{\partial I_t} \\ \frac{\partial I_{t+1}}{\partial k_t} & \frac{\partial I_{t+1}}{\partial I_t} \end{bmatrix}_{SS}$$

We denote the determinant and trace of  $J$  by  $D$  and  $T$  respectively. The discriminant,  $\Delta$ , is  $\Delta = T^2 - 4D$ . The elements of the Jacobian are given by:

$$\frac{\partial k_{t+1}}{\partial k_t} \Big|_{SS} = \alpha + \frac{\gamma}{1 - \frac{\pi}{k}} > 0$$

$$\frac{\partial k_{t+1}}{\partial I_t} \Big|_{SS} = \frac{1 - \theta}{\theta} \frac{k}{I} \gamma > 0$$

$$\frac{\partial I_{t+1}}{\partial k_t} \Big|_{SS} = \frac{I}{(k - \pi)} \frac{\left( \frac{1}{(1-\alpha)I} - \left(1 - \frac{\pi}{k}\right) \right) \alpha + \left(\gamma - \frac{\pi}{k}\right) \frac{\partial k_{t+1}}{\partial k_t} \Big|_{SS}}{\frac{1-\theta}{\theta} (1-\gamma)} \leq 0$$

$$\frac{\partial I_{t+1}}{\partial I_t} \Big|_{SS} = 1 + \frac{1 + \frac{(\gamma - \frac{\pi}{k})I}{(k-\pi)} \frac{\partial k_{t+1}}{\partial I_t} \Big|_{SS}}{\frac{1-\theta}{\theta} (1-\gamma)} > 0$$

Furthermore, the eigenvalues of  $J$  may be obtained by solving the following equation:

$$p(\lambda) = |J - \lambda I| = 0$$

$$|J - \lambda I| = \begin{vmatrix} \frac{\partial k_{t+1}}{\partial k_t} \Big|_{SS} - \lambda & \frac{\partial k_{t+1}}{\partial I_t} \Big|_{SS} \\ \frac{\partial I_{t+1}}{\partial k_t} \Big|_{SS} & \frac{\partial I_{t+1}}{\partial I_t} \Big|_{SS} - \lambda \end{vmatrix}$$

The possibility that inflation adds more uncertainty to the economy is illustrated in the following example:

*Example.* Let  $\alpha = 0.3$ ,  $\theta = 0.5$ ,  $\Psi = 5.4$ , and  $\pi = 1.2$ . At a constant rate of money growth, the low capital steady-state amount of capital is  $k = 2.370$  and the nominal interest rate is  $I = 1.013$ .  $T = 4.358$  and  $D = 3.397$ . The eigenvalues of the Jacobian lie outside the unit circle and the steady-state is a source. If  $\sigma = 1.05$ , the steady-state is also a source. However, if the rate of money growth is sufficiently high ( $\sigma = 1.34$ ),  $T^2 > 4D$  which implies that the eigenvalues are complex conjugates. Moreover, since the determinant of the Jacobian exceeds unity, the steady-state displays undamped oscillatory behavior.

The example shows that in relatively poor economies, high rates of money growth can lead to endogenous fluctuations that never disappear. Therefore, the



effects of high inflation policies may be particularly unpredictable in developing countries. Furthermore, while previous work emphasizes the role of strategic complementarities in generating multiple steady-states, our work also indicates that it is also a source of multiple dynamical equilibria.

## 5 Conclusions

There is a growing awareness that monetary policy is not super-neutral in many countries. In particular, in high inflation economies, a significant amount of evidence indicates that inflation is negatively related to economic activity. By comparison, inflation may be positively correlated with output in low inflation economies. This paper seeks to provide an explanation for the asymmetric effects of monetary policy across countries. In particular, our analysis is based on the idea that the degree of liquidity risk varies across different stages of economic development. Notably, in poor countries, individuals are more susceptible to events which cause them to liquidate their holdings of assets. Since the exposure to liquidity risk varies across countries, individuals respond differently to rates of return in low income countries than in advanced economies. As a result, the effects of monetary policy will also vary between developing and advanced countries.

We present a neoclassical growth model with money in which the incidence of liquidity risk is inversely related to aggregate capital formation. As the distribution of income in an economy with a high capital stock dominates the probability distribution in an economy with a low capital stock, there are positive spillovers from capital accumulation. Moreover, strategic complementarities from investment in capital are an important aspect of our modeling framework. In turn, there may be multiple monetary steady-states where the effects of monetary policy vary. In poor economies, the financial system is highly distorted and higher rates of money growth are associated with less capital formation. In contrast, in advanced economies, a Tobin effect is observed. Since inflation exacerbates distortions from a coordination failure in the low capital steady-state, individuals become much more exposed to liquidity risk. Consequently, optimal monetary policy depends on the level of development. The analysis concludes by investigating the behavior of dynamical equilibria. The results demonstrate that strategic complementarities lead to meaningful insights into the stability of economies with different levels of initial resources. Notably, the high capital steady-state is saddle-path stable. Therefore, the impact of a change in monetary policy along the transition can be determined since there is a unique path to the steady-state for advanced economies. This suggests that the effects of monetary policy should be easier to forecast in advanced countries.

## References

- Ahmed, S., and Rogers, J.H., 2000. Inflation and the Great Ratios: Long Term Evidence From the U.S. *Journal of Monetary Economics* 45, 3-35.
- Antinolfi, G. and T. Keister, 2006. Discount Window Policy, Banking Crises, and Indeterminacy of Equilibrium. *Macroeconomic Dynamics* 10, 1-19.
- Antinolfi, G., C.M. Landeo, and M. Nikitin, 2007. Dollarization and the Inflation Threshold. *Canadian Journal of Economics* 40, 628-649.
- Bae, S.K. and R.A. Ratti, 2000. Long-run Neutrality, High Inflation, and Bank Insolvencies in Argentina and Brazil. *Journal of Monetary Economics* 46, 581-604.
- Barro, R.J., 1995. Inflation and Economic Growth. *Bank of England Quarterly Bulletin*, 166-176.
- Bhattacharya, J., Haslag, J.H., Martin, A., 2007. Money, Output and the Payment System: Optimal Monetary Policy in a Model with Hidden Effort. Mimeo, University of Missouri.
- Bencivenga, V.R. and B.D. Smith, 1991. Financial Intermediation and Endogenous Growth. *Review of Economic Studies* 58, 195-209.
- Bishop, J.A., J.P. Formby, and P.D. Thistle, 1991. Rank Dominance and International Comparisons of Income Distributions. *European Economic Review* 35, 1399-1409.
- Boyd, J.H., R. Levine, and B.D., Smith, 2001. The Impact of Inflation on Financial Sector Performance. *Journal of Monetary Economics* 47, 221-248.
- Bruno, M. and W. Easterly, 1998. Inflation Crises and Long-Run Growth. *Journal of Monetary Economics* 41, 3-26.
- Bullard, J. and J.W., Keating, 1995. The Long-Run Relationship Between Inflation and Output in Postwar Economies. *Journal of Monetary Economics* 36, 477-496.
- Chetty, R. and A. Looney, 2005. Consumption Smoothing and the Welfare Consequences of Social Insurance in Developing Economies, NBER Working Paper #11709.
- Cooper, R. and A. John, 1988. Coordinating Coordination Failures in Keynesian Models. *Quarterly Journal of Economics* 103, 441-463.
- Diamond, D. and P. Dybvig, 1983. Bank Runs, Deposit Insurance, and Liquidity. *Journal of Political Economy* 91, 401-19.
- Diamond, P.A., 1965. National Debt in a Neoclassical Growth Model. *American Economic Review* 55, 1126-1150.
- \_\_\_\_\_, 1982. Aggregate Demand Management in Search Equilibrium. *Journal of Political Economy* 90, 881-894.

- Drazen, A. 1987. Reciprocal Externality Models of Low Employment. *European Economic Review* 31, 436-443.
- Foster, J.E. and A.F. Shorrocks, 1988. Poverty Orderings. *Econometrica* 56, 173-177.
- Gertler, P. and J. Gruber, 2002. Insuring Consumption against Illness, *American Economic Review* 92, 51-70.
- Hadar, J. and W.R. Russell, 1969. Rules for Ordering Uncertain Prospects. *American Economic Review* 59, 25-34.
- Haslag, J.H., and J. Koo, 1999. Financial Repression, Financial Development, and Economic Growth. Federal Reserve Bank of Dallas Working Paper #9902.
- Jacoby, H.G. and E. Skoufias, 1997. Risk, Financial Markets, and Human Capital in a Developing Country. *Review of Economic Studies* 64, 311-335.
- Laing, D., T. Palivos and P. Wang, 1997. Learning, Matching, and Growth. *Review of Economic Studies* 62, 115-129.
- Kochar, A., 2004. Ill-Health, Savings and Portfolio Choices in Developing Countries. *Journal of Development Economics* 73, 257-285.
- McPeak, J., 2006. Confronting the Risk of Asset Loss: What Role Do Live-stock Transfers in Northern Kenya Play? *Journal of Development Economics* 81, 415-437.
- Murphy, K.M., A. Shleifer, and R.W. Vishny, 1989. Industrialization and the Big Push. *Journal of Political Economy* 97, 1003-1026.
- Rosenzweig, M.R. and K. Wolpin, 1993. Credit Market Constraints, Consumption Smoothing and the Accumulation of Durable Production Assets in Low-Income Countries: Investments in Bullocks in India. *Journal of Political Economy* 101, 223-244.
- Rousseu, P.L. and P. Wachtel, 2001. Inflation, Financial Development and Growth, in *Economic Theory, Dynamics and Markets: Essays in Honor of Ryuzo Sato*, T. Negishi, R. Ramachandran and K. Mino (eds.). Kluwer Publishers.
- Schreft, S. and B.D. Smith, 1997. Money, Banking, and Capital Formation. *Journal of Economic Theory* 73, 157-182.
- Townsend, R., 1987. Economic Organization with Limited Communication. *American Economic Review* 77, 954-971.

## 6 Technical Appendix

1. **Proof of Lemma 1.** Taking the derivative of (16) with respect to  $k$  :

$$\frac{dI}{dk} = \frac{\Omega'(k) + \frac{\partial\gamma(k,I)}{\partial k}}{-\frac{\partial\gamma(k,I)}{\partial I}} \quad (24)$$

In the text, it has been shown  $\frac{\partial\gamma(k,I)}{\partial I} < 0$  and  $\frac{\partial\gamma(k,I)}{\partial k} < 0$ . Therefore, the sign of  $\frac{dI}{dk}$  depends on the sign of the term in the numerator. In particular,  $\frac{dI}{dk} < 0$  if  $-\frac{\partial\gamma(k,I)}{\partial k} > \Omega'(k)$ . This holds at low  $k$  since  $\Omega''(k) < 0$ . In contrast,  $\frac{dI}{dk} > 0$  if  $-\frac{\partial\gamma(k,I)}{\partial k} < \Omega'(k)$ , which holds at relatively high levels of capital. Finally,  $\frac{dI}{dk}|_{k=\hat{k}} = 0$  when  $-\frac{\partial\gamma(k,I)}{\partial k}|_{k=\hat{k}} = \Omega'(\hat{k})$ . Consequently, the supply curve has a convex shape as illustrated in Figures 1, 2, and 3. Alternatively, from the expression for  $\gamma(k, I)$ ,  $\frac{\partial\gamma(k,I)}{\partial k} = -\frac{1}{\pi_0} I^{\frac{1-\theta}{\theta}} \gamma^2$ . By (18),  $\frac{1}{\pi} I^{\frac{1-\theta}{\theta}} \gamma = \frac{1-\gamma}{k-\pi_0}$ . As a result,  $\frac{\partial\gamma(k,I)}{\partial k} = -\frac{(1-\gamma)}{k-\pi_0} \gamma$ . Finally, substitution from (16), yields:  $\frac{\partial\gamma(k,I)}{\partial k} = -\frac{\Omega(k)}{k-\pi_0} (1-\Omega(k))$ . Thus,  $\frac{dI}{dk} = 0$  when  $\frac{\Omega'(k)}{\Omega(k)} = \frac{(1-\Omega(k))}{k-\pi_0}$ . It is easy to verify that this polynomial has a unique positive real root,  $\hat{k} \in (\pi_0, \Omega^{-1}(1))$ . This completes the proof of Lemma 1.

2. **Proof of Proposition 1.** Lemma 1 describes the behavior from (16). Further, the demand for capital is strictly decreasing in  $I$ . A necessary condition for an equilibrium to exist is that the loci from (16) and (17) intersect. This occurs when the inflection point,  $\hat{k}$  is located below (17). That is,  $I^{16}(\hat{k}) < I^{17}(\hat{k})$ , where the superscripts 16 and 17 denote equations (16) and (17), respectively. Using the definition of  $\hat{k}$  and  $\gamma$  in (16),  $I^{16}(\hat{k}) = \left(\pi_0 \frac{\Omega'(\hat{k})}{(1-\Omega(\hat{k}))^2}\right)^{\frac{\theta}{1-\theta}}$ . A necessary condition for existence can be written as  $\left(\pi_0 \frac{\Omega'(\hat{k})}{(1-\Omega(\hat{k}))^2}\right)^{\frac{\theta}{1-\theta}} < \sigma f_k(\hat{k})$ .

As we are interested in cases where money is dominated in rate of return,  $I > 1$ , the location of the  $I = 1$  line determines the existence of steady-state equilibria. Specifically, if the  $I = 1$  line is below the supply curve, there are two steady-states. By definition of  $I^{16}(\hat{k})$ , this condition can be written as  $\left(\pi_0 \frac{\Omega'(\hat{k})}{(1-\Omega(\hat{k}))^2}\right)^{\frac{\theta}{1-\theta}} < 1$ . Alternatively it can be expressed as a condition on  $\pi_0$  as provided in case (a) of Proposition 1.

Next, suppose  $\pi_0 < \underline{\pi}$  where  $I^{16}(\hat{k}) > 1$ . In this case, the  $I = 1$  line intersects the capital supply curve twice at  $\underline{k}$  and  $\bar{k}$ , where  $\underline{k} < \bar{k}$  are the roots of the polynomial,  $\Omega(k) = (1 - \frac{\pi_0}{k})$ . Obviously, there exists a  $\sigma$ ,  $\underline{\sigma}_1$ , such that the intersection of (16) and (17) takes place at  $(\underline{k}, 1)$ , where  $\underline{\sigma}_1$  satisfies,  $\underline{\sigma}_1 = \frac{1}{f_k(\underline{k})}$ . Similarly, there exists a  $\sigma$ ,  $\underline{\sigma}_2$ , such that the intersection of (16) and

(17) takes place at  $(\bar{k}, 1)$ , where  $\underline{\sigma}_2$  satisfies,  $\underline{\sigma}_2 = \frac{1}{f_k(\bar{k})}$  and  $\underline{\sigma}_2 > \underline{\sigma}_1$ . Clearly, for all  $\sigma > \underline{\sigma}_2$ , the return to capital exceeds the return to money at  $B$  and two steady-states exist. In contrast, if  $\sigma \in (\underline{\sigma}_1, \underline{\sigma}_2]$ , there is a unique steady-state. This completes the proof of Proposition 1.

### 3. List of Numerical Results for Section 3.

$\sigma$	0.800	0.850	0.900	0.950	1.000	1.050	1.100	1.150	1.200	1.250	1.300	1.350	1.400	1.450	1.500	1.550	1.600
$k$	3.168	3.556	3.826	4.042	4.225	4.383	4.523	4.648	4.761	4.863	4.956	5.042	5.121	5.195	5.263	5.326	5.385
$m$	1.603	1.499	1.417	1.347	1.285	1.229	1.178	1.131	1.088	1.048	1.012	0.977	0.945	0.915	0.887	0.860	0.835
$w$	4.771	5.055	5.243	5.389	5.510	5.612	5.701	5.779	5.849	5.911	5.968	6.019	6.066	6.110	6.150	6.186	6.221
$\Omega(k)$	0.664	0.703	0.730	0.750	0.767	0.781	0.793	0.804	0.814	0.823	0.831	0.838	0.844	0.850	0.856	0.861	0.866
$\gamma$	0.336	0.297	0.270	0.250	0.233	0.219	0.207	0.196	0.186	0.177	0.169	0.162	0.156	0.150	0.144	0.139	0.134
$R$	1.506	1.422	1.371	1.333	1.304	1.280	1.260	1.243	1.229	1.216	1.204	1.194	1.185	1.176	1.169	1.162	1.155
$I$	1.205	1.208	1.233	1.267	1.304	1.344	1.387	1.430	1.474	1.520	1.565	1.612	1.658	1.705	1.753	1.800	1.848
$P_t/P_{t+1}$	1.250	1.176	1.111	1.053	1.000	0.952	0.909	0.870	0.833	0.800	0.769	0.741	0.714	0.690	0.667	0.645	0.625
$\pi/k$	0.379	0.337	0.314	0.297	0.284	0.274	0.265	0.258	0.252	0.247	0.242	0.238	0.234	0.231	0.228	0.225	0.223
<b>Welfare</b>	5.186	5.203	5.200	5.191	5.181	5.170	5.159	5.149	5.139	5.130	5.122	5.113	5.106	5.099	5.092	5.086	5.080

Table 1: High Capital Steady-State

$\sigma$	0.800	0.850	0.900	0.950	1.000	1.050	1.100	1.150	1.200	1.250	1.300	1.350	1.400	1.450	1.500	1.550	1.600
$k$	2.328	2.094	1.963	1.872	1.804	1.749	1.704	1.667	1.635	1.607	1.583	1.562	1.543	1.526	1.511	1.497	1.484
$m$	1.762	1.785	1.793	1.796	1.796	1.795	1.795	1.794	1.793	1.791	1.790	1.788	1.787	1.785	1.784	1.783	1.782
$w$	4.090	3.879	3.756	3.668	3.600	3.545	3.500	3.461	3.428	3.399	3.373	3.350	3.330	3.311	3.295	3.279	3.265
$\Omega(k)$	0.569	0.540	0.523	0.510	0.501	0.493	0.487	0.482	0.477	0.473	0.469	0.466	0.463	0.461	0.458	0.456	0.454
$\gamma$	0.431	0.460	0.477	0.490	0.499	0.507	0.513	0.518	0.523	0.527	0.531	0.534	0.537	0.539	0.542	0.544	0.546
$R$	1.757	1.852	1.913	1.959	1.996	2.027	2.053	2.076	2.096	2.114	2.130	2.145	2.158	2.170	2.181	2.191	2.201
$I$	1.406	1.575	1.722	1.861	1.996	2.128	2.259	2.388	2.516	2.643	2.769	2.896	3.021	3.147	3.272	3.396	3.521
$P_t/P_{t+1}$	1.250	1.176	1.111	1.053	1.000	0.952	0.909	0.870	0.833	0.800	0.769	0.741	0.714	0.690	0.667	0.645	0.625
$\pi/k$	0.515	0.573	0.611	0.641	0.665	0.686	0.704	0.720	0.734	0.747	0.758	0.768	0.778	0.786	0.794	0.802	0.809
<b>Welfare</b>	4.947	4.768	4.624	4.497	4.382	4.277	4.179	4.089	4.004	3.925	3.850	3.780	3.713	3.650	3.590	3.533	3.479

Table 2: Low Capital Steady-State

**4. Determining the Relationship Between  $I$  and  $k$  in the  $\Delta k_t = 0$  locus.** The proof follows directly from the proof of Lemma 1. Under a Cobb-Douglas production function of the form defined in the text,  $\frac{dI}{dk} = 0$  when  $(1 - \alpha) \left(1 - \frac{\pi_0}{k}\right) = [1 - \delta(k)]$ , where  $\delta(k) = \frac{k}{w(k)}$ . It is obvious that the term on the left-hand side is strictly increasing in  $k$ , while the term on the right-hand side is strictly decreasing in  $k$ . Therefore, the polynomial above has a unique positive real root,  $\hat{k} > \pi_0$ . In addition, for all  $k_t \geq (<) \hat{k}$ ,  $\frac{dI}{dk} \geq (<) 0$ . The  $\Delta k_t = 0$  locus has the shape illustrated in the text.

**5. Determining the Relationship Between  $I$  and  $k$  in the  $\Delta I_t = 0$  locus.** First, it can be easily shown that  $\frac{dI}{dk} > 0$  at  $(\pi_0, 1)$  and  $\frac{dI}{dk} < 0$  at  $\left(\delta^{-1}\left(\frac{\sigma\alpha}{(1-\alpha)}\right), 1\right)$ . Next, define the term in the denominator and numerator

of (23) by  $\mu(k, I)$  and  $\phi(k, I)$ , respectively. It can be verified that  $\mu(\pi_0, 1) < 0$  and  $\mu\left(\delta^{-1}\left(\frac{\sigma\alpha}{1-\alpha}\right), 1\right) < 0$ . In addition, it is clear that  $\mu(k, I) < 0$  when  $\phi(k, I) > 0$ . Further, at any  $I > 1$ , using the definition of  $\gamma$  and some algebra,  $\mu(k, I) < 0$  when

$$\frac{(1-\alpha)}{\sigma\alpha}\delta(k_t) - \frac{\pi_0}{k} \frac{1}{\left(\frac{\pi_0}{k} + \left(1 - \frac{\pi_0}{k}\right)I\right)^2} > -\frac{\pi_0}{k_t - \pi_0} \frac{1}{I^2}\delta(k) \quad (25)$$

Denote the term on the left-hand side of (25) by  $LHS$ . For all  $I_t > 1$ , the  $\frac{\partial LHS}{\partial k} > 0$  and  $\frac{\partial^2 LHS}{\partial k^2} < 0$ . Moreover,  $\lim_{k \rightarrow 0} LHS \rightarrow -\infty$  and  $\lim_{k \rightarrow \infty} LHS \rightarrow \infty$ . In addition,  $LHS < 0$  at  $k = \pi_0$ . Further, we examine the term on the right-hand side of (25), denoted  $RHS_d$ . It can be shown that  $\frac{\partial RHS_d}{\partial k} > 0$  for all  $k > \pi_0$ . Moreover,  $\lim_{k \rightarrow \infty} RHS_d \rightarrow 0$  and  $\lim_{k_t \rightarrow \pi_0} RHS_d \rightarrow -\infty$ . Finally, at  $k = \delta^{-1}\left(\frac{\sigma\alpha}{1-\alpha}\right)$ ,  $RHS < 0$ . In this manner, for all  $k \geq \pi_0$  and  $I \geq 1$ , the denominator is negative as  $LHS > RHS_d$  for all  $k \geq \pi_0$ .

We next show that the numerator flips signs at  $\tilde{k} \in \left(\pi_0, \delta^{-1}\left(\frac{\alpha\sigma}{1-\alpha}\right)\right)$ . In particular, for all  $I > 1$ , there exists a  $k, \tilde{k} > \pi_0$  at which  $\frac{dI}{dk} = 0$  and  $\tilde{k}$  is increasing in  $\sigma$ . If  $\phi(k, I) = 0$ ,  $\frac{dI}{dk} = 0$ . With some algebra,  $\phi(k, I) = 0$  when

$$\frac{(1-\alpha)}{\sigma\alpha}\delta(k) - \frac{\pi_0}{k} \frac{1}{\left(\frac{\pi_0}{k} + \left(1 - \frac{\pi_0}{k}\right)I\right)^2} = \alpha \frac{[1 - \gamma(I, k)]}{I} \quad (26)$$

where the term on the left-hand side of (26) is identical to  $LHS$  defined above in the denominator. Characterizing the term on the right-hand side of the numerator, denoted  $RHS_n$ , where  $RHS_n = \alpha \frac{[1 - \gamma(I, k)]}{I}$ . Unambiguously,  $\frac{\partial RHS_n}{\partial k} > 0$ ,  $RHS_n(\pi, 1) = 0$ , and  $\lim_{k \rightarrow \infty} RHS_n \rightarrow \alpha \frac{1}{I}$ . As a result, we have one inflection point,  $\tilde{k} \in \left(\pi_0, \delta^{-1}\left(\frac{\alpha\sigma}{1-\alpha}\right)\right)$ , above which the numerator is positive and the slope of (22) is negative. However, since  $I > 1$  and  $\alpha$  is generally less than  $\frac{1}{2}$  in the growth-accounting literature,  $\alpha \frac{1}{I}$  is relatively small and the level of capital at which the inflection point occurs is pretty close to  $\pi_0$ . Finally, it is clear that under higher  $\sigma$ ,  $\tilde{k}$  increases. This implies that the  $\Delta I_t$  locus resembles the one depicted in Figure 7 in the text. The dynamics for the nominal interest rate in Figure 7 can be determined in a similar manner.