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Abstract

The main concern of this report is to model the daily and weekly forecasting of the currency in circulation (CIC) for the State of Qatar. The time series of daily observations of the CIC is expected to display marked seasonal and cyclical patterns daily, weekly or even monthly basis. We have compared the forecasting performance of typical linear forecasting models, namely the regression model and the seasonal ARIMA model using daily data. We found that seasonal ARIMA model performs better in forecasting CIC, particularly for short-term horizons.

JEL classification: C45, E47, E58.

Keywords: Currency in Circulation, Linear Forecasting, ARIMA

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1 Introduction

Central banks have recently been maintaining price stability through different types of monetary policy instruments. To pursue its objectives effectively, a central bank needs an accurate estimate of money market liquidity. However, money market liquidity is influenced by several independent factors that are not under the full control of the central bank. One of the most important autonomous factors is currency in circulation, which is quite difficult to assess, as it is strongly influenced by many exogenous factors.

Monetary authorities use liquidity management policies to maintain stability in the money market. In this sense, projecting money market liquidity becomes necessary. One might ask the following question: what is money market liquidity and why it is so important to have stable liquidity of money? Broadly, money market liquidity refers to the balances held by banks on settlement accounts with the central bank. This term is also used to indicate the condition needed to maintain the stability in the monetary market needed to equalize supply and demand for the bank reserves. However, money market liquidity is influenced by several autonomous factors that are beyond the control of the central banks,-namely government deposits, the amount of banknotes and excess reserves held by the commercial banks. A considerable change of these factors increases or decreases liquidity, thereby leading to fluctuations in the money supply. Most importantly these fluctuations in money supply lead to volatility in daily interest rates.

Among those autonomous factors, forecasting the currency in circulation is always considered as the one of the hardest missions of the central banks. Given that the central bank has the monopoly for distributing the currency, it may not predict the demand for the currency accurately, since the amount of currency circulated in the economy is influenced by the non-bank public sector.

Indeed, central banks should focus on forecasting on the currency in circulation(CIC) because providing an accurate prediction for the CIC would enable the central bank to plan monetary policy strategies in advance so they can manage liquidity efficiently. In addition, having accurate forecasts of CIC helps to stabilize the money market in the short run, it definitely helps to decrease volatility in money market rates, thereby resulting in higher economic growth.[1] Given that Qatar economy has been growing enormously -parallel to the increase in oil price all around the world-, the demand for the CIC in the economy has been accelerated. Figure 1 shows the currency issued by Qatari Central Bank between the years 2002 and 2007. The increasing trend

[1] Lower volatility in the interest rate encourages domestic investors to participate more in the real sectors as well as helps foreign portfolio inflows to the domestic economy. Both channels will result in higher economic growth.

in CIC can be observed clearly. Nevertheless, forecasting the CIC creates important advantages, The QCB bank did not undertake a similar study earlier. This paper is the first study that forecasts the CIC for the State of Qatar.

In this paper, we attempt to forecast the daily CIC in the State of Qatar by using recent forecasting techniques. In particular, we investigate daily, and weekly liquidity forecasts for currency in circulation by employing linear forecasting techniques and, most importantly we consider the effect of Islamic calendar and Gregorian calendar together on forecasting the CIC for the state of Qatar.

Currency in circulation is the most important autonomous factor in the context of liquidity management, both in terms of size and volatility. Thus, researchers try to develop forecasting methods that will minimize the forecast errors. By innovations in the forecasting techniques, researchers have obtained accurate estimations in recent years.[2] For example, Cabrero *et al.* (2002) modeled the daily series of banknotes in circulation in the context of managing of the European monetary system. Empirical models in that paper relied on two liquidity forecasting approaches: seasonal ARIMA method and Structural Time series. Cabrero *et al.* (2002) noted that the error in forecasting banknotes in circulation never exceeds 1 billion Euro in either models mentioned above and they concluded that econometric models are able to explain an important part of the variation in the currencies in the circulation. Hlavacek *et al.* (2005) forecasted the CIC for Czech Republic using both linear (ARIMA) and non-linear techniques. Their study provided satisfactory forecasts using the ARIMA models both for long-term and short-term horizons.[3] In addition, they developed a new non-linear technique for forecasting a daily series for the CIC, the feed-forward neural network model. They concluded that ARIMA model provides satisfactory forecasts for the CIC, however, feed-forward neural network model is a better model for analysis of time series of CIC, compared to the ARIMA model.

Some papers forecasted the CIC based on the theory of transaction and portfolio demand for money. Black *et al.* (1997) studied the CIC by currency demand function during the early 1990s of New Zealand, by using seasonal ARIMA models. Although the number of their observations are limited, they obtained satisfactory results by employing the model. In a recent study, Dheerasinghe (2006) forecasted the currency in demand for Sri-Lanka with monthly, weekly and daily data sets for years 2000 to 2005. For different frequencies, the linear regression forecasting method provides satisfactory estimations for the currency circulation in Sri Lanka the results are extremely satisfactory for the out-of-sample forecasts.

[2]Although almost all central banks forecast the CIC in their economies, only a few central banks publish liquidity forecasts. The literature on this subject is, therefore limited.

[3]Short term horizon refers to forecasts are done for at most two weeks ahead, and long term horizons correspond the forecasts more than two weeks ahead.

The remainder of this paper is structured as follows. Section 2 documents how we derive the CIC and how we create dummy variables for calendar effects. In Section 3 we document the linear forecasting methodologies used for forecasting CIC. Section 4 evaluates the linear forecast models for different time horizons. Section 5 offers our conclusions.

2 Data

2.1 The Calendar Effect

In most of Islamic countries, socioeconomic and cultural events are arranged considering both the Islamic and Gregorian calendars.[4] Considering the State of Qatar which is a regular and secure applicant of the Islamic calendar to social and economic life, we expect that economic life will co-move with the islamic events. To give an example, the household consumption will increase during the month of Ramadan. In particular just before the Eid-ul-Fitr and the month of Dhul Hijja on the account of Islamic Pilgrimage (Hajj), household consumption is anticipated to increase substantially. Between Eid-ul Fitr and Eid-al-Adha and after Eid al-Adha during the month of Shawwal, consumption is expected to decline, however, even though we observe a marked rise in consumption again just before Eid-al-Adha.

Determining the effect of Islamic calendar on the economic life is even harder, because the Islamic calendar and Gregorian calendar are different. The Islamic calendar is lunar, having 354 or 355 days per year; however, the Gregorian calendar is having 365 or 366 days in every year.[5] Observations of the religious days or months on the Islamic calendar can not be converted to the same day of the Gregorian calendar for each year. Giving an example, if Ramadan starts on September 13 in Gregorian calendar this year, next year the first day of Ramadan will be September 2 or 3. Another important confusion between the calendars is that Islamic societies follow calendar based on observations; the religious authorities announce the beginning of the Islamic month Ramadan after sighting the new moon one day before. Thus, ex-ante conversion of the Islamic dates to Gregorian dates may produce some errors. Besides, the actual effect of the very beginning of Ramadan on the economy may not be observed accurately. For the State of Qatar, western countries which universally follow the Gregorian calendar, form the majority of Qatar's economic partners. Therefore, the State of Qatar considers the Gregorian

[4] However, the dominant is not same for all islamic societies. Some Islamic and secular countries, including Albania, Bosnia-Herzegovia and Turkey, have been using the Gregorian calendar extensively in arranging social and economic life(holidays, weekends); the religious holidays are also taken into account before the holidays are announced. Nevertheless, Middle Eastern and North African countries take references to the Islamic calendar to arrange the holidays as well as the weekends.

[5] The earth rotates around the sun in 365 days and 6 hours (1 year). These six hours are collected and counted as one extra day in every four years.

calendar to some extent but not as much as western countries. Furthermore, the State of Qatar is quite different when demographic factors are considered. State of Qatar is not a homogenous national state instead, Qatar's population is quite heterogenous.. According to 2004 Census data, 25% of the population of Qatar is Indian whereas 20% is from Pakistan, only 20% is the local residents. Qatar's population breakdown obliges us to consider the calendar effects in a more detailed way. To get a better estimate, we take into account the national and religious holidays of those non-citizen residents located in Qatar that might affect consumption in the economy.

2.2 Currency In Circulation

In Qatar, the currency outside the banks consists of all banknotes in the national currency that economic agents, i.e. residents, companies and non-resident workers, keep for a certain time for transaction purposes as well as the store value. This amount includes all banknotes and coins in Qatari Riyal(QR).[6] When the currency is returned to the banks, it is considered to be a part of a commercial bank's reserve with the QCB. From this point of view, an increase within bank's vaults reflects the increase in the reserves with the QCB. Accordingly, the CIC is calculated as follows;

Total printed banknotes and coins in Qatari Riyal
− Cash in the commercial and Islamic banks vaults
= Currency outside the banks

Figure 1 indicates the currency issued by QCB between years 2002 and 2007, expressed in logarithmic terms. As expected, CIC follows and increasing trend in this time interval, which can be attributed to the growing demand for cash in accordance with the positive economic growth of the State of Qatar. The seasonality of the CIC as being observed is identical for this period. In other words, the peaks and troughs of the CIC fall on almost same calendar days in those years. This finding suggests the existence of the seasonality in the CIC data. Figure 2 shows the log-linearized and lag-differenced CIC for State the of Qatar.[7] At first glance, we noted that the time series of CIC appears to be stationary. Most importantly, the fluctuations in the daily observations of CIC gets smaller in the most recent years, which prompts us to have

[6] Starting 1992, there were 4 different editions of Qatar Riyal, named Issue 1 through Issue 4. We obtained the amount of each currency edition and added them all to form the aggregate currency issued by QCB.

[7] Unless otherwise specified in this report, lag differenced the CIC is equal to $CIC_t - CIC_{t-1}$.

better forecasts.

3 Empirical Models

The time series of CIC with its regular patterns and large number of observations, is an ideal candidate for the application of time series analysis for forecasting its future values. Prior to this paper, the QCB had not used any formal statistical model for forecasting the CIC. First of all, it is appropriate, to set up a baseline model, that describes the dynamics of the CIC in the form of multiple linear regression model. Later, we apply standard ARIMA methodology to develop a second model that contains both the regression model variables and the seasonal ARIMA model.

We use both models to forecast the CIC. To create accurate forecasts, it is necessary to formulate the CIC's weekly, bi-weekly, monthly and annual patterns. It is also important to consider the effects of holidays, weekends, and religious and national days on currency holding. The variables employed in the models are selected according to these criteria.

3.1 Regression Model

To perform the regression model, we need to have a stationary series. Therefore, we use the Augmented Dickey-Fuller (1976) test to see if the CIC series is stationary or not. The following table contains the stationary test results for CIC.

Table 1: **The Stationary Test for Currency in Circulation**

	ADF Test
Currency in circulation	0.16
ΔCurrency in circulation	-10.11

The Augmented Dickey-Fuller test is implemented to test if the series is stationary or not. The null hypothesis is CIC has unit root.(not stationary) The alternative hypothesis is CIC has no unit root (stationary). The second row presents stationary test results of first differenced CIC variable. The critical values for 1%, 5%, and 10% for ADF test are –3.43, –2.80, and–2.57.

Since the CIC fails the stationarity test, we take the first difference of the variable (CIC_t-CIC_{t-1}) and use that variable as the left-hand side variable in the regression model. The regression model expressed in the first difference of the log of currency in circulation is;

$$\Delta y_t = \sum_{i=1}^{11} \alpha_i M_{i,t} + \sum_{i=1}^{4} \beta W_{i,t} + \sum_{i=1}^{k} \theta_i(B)\delta_{i,t} + \sum_{i=1}^{j} \gamma_i O_{i,t} + k_{i,t} + \varepsilon_t. \quad (1)$$

where y_t is the level of currency in circulation at time t, ε_t is the error term at time t, and M, W,δ, k, O refer to the seasonal effects in the following way:

- **Intra-weekly effect**(W): For each weekday (i=Sunday, Monday ... Thursday) we define a dummy variable at time t as having a value of 1 and if the day at time t is day i, or 0 otherwise.
- **Monthly effect** (M): In the model, we assume that for each calendar month changes of CIC are stable, but on different levels. According to that, for each calendar month i (i=Jan,...Dec) we define: M_i=1 $if\ month\ i\ is\ at\ time\ t,\ 0\ elsewhere.$
- **Holidays** (δ): This is matrix of dummy variables for holidays and important days in the calendar for the State of Qatar(Ramadan, Eid-al-Adha, Eid-ul-Fitr, New year...). $\theta_i(B)$ is the polynomial of the dummy variable B, where B is the standard backward shift operator. The Term $\theta_i(B)$ captures the change in the currency level before and after the holiday i.
- **Outliers** (O): In the analysis of the residuals, the largest outliers are identified and their possible effect on other parameters is removed with dummy variables. Formally, if there is an unexplained high residual at time t, we control that calendar date with a dummy and name it as an outlier.
- **Intra-monthly effect** (k): Previous literature has defined intra-monthly seasonality as a linear combination of trigonometric functions:

  $$k_t = \sum_{j=1}^{p} (a_j * sin\frac{2j\pi * m_t}{M_t} + b_j \cos\frac{2j\pi * m_t}{M_t}), \quad (2)$$

  where m_t stands for the day of the month and M_t stands for the days of a given month. The parameter p defines the number of different frequencies that we use in modelling the intra-monthly dynamics. Alternatively, we model the intra-monthly effect with dummy variables for each day of the month(total of 31 variables).

The parameters in the equation (1), α_i, β_i, δ_i, γ_i and coefficients of the polynomial $\theta_i(B)$ have been estimated by the ordinary least squares(OLS) methodology. The coefficients are listed in Table 5. We follow the General to Specific (GS) approach to determine the lags of holidays, national and religious days for the State of Qatar. Table 2 also contains the effects' of the holidays, national and religious days and the days before and after these important dates. For example, for Eid-al-Adha, we provide a polynomial that is multiplied by a backshift operator, $(\omega_0 I + \omega_1 B + \omega_2 B^2 ... + \omega_5 B^{11}) * B^{-7})$. This indicates that, starting from the seven working days before the first day of Eid-al-Adha, we include the preceding days that are statistically significant and we include up to four business days following Eid-al-Adha in the regression.[8] Lastly, Table 5 contains the parameters of the regression coefficients, and some test results for the goodness of fit of the regression model for forecasting. We provide results of the adjusted R-square, Akaike Information Criterion (Akaike (1974)), and the Schwarz information criterion (Schwarz (1978)) tests for the regression results. The results of these tests are optimal in accordance with GS methodology. When we run the regression model, we add fifteen days preceding and following of the important dates to the regression model. By dropping the preceding days and following days that are not statistically significant, we derive the final version of the regression model that has the best forecasting performance- has the highest Adjusted R-square and lowest AIC and SIC test results.

Table 2: **Seasonal Factors and Shocks Included in the Regression Model** & **Seasonal ARIMA Model**

Seasonal Factors and Shocks	$\omega_i(B)$
RAMADAN	$(\omega_0 I + \omega_1 B + \omega_2 B^2 ... + \omega_5 B^5) * B^{-4}$
EID AL-ADHA	$(\omega_0 I + \omega_1 B + \omega_2 B^2 ... + \omega_5 B^1 1) * B^{-7}$
EID AL-FITR	$(\omega_0 I + \omega_1 B + \omega_2 B^2 ... + \omega_5 B^7) * B^{-6}$
LAST DAY	$(\omega_0 I + \omega_1 B + \omega_2 B^2) * B^{-2}$
THANKSGIVING	$(\omega_0 I + \omega_1 B + \omega_2 B^2) * B^{-2}$
JULY 1	$(\omega_0 I + \omega_1 B) * B^{-1}$
AUG 1	$(\omega_0 I + \omega_1 B) * B^{-1}$

3.2 Box-Jenkins Methodology & the Seasonal ARIMA Model

Time series analysis has been widely used in economic forecasting, particularly by central bank researchers to forecast the main economic indicators in the very near future. As previously mentioned the times series of CIC, with its regular and seasonal patterns and large number of observations, is a perfect candidate for applying of times series techniques for forecasting the CIC for short-term and long-term horizons. The Box-Jenkins methodology used in analysis and forecasting is widely regarded to be an efficient forecasting technique, and is used extensively, especially for univariate time series. This methodology is actually based on Wald's theorem

[8] Please note that we construct the working days following Eid-al-Adha and Eid-ul-Fitr starting from the last day of the Eid-al-Adha and Eid-ul-Fitr. In other words, when we consider the first working day after Eid-al-Adha, this means one working day after the last day of the Eid-al-Adha vacation.

(Hamilton 1994) which states that any weak stationary process can be broken down into autoregressive (AR) and moving average (MA) processes,. According to Box-Jenkins model (also known as ARMA), a time series with an ARMA process of orders p and q can be written in the following form:

$$y_t = \sum_{i=1}^{p} \alpha_i y_{t-1} + \sum_{i=1}^{q} \beta_i \epsilon_{t-1} + \epsilon_t. \tag{3}$$

The ARMA process is not only applicable for the weak stationary process. Researchers may also apply the ARMA process including various trends, seasonal and other deterministic or stochastic components. Box and Jenkins (1976) and Bell and Hilmer (1983) suggested the integrated ARMA models, namely ARIMA models, following Box and Jenkins (1976). The integrated ARMA model has been used extensively for the non-stationary time series. To our best knowledge, recent central bank research papers including Cabrero *et al.* (2002), Hlavacek *et al.* (2005) and Basac *et al.* (2006) used the seasonal ARIMA models and obtained accurate estimations for the CIC.

Simply, the linear ARIMA model is represented as follows:

$$y_t = D_t + \frac{\theta(B)}{\phi(B)\Delta(B)}\epsilon_t. \tag{4}$$

In the model, y_t refers the currency in circulation in logarithmic form, and D_t represents the regression component:

$$D_t = \sum_{i=1}^{s} d_{i,t}, \tag{5}$$

whree s is equal to the number of calendar variation effects, and $d_{i,t}$ is a function of all seasonal factors. The second part of the Equation(4) contains the ARIMA components: B is the back-shift operator, and θ and ϕ are the moving average and autoregressive operators, respectively. Δ is a difference operator, depending on the frequency of the difference.[9] Finally, ϵ_t is assumed to be an independent and identically distributed(iid) stochastic process with zero mean and a variance of σ^2.

ARIMA models are, in theory, the most general class of models for forecasting a time series that can be made stationary by transformations such as differencing and logging. When we use quantity forecasting, such as the liquidity position, the model will generate valid results. Accordingly, we applied the integrated ARMA models, to eliminate this stationarity problem. In fact, the easiest way to think of ARIMA models is as fine-tuned versions of random-walk

[9] For example, (1-B) indicates one lag difference from the observation($Y_t - Y_{t-1}$), whereas $(1 - B^{261})$ indicates $Y_t{-}Y_{t-261}$

and random-trend models. Fine-tuning consists of adding lags of the differenced series and/or lags of the forecast errors to the prediction equation, as needed to remove any last traces of autocorrelation from the forecast errors.

When comparing the empirical performance of linear models for predicting seasonality in daily time series, the ARIMA-based approach should be used in short term forecasting. The forecasting performance of the ARIMA model has been assessed by the previous studies, indicating that it provides valid "out-of-sample" forecasting, particularly for the models that included seasonal patterns.

The regression model performed in the previous section documents the effects of the important calendar dates and the preceding and following business days on the amount of currency issued by QCB. However, the correlogram of the residuals of the regression model indicates that the residuals have some seasonal, autoregressive and moving average patterns. This motivates use of the seasonal ARIMA model.[10]

3.3 Building a Seasonal ARIMA Model

To build an accurate seasonal ARIMA model, we need to start with the raw CIC data itself. The ACF shows a linear decline while the PACFs show significant spikes at different periods. After taking the first difference of CIC, the ACF started to decline but some other patterns (weekly and seasonally) still exist. For example, starting from the first PACF observation, we observe significant spikes at every fifth observation. This refers to some weekly patterns. In this case, keeping the series integrated with the first difference (I(1)), we can try other ways to eliminate the patterns in the data. Next, we integrated the series with I(261). This means a 261-day period (annual) difference, allowing us to get rid of the seasonal differences in the dataset. The stationary ACF and PACF plot reveals that integration of order 1 (I(261)) and CIC solves the stationarity problem. Besides, the patterns on the plot are also eliminated . However, the significant spikes in the plots indicates that we need to work on the auto regressive and moving average part of the CIC. In detail, we observe that first seven of PACFs and ACFs are significant. We then decided to use; $\text{ARIMA}(6,1,6)^{261}$.[11] This representation indicates that this is an AR(6), MA(6) model with the time series being integrated by I(1) and I(261). The significant spikes remaining on the plot are on the order of 22^{th}, 36^{th}, 44^{th}, 48^{th}, 65^{th}. Some of the remaining spikes are very intuitive. When we look at the 22^{th} order of AR, it is nothing but the one-month lagged value of CIC(21-22 working days in a month). The 44^{th} order of AR is two

[10] For the sake of brevity, we did not report the correlogram of the residuals of the regression model. The correlogram figures would be available upon request.

[11] $\text{ARIMA}(6,1,6)^{261}$ corresponds to the series integrated once I(1) to make it stationary. To get rid of the seasonality, we integrated it one more time with I(261) and we added AR(6) and MA(6).

months' lag of CIC, similarly, the 65^{th} order of AR is the three month lagged value of CIC. We remodel the CIC by taking these spikes into account. Therefore, the model will be formed as: ARIMA$(6,1,6)^{261}$ with AR(22), MA(22), AR(36), MA(36), AR(44), MA(44), AR(48), MA(48), AR(65), and MA(65). Now, ACF and PACF do not contain the any significant spikes, and the residual are stationary as expected. In the end, we form a seasonal ARIMA model that we can apply to get valid forecasts. By the way, it is worth noting that we applied more than 100 different types of seasonal ARIMA model which were slightly different from the ARIMA model above. This model has been selected because it is most valid model with respected adjusted R-square, and Akaike Information criteria and Schwarz Information criteria results.

After selecting the correct seasonal ARIMA model, we apply the calendar effect dummy variables (previously used for the regression model) to the seasonal ARIMA model and get the following function:

$$y_t = \sum_{i=1}^{12} \alpha_i * M_{i,t} + \sum_{i=1}^{4} \beta * W_{i,t} + k_t + \sum_{i=1}^{k} \theta_i(B)\delta_{i,t} + \sum_{i=1}^{j} \gamma_{i,t} O_{i.t} + \eta_t, \tag{6}$$

where:

$$\eta = \frac{\theta(B)}{\phi(B)\Delta(B)}\epsilon_t. \tag{7}$$

The deterministic component has been explained in the previous section. According to the selected seasonal ARIMA model: the seasonal difference will be:

$$\Delta(B) = (1 - B)(1 - B^{261}). \tag{8}$$

The lags of the MA and AR processes were chosen with respect to the seasonal ACF and seasonal PACF diagrams. The moving average operated model for Equation 6 is as follows:

$$\theta(B) = 1 + \theta_1 B^1 + \theta_2 B^2 + \theta_3 B^3 + \theta_4 B^4 + \theta_5 B^5 + \theta_6 B^5 + \theta_7 B^{22} + \theta_8 B^{44} + \theta_9 B^{48} + \theta_{10} B^{65}. \tag{9}$$

Finally, the autoregressive operator model for forecasting currency in circulation:

$$\phi(B) = 1 + \phi_1 B^3 + \phi_2 B^6 + \phi_3 B^7 + \phi_4 B^4 + \phi_5 B^5 + \phi_6 B^6 + \phi_7 B^{22} + \phi_8 B^{44} + \phi_9 B^{48} + \phi_{10} B^{65}. \tag{10}$$

In fact, this representation is nothing a the re-written form of ARIMA$(6,1,6)^{261}$ with AR(2), MA(22), AR(36), MA(36), AR(44), MA(44), AR(48), MA(48), AR(65), and MA(65). Table 6 contains the parameters and test results for the goodness of the fit of Seasonal ARIMA model for forecasting the CIC. We will discuss the appropriateness of the model in the following section.

4 Evaluation of Forecasts

The main purpose of the models described above is to forecast CIC in time horizons. Therefore, it is so important to evaluate the out-of-sample forecasts. Accordingly the, models' parameters are estimated for the period 1/1/2002-12/31/2006. The out-of-sample forecasts were implemented for year 2007 and then compared with the actuals. We re-estimated the models with each new observation and forecasts were stored. The series of forecasts were then used to calculate the forecast errors in order to choose the best model. Figure 3 and Figure 5 contain the out-of-sample forecasts of the regression model and the ARIMA model, respectively. Figure 4 and Figure 6 contain the residuals difference between actual observations and forecasts for the sample period defined above. By simply comparing Figure 3 and Figure 5, at first glance it is observed that Figure 5 contains better forecasts than Figure 3. Similarly, in Figure 6, the forecast errors of the ARIMA model have obviously an narrower range compared to the forecast errors of the regression model held in Figure 5, which might be seen as an initial evidence that the ARIMA model is more appropriate for forecasting the CIC.

Table 3 contains the performance of the forecast estimations with the Diebold-Mariano test results. The first column contains the horizon of the forecasting. H=5 means that we forecast the CIC for five working days ahead using the given methods. The second column contains the actual amount of currency issued by the QCB in million QR. The ARIMA column represents the seasonal ARIMA forecast for the given horizon period. Similarly, the REG column represents the forecast amount for the given time horizon in million QR. In the table, we forecast the CIC for different time horizons starting from one day ahead to 261 days ahead. Table 3 helps us to comment on the ranges of the forecast errors. The ARIMA model implemented for the first five working days suggests that the forecast error has a range of 20 million QR to 65 million QR. For the same time horizons, the forecast error of the regression model has a range of 35 million to 131 million QR. When we implement the ARIMA model to forecast six to ten working days ahead, the forecast error has a range of 14 to 73 million QR whereas, the regression method's forecast errors have a range of 19 to 76 million QR at the same time horizon. The forecast errors of both methods increases significantly with a longer time horizon. The error in forecasting the CIC one year ahead(261 working days) is 121 millions QR with the ARIMA method and 112 million QR with the regression methods. In addition, the Diebold-Mariano test is used for testing the equivalence of the different forecasts. In the table, the last column contains the p-value of test results of the Diebold Mariano test statistics.[12] The table shows that the forecasts

[12]The null hypothesis for the Diebold Mariao (1995) test is Ho: The forecasts provide statistically same values and alternative hypothesis is that H1: The forecasts provide statistically different values. p-value in the last column contains this statistics. Intuitively, the lower the p-value, the higher the rejection rate of the null

are statistically different values from each other, although for the horizon period of 5 to 10 days we could not see that difference clearly (When H=5 p-value is 0.06, and when H=8, p-value is 0.07. For forecast horizons above 10, one can observe that the forecasts provide statistically significant different values from each other.

Table 3: **Evaluation Statistics for Regression Model and ARIMA Model**

Horizon (h)	Actual (million QR)	ARIMA (million QR)	REG (million QR)	D-M p-value
1	5265.45	5199.61	5134.33	0.01
2	5150.66	5114.19	5102.85	0.05
3	5037.41	5099.12	5115.22	0.05
4	5037.41	5056.88	5078.47	0.04
5	4928.02	4908.13	4893.11	0.01
6	4902.02	4916.22	4921.43	0.02
7	4883.13	4898.83	4904.92	0.04
8	4906.13	4879.37	4871.07	0.07
9	4928.06	4874.11	4852.28	0.03
10	4842.74	4878.19	4903.55	0.02
15	4803.65	4823.41	4832.05	0.02
20	4758.88	4724.44	4744.55	0.03
60	5339.14	5312.14	5208.21	0.06
130	5498.94	5438.55	5432.54	0.08
261	5819.98	5699.79	5708.22	0.01

*Notes.*In the first column, H refers to the the forecast horizon. H=5 means that, the forecasts are implemented for five working days ahead. The second column refers to the actual amount of currency issued by the QCB in million QR. The third column represents the forecast estimation with ARIMA Method for the given time horizon. The fourth column represents the forecast estimation with the regression method for the given time horizon. The last column contains the Diebold-Mariano test p-values. In this test we test if the forecasts are statistically different from each other.

Table 4 contains the RMSE estimations for the forecast models. As we can observe, the ARIMA model generally has lower forecast error rate than the regression model, which again confirms the appropriateness of the ARIMA model over the regression model for the CIC forecasting. Another finding is that, for both models, the forecast errors do not increase significantly with the forecast horizon when we limit the forecast horizon to 10 days. When forecast horizon is greater than 10 days, the forecast error increases remarkably for both forecast models.

5 Concluding Remarks

In this paper, we forecast the currency in circulation issued by the QCB by using linear forecasting methods. Comparing the linear methods, the seasonal ARIMA model provides better estimates for short-term forecasts. The range of forecast error is always less than 100 million QR with the seasonal ARIMA methodology for short-term CIC forecasts, the error terms of re-

hypothesis.

Table 4: **RMSE for Regression Model and ARIMA Model**

horizon(h)	ARIMA RMSE	REG RMSE
5	0.374	0.379
6	0.376	0.378
7	0.363	0.374
8	0.371	0.377
9	0.391	0.384
10	0.413	0.432
15	0.41	0.435
20	0.405	0.427
60	0.421	0.446
130	0.483	0.504
261	0.531	0.613

Notes. RMSE(Root of mean squared error) is a frequently-used measure of the differences between values predicted by a model or an estimator and the values actually observed from the thing being modeled or estimated.

gression methodology is higher though. For long-term forecasts, both models suffer from larger forecast errors. Since we did not use non-linear techniques to forecast the liquidity, we do not know how appropriate those models are for forecasting the CIC. In the future, it will be suitable to forecast the CIC, using a non-linear forecasting method, the feed-forward neural network method and to evaluate the forecasting performance of the linear and non-linear methods and document the best method for different forecast horizons.

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Table 5: **Determinants of the Currency Issued by QCB**

Constant	-0.003^{***}	SUNDAY	0.005^{***}
6BEIDFITR	0.006^{*}	THURSDAY	0.004^{***}
7AEIDADHA	0.002	TUESDAY	-0.003^{***}
7BEIDFITR	0.003^{*}	NOVEMBER	0.003^{***}
7BEIDADHA	0.0001^{***}	RAMADAN1	0.003
APRIL	0.001^{*}	SEPTEMBER	0.003^{***}
Dec 22	-0.002^{*}	1BAUG1	0.002
Dec 23	0.001^{*}	JULY1	0.001^{*}
Dec 24	-0.001	1BJULY1	0.004
Dec 25	0.0004	THANKSG	–0.0001
Dec 26	0.007^{**}	VALENTINE	–0.001
Dec 31	0.001^{***}	JANUARY	0.001
EIDALADHA1	0.01^{***}	6BEIDADHA	0.012^{***}
EIDALADHA2	0.004	5BEIDADHA	0.01^{***}
EIDALADHA3	0.001	4BEIDADHA	0.01^{***}
EIDALADHA4	0.004	3BEIDADHA	0.02^{***}
EIDALFITR1	0.02^{**}	2BEIDADHA	0.04^{***}
EIDALFITR2	0.01^{**}	1BEIDADHA	0.018^{***}
EIDALFITR3	0.004	5AEIDADHA	0.003
FEB	0.001	4AEIDADHA	-0.009^{***}
FIRST DAY	0.011^{***}	3AEIDADHA	-0.012^{***}
SEPT 3	0.001	2AEIDADHA	-0.022^{***}
JULY	0.001	1AEIDADHA	-0.02^{***}
JUNE	0.001	6AEIDFITR	−0.01
LAST DAY	0.02^{***}	5BEIDFITR	0.001^{*}
MARCH	0.004^{***}	4BEIDFITR	0.02^{**}
MAY	0.003^{***}	3BEIDFITR	0.03^{**}
MONDAY	-0.006^{***}	2BEIDFITR	0.03^{***}
1BEIDFITR	0.011^{***}	1AEIDFITR	–0.004
4BRAMADAN	0.01^{***}	3BRAMADAN	−0.001
2BRAMADAN	0.002^{**}	1BRAMADAN	0.0002^{*}
1ARAMADAN	0.003	4BTHANKSG	–0.001
3BTHANKSG	–0.002	2BTHANKSG	–0.007
1BTHANKSG	–0.001	3BVALENT	0.003
2BVALENT	–0.001	1BVALENT	–0.008
LAST DAY	0.013^{***}	2BLASTDAY	0.007^{***}
1BLASTDAY	0.013^{***}	JANUARY 1	–0.001
OUTLIER	0.232^{***}		
R-square	0.38	Adjusted R-square	0.37
Sum of Square RESID	0.16	Schwarz criterion	–6.14
Akaike Info Criterion	–6.36		

Notes: The dependent variable is the first difference of logarithms of currency issued by State of Qatar for the between 2002 and 2006. The regression equation is

$$\Delta y_t = \sum_{i=1}^{11} \alpha_i M_{i,t} + \sum_{i=1}^{4} \beta W_{i,t} + \sum_{i=1}^{k} \theta_i(B)\delta_{i,t} + \sum_{i=1}^{j} \gamma_{i,t} O_{i,t} + k_{i,t} + \varepsilon_t.$$

- All variables listed above are the binary dummy variables. These variables take 1 for the particular calendar day and take zero for elsewhere. For instance, JANUARY 1 takes one when it is January 1 (or the first business day after the January 1) and zero elsewhere. Likewise, the JANUARY dummy variable takes 1 for the business days of January and 0 elsewhere.
- *XB* stands for X working days before the particular holiday or important day. For instance, 2BEIDADHA means that two business days before the first day of Eid-al-Adha.
- *XA* stands for X working days after particular holiday or important day. For instance, 2AEIDADHA means that two business days after the last day of Eid-al-Adha.
- LAST DAY stands for the last business day of the month and, similarly FIRST DAY stands for the first business day of the month.
- THANKSG stands for Thanksgiving Day which only celebrated in US.(It is the third Thursday of every November.)
- VALENT stands for valentine Day (14 of February in the Gregorian calendar).
- OUTLIER corresponds the big amount of error terms in the regression that is not explained by the calendar effect variables.
- ***,** and * denote 1 %, 5%, and 10 % significance level respectively.

Table 6: **Determinants of the Currency Issued by QCB**

Constant	-0.003^{***}	SUNDAY	0.005^{***}
6BEIDFITR	0.006^{*}	THURSDAY	0.004^{***}
7AEIDADHA	0.001^{**}	TUESDAY	-0.003^{***}
7BEIDFITR	0.003	NOVEMBER	0.003^{***}
7BEIDADHA	0.0001^{*}	RAMADAN1	0.003
APRIL	0.001^{*}	SEPTEMBER	0.003^{***}
AUGUST	0.0001	AUG1	0.001
Dec 22	-0.004	1BAUG1	0.002
Dec 23	0.002	JULY1	0.001^{*}
Dec 24	-0.0004	1BJULY1	0.004
Dec 25	0.0004	THANKSG	-0.0001
Dec 26	0.007^{**}	VALENTINE	-0.001
Dec 31	0.001^{***}	JANUARY	0.001
EIDALADHA1	0.01^{***}	6BEIDADHA	0.012^{***}
EIDALADHA2	0.004	5BEIDADHA	0.01^{***}
EIDALADHA3	0.001	4BEIDADHA	0.011^{***}
EIDALADHA4	0.004	3BEIDADHA	0.013^{***}
EIDALFITR1	0.01^{*}	2BEIDADHA	0.017^{***}
EIDALFITR2	0.01	1BEIDADHA	0.018^{***}
EIDALFITR3	0.004	5AEIDADHA	0.003
FEB	0.001	4AEIDADHA	-0.009^{***}
FIRST DAY	0.011^{***}	3AEIDADHA	-0.012^{***}
SEPT 3	0.001^{*}	2AEIDADHA	-0.022^{***}
JULY	0.0006	1AEIDADHA	-0.019^{***}
JUNE	0.001	6AEIDFITR	-0.001
LAST DAY	0.02^{***}	5BEIDFITR	0.001^{*}
MARCH	0.004^{***}	4BEIDFITR	0.014^{***}
MAY	0.003^{***}	3BEIDFITR	0.017^{***}
MONDAY	-0.006^{***}	2BEIDFITR	0.026^{***}
1BEIDFITR	0.011^{***}	1AEIDFITR	-0.004
4BRAMADAN	0.01^{**}	3BRAMADAN	-0.001
2BRAMADAN	0.002^{*}	1BRAMADAN	0.0002^{*}
1ARAMADAN	0.003	4BTHANKSG	-0.001
3BTHANKSG	-0.003	2BTHANKSG	-0.007
1BTHANKSG	-0.003	3BVALENT	0.003
2BVALENT	-0.004	1BVALENT	-0.008
LAST DAY	0.007^{***}	2BLASTDAY	0.007^{***}
1BLASTDAY	0.010^{***}	JANUARY 1	-0.001
OUTLIER	0.30^{***}		
AR(1)	0.32^{***}	MA(1)	0.31^{**}
AR(2)	-0.05^{**}	MA(2)	0.02^{**}
AR(3)	-0.07^{**}	MA(3)	0.11^{**}
AR(4)	0.10^{**}	MA(4)	0.12^{**}
AR(5)	0.04^{*}	MA(5)	0.12^{*}
AR(6)	0.09^{**}	MA(6)	0.12^{**}
AR(22)	0.06^{***}	MA(22)	-0.04^{**}
AR(44)	0.22^{**}	MA(44)	0.06^{***}
AR(48)	0.12^{***}	MA(48)	0.03^{**}
AR(65)	0.04^{***}	MA(65)	0.07^{***}
R-square	0.65	Adjusted R-square	0.653
Sum of Square RESID	0.11	Schwarz criterion	-6.49
Akaike Info Criterion	-6.71		

Notes: The dependent variable is the log of banknotes and coins issued by State of Qatar for the period between 2002 and 2006. The remaining coefficients are not listed in the table due to space constraint.

$$y_t = \sum_{i=1}^{11} \alpha_i * M_{i,t} + \sum_{i=1}^{4} \beta * W_{i,t} + k_t + \sum_{i=1}^{k} \theta_i(B)\delta_{i,t} + \sum_{i=1}^{j} \gamma_{i,t} O_{i.t} + \eta_t$$

$$\eta = \frac{\theta(B)}{\phi(B)\Delta(B)} \epsilon_t.$$

$$\Delta(B) = (1 - B)(1 - B^{261}).$$

The moving average operated model for the equation as follows;

$$\theta(B) = 1 + \theta_1 B^1 + \theta_2 B^2 + \theta_3 B^3 + \theta_4 B^4 + \theta_5 B^5 + \theta_6 B^5 + \theta_7 B^{22} + \theta_8 B^{44} + \theta_9 B^{48} + \theta_{10} B^{65}$$

$$\phi(B) = 1 + \phi_1 B^3 + \phi_2 B^6 + \phi_3 B^7 + \phi_4 B^4 + \phi_5 B^5 + \phi_6 B^6 + \phi_7 B^{22} + \phi_8 B^{44} + \phi_9 B^{48} + \phi_{10} B^{65}.$$

For the definition of the variables see Table 5.

Figure 1

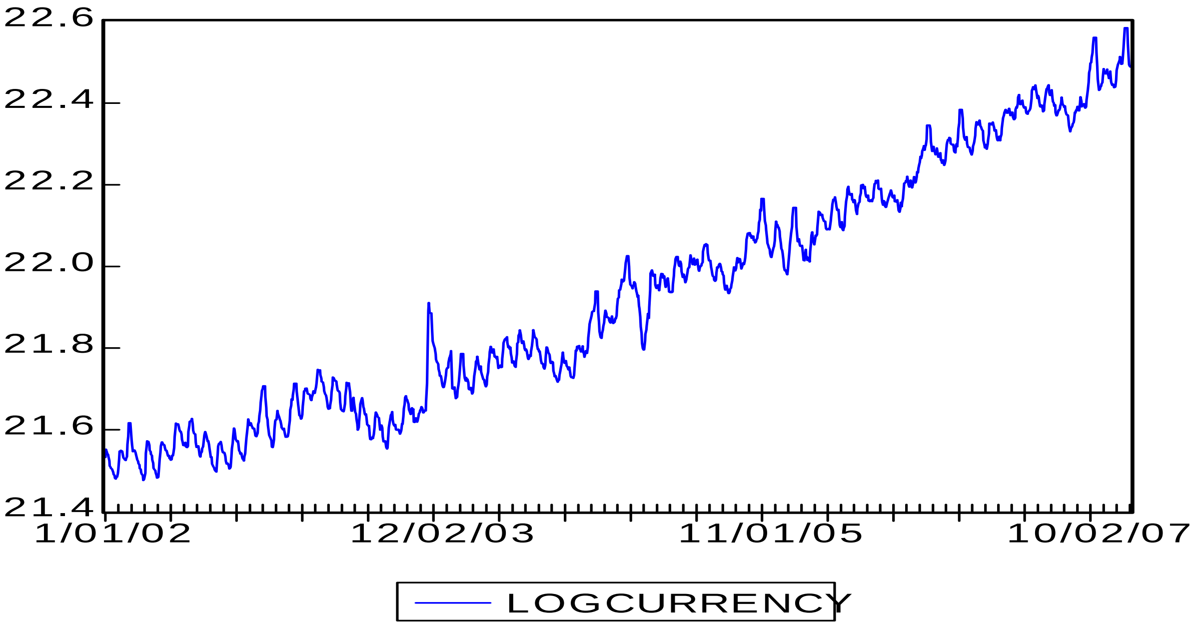

Figure 2

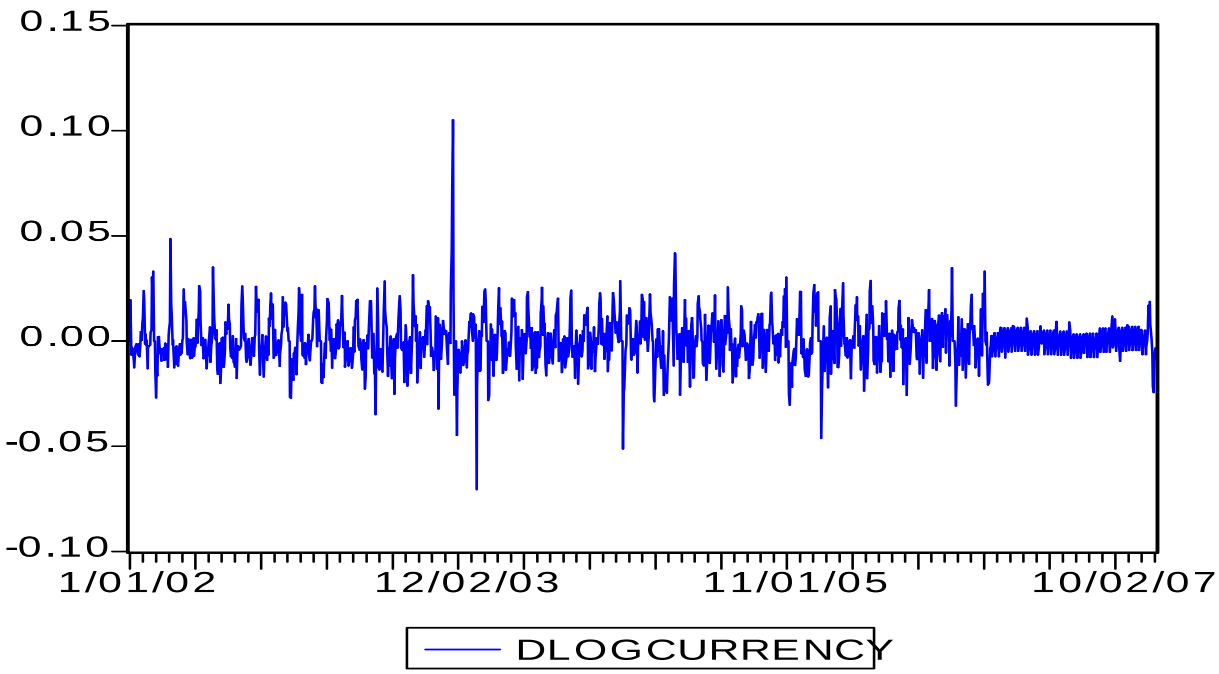

Figure 3

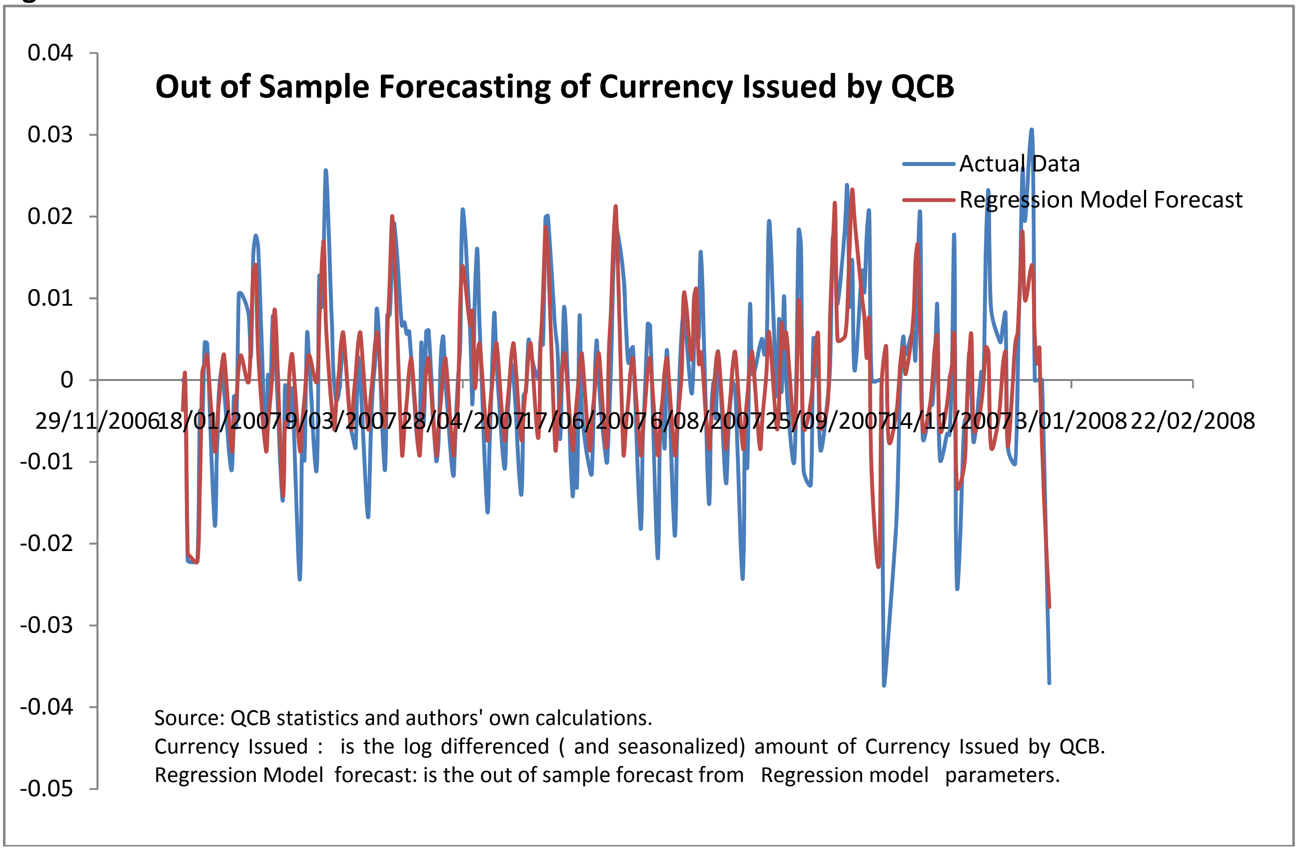

Figure 4

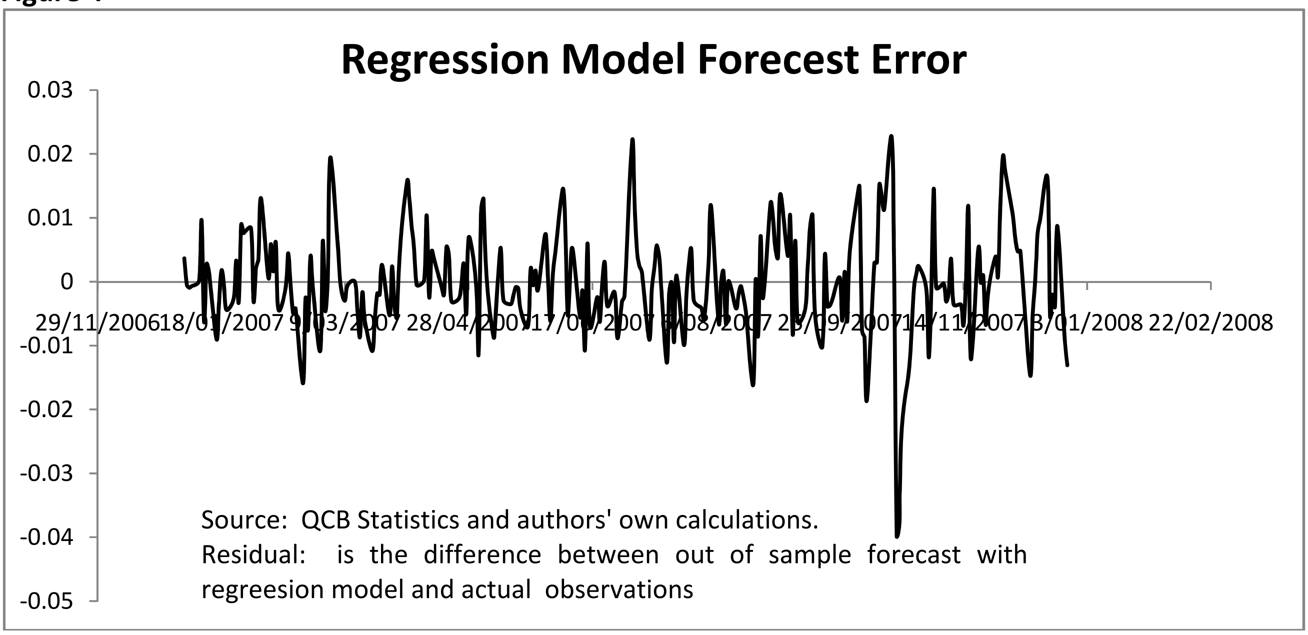

Figure 5

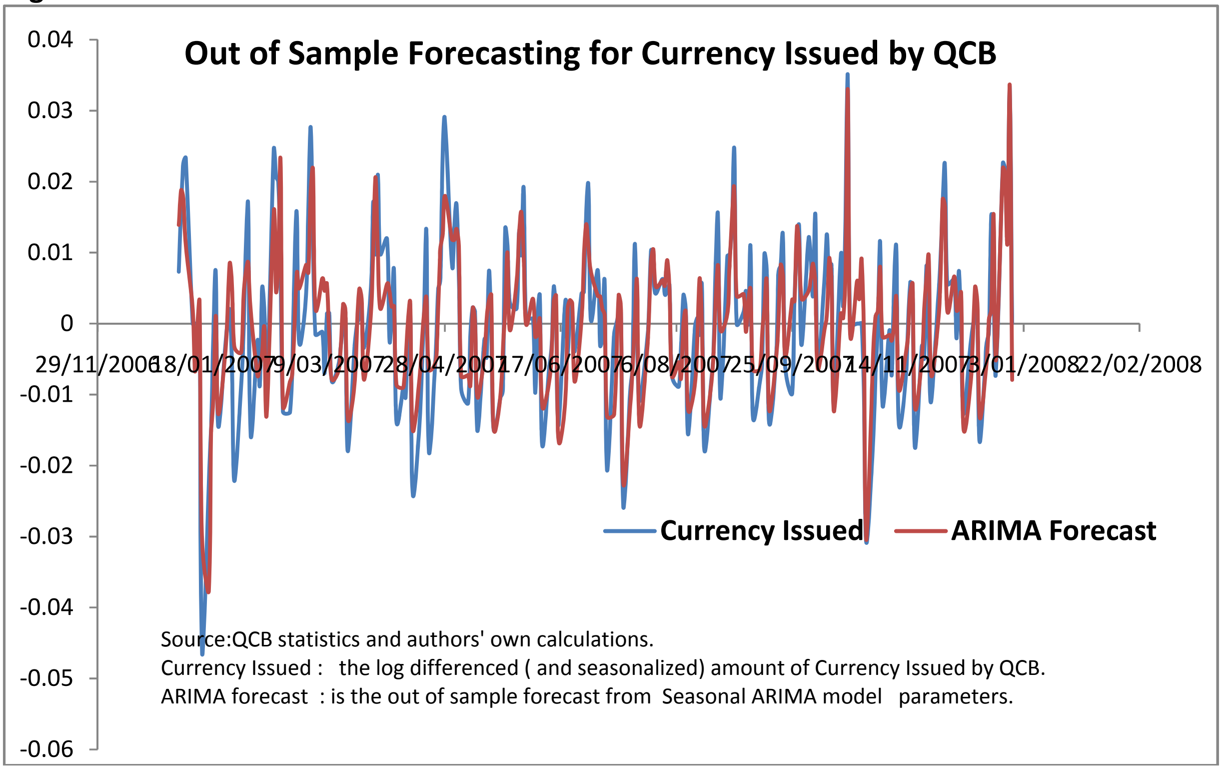

Figure 6