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MORE ON ESTIMATING ELASTICITIES USING LAGGED ENDOGENOUS VARIABLES

by

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Introduction

In an article appearing in this journal, Bopp and Pendley[1], hereafter abbreviated B-P, have considered the problem of constructing confidence intervals for long-term elasticities estimated from logarithmic equations containing lagged endogenous variables as explanatory variables. The estimated long-term elasticity for explanatory variable i is

$$\hat{\eta}_i = \frac{\hat{B}_i}{(1-\hat{\lambda})} \quad , \tag{1}$$

where $\hat{B}_i$ is the estimated regression coefficient for explanatory variable i, or equivalently, the short-term elasticity for explanatory variable i, and $\hat{\lambda}$ is the estimated coefficient for the lagged endogenous variable. Bopp and Pendley use an estimate of the asymptotic variance of $\hat{B}_i/(1-\hat{\lambda})$ to construct approximate confidence intervals for η_i. The purpose of this note is to present a simple technique for constructing <u>exact</u> confidence intervals for η_i. This technique is also illustrated by application to the data used by B-P.

Exact Confidence Intervals for Long-Term Elasticities

The ratio $\hat{B}_i/(1-\hat{\lambda})$ is, under the usual assumptions, a ratio of two normally distributed random variables with the numerator having an expected value of B_i and variance $\sigma^2_{\hat{B}_i}$, and the denominator having expected value of $1-\lambda$ and variance $\sigma^2_{\hat{\lambda}}$. The covariance of the numerator and denominator is $-\sigma_{\hat{B},\hat{\lambda}}$. Although Hinkley[2] has derived an exact expression for the exact cumulative distribution function (c.d.f.) for such a ratio, evaluation of that function involves use of tabulations of the bivariate normal distribution function. Since use of these tabulations usually requires trivariate interpolation, construction of exact confidence intervals for η_i using this approach would be at best inconvenient.

However, a result due to Fieller[3] allows construction of exact confidence intervals for η_i and requires no more than solutions to quadratic equations. This result makes use of the fact that the ratio R = $\hat{B}_i/(1-\hat{\lambda})$ is equivalent to

$$\hat{B}_i - R(1-\hat{\lambda}) = 0 \ . \tag{2}$$

A 1-α percent confidence interval for R is given by those values of R for which

$$\text{Prob}\left\{\frac{(\hat{B}_i - R(1-\hat{\lambda}))^2}{s^2_{(1-\hat{\lambda})} - 2R\left(\hat{B}_i(1-\hat{\lambda}) - t^2_{\alpha/2}s_{\hat{B}_i,(1-\hat{\lambda})}\right) + \hat{B}_i^2 - t^2_{\alpha/2}s^2_{\hat{B}_i}} \leqq t^2_{\alpha/2}\right\} = 1-\alpha \tag{3}$$

where $s^2_{\hat{B}_i}$ is the sample variance of $\hat{B}_i$; $s^2_{(1-\hat{\lambda})}$ is the sample variance of $(1-\hat{\lambda})$; $s_{\hat{B}_i,(1-\hat{\lambda})}$ is the sample covariance of $\hat{B}_i$ and $(1-\hat{\lambda})$; and $t^2_{\alpha/2}$ is the squared value of the upper $\alpha/2$ percentage point of the t-distribution with appropriate degrees of freedom, or equivalently the upper α percentage point of the F-distribution with one degree of freedom in the numerator and the appropriate degrees of freedom in the denominator. For the problem at hand, the appropriate degrees of freedom for the t-distribution or the denominator of the F-distribution are the error degrees of freedom of the regression used to estimate $\hat{B}_i$ and $\hat{\lambda}$. For the confidence interval to be closed, $(1-\hat{\lambda})$ must be significantly different from zero. The exact $1-\alpha$ confidence limits for η_i are those values of R for which

$$R^2\left((1-\hat{\lambda})^2-t^2_{\alpha/2}s^2_{(1-\hat{\lambda})}\right)-2R\left(\hat{B}_i(1-\hat{\lambda})-t^2_{\alpha/2}s_{\hat{B}_i(1-\hat{\lambda})}\right)+B^2_i-t^2_{\alpha/2}s^2_{\hat{B}_i} = 0. \quad (4)$$

Note that the solution to this quadratic equation in R need not, and generally will not, be symmetric about $\hat{\eta}_i$. Thus, the use of the asymptotic variance of $\hat{\eta}_i$ to construct approximate confidence intervals for η_i centered upon $\hat{\eta}_i$ can be misleading.

An Application

In order to illustrate the construction of exact confidence intervals for long-term elasticities, we make use of the data analyzed by B-P. They estimated a logarithmic equation expressing residential electrical consumption in Connecticut (Q_t) as a function of real disposable income (Y_t), own marginal price (P_t), and lagged consumption (Q_{t-1}) using annual observations from 1960 to 1975. The estimated short- and long-term elasticities and their respective variances are summarized in Table 1. The estimated coefficient of adjustment $(1-\hat{\lambda})$ was 0.152, and the relevant covariance terms were estimated as follows: $s_{\hat{Y},(1-\hat{\lambda})} = -s_{\hat{Y},\hat{\lambda}} = 0.005$; and $s_{\hat{P},(1-\hat{\lambda})} = -s_{\hat{P},\hat{\lambda}} = 0.0008$. The error degree of freedom is 12. With this information, exact confidence intervals can be constructed for both the short- and long-term elasticities. These are also displayed in Table 1.

Note that the exact confidence intervals for the long-term elasticities are much wider than their respective short-term counterparts. Also, the exact confidence intervals for the long-term elasticities are not symmetric about their respective point estimates, a result not

Table 1. Exact Confidence Intervals for Short- and Long-Term Elasticities

	Estimated Elasticity[a]		Exact 95% Confidence Intervals	
Variable	Short-term	Long-term	Short-term	Long-term
Income	0.17 (0.1221)	1.13 (0.474)	-0.10 to 0.44	-1.51 to 1.78
Price	-0.21 (0.0480)	-1.4 (0.587)	-0.31 to -0.11	-4.08 to -0.52

a. Standard errors of estimated elasticities are shown in parentheses. The standard errors for long-term elasticities are asymptotic.

detected by the use of the asymptotic variance of $\hat{\eta}_i$ in constructing appropriate confidence intervals for η_i. Although the exact confidence interval for the short-term income elasticity is wider than the interval for short-term own price elasticity, the relative widths of the long-term elasticity intervals are reversed, a result agreeing with the findings of B-P.

Summary and Conclusions

The purpose of this note is to demonstrate a technique for constructing _exact_ confidence intervals for long-term elasticities from logarithmic equations containing lagged endogenous variables as regressors. This technique involves only simple computations based upon results generated in the course of parameter estimation.

Notes

[1]Bopp, Anthony E., and Robert E. Pendley. "A Note on Estimating Economic Elasticities Using Lagged Endogenous Variables," Intermountain Economic Review (Fall 1978), pp. 111-7.

[2]Hinkley, D. V. "On the Ratio of Two Correlated Normal Random Variables," Biometrika 56(1969), pp. 635-9.

[3]Fieller, E. C. "The Distribution of the Index in a Normal Bivariate Population," Biometrika 24(1932), pp. 428-40.

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