Economics Bulletin

Volume 31, Issue 4

Harrod's cumulative growth circle

Giulio Guarini
Italian Ministry of Economic Development

Abstract

In this paper I aim to show through an analysis from a history of economic thought perspective that Harrod's famous model, An Essay in Dynamic Theory (1939), can be considered a fundamental example of cumulative growth circle. Having described the original model, I then go on to examine the meaning of dynamic analysis and instability in Harrod's approach, analyzing respectively the debates between Asimakopulos and Kregel and between Harrod and Keynes. Finally, I describe the main properties of Harrod cumulative growth circle by comparing Harrod's model to Domar's (1946). I argue that Harrod's growth approach is very different from the mainstream version.

1. Introduction

The aim of this paper is to demonstrate with an analysis from a history of economic thought perspective, that Harrod's model, in the original rather than the received version, can be seen as a significant example of cumulative growth circle, illustrating its characteristics. I must point out from the start that Harrod himself made no explicit reference to cumulative growth circles, but in practice instability in the commodity and labour markets arises precisely through demand and supply interaction driving the system away from equilibrium. This link in unstable cumulative circularity is studied in the fundamental model with 'instant in time' analysis in which a decisive role is played by the state of uncertainty characterising the entrepreneurs' choices. I begin by outlining the original model and go on to examine the Harrodian meaning of dynamic analysis and instability through the interpretative debate between Asimakopulos and Kregel. I then look into how Harrod conceived of his 'instant in time' analysis, taking into consideration the correspondence between Harrod and Keynes, and highlight the peculiarities of Harrod's cumulative circle link that subsequently disappeared in the canonical version of the what is known as the Harrod-Domar model, tracing out the main differences distinguishing Domar's version from Harrod's original model. In conclusion I will illustrate how, beginning with Solow, the Neoclassical approach was held to have solved the problem of Harrodian cumulative growth circle instability.

2. Harrod's original model

In 1939 the economist Roy Harrod published *An Essay in Dynamic Theory* with the aim of laying the foundations for a dynamic theory which could take its place alongside (and supersede) the traditional static theory. In modern economic literature Harrod's is seen as the archetypal economic growth model; indeed, Harrod may with reason be described as the founder of modern growth theory. What Harrod produced was, in fact, a new, dynamic version of the Keynesian approach. In the commodity market equality still obtained between *ex-post* investments and total savings[1]. With the symbols adopted by Harrod:

$$S \equiv sx_0 = I \equiv C_p(x_1 - x_0) \quad (1)$$

where $C_p \equiv I/(x_1 - x_0)$.[2]

The total savings, S, are given by the product of the average propensity to save, s, and production at time 0, x_0, C_p represents the value of increase in capital in the period divided by the increase in the effective total output,; consequently, the total investments, I, correspond to the product between C_p and the increase in effective total output $(x_1 - x_0)$. Hence the effective income growth rate is obtained thus:

$$G \equiv (x_1 - x_0)/x_0 = s/C_p \quad (2).$$

Equality (1) continues to apply as long as the entrepreneurs, at the end of the period, adjust the supply decided upon at the beginning of the period in accordance with the effective market demand,

---

[1] Reference is to a closed economy with no public sector; otherwise, the equality would be $S + T + M = I + G + E$ in which T, M, G, E stand respectively for taxes, imports, public spending and exports. Moreover, the investments are net of amortisation.

[2] Equation (1) implies that that there is no autonomous consumption, since $S \equiv x - Z - T$, where x is the product, Z the consumption and T taxes, and in this case $T = 0$ and $S = sx$, so that we have $sx = x - Z$, and therefore $Z = (1-s)x$, implying that consumption is proportional to income (i.e. there is a constant average as well as marginal propensity to consume).

modifying stocks in such a way as to achieve equality between investments and total savings (in other words, any excess or shortage that may occur translates into increase – or reduction – in stocks; by definition, modifications in stocks are included in investments; essentially, therefore, equality between *ex post* investments and total savings constitutes an accounting identity). On the other hand, commodity market equilibrium obtains when *ex ante* investments are equal to the total savings, or in other words when the increases in capital programmed by the entrepreneurs on the basis of their evaluations are such as to match aggregate supply and demand. In other words, the level of income is in equilibrium when the inputs (*I*) into the circular income flow are equal to the subtractions (*S*). In this case no undesired variations occur in the stocks and the firms can go on producing, following a path corresponding to a constant growth rate, which Harrod called the "warranted rate of growth":

$$G_W = s/C \ (3)$$

where C is the increase in capital desired by the entrepreneurs *ex ante*, which corresponds to C_p only in equilibrium. This rate "warrants" that the system will keep to the equilibrium growth rate over time. The crucial variable in the model are the investments, which have a twofold function. On the one hand, together with consumption, planned (*ex ante*) investments constitute aggregate demand. On the other hand, by virtue of Clark's acceleration principle $I = C_p(x_1 - x_0)$, actual (*ex post*) investments constitute an essential element for growth in aggregate supply, providing for expansion of productive capacity in proportion to the expanding demand. Also, by virtue of Kahn's multiplier principle $x_0 = I/s$, *ex post* investments determine income. In this way, if income grows at the rate G_W, the increase in demand and in productive capacity will combine harmoniously, leaving the entrepreneurs satisfied with their choices to the extent that, *ceteris paribus*, they will have no inclination to change their production and investment plans. In the type of equation described (3) G_W depends on certain "fundamental conditions" – the average propensity to save and the state of technology – determined by simple parameters (respectively *s* and C), and referring to a point in time. This marks Harrod's conception of dynamics out from the conception dominant at the time; in fact, he wrote: *"[…] I prefer to define dynamic as referring to propositions in which a rate of growth appears as an unknown variable. This equation is clearly more fundamental than those [in the traditional approach, A/N] expressing lags of adjustment"* (Harrod, 1939 p. 17). The economists among his contemporaries, who in general approached economic dynamics in terms of lags, took a dim view of the 'instant-in-time' method. Indeed, they took a contrasting approach which can be traced back to Ragnar Frisch, and which had it that a system is dynamic only if at least one of its equations refers to different points in time[3].

The main problem for Harrod was that the flow of income is not very likely to grow at the equilibrium rate since centrifugal forces are at work on it from outside; in fact, the market mechanisms tend to increase the divergence between the effective rate of growth, *G*, and the warranted rate of growth G_W. If $G > G_W$, we have $C > C_p$ and the *ex ante* investments exceed the *ex post* investments since an undesired reduction of stocks has occurred. In this case the aggregate demand, consisting of consumption plus *ex ante* investments, exceeds supply, which is equal to consumption plus *ex post* investments (= total savings). The system will then require greater investments to boost supply; in other words, the entrepreneurs will have to step up production to reconstitute the stocks and/or plant to the extent determined by the acceleration principle. As we

---

[3] Kregel wrote *"As is now well known, Harrod had become interested in the problems of dynamics, which he first called "steady advance" (...) and then more clearly defined as the study of rates of change at a point of time* (Harrod 1939)" (Kregel 1980, p.98) and Besomi, underlining Harrod's originality in pursuing his 'instant-in-time' analysis, wrote: *"Harrod distinguishes his own approach from the econometricians - who define dynamics as having a cross-reference to two points of time - (1939,21)*, (Besomi 1996, p.289).

have said, however, investments also form part of demand; thus, through the multiplier, the increase in investments will lead to a further increase in income, with the result that the divergence between the two rates will continue to grow. If $G < G_W$, we have $C < C_p$, which means an increase in stocks, given the positive difference between *ex post* investments and *ex ante* investments; thus there will be an excess in supply with consequent pressure to reduce production and, through the acceleration principle, investments, too. By virtue of the multiplier, the reduction in investments will generate a further fall in aggregate demand, and the difference between the two growth rates will consequently widen. According to Harrod, as the system approaches full employment certain elements can slow down the growth of the productive system. In the first place, labour mobility is deemed to be decreasing; moreover, expansion of productive capacity is compromised by bottlenecks caused by specific capital shortages. And, finally, the entrepreneurs will compare the current rate of growth in demand with the average rate and, should they judge it unsustainable, they will tend to hold the growth of orders in check. As Harrod sees it, the market is unable to keep the system in equilibrium due to lack of coordination: *"Thus in the dynamic field we have a condition opposite to that which holds in a static field. A departure from equilibrium, instead of being self-righting, will be self-aggravating. G_W represents a moving equilibrium, but a highly unstable one."* (Harrod, 1939, p. 22) Thus Harrod found the dynamic condition for equilibrium between demand and supply; in keeping with Keynes's teaching, this condition may not be optimal from the point of view of social life. In other words, commodity market equilibrium can coexist with involuntary unemployment. In getting to grips with this problem, Harrod defines the "natural growth rate" as *"the maximum rate of growth allowed by the increase of population, accumulation of capital, technological improvement and the work/leisure preference schedule, supposing that there is always full employment in some sense"* (Harrod, 1939, p. 30). To put the matter in a few words, the natural rate of growth can be considered that particular rate that would allow for full employment of the available workforce, and which is given by the sum of the rate of growth in labour supply, n , and the rate of labour productivity, π. In short,[4] we have:

$$G_N = n + \pi \quad (4).$$

According to Harrod, the two rates G and G_N are unlikely to converge and *"there is no inherent tendency for these two rates to coincide"* (Harrod, 1939 p.30) since the market mechanisms are unsuccessful in coordinating the choices of the various economic agents. In fact, if $G < G_N$ unemployment will tend to grow because the parameters (n, π, s, C) that make up the rates in the Harrodian model are independent of one another. In this respect the economic and social system contains no self-regulators.

At the end of his paper Harrod reduces the influence of the acceleration principle, acknowledging that not all outlays in capital account can be directly affected by variations and level of short period income, for they may be related to technological innovations or activities having to do with long-term processes. Moreover, the fundamental equation is supplemented with further important but not crucial equations including the rates of import and export growth, but they are not relevant to our purposes in this paper. Harrod's model is very simple, but it brings some strikingly original and

---

[4] Expressing in rates of growth the identity $Y \equiv Y / L \equiv \Pi L$ we obtain $y \equiv \pi + n + \pi n$, but the practice is to omit the term πn since it takes on in insignificant values. In fact, if we interpret Harrod's model as operating in continuous time, then we can show that, since $Y \equiv Y / L \equiv \Pi L$, (where Π is the productivity of labour Y / L), then $G_N = \dot{Y} / Y = (L\dot{\Pi} + \dot{L}\Pi)/\Pi L = \dot{\Pi}/\Pi + \dot{L}/L = \pi + n$ (where $\dot{x}$ denotes the time derivative of x). If we wish for a discrete time interpretation, firstly we have $G_{Nt} = (Y_{t+1} - Y_t)/Y_t$, thus $G_{Nt} = (\Pi_{t+1}L_{t+1} - \Pi_t L_t)/\Pi_t L_t$, then we obtain $G_{Nt} = [\Pi_t (L_{t+1} - L_t) + (\Pi_{t+1} - \Pi_t)L_t + (L_{t+1} - L_t)(\Pi_{t+1} - \Pi_t)]/ L_t \Pi_t$; finally we have $G_{Nt} = [\Pi_t (L_{t+1} - L_t)/L_t\Pi_t] + [(\Pi_{t+1} - \Pi_t)L_t/L_t\Pi_t] + [(L_{t+1} - L_t)(\Pi_{t+1} - \Pi_t)/L_t\Pi_t] \approx n + \pi.$

specific elements into economic thinking on growth, as we will see on turning our attention to the interpretative dispute between Asimakopulos and Kregel, Keynes's papers and, finally, the differences between the models proposed by Harrod and Domar. This will bring into sharper and clearer focus the innovative pillars of Harrod's model: dynamics and instability.

3. The interpretative dispute between Asimakopulos and Kregel

Asimakopulos and Kregel differ widely in their interpretation of Harrod's thought in relation to the characteristics of the warranted rate of growth[5]. Asimakopulos argues that central to Harrod's thinking is the path of steady growth with a warranted rate of growth consistent with the producers' equilibrium, and that Harrod's analysis has to do with the possible fluctuations around that rate of growth. According to Kregel, on the other hand, Harrod's intention is to lay the foundations for a dynamic theory, and he studies the G_W trend in the trade cycle. Both positions have some truths to tell us, approaching Harrod's analysis from different perspectives. Kregel takes two texts into consideration together, The Trade Cycle (1936) and An Essay in Dynamic Theory (1939), and finds a common thread running through them[6]. According to Kregel, the 1939 essay develops the axiomatic bases for a dynamic theory that departs from the traditional theory. The focus is also brought to bear on the need for closer definition of what is meant by stability, since in static macroeconomics it applies to matching demand and supply, while in Harrod's new macroeconomics it refers to the relationship between the effective rate of growth and the warranted, or natural, rate of growth. Kregel also argues that, although it antedates the other text, in The Trade Cycle the axiomatic apparatus is used for analysis of the trade cycle.

Harrod attached considerable importance to the theoretical path upon which he had embarked since it was not only a matter of attempting to solve a technical problem but also a new way of thinking about and addressing the issues involved in dynamics. Harrod wrote: *"The significance of what follows should not be judged solely by reference to the validity or convenience of the particular equations set forth. It involves something wider: a method of thinking, a way of approach to certain problems. It is necessary to "think dynamically".* (Harrod, 1939, p.15) Harrod was not contemplating anything like a real theoretical revolution, for he did not see dynamics thus construed as contrasting with the orthodox static theory, and each of the two approaches has a specific field of application. As he wrote, in fact: *"Once the mind is accustomed to thinking in terms of trends of increase, the old static formulation of problems seems stale, flat and unprofitable. This is not to deny to static theory its own appropriate sphere. It will become apparent which kind of problem belongs to each branch of study."* (Harrod, 1939, p.15).

In The Trade Cycle, which antedates the other essay, Harrod dwelt upon both the multiplier and acceleration mechanisms, and upon the importance of dynamic analysis, concentrating on variations in the rate of growth in the various stages of the cycle and for the various different levels of income. Of course, Harrod's thought shows a fundamental consistency, and in An Essay in Dynamic Theory, too, reference is made to variations in *s* and *C*, but it is not central to the analysis here. As Kregel saw it, by concentrating solely on the matter of settling Harrodian instability, the Neoclassicals, the Neo-Keynesians and the Neo-Malthusians not only abandoned analysis of the

---

[5] Asimpakopulos himself illustrates the contrast between the two interpretations: *"The position taken in this paper is that Harrod tries to use the "normal" warranted rate of growth to define an equilibrium growth path, "a moving equilibrium", that provides the required line of steady growth"* around which to build his trade cycle analysis. A different view is expressed by Kregel who writes that…the analysis is no longer primarily concerned with explanation of fluctuations around the trend rate of growth, but with the explanations of the possibility of fluctuations in the trend rate itself *"* (Asimakopulos, 1986, p.280).

[6] He sees Harrod's theoretical work as divisible into two parts, thus:*"Thus, Harrod's approach could be divided into two parts; one part concerned with the theory of dynamics, the derivation of the basic axioms of the theory, and a second part which used these axioms to analyse the trade cycle*" (Kregel, 1980, p.114).

trade cycle but also identified the long-period rate of growth with the stable rate of growth at full employment, and here was the crux of the "bad" interpretations of Harrod's model[7]. Let us look more closely into this point.

Each of the theoretical schools that has "got to work" on Harrod's model has distorted the sense of it. The three schools all take the fundamental equation to apply to the long period; they then go on to find fault with keeping the dynamic parameters constant over time; finally, they attempt to solve the problem of the instability of the economic system by proposing automatic centripetal mechanisms. As the textbooks make amply clear, each strand takes "adjustment" of a particular parameter to be crucial – the rate of growth in labour supply, n, for the Neo-Malthusians, the average propensity to save, s, for the Neo-Keynesians, and, finally, the capital-income ratio, C, for the Neoclassicals. All these theoretical endeavours misrepresent Harrod's thought.

To begin with, they drop Harrod's reflections on variations over time of the parameters s and C which are analysed in The Trade Cycle and referred to in An Essay in Dynamic Theory, and which differ greatly from those of the three schools. In fact, on the variations of C Harrod wrote: *"The value of C depends on the state of technology and the nature of the goods constituting the increment of output. It may be expected to vary as income grows and in different phases of the trade cycle ; it may be somewhat dependent on the rate of interest."* (Harrod, 1939 p.17). As regards changes of s , Harrod: *"This may be expected to vary, with the size of income, the phase of the trade cycle, institutional changes, etc."* (Harrod, 1939, p.16). Secondly, the issue of the intrinsic instability of the economic system is totally eliminated, despite the fact that it remains the fundamental message of An Essay in Dynamic Theory. Thus the knife-edge does not apply to the warranted rate of growth, G_W, but to the instability of the system. In Scope and Method of Economics (1938) Harrod explained how, from the methodological point of view, the study of dynamics begins with analysis of a system in an instant of time, building up *"a simultaneous chart or survey of the economic field"* (Harrod, 1938, p.387), and then goes on to study the integration of these points in time as a sequence of events, analysing the *"general laws concerning the succession of events"* (Harrod 1938, p. 386). He meant to keep these two phases of study separate: thus, in An Essay in Dynamic Theory, he took the coefficients *s* and *C* as given parameters to derive the rate of growth, while in The Trade Cycle, when the focus is on the cycle itself, *s* and *C* became variables. Asimakopulos's analysis, then, underlines the central argument of the *Essay*, while Kregel takes a broader view of Harrod's thought and makes it available for construction of an approach alternative to the mainstream theory of growth.

4. The theoretical relationship between Harrod and Keynes

Over the two years 1937-38 Keynes and Harrod exchanged a great deal of correspondence discussing their respective theories. 1936 had seen the publication of the General Theory; six months later The Trade Cycle came out and in '37 Harrod passed on to Keynes the first draft of An Essay in Dynamic Theory. Harrod's analysis was dynamic, the unknown quantity in his fundamental equation being the rate of growth of income or of investments, while the framework for Keynes's analysis was static. In practice Keynes's analysis had as implicit point of reference an

---

[7] As is confirmed by Kregel himself: *"It is this shift in emphasis that lays the groundwork for the misinterpretation of Harrod's dynamic theory which produced the problem of the "knife-edge"."* (Kregel, 1980, p.102). And again *"It is this distinction between economic dynamic and trade cycle analysis that most of Harrod's readers have been unable to discern."* (Kregel, 1980, p.104). Kregel makes it clear that reference here is to the Neoclassical, Neo-Keynesian and Neo-Malthusian interpretations of Harrod's model, observing: *"The resolution of this problem of the knife-edge destroyed Harrod's method as well as the particular meaning that Harrod attached to the concept of a warranted rate of change. It also removed the analysis of cyclical fluctuations around a trend growth rate from the concerns of growth theory, which became solely concerned with long-period equilibrium growth paths ... a proper understanding of Harrod's original proposal concerning dynamics would have prevented the current tendency to identify long-period equilibrium with stable equilibrium growth at full employment (golden age)."* (Kregel, 1980, p.120).

advanced state of economic development in which the productive capacity has reached or at any rate can easily reach, if the entrepreneurs so decide, levels of available productive capacity sufficient to employ the workforce available, such that the authorities dealing with economic policy (and the theoretical economists who point out to them the path to follow) will be justified in turning their attention to stimuli for aggregate demand leading to growth in income with greater use of the available productive capacity, which is considered given.

Harrod looked into contemporaneous, parallel increases in effective demand and supply. In this context the part played by investments becomes fundamental for growth thanks to their twofold function with a role both in demand, through the multiplier principle, and in supply, through the acceleration principle. In Keynes, on the other hand, productive capacity being fixed, investments are a component of demand alone. Finally, while in Keynes investments are taken to be gross, in Harrod they are net, precisely because in Harrod's equations they are taken in relation to increases in production, with which costs for replacement of productive capacity deteriorating and falling into disuse over time logically have nothing to do. In the correspondence between the two, before the publication of Harrod's essay Keynes was not convinced that the warranted rate of growth remained constant over the long period, suggesting to Harrod that he should introduce a "temporary" warranted rate of growth depending on the level of production and employment, given the propensity to save, the level of expectations, the rate of interest and the state of technology.

In March 1939 Harrod published the final version of his essay, revising the first draft of August 1938 in the light of Keynes's criticisms. Among the points that Harrod made clearer were the term *C*, the meaning of warranted rate of growth and the difference between it and the natural rate. Moreover, he offered more precise definition of the term *ex ante* in relation to *C* ; in connection with this variable, in An Essay in Dynamic Theory he pointed out that the term "investments" as used in Keynes's Treatise on Money is to be understood as investments *ex ante*[8], and that Keynes's proposition stating that if I >S expansion will occur "*may still be a useful aid to thinking*" (Harrod 1939, p.19). Subsequently, in his Economic Dynamics Harrod introduced "temporary" warranted rates of growth, on the one hand reducing the effects of the instability principle, and on the other hand containing fluctuations between the lower and upper limit. As Besomi, the editor of the critical edition of the works of Harrod, interpreted the situation, Keynes played an important role in Harrod's theoretical formulation, but on one point the two economists stuck to their differing opinions. Keynes interpreted Harrod's analysis in a long-period perspective in which equilibrium must arrive at a stable position, as is the case with static analysis, and accordingly suggested to Harrod that he introduce into his analysis assumptions allowing for variations of s and C. Indeed, he wrote:*"[it is not correct to] assume absolute rigidity of s and v and a departure from warranted growth. You have to make some assumptions as to the changes in s and v in unwarranted conditions."* (Keynes, 1973, CW 14:334).

As Besomi points out, this was a misunderstanding that persisted throughout the correspondence, and the issue as to whether Harrod's analysis of economic growth referred to an instant in time or the long period remained the major difference[9]. Although Harrod introduced some modifications to the first version in response to Keynes's suggestions, the two economists continued to be divided on the major issue of a variation of parameters in the fundamental equation: Harrod accepted Keynes's

---

[8] As he writes: "C is rather that addition to capital goods in any period, which producers regard as ideally suited to the output which they are undertaking in that period" (Harrod, 1939, p.19) while, as we have seen, C_p is the net investment effectively made taking also into account undesired variations of stocks.

[9] Besomi writes:"*Keynes thus noticed the weakness of the analytical structure of the "Essay", but, instead of locating its cause in the peculiarity of Harrod's method, he thought Harrod was implicitly assuming constancy for his coefficients*" (Besomi, 1995, p.338).

advice to make his exposition clearer in the conviction that Keynes had only misinterpreted his theses, while Keynes appreciated Harrod's clarifications but still stood by his objections[10].

5. The main differences between Harrod's and Domar's models

In the dominant economic literature, Harrod's model is never presented in its original version, but as what is known as the Harrod-Domar model, far closer in spirit to the Neoclassical approach. It is worth taking a look, briefly, into the Domar model, presented in "Capital Expansion, Rate of Growth and Employment" of 1946, for it is interesting to see whether the two economists really did present one and the same model. In the following analysis I adopt the symbols used by Domar. It may seem inconsistent, and it will certainly not make things easier for the reader, not to have adapted the symbols in presentation of Domar's model to conform with those of Harrod's, but it does help to bring out the different approaches of the two models in which, in particular with respect to the rate of growth, variables indicated with the same name can have different implications. Domar's model consists of three principal relationships. On the supply side we have

$$\Delta P = I\sigma \ (5).$$

According to this relationship, an increase in productive capacity P is due to level of investments *I* multiplied by σ, which is the average potential social productivity of the investments. On the demand side we have our multiplier principle

$$\Delta Y = \Delta I / \alpha \ (6)$$

where α this average propensity to save. Finally, the position of equilibrium is

$$P = Y \ (7).$$

Working out the system of equations, Domar finds that the equilibrium rate of growth of the investments is $\alpha\sigma$. From here, applying the Keynesian equality $I = \alpha Y$, we arrive directly at the fundamental equation of Domar's model, which turns out to be similar to Harrod's:

$$\Delta Y / Y = \alpha\sigma \ (8).$$

Nevertheless, representing the different analyses of the two authors with a single model, summed up in this fundamental equation, remains a simplification that may prove misleading. In fact it will be seen that the two authors take substantially different paths to arrive at the same final equation. For Domar the core of the matter is the twofold function of investments, present on the supply side and on the demand side, while Harrod builds his model around firm equilibrium and the entrepreneurs' choices.[11]

---

[10] Besomi writes: "*From the Harrod-Keynes correspondence it emerges quite clearly that Harrod considered the first series of changes only as verbal amendments, introduced to avoid misunderstandings by clarifying some passages of the text that were obscure. In fact, at first Harrod accepted without discussion some of Keynes's suggestions, while he interpreted Keynes's fundamental criticism as being due to a misinterpretation of this thesis. Keynes, in turn, accepted some of Harrod's qualifications but did not abandon his line of attack.*" (Besomi, 1996, p.250).

[11] It is worth noting that, while having entrepreneurial decisions at the core of his analysis, Harrod does not take a Neo-classical approach of methodological individualism. Indeed, with reference to the equality between effective and warranted rate of growth he writes: "*Of course what applies to the system in general may not apply to each individual separately. But if one feels he has over-produced or over-ordered, this will be counterbalanced by an opposite experience of an equal importance in some other part of the field*" (Harrod, 1939, p.22).

The aim of Harrod's essay is to establish a dynamic analysis, while Domar sets out to find the conditions through which the economic system is maintained at a full employment level of growth. *"Our first task is to discover the conditions under which this equilibrium can be maintained, or more precisely, the rate of growth at which the economy must expand in order to remain in a continuous state of full employment."* (Domar 1946, p.138).

Domar's aim is to relate the rate of growth of investments and full employment in a long-term perspective in contrast with Harrod's instant-in-time analysis. According to Harrod, as we have seen, the warranted rate of growth can coexist with involuntary unemployment, while for Domar the equilibrium rate of growth is that of full employment. For Harrod the rate of income growth finds a rightful place in his pursuit of dynamic analysis while it finds no place in Domar, although he recognises its relevance to economic analysis. *"The central theme of the paper was the rate of growth, a concept which has been little used in economic theory, and in which I put much faith as an extremely useful instrument of economic analysis"* (Domar, 1946 p.147).

In Domar the system is not intrinsically unstable, and the commodity market is in equilibrium contemporaneously with the labour market; as in Neoclassical analysis, the problem he raises has to do solely with the failure to reach full employment equilibrium, and the starting point for his analysis is the hypothesis of equality between productive capacity and income (in other words, the full exploitation of the available productive capacity). While Harrod sees the instability of the economic system deriving mainly from the divergence between the effective rate of growth and the warranted rate of growth, the only divergence that seems really to matter in Domar is between the potential average productivity of social investments σ and the effective average productivity of social investments s. This is the only case of possible instability in the system but, as Asimakopulos points out, Domar's analysis can certainly not be described as a study of instability[12].

As in Keynes, so in Harrod, uncertainty pervades all economic activity, generating the instability of the system. Essentially, in fact, there is a divergence between G_W and G since, at the end of the financial year, the level of available plant and stocks does not correspond to the entrepreneurs' wishes. In Domar, on the other hand, the state of uncertainty has no particular part to play. In short, the two original models differ with respect to stability, commodity market equilibrium, the actual aim of the analysis and, of course, uncertainty.

6. The neoclassical solution to Harrodian instability

Dynamic neoclassical theory takes equality between the effective rate of growth G and the warranted rate G_W to apply in all cases, consistently with traditional static analysis, in which the commodity markets is always in equilibrium through the adjustment of prices. Moreover, this approach solves the problem of divergence between G_W and the natural rate of growth G_N by varying parameter C. Thanks to the variations of this term (henceforth indicated with v, as is the practice in the mainstream literature) the system can automatically return to a situation of equilibrium, thereby ensuring validity for the rates of growth (in the sense that if the rate of growth diverges from the equilibrium rate it is driven by market mechanisms to converge towards it). The incremental ratio between capital and equilibrium income v can be defined thus:

$$v = (K_1 - K_0)/(Y_1 - Y_0) = \Delta K/\Delta Y \quad (9).$$

---

[12] As Asimakopulos writes: "*Domar does not have any comparable discussion of the stability of his equilibrium, although there is the hint that failure of investment to grow at the equilibrium rate might lead to depression, even when σ is equal to s, because of unused capacity and employment*" (Asimakopulos, 1986, p. 288).

We indicate with $v*$ the value of the variable v which brings about the equality between warranted rate of growth and natural rate of growth, with the result that $G_W = s/v$ and $G_W = s/v* = G_N$. Moreover, under the hypothesis of a technology with constant returns, the capital-income incremental ratio is equal to the average capital-income ratio, which in turn is equal to the average ratio between capital intensity $k = K/L$ and average labour productivity $y = Y/L$; thus we have

$$v = \Delta K/\Delta Y = K/Y = k/y \text{ (10).}$$

Let us now briefly see how convergence is obtained between the warranted rate of growth and the natural rate of growth according to the neoclassical approach, assuming that the effective rate of growth coincides with the warranted rate of growth.

If $G_W < G_N$, then we have an increasing excess of supply on the labour market, and so a drop in wages. This, in turn, leads to a decrease in capital intensity[13], for with lower wages it becomes more advantageous to apply techniques that make use of relatively more labour. Thus the variable v, which is at the denominator of G_W, is reduced and the warranted rate of growth therefore increases, tending to come equal to the natural rate. If $G_W > G_N$, labour market demand grows more than does the supply, firms are driven to raise wages, capital intensity increases, v rises, and consequently the warranted rate of growth falls, converging towards the natural rate of growth.

From this it already becomes apparent, to begin with, that the direct relationship between capital intensity and real wage (or the corresponding inverse relationship between capital intensity and rate of interest) is fundamental for convergence towards equilibrium. In the second place, it can be seen that v depends solely on variations in the real wage (or rate of interest). As we have seen, however, according to Harrod other factors can determine variations in v, and in any case Harrod shows a certain scepticism, not only in An Essay in Dynamic Theory, about the effectiveness of the rate of interest as determinant of variations in v. As he wrote, in fact[14]: *"the rate of interest should be considered the barometric indicator of the limited part of the market that is perfect (stock exchange, market discount, etc.)."* (Harrod, 1960 p. 227) and, moreover, *"Consequently, one might draw the wrong impression by asking the question to the producers in this form:-Are you influenced by the rate of interest? - This involves a difficult calculation based on comparing the rate of interest with the expected return on the cost of production of the planned activities. Many producers who refuse to make such a calculation may nevertheless be influenced for or against a reorganization of production towards more capital intensive, because the necessary financing is easy or difficult to obtain."* (Harrod, 1960 p. 227)

7. Conclusion

We have demonstrated that Harrod's model in An Essay in Dynamic Theory can be seen as an example of cumulative causation between supply and demand, that this circular link is characterised by instability and uncertainty, and that there are significant differences between Harrod's conception of dynamic analysis and that one of Domar, with which it is generally associated although it lacks certain important elements of Harrods thought such as instability and uncertainty. While the relationship between effective and warranted rate of growth lies at the heart of Harrod's analysis of instability, three strands of research – Neo-Malthusian, Neo-Keynesian, Neoclassical – have come up with solutions for instability resulting from the divergences between the warranted

---

[13]According to Pasinetti, it would be more correct to define as "degree of mechanisation" the capital-labour ratio, and as "capital intensity" the capital-income ratio. See Pasinetti 1981, p. 201.

[14] The page references for the quotations from Harrod (1960) is to the Italian translation.

and natural rate of growth, varying respectively the labour supply rate of growth, the propensity to save, and the capital-product relationship. In so doing they take the fundamental Harrodian equation to refer to the long term, while, as emerges from the history of economic thought debate (Kregel, Besomi), it applies to the instant in time. Furthermore, these three strands follow different lines from the paths taken by Harrod in analysing variations in the capital-product relationship and the propensity to save, which Harrod analysed in the book The Trade Cycle, examining the variations in the warranted rate of growth during the trade cycle. The three schools also failed to take into account the "temporary" warranted rates of growth introduced, as advised by Keynes, in Economic Dynamics (1973), and left out the main cause of Harrodian instability, i.e. the divergence between effective and warranted rate of growth relative to the commodity market conditions of disequilibrium. In conclusion, Harrod's model can be said to afford original insights into the significance of dynamics and instability which, unfortunately, have found little comprehension and some distortion in the mainstream growth literature.[15]

References

Asimakopulos A. (1986) "Harrod and Domar on Dynamic Economics", *Banca Nazionale del Lavoro Quarterly Review*, 158: 275-290.

Besomi D. (1995) "From The Trade Cycle to the 'Essay in Dynamic Theory'. The Harrod-Keynes correspondence, 1937-1938" *History of Political Economy*, 272: 309-343.

Besomi D. (1996) "An Additional Note on the Harrod-Keynes Correspondence", *History of Political Economy,* 28:281-294.

Domar, E. (1946) "Capital Expansion, Rate of Growth and Employment", *Econometrica*, 14: 137-147.

Guarini G. (2007) "Instabilità harrodiana e teoria della crescita endogena", *Studi Economici*, 91: 91-116.

Harrod R.F. (1936) *The Trade Cycle*, Oxford: Clarendon Press.

Harrod R.F. (1938) "Scope and Method of Economics", *Economic Journal*, 48: 383-412.

Harrod R.F. (1939) "An Essay in Dynamic Theory", *Economic Journal*, 49: 14-33.

Harrod R.F. (1960) "A Second Essay in Dynamic Theory", *Economic Journal*, 278: 277-293. it. trans., *Secondo saggio sulla teoria dinamica*, in R.F. Harrod, *Dinamica Economica*, Il Mulino, Bologna 1990: 225-250.

Harrod R.F. (1973) *Economic Dynamics*, Macmillan, London.

Harrod R.F. (1990) *Dinamica Economica*, Il Mulino, Bologna.

Kregel J.A. (1980) "Economic dynamic and the theory of steady growth: an historical essay on Harrod's "knife-edge"", *History of Political Economy*, 12: 97-123.

Keynes, J.M. (1930) *A Treatise on Money*. MacMillan, London.

Keynes, J.M. (1936) *The General Theory of Employment, Interest and Money*. Macmillan London.

Keynes, J.M. (1973) *The General Theory and After, Part I: Preparation*, C.W., vol. XIII, Macmillan, London.

Keynes, J.M. (1973) *The General Theory and After, Part II: Defence and Development*, C.W., vol. XIV, Macmillan, London.

Pasinetti L. (1981) *Structural Change and Economic Growth. A Theoretical Essay on the Dynamics of the Wealth of Nations,* Cambridge University Press.

---

[15] See Guarini (2007).