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The relationship between inflation, output growth, and their uncertainties: Nonlinear Multivariate GARCH-M evidence

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Abstract

In this paper, we propose a nonlinear multivariate GARCH-M model. We have illustrated the actual modelling by applying the models to inflation and output growth variables and found that the effects of real and nominal uncertainties are regime-dependent.

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1. Introduction

After Friedman's Nobel Lecture (1977), the effects of uncertainties on macroeconomic variables have attracted a considerable interest of macroeconomists, and a huge amount of both theoretical and empirical work on the relationship between uncertainty and macroeconomic variables has been accumulated. Main conclusion of Friedman (1977) is that inflation uncertainty may lower output growth rate. In addition, other researchers have explored all possible interrelationships between inflation rate, inflation uncertainty, output growth rate and output uncertainty. See, for example, Mirman (1971), Cukierman and Meltzer (1986), Black (1987), Devereux (1989), Pindyck (1991), Holland (1995), Dotsey and Sarte (2000), Cukierman and Gerlach (2003), and Blackburn (1999) [1].

In this paper we propose a smooth transition multivariate GARCH-M model to test all possible interrelationships among inflation rate, output growth rate and their uncertainties in a nonlinear framework. To the best of our knowledge, all previous empirical studies that have focused on the effects of real and nominal uncertainty have examined these relationships in a linear framework (e.g., Holland, 1995; Grier and Perry, 2000, Hasanov and Omay, 2011). However, it is well known that many economic and financial series are characterized by nonlinearities (e.g. Granger and Teräsvirta, 1993). Therefore, modeling nonlinear relationships with a linear model may give spurious results. In addition, it is shown that the inflation rate itself and the effects of inflation on macroeconomic variables are subject to regime shifts (e.g. Omay and Hasanov, 2010). This paper fills a gap in the empirical literature by taking into account the nonlinear behavior inherent in the variables under consideration.

The paper is organized as follows. We describe the model and discuss specification and estimation procedures in section 2. The estimation results are reported in Section 3, and finally Section 4 contains a brief conclusion.

2. Model and Empirical Results

2.1. The Model

In this section we extend the empirical specification procedure of STAR-GARCH model of Lundbergh and Teräsvirta (1998) and Omay and Hasanov (2010) to a multivariate setting[2]. As noted by Lundbergh and Teräsvirta (1998), misspecification of conditional mean could lead to misspecification of the conditional variance. Therefore, before proceeding to model the conditional variance and covariance of the equation system, one must test whether the specified equation for the conditional mean captures the dynamics of the system reasonably well[3]. For this

[1] An extensive literature review can be found in Fountas and Karanosos(2007) and Hasanov and Omay (2011).

[2] The detailed steps can be followed from above mentioned papers. Moreover, Omay (2010) provide a detailed discussion of statistical techniques which are used in STAR-STGARCH estimation.

[3] The information matrix of a STAR-GARCH model is block-diagonal if error term follows a symmetric distribution (see Lundbergh and Teräsvirta, 1998), so that a STAR-GARCH model estimated using a two stage procedure. In the first stage, the conditional mean is estimated by NLS (Nonlinear Least Squares), which is equivalent to quasi maximum likelihood based on a normal distribution.

purpose we employ the procedure proposed by van Dijk (1999) to test whether there is smooth transition type nonlinearity in a VAR model[4].

Let π_t and y_t denote the inflation rate and output growth rate, respectively. Then, a nonlinear VAR model for inflation rate and output growth can be written as follows:

$$x_t = \phi_1 + \sum_{i=1}^{p-1} \varphi_{1i} x_{t-i} + \left\{ \phi_2 + \sum_{i=1}^{p-1} \varphi_{2i} x_{t-i} \right\} . \left(F(s_t;\theta,c) \right) + \varepsilon_t \qquad (1)$$

where x_t is a (2x1) column vector given by $x_t = (\pi_t, y_t)'$, ϕ_j j=1,2 are (2 x 1) vector of constants, $\varphi_{j,i}$, $j=1,2$, $i=1,...,p$ are (2 x 2p) matrix of parameters, and $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})$ is a (2x1) vector of residuals. The transition function $F(s_t;\theta,c)$ is assumed to be a continuous function between zero and one, with parameters θ and c determining the smoothness and location of the change in the value of $F(s_t;\theta,c)$, respectively. Here we focus on the logistic function[5].

$$F(s_t;\theta,c) = \frac{1}{1+\exp\left\{-\theta\left[s_t - c\right]\right\}} \;,\quad \theta > 0 \qquad (2)$$

The specific-to-general approach of specifying multivariate STR model starts with a specification of a linear vector autoregressive model. In the next step of the specification procedure, one must test linearity against STR-type nonlinearity as given in (1), with (2). For the linearity test, we use the below auxiliary regression following van Dijk (1999):

$$x_t = A_0 + \sum_{i=1}^{p-1} B_i x_{t-i} + A_1 s_t + \sum_{i=1}^{p-1} B_{1,i} x_{t-i} s_t + e_t \qquad (3)$$

where e_t comprises the original shocks ε_t as well as the error arising from the Taylor approximation. To identify an appropriate transition variable s_t, the LM-type statistic can be computed for several candidates, and the one for which the associated p-value of the test statistic is smallest, can be selected.

We assume that the vector of residuals $\boldsymbol{\varepsilon}_t$ is conditionally normal with mean vector $\mathbf{0}$ and covariance matrix $\mathbf{H}_t$, that is, $\left(\boldsymbol{\varepsilon}_t | \Omega_{t-1}\right) \sim \mathrm{N}\left(\mathbf{0}, \mathbf{H}_t\right)$ where Ω_{t-1} is the information set available at time t-1. We assume that the conditional covariance matrix $\mathbf{H}_t$ has the GARCH(1,1) structure proposed by Bollerslev (1990)[6]. In particular, we assume that

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[4] A good application of this proposed procedure can be find in Araz-Takay *et al.* (2009).

[5] Ideally, one may use a sequence of F tests in the auxiliary regression (3) to choose the appropriate transition function. See, for example, van Dijk (1999). From the recursive F test we obtained (F1=64.691(0.000), F2=82.907(0.000), F3=94.420(0.000)) which shows logistic function is suitable.

[6] In addition to diagonal CCC-GARCH(1,1) model of Bollerslev (1990), we estimated other types of multivariate GARCH models. The AIC criteria suggest that the suitable model is CCC-GARCH(1,1). Besides, the assumption of a constant correlation matrix represents a major reduction in terms of computational complexity and therefore is commonly used in multivariate GARCH models (e.g., Grier and Perry 2000). Thus, this model specification leads

$$\begin{aligned} h_{\pi t} &= \alpha_\pi + \beta_\pi h_{\pi,t-1} + \gamma_\pi \varepsilon^2_{\pi,t-1}, \\ h_{yt} &= \alpha_y + \beta_y h_{y,t-1} + \gamma_y \varepsilon^2_{y,t-1}, \\ h_{\pi yt} &= \rho \sqrt{h_{\pi t}} \sqrt{h_{yt}} \end{aligned} \quad (4)$$

where $h_{\pi t}$ and h_{yt} are the conditional variances of inflation rate and output growth rate, respectively, and $h_{\pi yt}$ is the conditional covariance between inflation residuals $\varepsilon_{\pi t}$ and output residuals ε_{yt}. We use this estimated variance $h_{\pi t}$ and h_{yt} as proxy for inflation uncertainty and output uncertainty, respectively. It is assumed that α_i and $\gamma_i > 0$, $\alpha_i \geq 0$ for $i = \pi, y$ and $-1 \leq \rho \leq 1$ in (4).

The nonlinear multivariate GARCH model given in (1), (2) and (4) is estimated using the maximum likelihood estimation method by using the advices which are given in Chan and McAleer (2002). In their research they have shown the efficient estimation method and asymptotic properties for STAR-GARCH models.

2.2 Data and model specification

Our sample includes monthly data from 1980.M1 through 2009.M3 on producer prices (PPI_t) and industrial production (IP_t) indices for the USA. We compute inflation rate as $\pi_t = \log(PPI_t / PPI_{t-1})*1200$ and output growth rate as $Y_t = \log(IP_t / IP_{t-1})*1200$. Following Grier and Perry (2000), we include the spread (sp_t) variable defined as the difference between the 6-month and 3-month treasury bill rates as a measure of term structure of interest rate since it has been shown to be a good predictor of real output growth.

Our econometric methodology outlined in the previous section relies on the assumption that both the inflation rate and output growth rates are *I(0)* processes. We have found out that both variables are stationary in level by using both linear and nonlinear unit root tests. As a first step in the specification procedure, we estimated a linear VAR model for inflation and output growth rates. In the second step, we obtained the results of system-wide linearity tests by using auxiliary regression in Eq. (3), which suggests that the tenth lag of inflation rate is π_{t-10} (with test statistic $LM_3 = 102.233(0.000)$) is the most appropriate transition variable in the nonlinear model. All the test results are summarized in the below Table 1.

to less convergence problem in estimation stages. Moreover, we have found similar results from different GARCH specifications. Estimation results with other specifications are available upon request.

Table 1. Nonlinear Model Identification Tests

	Unit Root Tests*					
	ADF		**PP**		**KSS**	
Y_t	-5.837		-15.944		-9.754	
	(0.000)		(0.000)		(0.000)	
π_t	-13.403		-13.428		-5.728	
	(0.000)		(0.000)		(0.000)	
	Linearity Test**					
Transition Variables	Y_t	π_t			Y_t	π_t
Lag1	79.158	72.544	Lag7		41.515	68.885
	(0.000)	(0.000)			(0.000)	(0.000)
Lag2	67.472	65.376	Lag8		46.319	64.419
	(0.000)	(0.000)			(0.000)	(0.000)
Lag3	60.574	57.318	Lag9		53.962	93.523
	(0.000)	(0.000)			(0.000)	(0.000)
Lag4	62.887	74.122	Lag10		62.087	**102.233**
	(0.000)	(0.000)			(0.000)	**(0.000)**
Lag5	41.463	56.454	Lag11		50.125	96.711
	(0.000)	(0.000)			(0.000)	(0.000)
Lag6	54.997	60.852	Lag12		45.126	62.625
	(0.000)	(0.000)			(0.000)	(0.000)

*Prior to estimation, we test for stationarity of the variables under investigation. For this purpose, we use augmented Dickey-Fuller (ADF), Phillips-Perron (PP) linear unit root tests and Kapetanois Shin and Snell (KSS) nonlinear unit root test.
** Tests against STR type nonlinearity. By using this system wide test procedure, we are selecting transition variable as well.

Considering the convergence problem in the estimation of the STVAR-GARCH model, we have initially preferred to estimate a parsimonious model[7]. However, the estimated model suffered from serial correlation problem in the residuals. Having found that parsimonious model leads to serial correlation problem and over parameterized model creates convergence problem, we tried an alternative strategy which creates minor problems in estimation of nonlinear multivariate GARCH-M model. The linear and nonlinear models were initially specified with maximum lag order of 12, with intermediate lags then deleted one by one (starting with the least statistically significant according to the t-ratio) provided that such deletions reduce the AIC.

2.3. Empirical results

Table 2 shows the estimation results of linear and nonlinear multivariate GARCH-M models.

[7] The estimation results of STVAR-GARCH models are available upon request.

Table 2. Linear and Nonlinear Bivariate –GARCH –M model

	Bivariate GARCH-M		Logistic smooth transition bivariate GARCH-M			
			Low Inflationary Regime		*High Inflationary Regime*	
Mean Eq.	y_t	π_t	y_t	π_t	y_t	π_t
y_{t-2}	0.209*	-0.081**	0.206*	-0.004	0.003	-0.001
y_{t-3}	0.203*		0.204*		0.019	
y_{t-4}	0.045		0.089**		-0.007	
y_{t-6}	0.081***		0.054		0.035	
sp_t	0.509**		0.702*		0.017	
π_{t-1}	0.008	0.220*	0.122*	0.242*	-0.048	0.008
h_{yt}	-0.014		-0.013*		0.002	
$h_{\pi t}$	-0.010***		-0.006**		-0.003	
Intercept		2.216*		2.498*		0.299
π_{t-6}		0.073		0.018		0.068***
π_{t-9}		0.053		0.116*		-0.067***
π_{t-10}		0.119**		0.100**		0.035
h_{yt}		-0.008		-0.024*		0.013*
$h_{\pi t}$		-0.019		-0.013		-0.013
θ				19.923		
c				6.791*		
Variance equation						
α	30.793*	1.904***	27.898*		1.164*	
β	0.025	0.749*	0.027		0.815*	
γ	0.357*	0.221*	0.486*		0.169*	
Covariance equation						
ρ		-0.059		-0.045		
Log Likelihood		-2155.789		-2139.638		

*,**,*** denote significance at 1%, 5%, and 10% significance levels, respectively.

Following earlier studies employing nonlinear STR models, we performed diagnostic tests for the estimated mean equations and found that the estimated equations capture the nonlinear dynamics in the data quite well. The misspecification tests of linear and nonlinear models are summarized in the below Table 3.

Table 3. Diagnostic Check for Linear and Nonlinear Models

Linear Model*

Equations	$Q(4)$	$Q(12)$	$Q^2(4)$	$Q^2(12)$	$Q(4)$	$Q(12)$
Δy_t	6.461	15.461	4.353	7.758	6.801	12.451
π_t	1.698	12.771	3.771	17.010		

Nonlinear Model

Test Against Autocorrelation**

Equations	Δy_t	π_t		Δy_t	π_t
Lag1	1.448	1.895	Lag7	3.229	1.239
	(0.229)	(0.169)		(0.002)	(0.266)
Lag2	1.176	0.977	Lag8	3.959	1.488
	(0.309)	(0.323)		(0.000)	(0.223)
Lag3	1.857	0.764	Lag9	6.066	1.730
	(0.136)	(0.382)		(0.000)	(0.189)
Lag4	2.595	0.848	Lag10	5.450	1.651
	(0.036)	(0.357)		(0.000)	(0.199)
Lag5	2.963	1.339	Lag11	5.445	1.517
	(0.012)	(0.247)		(0.000)	(0.218)
Lag6	3.374	1.270	Lag12	5.065	1.487
	(0.003)	(0.260)		(0.000)	(0.223)

Heteroskedasticity-consistent HCC Version of Test Against Autocorrelation***

Equations	Δy_t	π_t		Δy_t	π_t
Lag1	0.310	0.053	Lag7	0.972	0.464
	(0.577)	(0.817)		(0.324)	(0.495)
Lag2	0.319	0.005	Lag8	1.588	1.099
	(0.571)	(0.938)		(0.207)	(0.281)
Lag3	1.567	0.001	Lag9	1.605	0.691
	(0.210)	(0.968)		(0.193)	(0.405)
Lag4	0.247	0.074	Lag10	0.011	0.536
	(0.619)	(0.784)		(0.913)	(0.463)
Lag5	0.397	1.520	Lag11	1.102	0.087
	(0.528)	(0.217)		(0.293)	(0.766)
Lag6	0.389	0.208	Lag12	0.069	0.522
	(0.532)	(0.648)		(0.791)	(0.469)

* Diagnostic tests for the estimated multivariate GARCH model. We employ the Ljung Box Q, Q^2 statistics for residuals as well as the Q statistics for cross equation. These test are not valid for the nonlinear model, see Eitrheim and Teräsvirta (1996) for further discussion.

** Diagnostic tests for the estimated nonlinear multivariate-GARCH model for output growth and inflation equations.

*** HCC values are robust versions of the standard test. This test statistics is obtained from Eitrheim and Terasvirta (1996), and robustified by using the test procedure Wooldridge (1991, procedure 4.1, see also Granger and Teräsvirta (1993, pp.69-70)). The result of the misspecification test of remaining autocorrelation for output growth and inflation equation shows that there is no remaining autocorrelation.

Furthermore, to investigate the possible interrelationships between uncertainties and macroeconomic variables, we augment the baseline linear and nonlinear multivariate GARCH model with inflation and output uncertainty which gives rise to GARCH-in-mean model. Competing economic theories bear different implications for the correlation between these variables. All the theories and implied causal relationships between the variables under consideration are provided in Table 4.

Table 4. The results of estimation from multivariate GARCH models

Theories	*Signs*	Linear Model	Nonlinear Model	
			Low Inflationary Regime	*High Inflationary Regime*
$h_{yt} \rightarrow y_t$		Insignificant		Insignificant
Pindyck (1991)	(-)		Significant	
Mirman (1971), Black (1987), Blackburn (1999)	(+)			
$h_{\pi t} \rightarrow y_t$				Insignificant
Friedman (1977)	(-)	Significant	Significant	
Pourgerami and Maskus (1987), Ungar and Zilberfarb (1993)	(+)			
$h_{yt} \rightarrow \pi_t$		Insignificant		
Devereux (1989), Cukierman and Gerlach (2003)	(+)			
Taylor effect, Cukierman and Meltzer (1986)	(-)		Significant	Significant
$h_{\pi t} \rightarrow \pi_t$		Insignificant	Insignificant	Insignificant
Cukierman and Meltzer(1986)	(+)			
Holland (1995)	(-)			

The estimated linear model suggests that inflation uncertainty reduces output growth rate. The causal relationships among other variables are not statistically significant according to the linear regression model. On the other hand, the nonlinear multivariate GARCH-M model suggests that output uncertainty and inflation uncertainty reduces average output growth rate in low inflation periods whereas both uncertainties have no statistically significant effects on output in high inflation periods. In addition, we find that output uncertainty reduces average inflation rate both in low and high inflation periods, whereas the effects in high inflation periods are marginally significant and less in absolute value. The results of the linear model are compatible with those of Grier and Perry (2000). However, the results of the nonlinear multivariate GARCH-M model are quite different from the results of the linear model and as well as from previous empirical findings. Our findings suggest that the models which do not consider nonlinearities (regime-wise effects) in these relationships can lead to misleading results about the effects of uncertainties.

3. Concluding Remarks

In this paper, we proposed a procedure for specification, estimation and diagnostic check of a nonlinear multivariate-GARCH-M model in the form of smooth transition methodology. We then applied the proposed model to re-examine the empirical relationships among real and nominal uncertainties and macroeconomic variables. The results suggest that the causal relationships among these variables are regime dependent which has not been observed in the previous literature due to the insufficiency of restricted linear models. Especially, if we interpret the linear models as the weighted average of these regimes which are obtained from nonlinear models, it is easy to conclude that the significantly estimated less weighted regime effect can easily be nullified by the insignificantly estimated more weighted regime. Therefore, this restricted way of estimation veils the true relationships.

On the other hand, we have employed several diagnostic tests. From these misspecification tests we can conclude that both linear and nonlinear models are acceptably specified. However, from the linearity test we have shown that the linear model specification is inadequate. In the previous literature nonlinearity feature of the relationship between output growth and inflation is not examined. Thus, we extended the existing literature by using STR methodology. The methodology that we have proposed may contain some shortcomings that can be further improved such as the identification, estimation and misspecification tests stages for the mean and the variance equation. Particularly, for the variance equation different specification and misspecification tests can be developed following the previous literature which is obtained for STAR-ST-GARCH models. These model improvements can lead researchers to investigate wider perspectives of the effects of real and nominal uncertainties on inflation and output growth rate.

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