MAGYAR NEMZETI BANK

MNB WORKING PAPER

2004/5

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Dual inflation and the real exchange rate in new open economy macroeconomics

November, 2004

I would like to thank Péter Benczúr, Hugo Rodríguez Mendizabal, András Simonovits, János Vincze and seminar participants at the Budapest University of Economic Sciences and Public Administration and the Magyar Nemzeti Bank. All errors are, of course, my own.

Online ISSN: 15 855 600
ISSN 14195 178
ISBN 963 9383 55 4

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Abstract

This paper studies how the models of the new open economy macroeconomics, which usually focuses on the relationship between the nominal exchange rate and the external real exchange rate, can explain the coexistence of permanent dual inflation, i.e. diverging inflation rates for tradable and non-tradable goods, and real appreciation in emerging market economies.

It is shown that the impact of asymmetric sectoral productivity growth on the real exchange rate heavily depends on the market structure, and that the models of new open economy macroeconomics can be reconciled with the Balassa - Samuelson effect only if pricing to market is added to models.

It is demonstrated that in the presence of nominal rigidities and investments adjustment costs firms' marginal cost is influenced by demand factors even if technology exhibits constant returns to scale. As a consequence, the effect of asymmetric productivity growth becomes weaker. Furthermore, in this case alternative factors can influence dual inflation as well. But according to the numerical simulations, these factors hardly explain the empirically observable dual inflation and real appreciation by themselves without asymmetric productivity growth.

Keywords: dual inflation, real exchange rate, new open economy macroeconomics, Balassa - Samuelson effect.
JEL classification number: E31, F41.

Contents

1 Introduction

The traditional approach in international macroeconomics has attempted to explain real exchange rate behavior by the movements of domestic relative prices, that is, by the *internal real exchange rate*. This was a consequence of the assumptions they employed: strong homogeneity in international goods markets, where *purchasing power parity* (PPP) is dominant and the only source of heterogeneity is the distinction between *tradables* and *non-tradables*. In recent years, however, the literature has switched sides. According to the recent approach consumer markets are segmented, PPP has little explanatory power, and the main determinant of real exchange rate movements is the *external real exchange rate*, which is the relative price of domestic and foreign tradables. This new focus of research was initiated by empirical findings, see, e.g., the papers of Engel (1999) and Rogoff (1996). It appeared that, as Obstfeld (2001) put it "apparently, consumer markets for tradables are just about as segmented internationally as consumer markets for non-tradables."

After the collapse of the Bretton - Woods system, floating exchange rate regimes became widespread. This enabled scrutiny of the relationship between nominal and real exchange rate behavior: It turned out, as first forcefully documented by Mussa, that nominal and real exchange rates were strongly correlated, and moving from fixed to floating exchange rate regimes resulted in a dramatic rise in the variability of the real exchange rate. The need for a comprehensive explanation for the aforementioned empirical findings stimulated the birth of *new open economy macroeconomics* (NOEM), initiated by the seminal paper of Obstfeld and Rogoff (1995), which combines the heterogeneity of goods with *nominal rigidities* in models with microfoundations.

Although the empirical literature related to NOEM revealed the importance of the external real exchange rate, still in fast-growing and emerging market countries there are considerable movements of the internal real exchange. Permanent *dual inflation*, i.e. a significant divergence of inflation rates for tradable and non-tradable goods, is a frequent phenomenon of such markets: the inflation rate of non-tradables is permanently higher than that of tradables, which results in long-run real appreciation. This phenomenon was documented by Ito et al. (1997) for the case of Japan and some Southeast Asian countries as well as by Halpern and Wyplosz (2001) and Kovács (2002) for European post-communist countries. Of course, this does not mean that in these countries the empirical phenomena emphasized by the NOEM literature are not present. For example, the required disinflation efforts, related to future EMU accession, have revealed that the connection between the consumer price index and the nominal exchange rate is weak, which, of course, violates the PPP and implies the strong co-movement of nominal and real exchange rates.

The objective of this paper is to build a NOEM model which is able to replicate both sets of empirical facts observable in emerging markets: the strong correlation of the nominal and real exchange rate, and the dual inflation accompanying with real appreciation.

The problem is the following: The majority of empirical studies explain emerging markets' dual inflation by the *Balassa - Samuelson* (BS) effect, i.e. the relatively rapid productivity growth in the tradable sector. But dual inflation accompanies real appreciation only if growth in tradable productivity does not result in a significant depreciation of the external real exchange rate. The external real exchange rate does not depreciate considerably if the common currency prices of domestically produced and foreign tradables cannot strongly deviate from each other, i.e. if domestically produced and foreign tradables are close substitutes. On the other hand, the strong co-movement of the nominal and real exchange rates stressed by the NOEM literature requires considerable deviations in the short run between domestic and foreign tradable prices (denominated in the same currency). But this requirement can be fulfilled only if the products of the aforementioned sectors are distant substitutes and/or *pricing to market* (PTM) is possible.

The paper demonstrates that no intermediate degree of international substitution exists that simultaneously guarantees the operation of the BS effect and strong co-movement of nominal and real exchange rate. One possible remedy is an assumption of PTM. In this case it is possible that domestically produced export goods are close substitutes of foreign tradables, which ensures the existence of the BS effect. On the other hand, with PTM the common currency price of the exported and locally sold domestically produced goods can be substantially different over the short run. Hence, nominal-exchange-rate movements can influence the behavior of the real exchange rate.

Another key problem investigated by this study is whether non-technological factors can induce significant, long-run dual inflation. This is not simply a theoretical curiosity: several empirical studies have documented that other, principally demand, factors influence the difference between sectoral inflation rates. If technology exhibits constant returns to scale, then demand factors can influence the difference between sectoral inflation rates only if firms are not able to form their optimal input combinations continuously, which implies that scarcity occurs and results in decreasing returns to scale.

The paper shows that a combination of investments adjustment costs and nominal rigidities may lead demand factors to have a significant impact on the difference between sectoral inflation rates. As a consequence, the size of the effect of asymmetric sectoral productivity growth, in line with empirical observations, becomes smaller than predicted by the models of the traditional approach. It is also demonstrated that in NOEM models it is impossible to replicate the size and persistence of dual inflation in emerging markets by demand factors alone. Thus, alternative explanations can play only a supportive role beyond asymmetric productivity growth.

The paper is structured as follows. *Section 2* reviews the main problems in a non-technical manner. *Section 3* presents the model and the solution technique employed. *Section 4* surveys the empirical literature which initiated the research of this study. In *section 5* it is studied how the model can reproduce the co-existence of dual inflation and real appreciation. *Section 6* studies the relationship between asymmetric productivity growth and the magnitude of the difference of sectoral inflation rates. In *section 7* some alternative factors which are able to generate dual inflation are considered. *Section 8* presents the conclusions.

2 Review of studied problems

Before setting up the formal model it is worthwhile to review the problems being analyzed by this study in a non-technical way.

The first important problem is how a NOEM model can generate the Balassa - Samuelson effect, the usual explanation for the coexistence of dual inflation and real appreciation.

Let $\mathcal{Q}_t$ denote the natural logarithm of the real exchange rate. By definition $\mathcal{Q}_t = \mathcal{E}_t + \mathcal{P}_t^{F*} - \mathcal{P}_t$, where $\mathcal{P}_t$ is the logarithm of the domestic consumer price index in domestic currency terms, $\mathcal{P}_t^{F*}$ is the logarithm of the foreign consumer price index in foreign currency terms, $\mathcal{E}_t$ is the logarithm of the nominal exchange rate, and t is the time index.[1] Let us assume that the price indices can be decomposed as

$$\mathcal{P}_t = a\mathcal{P}_t^T + (1-a)\mathcal{P}_t^N, \quad \mathcal{P}_t^{F*} = b\mathcal{P}_t^{FT*} + (1-b)\mathcal{P}_t^{FN*},$$

where $\mathcal{P}_t^T$ and $\mathcal{P}_t^{FT*}$ are the logarithms of the domestic and foreign price indices of tradables, $\mathcal{P}_t^N$ and $\mathcal{P}_t^{FN*}$ are the same indices of non-tradables and a and b are parameters. Then the real exchange rate can be expressed as

$$\mathcal{Q}_t = \mathcal{Q}_t^T + \mathcal{Q}_t^R,$$

where $\mathcal{Q}_t^T = \mathcal{E}_t + \mathcal{P}_t^{FT*} - \mathcal{P}_t^T$, i.e. the logarithm of the external real exchange rate, and $\mathcal{Q}_t^R$ is the logarithm of the internal real exchange rate, which is

[1] Throughout this study prices indicated by $*$ are measured in foreign currency.

related to sectoral relative prices, i.e. $\mathcal{Q}_t^R = (1-b)\mathcal{P}_t^{FR} - (1-a)\mathcal{P}_t^R$, where $\mathcal{P}_t^R = \mathcal{P}_t^N - \mathcal{P}_t^T$ and $\mathcal{P}_t^{FR} = \mathcal{P}_t^{FN*} - \mathcal{P}_t^{FT*}$. The BS effect is based on two assumptions:

- First, the two sectors use the same production inputs, but the *total factor productivity* (TFP) of the sectors can be different.
- Second, PPP is fulfilled, that, is $\mathcal{P}_t^T = \mathcal{E}_t + \mathcal{P}_t^{FT*}$.

The first assumption implies that $\mathcal{P}_t^R = \mathcal{A}_t^T - \mathcal{A}_t^N$ and $\mathcal{P}_t^{FR} = \mathcal{A}_t^{FT} - \mathcal{A}_t^{FN}$ if the sectors have the same constant-returns-to-scale technologies. $\mathcal{A}_t^T$, $\mathcal{A}_t^N$, $\mathcal{A}_t^{FT}$ and $\mathcal{A}_t^{FN}$ denote the logarithms of the sectoral TFP measures. The second assumption implies that the external real exchange rate is constant if the foreign price index is fixed. Hence, if it is assumed that the foreign productivity difference is zero, then

$$d\mathcal{Q}_t = \pi_t^N - \pi_t^T = d\mathcal{A}_t^T - d\mathcal{A}_t^N,$$

where d is the difference operator and π_t^s $(s = T,\, N)$ are the sectoral inflation rates. That is, if the productivity growth of tradables is higher than that of non-tradables, then the inflation rate of the non-tradables will be higher, and the real exchange rate will appreciate.

Obviously, if PPP is fulfilled and the external real exchange rate is constant, then the main propositions of the NOEM cannot be valid. That is, real exchange rate behavior cannot essentially be determined by the movements of the external real exchange rate, which correlates with the nominal exchange rate. Illustrating this contradiction, let us sketch how a typical NOEM model explains the co-movement of the nominal and real exchange rate. Since usually in these models the distinction between tradables and non-tradables is missing, I set $\mathcal{P}_t^R = \mathcal{P}_t^{FR} = 0$. The correlation of the nominal and real exchange rates is guaranteed by the following two conditions:

- It is allowed that $\mathcal{P}_t^T \neq \mathcal{E}_t + \mathcal{P}_t^{FT*}$. This can occur only if the markets of the domestic and foreign tradables are segmented, that is, PPP is not guaranteed since international goods arbitrage is impossible.
- Prices are sticky.

For the sake of clarity, the simplest form of nominal rigidity is used in this example: prices are set one period in advance. Let us assume that at date t an unexpected nominal-exchange-rate shock occurs, which was not accommodated at date $t-1$ when the prices were set. Then the real exchange rate is given by

$$\mathcal{Q}_t = \mathcal{E}_t + \mathcal{P}_{t-1}^{FT*} - \mathcal{P}_{t-1}^T.$$

This expression is not necessarily constant by the first assumption, and the preset prices imply that nominal and real exchange rates are perfectly correlated. Thus, the essential distinction between the traditional and the NOEM approach is not that the latter has usually one sector. One can build two-sector NOEM models as well. Rather it is that they describe differently the behavior of the external real exchange rate $\mathcal{Q}_t^T$.[2]

This paper studies how the contradiction described above can be solved. That is, how it is possible to build a NOEM model in which asymmetric sectoral productivity growth results in dual inflation and real appreciation since the external real exchange rate does not depreciate so much as to neutralize or suppress the appreciation of the internal real exchange rate.[3]

NOEM models guarantee the $\mathcal{P}_t^T \neq \mathcal{E}_t + \mathcal{P}_t^{TF*}$ requirement in two ways. The first way is that they assume that domestic export goods and their foreign rivals are not perfect substitutes. Then, the price of these export goods and their foreign rivals do not need to coincide when expressed in the same currency. The other way is the assumption of *pricing to market* (PTM), which is often the consequence of third degree international price discrimination. Then it is possible that in the short run the same good have diverging prices in common currency terms at home and abroad.[4]

According to the imperfect substitutability approach, external demand for domestically produced goods is expressed by a formula similar to the following:

$$\mathcal{X}_t = \eta^* \left(\mathcal{E}_t + \mathcal{P}_t^{TF*} - \mathcal{P}_t^T \right) + \mathcal{X}_t^*, \tag{1}$$

where $\mathcal{X}_t$ is the logarithm of exports, $\mathcal{X}_t^*$ is a variable related to the volume of external demand, and η^* is an exogenous parameter. The models of Obstfeld and Rogoff (2000), Galí and Monacelli (2002), and Monacelli (2004) represent this approach.

The parameter η^* measures the substitutability between domestic exports and their rival goods. If $\eta^* = +\infty$, they are perfect substitutes as the traditional approach assumes. Then the expression (1) takes the simpler form

[2] In a short review like this one, of course, it is impossible to provide an exact classification of pre-NOEM models. But it is important to note that the external real exchange rate is not fixed in all models in the traditional approach. But this does not influence the validity of my argument since external-real-exchange rate movements are independent from the nominal exchange rate even in these models.

[3] Fagan et al. (2003) study problems related to the BS effect with a two sector NOEM-like model. Although they assume price stickiness in the non-tradable sector, the markets of tradables are internationally homogenous and competitive. In my opinion this is not a solution, but a bypass of the problem.

[4] If PTM occurs, then the assumption of the imperfect substitutability of domestic and foreign tradables is not necessary but possible.

$\mathcal{P}_t^T = \mathcal{E}_t + \mathcal{P}_t^{TF*}$. However, the strong correlation between the nominal and real exchange rate requires that the goods are far substitutes, i.e. η^* is small. But in this case, if the TFP of domestic tradables increases, then $\mathcal{P}_t^T - \mathcal{E}_t$ will decrease, resulting in depreciation of external real exchange rate in a small open economy since foreign prices are not influenced by domestic factors. The problem is whether there exists an intermediate value of η^*, which guarantees a rather strong correlation between nominal and real exchange rates, but the BS effect remains still valid, as increasing productivity does not cause such a large decrease of $\mathcal{P}_t^T - \mathcal{E}_t$, that neutralizes the appreciation of the internal real exchange rate.

In NOEM models with PTM it is usually assumed that the prices of domestic export goods are sticky in the currency of the destination country. This price setting practice is called *local currency pricing* (LCP). Betts and Devereux (1998), Chari et al. (2002), Devereux and Engel (1999), and Laxton and Pesenti (2003), for example, apply this price setting strategy.[5] If PTM is valid, one can imagine that export prices are sticky in the domestic currency, i.e. *producer currency pricing* (PCP) is performed. Bergin (2004) considers this case as well. But usually the PCP assumption is applied without PTM, which is nothing but the imperfect substitutability approach represented by formula (1).

In NOEM models with PTM it is less problematic to reconcile the co-movement of nominal and real exchange rate and the BS effect than in models with imperfect substitutability of domestic and foreign tradables. Let us briefly illustrate why: For the sake of simplicity let us assume that domestic export goods and their foreign rivals are perfect substitutes. Furthermore, assume that domestic firms are price takers abroad (in this case the LCP versus PCP distinction becomes meaningless). Let us denote by $\mathcal{P}_t^{T*}$ the logarithm of the foreign currency price of the exported domestic goods. The assumption of price taking guarantees that $\mathcal{P}_t^{T*} = \mathcal{P}_t^{FT*}$. Furthermore, assume that the economy is in its long run equilibrium, when $\mathcal{P}_t^T - \mathcal{E}_t = \mathcal{P}_t^{T*}$. Assume again that $\mathcal{P}_t^T$ and $\mathcal{P}_t^{FT*}$ are set one period in advance. If an unexpected nominal-exchange-rate shock hits the economy, then PTM implies, at least in the short run, that

$$\mathcal{P}_{t-1}^T - \mathcal{E}_t = \mathcal{P}_t^T - \mathcal{E}_t \neq \mathcal{P}_t^{T*} = \mathcal{P}_{t-1}^{T*}.$$

Thus, as previously, the nominal and the external real exchange rate correlates. On the other hand, in models with PTM the BS effect remains valid

[5] A variant of this approach assumes a transportation sector to guarantee the existence of PTM. See, e.g., Benigno and Thoenissen (2002), Monacelli (2003, 2004), and Smets and Wouters (2002).

since in the longer run, which is relevant for the BS effect, $\mathcal{P}_t^T - \mathcal{E}_t = \mathcal{P}_t^{T*} = \mathcal{P}_t^{FT*}$. That is, the external real exchange rate is fixed. This implies that higher productivity growth results in real appreciation.

One of the objectives of the paper is to investigate whether PTM is a necessary condition for reproducing the BS effect in a NOEM model. My results verify the previously described conjecture: the assumption of PTM, both with LCP and PCP, is consistent with the BS effect. However, it is demonstrated that without this assumption the model cannot generate the BS effect: there is no intermediate value of parameter η^* which guarantees the strong relationship between the nominal and real exchange rate and reproduces the BS effect simultaneously, if PTM does not exist.

So far I have discussed which conditions can guarantee a relative stable external real exchange rate without suppressing appreciation of the internal real exchange rate. But it is also worth discussing which other factors can cause the real appreciation of the internal real exchange rate in the presence of nominal rigidities. In a particular group of small open economy models of the traditional approach the only possible explanation is asymmetric sectoral productivity growth, that is, the BS effect. This result is based on the following assumptions:

- Constant returns to scale;
- the capital goods in both sectors are produced by tradable goods without any adjustment costs; and
- tradable sectors are internationally homogenous, and the PPP is valid.

Constant returns to scale implies that sectoral prices and inflation rates are not influenced by *demand directly*, only by input prices. The second assumption guarantees that the adjustment of physical capital is frictionless, and the real rental rate of capital is the same for both sectors. Finally, the third assumption implies that both the external real exchange rate and the real rental rate are determined by exogenous foreign factors. Moreover, the real wage is determined by the same factors. Since both the real rental rate and the real wage is determined only by exogenous factors, *indirect-demand effects* are excluded as well.

Contrary to this, if the markets for domestic and foreign tradables are segmented, and if capital goods are not perfect substitutes of consumption goods, i.e. if investments adjustment costs exist, and the adjustment of capital goods is not instant and frictionless, then the real wage and the real rental rate are not exclusively determined by foreign factors. Hence, demand can indirectly influence sectoral prices setting.

If prices are sticky direct demand effects can also play a role even if the assumption of constant returns to scale is not relaxed. As Woodford (2003, chapter 5) shows, if there is no rental market for physical capital, investments have adjustment costs, and price setting is sticky and asynchronized, then firms are not able to form their optimal input combinations continuously. As a consequence, scarcity occurs, and decreasing returns to scale prevails in the short run. Hence, direct demand effects becomes effective.

In this study it is analyzed by simulations the following two related questions: First, how does the presence of demand factors in price setting modify the effect of asymmetric sectoral productivity growth on the difference between sectoral inflation rates? Second, is it possible to explain the empirically observable large and persistent dual inflation of post-communist countries by alternative, principally demand factors? This latter problem is not a mere theoretical curiosity. For example, De Gregorio and Wolf (1994) and Halpern and Wyplosz (2001) detected demand effects in the determination of sectoral relative prices. Moreover, Arratibel et al. (2002) do not simply provide alternative explanations for dual inflation, but they deny the role of productivity factors.

My numerical simulations demonstrate that sticky and asynchronized price setting, the lack of rental markets for capital goods, and investments adjustment costs significantly weaken the impact of asymmetric sectoral productivity growth on the difference between sectoral inflation rates, unless exaggerated asymmetry of sectoral technologies is assumed. The difference between sectoral inflation rates becomes approximately half of the difference between productivity growth rates. This finding is in line with empirical results, such as those of Halpern and Wyplosz (2001).

On the other hand, it is also demonstrated that alternative, especially demand, factors by themselves, without productivity factors, are not able to explain the size and duration of empirically observable dual inflation rates. Beyond productivity factors only one factor proved to be important: price liberalization.

3 The model

As was discussed in the previous section, one of the main focuses of this paper is how to construct a model which can simultaneously guarantee the co-movement of the nominal and real exchange rates and generate the Balassa-Samuelson (BS) effect, that is the co-existence of productivity based dual inflation and real appreciation.

To guarantee the correlation between the nominal and real exchange rates the model needs sticky prices and internationally segmented tradable markets. Obviously, to consider the BS effect it is necessary to have at least two sectors with different total factor productivities (TFP).

In *section 2* it was noted that international market segmentation can be captured in different ways. I therefore compare whether model versions with different descriptions of market segmentation can generate the BS effect. I consider a version (version A) without pricing to market (PTM) and with the assumption that domestic and foreign tradables are imperfect substitutes. In the other two versions PTM is added to the models. In version B PTM is combined with local currency pricing (LCP), in version C with producer currency pricing (PCP).

The other main topic of the paper is an investigation of the factors and mechanisms generating dual inflation. As discussed in detail in *section 2*, if the adjustment of physical capital is frictionless then, practically speaking, beyond productivity there is not much role for other factors to induce dual inflation. Therefore some imperfections in capital formation are considered: it is assumed that there is no rental market for physical capital.[6] This imperfection of input allocation combined with sticky asynchronized price setting results in scarcity and decreasing returns to scale in the short run. As a consequence, the effect of productivity factors on dual inflation will be modified and alternative factors can influence dual inflation as well.

The above features of the model, that is, sticky prices, internationally segmented goods markets, several sectors with different TFPs, and imperfections in capital formation are essential ingredients from the point of view of the main topics of this study. However, some other imperfections are added to the model: *habit formation*, *sticky wages* and *implicit indexation* in price and wage setting. These factors modify the shape of the impulse responses of the model in such way that it better reproduces the empirical impulse responses.

3.1 Households

The domestic economy is populated by a continuum of infinitely-lived identical households. The utility accrued to household j at date t is the following function:

$$\mathcal{U}\left(H_t(j), l_t(j)\right) = u(H_t(j)) - v(l_t(j)),$$

[6]There can be different explanations for the lack of a rental market for physical capital. One is based on the existence of *firm-specific* investments and capital goods. The literature of the theory of firms considers this factor very important, one can explain with this phenomenon the size and integration of firms, as Hart (1995) discusses.

for all $j \in [0,1]$, $H_t = c_t(j) - hc_{t-1}$, where $c_t(j)$ is the consumption of household j at date t, c_{t-1} is the aggregate consumption of the previous period, $h \in [0,1)$ measures the strength of habit formation,[7] and $l_t(j)$ is the labor supply of household j. Furthermore, $u(H) = H^{1-\sigma}/(1-\sigma)$ and $v(l) = l^{1+\varphi}/(1+\varphi)$, σ, $\varphi > 0$. Households discount the future at the rate $0 < \beta < 1$.

The consumption good $c_t(j)$ is composed of *tradable* and *non-tradable* consumption goods:

$$c_t(j) = \left[\left(a_T \chi_t^T\right)^{\frac{1}{\eta}} c_t^T(j)^{\frac{\eta-1}{\eta}} + \left(a_N \chi_t^N\right)^{\frac{1}{\eta}} c_t^N(j)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $c_t^T(j)$ is the tradable, $c_t^N(j)$ is the non-tradable consumption good, η and $a_T = 1 - a_N$ are non-negative parameters, while χ_t^T and χ_t^N are non-negative exogenous shocks, such that

$$a_T \chi_t^T + a_N \chi_t^N = 1. \tag{3}$$

The intertemporal budget constraint of a given household is the following:

$$P_t^T c_t^T(j) + P_t^N c_t^N(j) + P_t^B(j) B_t(j) = \zeta_t(j) B_{t-1}(j) + (1 - \tau_t^w) W_t(j) l_t(j) + T_t,$$

where P_t^T and P_t^N are the price indices of tradables and non-tradables, $B_t(j)$ is the household's nominal portfolio at the beginning of date t, $P_t^B(j)$ is its price, and $\zeta_t(j)$ is its stochastic payoff. $W_t(j)$ is the nominal wage paid to household j, τ_t^w represents labor market taxes and transfers, and T_t is a lump-sum tax/transfer variable. Households supply differentiated labor, hence the wage paid to individual households can be different. On the other hand, it is assumed that the asset markets are complete and it is possible to eliminate the risk of heterogeneous labor supply and income.[8] As a consequence, all households have the same income, consumption is uniform, that is, $c_t(j) = c_t$, and they have the same portfolio, that is, $B_t(j) = B_t$, for all j and t.

It is well known that the linear homogeneity of function (2) implies that the households' problem can be solved in two steps: First they maximize the objective function

$$\sum_{t=1}^{\infty} \beta^{t-1} \mathrm{E}_0 \left[\mathcal{U} \left(H_t(j), l_t(j) \right) \right],$$

[7] I follow Smets and Wouters (2003) in that the consumption habit is defined by past aggregate consumption and not by past individual consumption. This assumption makes the model technically more tractable.

[8] It is assumed that the government's budget is balanced every period. The labor tax/transfer policy represented by τ_t^w is compensated by the non-distortive T_t lump-sum tax/transfer.

with respect to c_t subject to the following modified budget constraint:

$$P_t c_t + P_t^B B_t = \zeta_t B_{t-1} + (1-\tau_t^w) W_t(j) l_t(j) + T_t, \tag{4}$$

non-negativity constraints on consumption, and no-Ponzi schemes. In the budget constraint (4) the consumer price index P_t is defined by the following expression:

$$P_t = \left[a_T \chi_t^T \left(P_t^T\right)^{1-\eta} + a_N \chi_t^N \left(P_t^N\right)^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{5}$$

Second, knowing c_t it is possible to determine c_t^T and c_t^N by the demand functions

$$c_t^T = a_T \chi_t^T \left(\frac{P_t}{P_t^T}\right)^{\eta} c_t, \quad c_t^N = a_N \chi_t^N \left(\frac{P_t}{P_t^N}\right)^{\eta} c_t. \tag{6}$$

The assumption of complete asset markets implies that the optimal intertemporal allocation of consumption is determined by the following condition in all states of the world:

$$\beta \frac{\Lambda_{t+1} P_t}{\Lambda_t P_{t+1}} = D_{t,t+1}, \tag{7}$$

where Λ_t is the marginal utility of consumption,

$$\Lambda_t = (c_t - h c_{t-1})^{-\sigma},$$

and $D_{t,t+1}$ is the stochastic discount factor, which satisfies the condition

$$P_t^B = \mathrm{E}_t \left[D_{t,t+1} \zeta_{t+1}\right].$$

Because in this economy the asset markets are also complete internationally, the foreign equivalent of equation (7) is also held:

$$\beta \frac{\Lambda_{t+1}^* e_t P_t^{F*}}{\Lambda_t^* e_{t+1} P_{t+1}^{F*}} = D_{t,t+1}, \tag{8}$$

where Λ_t^* is the marginal utility of foreign households, and P_t^{F*} is the foreign consumer price index in foreign currency terms, e_t is the nominal exchange rate. For simplicity P_t^{F*} is assumed to be constant. Combining equations (7) and (8) and applying recursive substitutions yields formula

$$\frac{\Lambda_t e_t P_t^{F*}}{\Lambda_t^* P_t} = \iota, \tag{9}$$

where ι is a constant, which depends on initial conditions.

There is monopolistic competition in labor markets with the nominal wage $W_t(j)$ set by household j. It is assumed that wage setting is sticky as in the paper by Erceg et al. (2000). Similarly, as in Calvo (1983), every individual household at a given date changes its wage in a rational, optimizing forward-looking manner with probability $1 - \gamma_w$. All those households which do not behave like this at the given date follow a *rule of thumb*, as in the model of Christiano et al. (2001) and Smets and Wouters (2003), and, update their wages according to the past inflation rate, i.e.

$$W_t(j) = W_T(j) \left(\frac{P_{t-1}}{P_{T-1}} \right)^{\vartheta_w}, \tag{10}$$

where $\vartheta_w \in [0, 1]$ measures the degree of implicit indexation and T is the last date when the wage was set rationally.

If household j rationally sets its wage at date T, it will take into account that the chosen nominal wage $W_T(j)$ will survive until date t with probability γ_w^{t-T}. Thus, to find the optimal solution the household maximizes the objective function

$$\sum_{t=T}^{\infty} (\beta \gamma_w)^{t-T} \mathrm{E}_T \left[\mathcal{U} \left(H_t(j), l_t(j) \right) \right]$$

with respect to $W_T(j)$ subject to constraints (4), (10), and the demand function

$$l_t(j) = \left(\frac{W_t}{W_t(j)} \right)^{\theta_w} l_t, \tag{11}$$

which is a consequence of firms' optimization behavior discussed later, and where the aggregate wage index W_t is defined by

$$W_t = \left(\int_0^1 W_t(j)^{1-\theta_w} \, dj \right)^{\frac{1}{1-\theta_w}}. \tag{12}$$

The log-linear approximation of the solution of the wage setting problem is presented in *Appendix A.2*.

3.2 Production

Final and intermediate goods production

There are two stages of production in the model: in the first step import goods and labor are transformed into differentiated intermediate goods in each sector,[9] while in the second step a homogenous final good is produced in each sector by intermediate products.

As mentioned above, one objective of this paper is to study how the different descriptions of international goods markets segmentation influence the operation of the BS effect. Therefore, three different model versions are considered and compared. In version A it is assumed that there is no PTM. That is, the domestically produced export goods and the domestically consumed tradable goods have the same prices, if they are measured in the same currency. In versions B and C there is pricing to the market, i.e. the price of the domestically produced export goods and the domestically consumed tradable goods can be different, even if they are measured in the same currency.

To capture these characteristics in version A the assumption is made that the domestically produced export goods and the locally traded tradable goods are the same and produced by the same sector. Hence, two sectors are distinguished in version A: a tradable and a non-tradable one.

In versions B and C there are two types of tradable goods: goods which are traditionally classified as tradable, but in practice they are *local goods*, and another type of tradables that are produced for export. As a consequence, prices of local tradables and export goods denominated in the same currency can be different. Local tradables and the export goods are produced by different sectors.[10]

Let us denote by y_t^s the production of a given sector, where $s = T$, x, N, with T referring to the tradable sector in version A and to the sector of local tradables in version B and C, x to the exports sector in version B and C, and N to non-tradables. The final goods are produced in competitive markets by constant-returns-to-scale technologies from a continuum of differentiated inputs, $y_t^s(i)$, $i \in [0, 1]$. The technology is represented by the following CES production function:

$$y_t^s = \left(\int_0^1 y_t^s(i)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}},$$

[9]Thus, I apply the approach of McCallum and Nelson (2001), Smets and Wouters (2002) and Laxton and Pesenti (2003), who consider imports as a production input.

[10]To guarantee PTM, of course, the distinction of local tradables and export goods is not necessary. My approach is similar to that of Burnstein et al. (2002). They also assumed the existence of local and real tradables. But unlike me, they assumed quality difference between the two groups: local goods are inferior.

where $\theta > 1$. As a consequence, the output price P_t^s is given by

$$P_t^s = \left(\int_0^1 P_t^s(i)^{1-\theta} \, di \right)^{\frac{1}{1-\theta}}, \tag{13}$$

where $P_t^s(i)$ denotes the prices of differentiated goods. The demand for good $y_t^s(i)$ is determined by

$$y_t^s(i) = \left(\frac{P_t^s}{P_t^s(i)} \right)^{\theta} y_t^s. \tag{14}$$

In each sector the continuum of good $y_t^s(i)$ is produced in a monopolistically competitive market. Each $y_t^s(i)$ is made by an individual firm using the following uniform technology:

$$y_t^s(i) = A_t^s k_t^s(i)^{\alpha} z_t^s(i)^{1-\alpha}, \tag{15}$$

where $0 < \alpha < 1$, A_t^s is total factor productivity of sector s, $k_t^s(i)$ is the stock of physical capital available for firm i at date t (it was produced in the previous period), and $z_t^s(i)$ denotes an individual firm's utilization of the composite input z_t^s defined in the following way:

$$z_t^s(i) = \left[n_s^{\frac{1}{\rho_s}} l_t^s(i)^{\frac{\rho_s - 1}{\rho_s}} + (1 - n_s)^{\frac{1}{\rho_s}} m_t^s(i)^{\frac{\rho_s - 1}{\rho_s}} \right]^{\frac{\rho_s}{\rho_s - 1}}, \tag{16}$$

where $l_t^s(i)$ an individual firms' utilization of composite labor l_t is defined by

$$l_t = \left(\int_0^1 l_t(j)^{\frac{\theta_w - 1}{\theta_w}} \, dj \right)^{\frac{\theta_w}{\theta_w - 1}},$$

and $l_t(j)$ is the labor supply of household j, and $\theta_w > 1$. Furthermore, $m_t^s(i)$ is the utilization of imported good m_t, and ρ_s, n_s are given non-negative parameters. The price of z_t^s is given by

$$W_t^{z,s} = \left[n_s W_t^{1-\rho_s} + (1 - n_s) \left(e_t P_t^{m*} \right)^{1-\rho_s} \right]^{\frac{1}{1-\rho_s}}, \tag{17}$$

where P_t^{m*} is the foreign currency price of the imported good.

Cost minimization and input demand

It is assumed that there is no rental market for physical capital. The necessary capital goods are produced by the firms themselves. As a consequence, firms' optimal input allocation problem cannot be separated from the problem of capital accumulation and cannot be derived from a sequence of static cost minimization problems.

Instead they solve the following dynamic cost minimization problem: Suppose the trajectories of $y_t(i)$, P_t, $W_t^{z,s}$ and $D_{T,t}$ are given. Then a firm should minimize the objective function

$$\sum_{t=T}^{\infty} \mathrm{E}_T \left[D_{T,t} \left(W_t^{z,s} z_t^s(i) + P_t I_t^s(i) \right) \right],$$

with respect to $z_t^s(i)$, $I_t^s(i)$, $k_{t+1}^s(i)$, subject to the technological constraint (15) and the investment constraint

$$k_{t+1}^s(i) = (1-\delta) k_t^s(i) + \Phi_s \left(\frac{I_t^s(i)}{k_t^s(i)} \right) k_t^s(i), \tag{18}$$

where $I_t^s(i)$ is the investment of firm i at date t. Function Φ_s represents the adjustment costs for investments, and δ is the depreciation rate. As is common in the literature, it is assumed that $\Phi_s' > 0$, $\Phi_s'' < 0$, and that in the steady-state adjustment costs do not exist, i.e. $\Phi_s(I^s/k^s) = I^s/k^s$ and $\Phi_s'(I^s/k^s) = 1$, where variables without time indices refer to the steady-state values.

The first-order conditions of the cost minimization problem are

$$\frac{D_{T,t} P_t}{\nu_t^s(i)} = \Phi_s' \left(\frac{I_t^s(i)}{k_t^s(i)} \right), \tag{19}$$

where $\nu_t^s(i)$ is the Lagrange multiplier of the investment equation,[11] and

$$\nu_t^s(i) = \mathrm{E}_T \left[\nu_{t+1}^s(i) \left\{ (1-\delta) + \phi_s \left(\frac{I_t^s(i)}{k_t^s(i)} \right) \right\} + D_{T,t+1} P_{t+1} r_{t+1}^s(i) \right], \tag{20}$$

where $\phi_s(y) = \Phi_s(y) - y\Phi_s(y)$, and

$$r_{t+1}^s(i) = \frac{\alpha}{1-\alpha} w_{t+1}^{z,s} \frac{z_{t+1}^s(i)}{k_{t+1}^s(i)}. \tag{21}$$

In models with a rental market for physical capital $r_{t+1}^s(i)$ in equation (20) represents the rental rate of capital.[12]

The solution of the cost minimization problem provides equations (15), (18), (19) (20) and (21), which determine the paths of $z_t^s(i)$, $k_t^s(i)$, $I_t^s(i)$,

[11] That is, it is the shadow price of investment. $\nu_t^s(i)/P_t$ is the equivalent of Tobin's q in this model.

[12] If there is no adjustment costs for investments, then condition (20) becomes $P_t = \mathrm{E}_t \left[D_{t,t+1} P_{t+1} \left((1-\delta) + r_t^s(i) \right) \right]$. As a consequence, $r_t^s(i) = r_t^s = r_t$. In a deterministic setting the previous equation takes the form $1/\beta = r + 1 - \delta$, which is a simple arbitrage condition.

$r_t^s(i)$, and $\nu_t^s(i)$ given the paths for $y_t^s(i)$, P_t, $w_t^{z,s}$ and $D_{T,t}$. Knowing $z_t^s(i)$ one can determine the labor and import demand of a particular firm by

$$l_t^s(i) = n_s \left(\frac{W_t^{z,s}}{W_t}\right)^{\rho_s} z_t^s(i), \tag{22}$$

$$m_t^s(i) = (1-n_s)\left(\frac{W_t^{z,s}}{e_t P_t^{m*}}\right)^{\rho_s} z_t^s(i). \tag{23}$$

Firms' investment good is a composition of (local) tradables and non-tradables. The investment good and aggregate consumption good c_t are defined by the same function:

$$I_t^s(i) = \left[\left(a_T \chi_t^T\right)^{\frac{1}{\eta}} \mathcal{I}_t^{Ts}(i)^{\frac{\eta-1}{\eta}} + \left(a_N \chi_t^N\right)^{\frac{1}{\eta}} \mathcal{I}_t^{Ns}(i)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{24}$$

where $\mathcal{I}^{Ts}$ is the demand for (local) tradables of firm i in sector s, and $\mathcal{I}^{Ns}$ is the demand for non-tradables. The particular form of function (24) implies that

$$\mathcal{I}_t^{Ts}(i) = a_T \chi_t^T \left(\frac{P_t}{P_t^T}\right)^{\eta} I_t^s(i), \quad \mathcal{I}_t^{Ns}(i) = a_N \chi_t^N \left(\frac{P_t}{P_t^N}\right)^{\eta} I_t^s(i). \tag{25}$$

Price setting

So far, it has been shown how to find the optimal paths of $z_t^s(i)$, $k_t^s(i)$, $l_t^s(i)$, $m_t^s(i)$ conditional on the trajectories of $y_t^s(i)$ and $P_t^s(i)$. Now the optimal paths of the latter two variables will be determined.

Intermediate goods producers follow a sticky price setting practice. As in the model of Calvo (1983) each individual firm in a given time period changes its price in a rational, optimizing, forward looking manner with probability $1-\gamma_s$. Those firms which do not optimize at a given date follow a rule of thumb, as in Christiano et al. (2001), and Smets and Wouters (2003), and update their prices according to the past sectoral inflation rate.

All firms in sector $s = T$, N which follow the simple indexation rule at date T update their prices according to formula

$$P_t^s(i) = P_T^s(i)\left(\frac{P_{t-1}^s}{P_{T-1}^s}\right)^{\vartheta_s}.$$

Those which set their prices rationally take into account, that $P_T^s(i)$ (the price they set at date T) will exist with probability γ_s^{t-T} at date t. Thus, they maximize the expected profit function

$$\sum_{t=T}^{\infty} \mathrm{E}_T\left[\gamma_s^{t-T} D_{T,t}\left\{(1-\tau_t^s)\, P_T^s(i)\left(\frac{P_{t-1}^s}{P_{T-1}^s}\right)^{\vartheta_s} - MC_t^s(i)\right\}\right]$$

with respect to $P_T^s(i)$ and $y_t^s(i)$ subject to constraint (14), where τ_t^s is tax/transfer variable which modifies firms' markup,[13] and $MC_t^s(i)$ is the marginal cost of firm i. In version B of the model the output price of the exports sector in foreign currency terms $P_T^{x*}(i)$ is sticky. Thus, the problem of the firms in the sector is

$$\max_{P_T^{x*}(i), y_t^x(i)} \sum_{t=T}^{\infty} \mathrm{E}_T \left[\gamma_x^{t-T} D_{T,t} \left\{ (1-\tau_t^x)\, e_t P_T^{x*}(i) \left(\frac{P_{t-1}^{x*}}{P_{T-1}^{x*}} \right)^{\vartheta_x} - MC_t^x(i) \right\} \right],$$

subject to constraint (14). In version C the output price of the export sector in domestic currency terms $P_T^x(i)$ is sticky. Thus, the problem of the firms is

$$\max_{P_T^{x}(i), y_t^x(i)} \sum_{t=T}^{\infty} \mathrm{E}_T \left[\gamma_s^{t-T} D_{T,t} \left\{ (1-\tau_t^x)\, P_T^{x}(i) \left(\frac{P_{t-1}^{x}}{P_{T-1}^{x}} \right)^{\vartheta_x} - MC_t^x(i) \right\} \right],$$

subject to constraint (14). The log-linear approximations of the solutions of the above price setting problems can be found in *Appendix A.2*.

Since the capital stock available at a given date is predetermined, the variable cost of a firm is $W_t^{z,s} z_t^s(i) + P_t I_t^s(i)$. Thus, its marginal cost is

$$MC_t^s(i) = W_t^{z,s} \frac{\partial z_t^s(i)}{\partial y_t^s(i)}.$$

Expressing $z_t^s(i)$ by the technological constraint (15), and differentiating it with respect to $y_t^s(i)$ yields

$$MC_t^s(i) = W_t^{z,s} \left(\frac{y_t^s(i)}{k_t^s(i)} \right)^{\frac{\alpha}{1-\alpha}} (A_t^s)^{\frac{-1}{1-\alpha}}. \tag{26}$$

3.3 Exports demand

Foreign behavior is not modelled explicitly. It is assumed that the following *ad hoc* equation determines demand for exports:

$$x_t = \left(\frac{P_t^{FT*}}{P_t^{x*}} \right)^{\eta^*} x_t^*, \tag{27}$$

where x_t, P_t^{x*} is the foreign currency price of the export goods, P^{FT*} is the foreign currency price of the rival goods (which is constant by assumption),

[13]Since in this model the government's budget is balanced, the tax/transfer represented by τ_t^s is compensated by T_t lump-sum tax/transfer variable in equation (4).

x_t^* is an exogenous shock representing the volume of demand, and $\eta^* > 0$ is an exogenous parameter.

In version A of the model, exported goods are produced by the tradable sector, and $P_t^{x*} = P^T/e_t$. While in versions B and C the local tradables and export goods are different, hence their prices denominated in the same currency can be different, i.e. it is possible that $P_t^{x*} \neq P^T/e_t$.

3.4 Equilibrium conditions

In version A the equilibrium of the tradable sector is given by

$$y_t^T = c_t^T + \sum_{s=T,N} \mathcal{I}_t^{Ts} + x_t. \tag{28}$$

In versions B and C the equilibrium conditions of the sector of local tradables and of the exports sector is given by

$$y_t^T = c_t^T + \sum_{s=T,x,N} \mathcal{I}_t^{Ts}, \quad y_t^x = x_t, \tag{29}$$

where $\mathcal{I}_t^{Ts} = \int_0^1 \mathcal{I}_t^{Ts}(i)\,di$. The equilibrium condition of the non-tradable sector is

$$y_t^N = c_t^N + \sum_s \mathcal{I}_t^{Ns}, \tag{30}$$

where $\mathcal{I}_t^{Ns} = \int_0^1 \mathcal{I}_t^{Ns}(i)\,di$. Finally the labor market equilibrium condition is

$$l_t = \sum_s \int_0^1 l_t^s(i)\,di. \tag{31}$$

3.5 Real exchange rate indices

In this study the following real exchange indices will be considered:

$$q_t = \frac{e_t P_t^{F*}}{P_t}, \quad q_t^T = \frac{e_t P_t^{FT*}}{P_t^T}, \quad P_t^R = \frac{P_t^N}{P_t^T}, \tag{32}$$

where q_t is the CPI-based real exchange rate and q_t^T is the external real exchange rate. The movements of P_t^R, the domestic relative price of non-tradables to tradables, unambiguously determine the fluctuation of the internal real exchange rate, since it is assumed that P^{FT*} and P^{FN*} are constant.

3.6 The log-linearized model

To solve the model its log-linear approximation around the steady state is taken. The complete description of the log-linearized model and the derivation of its equations can be found in *Appendix A.2*. In this section, the most important equations of the system are reviewed. Variables without time indices refer to their steady-state values, and the tilde denotes the log-deviation of a variable from its steady-state value.

Aggregate demand

The path of the aggregate consumption is described by

$$\tilde{c}_t = h\tilde{c}_{t-1} + \frac{1-h}{\sigma}\tilde{q}_t + \tilde{c}_t^*, \tag{33}$$

where $\tilde{c}_t^*$ is an exogenous variable, which represent the foreign business cycle.

In version A exports demand is represented by

$$\tilde{x}_t = \eta^* \tilde{q}_t^T + \tilde{x}_t^*, \tag{34}$$

since in this version $\tilde{q}_t^T = \widetilde{P}_t^{x*}$. In versions B and C the log-linearized exports demand becomes

$$\tilde{x}_t = -\eta^* \widetilde{P}_t^{x*} + \tilde{x}_t^*. \tag{35}$$

Demand for tradable goods depends on exports demand, aggregate consumption and investments, and the sectoral relative price. In version A it takes the form

$$\tilde{y}_t^T = \frac{x x_t + c\tilde{c}_t + I\tilde{I}_t + (c+I)\left(\eta a_N \widetilde{P}_t^R - \frac{a_T}{a_N}\widetilde{\chi}_t^N\right)}{c+x+I}, \tag{36}$$

where I_t denotes aggregate investments, and $\widetilde{\chi}_t^N$ is and exogenous shift of relative sectoral demand. In version B and C the demand for tradables is given by

$$\tilde{y}_t^T = \frac{c}{c+I}\tilde{c}_t + \frac{I}{c+I}\tilde{I}_t + \eta a_N \widetilde{P}_t^R - \frac{a_T}{a_N}\widetilde{\chi}_t^N. \tag{37}$$

Demand for non-tradables depends on the same factors:

$$\tilde{y}_t^N = \frac{c}{c+I}\tilde{c}_t + \frac{I}{c+I}\tilde{I}_t - \eta a_T \widetilde{P}_t^R + \widetilde{\chi}_t^N. \tag{38}$$

Price and wage setting

Following Woodford (2003, chapter 5), *Appendix A.2* presents the solution of the price setting problem of *section 3.2*. The path of the tradable inflation rate is given by

$$\bar{\pi}_t^T = \psi_T^1 \mathrm{E}_t\left[\bar{\pi}_{t+1}^T\right] - \psi_T^2 \mathrm{E}_t\left[\bar{\pi}_{t+2}^T\right] + \xi_T^0 \widetilde{mc}_t^T - \xi_T^1 \mathrm{E}_t\left[\widetilde{mc}_{t+1}^T\right], \tag{39}$$

where $\bar{\pi}_t^T = \pi_t^T - \vartheta_s \pi_{t-1}^T$, and $\pi_t^T = \widetilde{P}_t^T - \widetilde{P}_{t-1}^T$ is the sectoral inflation rate. Furthermore, $\widetilde{mc}_t^T$ is the average real marginal cost of sector. The coefficients ψ_T^1, ψ_T^2, ξ_T^0 and ξ_T^1 are defined in the *Appendix*. To derive the above equations it is assumed that the sectoral tax/transfer variable τ_t^T is constant, hence it does not appear in the log-linear equation. The evolution of the inflation rate in the non-tradable sector is described by

$$\begin{aligned}\bar{\pi}_t^N &= \psi_N^1 \mathrm{E}_t\left[\bar{\pi}_{t+1}^N\right] - \psi_N^2 \mathrm{E}_t\left[\bar{\pi}_{t+2}^N\right] \\ &+ \xi_N^0 \left(\widetilde{mc}_t^N + \frac{\tau^N}{1-\tau^N}\tilde{\tau}_t^N\right) - \xi_N^1 \mathrm{E}_t\left[\widetilde{mc}_{t+1}^N + \frac{\tau^N}{1-\tau^N}\tilde{\tau}_{t+1}^N\right], \end{aligned} \tag{40}$$

where similar notations are used as in the tradable equation. The equation for the inflation rate of the exports sector in version B is

$$\bar{\pi}_t^{x*} = \psi_T^1 \mathrm{E}_t\left[\bar{\pi}_{t+1}^{x*}\right] - \psi_T^2 \mathrm{E}_t\left[\bar{\pi}_{t+2}^{x*}\right] + \xi_T^0 \widetilde{mc}_t^x - \xi_T^1 \mathrm{E}_t\left[\widetilde{mc}_{t+1}^x\right], \tag{41}$$

while in version C it is

$$\bar{\pi}_t^{x} = \psi_T^1 \mathrm{E}_t\left[\bar{\pi}_{t+1}^{x}\right] - \psi_T^2 \mathrm{E}_t\left[\bar{\pi}_{t+2}^{x}\right] + \xi_T^0 \widetilde{mc}_t^x - \xi_T^1 \mathrm{E}_t\left[\widetilde{mc}_{t+1}^x\right], \tag{42}$$

again the same notation is applied as previously. To derive the above two equations it is assumed that the technology and the price setting parameters of the tradable and the exports sector are the same. Hence, the coefficients of these two equations are the same as in equation (39).

It is shown in *Appendix A.2* that the solution of the wage setting problem of *section 3.1* is given by

$$\begin{aligned}&\pi_t^w - \vartheta_w\left(a_T \pi_{t-1}^T + a_N \pi_{t-1}^N\right) = \beta \mathrm{E}_t\left[\pi_{t+1}^w - \vartheta_w\left(a_T \pi_t^T + a_N \pi_t^N\right)\right] \\ &+\xi_w\left[\varphi \tilde{l}_t + \frac{\sigma}{1-h}\left(\tilde{c}_t - h\tilde{c}_{t-1}\right) - \widetilde{w}_t + \frac{\tau^w}{1-\tau^w}\tilde{\tau}_t^w\right],\end{aligned} \tag{43}$$

where ξ_w is a parameter defined in the *Appendix*, $\widetilde{w}_t = \widetilde{W}_t - \widetilde{P}_t$ is the real wage, $\pi_t^w = \widetilde{W}_t - \widetilde{W}_{t-1}$ is the rate of wage inflation, $\pi_t = \widetilde{P}_t - \widetilde{P}_{t-1}$, is the rate of CPI inflation.

Marginal costs

The previous equations reveals that sectoral real marginal costs play a key role in the price setting process. I therefore summarize the determinants of such costs. The average real marginal cost of the tradable sector is given by

$$\begin{aligned} \widetilde{mc}_t^T &= \frac{\alpha}{1-\alpha}\left(\tilde{y}_t^T - \tilde{k}_t^T\right) - \frac{1}{1-\alpha}\widetilde{A}_t^T \\ &+ n_T\widetilde{w}_t + (1-n_T)\left(\widetilde{P}_t^{m*} + \tilde{q}_t\right) + a_N\widetilde{P}_t^R, \end{aligned} \tag{44}$$

while that of the non-tradable sector is described by

$$\begin{aligned} \widetilde{mc}_t^N &= \frac{\alpha}{1-\alpha}\left(\tilde{y}_t^N - \tilde{k}_t^N\right) - \frac{1}{1-\alpha}\widetilde{A}_t^N \\ &+ n_N\widetilde{w}_t + (1-n_N)\left(\widetilde{P}_t^{m*} + \tilde{q}_t\right) - a_T\widetilde{P}_t^R. \end{aligned} \tag{45}$$

Finally, in the exports sector it is

$$\begin{aligned} \widetilde{mc}_t^x &= \frac{\alpha}{1-\alpha}\left(x_t - \tilde{k}_t^x\right) - \frac{1}{1-\alpha}\widetilde{A}_t^T \\ &+ n_T\left(\widetilde{w}_t - \tilde{q}_t\right) + (1-n_T)\widetilde{P}_t^{m*} - \widetilde{P}_t^{x*}. \end{aligned} \tag{46}$$

Policy rule

In this model monetary policy is represented by the following simple log-linear nominal exchange rate rule:

$$d\tilde{e}_t = -\omega\left(a_T\pi_t^T + a_N\pi_t^N\right) + \mathcal{S}_t^{de}, \tag{47}$$

where $d\tilde{e}_t = \tilde{e}_t - \tilde{e}_{t-1}$ is the nominal depreciation rate, and $\mathcal{S}_t^{de}$ is an exogenous nominal depreciation shock.

Exogenous shocks

It is assumed that the log-deviation of the exogenous shocks are *independent* random variables and each follows a *first order autoregressive process*. The 9 exogenous shock of the model are summarized in *Table 1*.

Table 1

Exogenous shocks of the model

Shock	Description	The log-linear equations of the main text it appears in
$\mathcal{S}_t^{de}$	Nominal depreciation rate	(47)
$\widetilde{A}_t^T$	Tradable productivity	(44), (46)
$\widetilde{A}_t^N$	Non-tradable productivity	(45)
$\tilde{c}_t^*$	Foreign business cycle	(33)
$\tilde{x}_t^*$	Exports demand	(34), (35)
$\widetilde{P}_t^{m*}$	Imports price	(44), (45), (46)
$\tilde{\tau}_t^w$	Nominal wage	(43)
$\tilde{\tau}_t^N$	Non-tradable price	(40)
$\widetilde{\chi}_t^N$	Relative demand for non-tradables	(36), (37), (38)

The autoregressive parameters of the shocks are set to 0.95; this choice was motivated by Ireland (2004) and Smets and Wouters (2003). The exceptions are the nominal-depreciation shock $\mathcal{S}_t^{de}$, which is considered as a white noise (i.e. the nominal exchange rate follows a random walk if $\mathcal{S}_t^{de}$ is the only source of nominal-exchange-rate movements), $\tilde{\tau}_t^N$ and $\widetilde{\chi}_t^N$, which are assumed to be permanent shocks.

Let us briefly review the role of the different shocks in the model. One of the main aims of this paper is to construct a model which can replicate the empirical results for the connection between the nominal and real exchange rates and for the behavior of the internal real exchange rate in developing economies. To address these issues in model simulations the nominal-exchange-rate shock variable and the sectoral-productivity shock variables are used, since the most widespread explanation for the internal real exchange rate movements in these economies is based on asymmetric sectoral productivity growth. However, alternative explanations for the internal-exchange-rate movements are also considered. As a consequence, some other shock variables are needed to study the role of different types of demand and cost-push shocks.

The productivity variables of the model, $\widetilde{A}_t^T$ and $\widetilde{A}_t^N$ represent the total factor productivity (TFP) of the sectors relative to the TFP variable of the (developed) rest of the world, which is normalized to zero (it is assumed that the productivity factors of the tradable and the exports sector are the same).

These are used to induce the BS effect in the simulation exercises. It is important to note that the BS effect prevails over a relatively long time period, but is not a permanent phenomenon, since BS is related to economies in transition or to periods of higher-than-average growth rates. That is, it is a transitory event, although it often lasts longer than usual business-cycle phenomena. I therefore assume that the productivity variables of the model are stationary.

The productivity levels of all sectors in the post-communist countries, which are my primary subject, were lower than that of the developed reference countries in the beginning of the post-communist transition, that is, $\widetilde{A}_1^T < 0$, $\widetilde{A}_1^N < 0$. It is also assumed that the productivity growth of each sector is positive, and it is higher in the tradable and the exports sector. Expressing this formally:

$$d\widetilde{A}_t^T > 0, \quad d\widetilde{A}_t^N > 0, \quad d\widetilde{A}_t^T - d\widetilde{A}_t^N > 0,$$

where d is the difference operator. Trajectories satisfying the above conditions can be generated by two first-order autoregressive processes with the same autoregressive parameters, if it is assumed that $\widetilde{A}_1^T < \widetilde{A}_1^N < 0$.

But it would be misleading if one would generate the above discussed productivity processes by two unexpected negative shocks at date 1 since the divergence between the technology of the post-communist countries and that of the developed countries started long before the beginning of their transitions. The surprise caused by unexpected shocks would generate such trajectories that are not peculiar to the transition processes of the studied countries, where the economic variables had been adapted to the negative values of $\widetilde{A}_t^T$ and $\widetilde{A}_t^N$ before the transition processes started. The impulse responses of the endogenous variables to unexpected shocks display characteristic peaks and humps and the state variables have zero values at date $t = 0$. In the simulation exercises I instead choose non-zero values for the state variables at date $t = 0$, in such a way that the trajectories of the endogenous variables become smooth. This approach captures better the impacts of the productivity processes of transition economies.

3.7 Model solution and parameterization

To solve the model Uhlig's (1999) implementation of the *undetermined coefficients* method is used, the numerical results are generated by the aforementioned author's MATLAB algorithm.

The benchmark values of the basic parameters are found in *Table 2*. The value of β is taken from King and Rebello (1999). The value α is chosen in such a way that the capital's share in GDP is 1/3.[14] The values of a_T, η, ρ_s, δ, and θ are widely accepted in the literature. The values of ε_s are taken from Woodford (2003, ch. 5). The values of σ, h, φ, θ_w, γ_w, ϑ_w are Euro area

[14] In this model α is not equal to the capital's share in GDP since one has to subtract the value of imports from the value of total output to obtain GDP.

estimates, taken from the paper of Smets and Wouters (2003). Since in that paper the pricing equation is estimated under the assumption of constant returns to scale, I take the values of γ_s and ϑ_s from the study of Galí et al. (2001), which also contains Euro area estimates.

Table 2
Parameter values of the benchmark economy

Parameter		
Name	Value	
	version A	version B, C
β	0.984	0.984
σ	1.607	1.607
h	0.541	0.541
φ	0.755	0.755
a_T	0.500	0.500
η	1.000	1.000
η^*	10.00	30.00
α	0.208	0.208
ρ_s	1.000	1.000
δ	0.025	0.025
ε_s	3.000	3.000
θ	6.000	6.000
θ_w	3.000	3.000
γ_s	0.787	0.787
ϑ_s	0.365	0.365
γ_w	0.763	0.763
ϑ_w	0.656	0.656
ω	2.500	2.500

Note: $s = T$, x, N.

In that study they interpret inflation persistency differently from the approach I use. They use the model of Galí and Gertler (2000), and assume that each firm updates its price in a given period by probability $1 - \gamma$. Hence, according to the law of large numbers in a given period $1 - \gamma$ fraction of the firms change their prices. But only $1 - \vartheta$ fraction of the price setters choose their prices in an optimal forward-looking manner, the rest update their prices according to the past inflation rate. If $\beta = 1$, then the approach I use and the one used by Galí and Gertler coincides, if $\vartheta_s = \vartheta/\gamma$. Although in our case $\beta \neq 1$, as an approximation I used the above mentioned formula to determine the value of ϑ_k. The choice of η^* will be discussed in detail in *section 5*. Finally, ω was chosen in such a way that the model fits to the empirical findings of *section 4*.

4 Previous empirical results

This section briefly reviews the empirical literature which initiated the research of this paper. First, the findings on the strong relationship between the nominal and real exchange rates are considered, which are relevant in both developed and emerging economies. Second, the findings related to the internal real exchange rate are surveyed. On this issue the evidence is ambiguous. In developed economies, internal-real-exchange-rate movements are negligible, while in several emerging economies dual inflation is an important phenomenon.

4.1 The co-movement of the nominal and real exchange rates

To evaluate the degree of co-movement of the nominal and real exchange rates the following statistics are used: The correlation between the nominal and real depreciation rates, which reveals whether a nominal depreciation is accompanied with a real depreciation. The relative variance of the nominal and real depreciation, which indicates the relative size of the nominal and real-exchange-rate movements. Finally, the autocorrelation function of the real exchange rate, which shows the time pattern of the reaction of the real exchange rate to the nominal-exchange-rate, or other shock.

As discussed in the *Introduction*, the NOEM literature was partly initiated by the empirical findings of Mussa (1986), who first documented the strong connection between the nominal and real exchange rates. Using Monacelli (2004), I summarize some important findings. The post-1971 data from 12 developed countries reveal that the unconditional correlation of real and nominal depreciation rates is 0.98. The correlation of the nominal depreciation rate and the inflation rate is practically zero. In managed floating exchange rate regimes the unconditional variance of the real depreciation rate is two times greater than the unconditional variance of the nominal depreciation rate. In flexible exchange rate regimes the two measures are nearly equal.

The violation of purchasing power parity (PPP) is a necessary condition for the above findings. Moreover, the violation of PPP is not a transitory phenomenon as several empirical studies have shown. Chari, Kehoe and McGrattan (2002), hereinafter 'CKM', studied the persistency of the real-exchange-rate shocks using HP-filtered quarterly data for the USA and 11 developed European countries for the period 1973:1-2000:1. Their estimated quarterly autocorrelation is 0.84.[15] Though the above empirical results are all related to developed countries, the violation of PPP can also be detected in European post-communist countries, which are the primary focus of this study,[16] although the supporting evidence is mainly only stylized facts.

4.2 The external and internal real exchange rates

Developed economies

As mentioned in the *Introduction* and discussed in *section 2*, NOEM literature focuses on the behavior of the external real exchange rate, instead of the internal one, which was mainly studied by the previous traditional literature. This switch of interest is was partly initiated by the findings of Engel (1999), who using US data showed that the volatility of the real exchange rate can be explained nearly perfectly by the movements of the external real exchange rate.

The Balassa - Samuelson effect

Balassa (1964) and Samuelson (1964) formulated the hypothesis that the difference of productivity growth rates of tradable and non-tradable sector results in dual inflation, and as a consequence real appreciation.[17]

Although the effect of unequal sectoral productivity on the real exchange rate can be detected in developed countries, as De Gregorio and Wolf (1994) documented, the real importance of this phenomenon is manifested in high growth and emerging market countries. Several empirical studies demonstrate that the Balassa - Samuelson (BS) effect plays a significant role in these countries. Ito et al. (1997) showed that mainly in Japan, Korea, and Taiwan, but to some extent in other Southeast Asian countries as well, the BS effect was determinant at particular stages of their development process. It also plays an important role in the transition of European post-communist coun-

[15] Diebold et al. (1991) and Lothian and Taylor (1996) using long annual time series of different currencies found much more persistent real-exchange-rate shocks than CKM. It is difficult to explain their findings purely by nominal rigidities. Rogoff (1996) refers to this phenomenon as the '*PPP puzzle*'. Engel and Morley (2001) built an empirical model, which may help to resolve this puzzle.

[16] Hornok et al. (2002) tried to perform econometric estimations on very short time series and the half-time they found is approximately 2.8 years. On the other hand, Darvas (2001) using the data of the Czech Republic, Hungary, Poland, and Slovenia found very short, less than one year, half-lives. But in the studied time periods narrow-band crawling peg regimes were typical in these countries, which may explain his results.

[17] On the Balassa - Samuelson effect see Obstfeld and Rogoff (1996, chapter 4).

tries, as the empirical studies of Halpern and Wyplosz (2001) and Kovács (2002) have documented.

Halpern and Wyplosz (2001) studied the relevance of the BS effect in nine European post-communist countries by estimating a panel regression for the period 1991-98.[18] The estimated coefficients of sectoral productivity factors are significant and have the correct sign, confirming the presence of the BS effect: If tradable productivity rises by 1 percent, the sectoral relative price rises by 0.24 percent in the short run and by 0.43 percent in the long run. A 1 percent rise of non-tradable productivity results in a 0.18 percent decrease of the relative price in the short run and a 0.32 percent decrease in the long run.

The central banks of Central European accession countries have also studied the role of the BS effect in the post-communist transition.[19] The paper edited by Kovács (2002) summarizes the results. They found that the real exchange rate appreciation in each country can be explained to some extent by the BS effect, although its importance was different for the various countries.

Other evidence on dual inflation

Empirical studies analyzing the BS effect have often detected other non-productivity factors in the determination of the sectoral relative price. See, e.g., De Gregorio and Wolf (1994) and Halpern and Wyplosz (2001). Moreover, Arratibel, Rodríguez - Palenzuela and Thimann (2002), hereinafter 'ART', do not simply provide alternative explanations for dual inflation, but they deny the role of productivity factors in the determination of the examined countries. In their paper they studied the inflation processes in 10 European post-communist countries.[20] Their results support the existence of dual inflation in these countries: the average difference of sectoral inflation rates is 4.9 percentage points.

Though ART did not build a theoretical model to test, they used the New Keynesian Phillips curve literature as an analytical benchmark to derive alternative explanations for dual inflation, and to test these explanations econometrically. They did not consider the real appreciation accompanying

[18] The countries in the sample were the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Russia, Romania, and Slovenia.

[19] The examined countries and the length of the data set: Czech Republic (1994-2001), Hungary (1992-2001), Poland, (1990-2001), Slovakia (1995-2000), and Slovenia (1992-2001).

[20] The countries examined are Bulgaria, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Slovakia, and Slovenia. They used time series of the *Harmonised Index of Consumer Prices* (HICP) available from 1995-98 in these countries.

dual inflation. They found that the main factor impacting dual inflation was the fact that different sectors respond to particular shocks differently. According to their results the tradable sector is sensitive to foreign demand, terms of trade, and the oil price, while the non-tradable sector is basically influenced by domestic demand, the budget deficit, the nominal wage, and price regulations. Contrary to other empirical studies ART found the productivity factor irrelevant in the determination of dual inflation. According to their estimations a positive productivity shock negatively influences the inflation rate in the non-tradable sector, although the authors admit that one should interpret this latter result with caution because of the poor quality of productivity data.

Finally, I should like to mention the role of the nominal-exchange-rate shocks in the determination of dual inflation. Burnstein et al. (2002) and Halpern and Wyplosz (2001) documented that *large devaluations* reduced the size of the difference between sectoral inflation rates, since in this case tradable inflation rose more than non-tradable inflation.

5 Productivity induced dual inflation and real appreciation

It was noted in the previous section that there is a strong relationship between the nominal and real exchange rates, and asymmetric sectoral productivity growth results in dual inflation and real appreciation in developing countries. In this section it is illustrated how it is possible to reproduce both sets of evidence in a NOEM model.

5.1 Theoretical background

Usually the productivity induced coexistence of dual inflation and real appreciation, i.e. the BS effect is analyzed with models of the traditional approach assuming the PPP, which prevents external real exchange rate movements. In this case, of course, appreciation of the internal real exchange rate yields real appreciation. The problem is how it is possible to reproduce this phenomenon by a NOEM model which violates the PPP. In this case it is necessary to guarantee that the possible depreciation of the external real exchange rate cannot neutralize or suppress the appreciation of the internal real exchange rate. However, if dual inflation is induced by asymmetric sectoral productivity processes, then the latter condition may be violated. If the productivity growth of the domestic tradable sector is higher than those of the non-tradable sector and foreign tradable sector, then domestic tradables become cheaper than their foreign rivals and this may cause considerable depreciation of the external real exchange rate.

This possibility is especially important in version A. Consider the exports demand equation (34). If the international substitution parameter $\eta^* + \infty$ then $\tilde{q}_t^T = 0$, i.e. the external real exchange rate becomes constant, and there will not be any relationship between the nominal and the real exchange rate, which contradicts the empirical results. On the other hand, if η^* is low, and $\widetilde{P}_t^T$ is sticky, i.e. it responds to shocks slowly, then $\tilde{q}_t^T = \widetilde{P}_t^T - \tilde{e}_t$ will move together with the nominal exchange rate. But in this case high tradable-productivity growth may cause strong external-real-exchange depreciation. The question is whether there is an intermediate value of η^* when version A of the model can replicate both sets of empirical findings.

In versions B and C even a high value of η^* can guarantee a strong co-movement of the nominal and real exchange rates. On the other hand, in this case the foreign currency price of domestically produced export goods, i.e. P_t^{x*}, does not deviate much from the prices of their foreign rivals. As a consequence, if other factors are kept fixed, then the marginal costs of the domestic exports and the tradable sector is similar, hence $P_t^{T*} = P_t^T / e_t$ remains relatively stable. Thus, the conjecture is that in versions B and C it is possible to find appropriate values for the substitution parameter such that the asymmetric sectoral productivity growth does result in real appreciation.

5.2 Simulation results

First, it is studied which value of the substitution parameter η^* is consistent with the strong co-movement of the nominal and real exchange rates in the three different model versions. In the simulation exercises the depreciation shock $\mathcal{S}_t^{de}$ is the only source of nominal-exchange-rate movements. This approach is supported by several empirical studies. In a closed economy context Ireland (2004) and Smets and Wouters (2003) demonstrated by their estimated models that nominal shocks have a primary role while technological shocks have only an auxiliary role in explaining business cycles. Clarida and Galí (1994) showed that in open economies 35-41 percent of real exchange rate movements can be attributed to nominal shocks. The prominent importance of the nominal-exchange-rate shocks in emerging markets is documented by Calvo and Reinhart (2002).

In the following simulations all parameters, except η^*, are set to their benchmark values (see *Table 2*). *Table 3* displays the results. The empirical values of the statistics in the table are taken from *section 4*. The first measure of the strength of the co-movement is the correlation coefficient of real and nominal depreciations. It is insensitive to the change of the substitution

Table 3

The relationship between the nominal and real exchange rates in the model economy

Version *A*

Statistics	Data	Parameter values of η^*			
		1	10	20	30
The correlation of the real and nominal depreciations	0.98	0.996	0.995	0.995	0.994
The relative variance of the real and nominal depreciations	1-4	0.96	0.96	0.95	0.94
Autocorrelation of the real exchange rate					
1 quarter	0.84	0.83	0.78	0.74	0.71
1 year	0.50	0.47	0.34	0.26	0.21
2 years	0.25	0.26	0.15	0.09	0.06
3 years	0.12	0.17	0.09	0.06	0.04

Version *B*

Statistics	Data	Parameter values of η^*			
		1	10	20	30
The correlation of the real and nominal depreciations	0.98	0.997	0.996	0.996	0.996
The relative variance of the real and nominal depreciations	1-4	0.98	0.98	0.98	0.98
Autocorrelation of the real exchange rate					
1 quarter	0.84	0.84	0.83	0.82	0.82
1 year	0.50	0.48	0.46	0.44	0.44
2 years	0.25	0.27	0.25	0.23	0.22
3 years	0.12	0.18	0.15	0.14	0.13

Version *C*

Statistics	Data	Parameter values of η^*			
		1	10	20	30
The correlation of the real and nominal depreciations	0.98	0.996	0.996	0.995	0.995
The relative variance of the real and nominal depreciations	1-4	0.98	0.98	0.98	0.98
Autocorrelation of the real exchange rate					
1 quarter	0.84	0.83	0.80	0.78	0.77
1 year	0.50	0.47	0.39	0.34	0.32
2 years	0.25	0.26	0.18	0.14	0.12
3 years	0.12	0.17	0.11	0.07	0.06

parameter in all model versions. This comes as no surprise, since independently of η^* a nominal depreciation accompanies with real depreciation, only the size of the real depreciation decreases as η^* increases. This latter effect can be captured by the relative variance of the nominal and real depreciations. This statistic decreases as η^* increases in version A, but the change is not considerable. On the other hand, in versions B and C the relative variance does not react to the change of the substitution parameter. The time pattern of the reaction of the real exchange rate to the nominal-exchange-rate shock can be captured by the autocorrelation function of the real exchange rate. In version A all of the autocorrelation coefficients diminish as η^* increases. Especially, if $\eta^* \geq 20$, then the decrease of the 2-year and 3-year coefficients becomes significant. On the other hand, in version B the autocorrelation coefficients are insensitive to the substitution parameter. In version C the autocorrelation coefficients are more sensitive than in version B, but their changes are less important than in version A.[21]

To summarize: while model version B is insensible, and version C weakly reacts to the change of η^*, version A is sensible to the variation of the substitution parameter. It reproduces the empirical results only if η^* has low values, i.e. the domestically produced export goods and their foreign rivals are far substitutes.

The next issue is whether the dual inflation induced by asymmetric productivity growth is accompanied with real appreciation. As discussed, real appreciation occurs if domestically produced export goods and their foreign rivals are close substitutes. That is, if the value of η^* in equations (34) and (35) is high. On the other hand, as it was shown above, version A of the model can reproduce the empirical regularities associated with the nominal and real exchange rates only if parameter η^* has a relatively low value. Thus, in version A I set $\eta^* = 10$, which is the highest value that provides relatively good results in terms of the co-movement of the nominal and real exchange rates. Contrary to this, in versions B and C I set $\eta^* = 30$ since according to *Table 3* in these versions the relevant statistics do not depend on the substitution parameter.[22]

[21]It is interesting to note that in all versions with appropriate parameter values the model is able to reproduce the empirical autocorrelation values. This contradicts the simulation results of CKM. However, Benigno (2004) demonstrated that if monetary policy is described by a rule with inertia, and the foreign and home country are asymmetric in such a way that monetary shocks result in terms of trade changes, then the required persistence can be attained by the model. These conditions are fulfilled in my model.

[22]One may criticize the choice of the substitution parameter in version B and C. The value of $\eta^* = 30$, is much higher than in other open economy models. E.g., Backus et al. (1994) use much lower substitution parameter to replicate the empirically observable responses of the trade balance to productivity shocks. My conjecture is that if the inertia

Figure 1 displays the results for the benchmark economy in version A. The first panel plots the difference between the growth rates of sectoral productivity factors $d\widetilde{A}_t^T - d\widetilde{A}_t^N$, and the difference between sectoral inflation rates $\pi_t^R = \pi_t^N - \pi_t^T$. The latter determines the movements of the internal real exchange rate. If π_t^R is positive, then the internal real exchange rate appreciates. The second panel plots the depreciation of the real exchange rate $d\tilde{q}_t$, and the external real exchange rate $d\tilde{q}_t^T$. The third panel displays the growth rates of relative sectoral output and of capital, that is, $d\tilde{y}_t^N - d\tilde{y}_t^T$ and $d\tilde{k}_t^N - d\tilde{k}_t^T$. Finally, panel four plots the growth rates of the real wage and of exports. All growth rates are expressed in annualized terms. The paths of the productivity variables are chosen such that in the initial period the difference of the annualized growth rates is 1 percentage point.

The simulation results reveal that although the internal real exchange rate appreciates, the real exchange rate depreciates since the effect of the depreciating external rate is stronger then that of the internal rate. The reason is that the productivity growth of the tradable sector is higher than those of the non-tradable sector and foreign tradable sectors. As a consequence, the relative price of domestically produced tradables to foreign tradables decreases. That is, the external real exchange rate depreciates. If domestically produced and foreign tradables were perfect substitutes, then the reduced relative price would induce a large instant increase of demand for domestic tradables. Hence, domestic real wages and tradable prices would increase and the prices of domestic and foreign tradables denominated in the same currency would equalize immediately. But in the studied case domestic and foreign tradables are far substitutes, hence increasing demand does not result in equalized prices. The fourth panel shows that even with a constant increase in export, real wages increase little.

The trajectories of *Figure 2* demonstrate that the above results do not depend on the presence of nominal rigidities and investment adjustment costs. Flexible prices and flexible capital allocation yield similar results if domestic and foreign tradables are far substitutes. The lack of price stickiness and the lack of frictions in capital accumulation imply that only productivity factors influence the sectoral relative price, and the difference of sectoral productivity growth rates, and the difference of sectoral inflation rates are equal, as in the models of the traditional approach. Although the appreciation of the internal real exchange rate is stronger than in the previous case with nominal rigidities, the depreciation of the external real exchange rate is larger,

of the exports demand is increased, as in Laxton and Pesenti (2003), or the import requirement of exports production is increased, then my model would also be able to reproduce the short run behavior of the trade balance.

which causes the real exchange rate to depreciate. Panel four displays that without frictions the real wage adjustment is also weak, thus it was not the consequence of sticky wages in the previous case.

Compare these results with the ones from versions B and C displayed in *Figures 3 - 4*. In both versions the appreciation of the internal real exchange rate is smaller than difference of sectoral productivity growth rates. However, the real exchange rate appreciates since the depreciation of the external real exchange rate is negligible. The reason is the following: The products of the exports and the foreign tradable sector are close substitutes. As a consequence, the increase of the demand for exports induced by the exports price reduction is stronger than in version A, and this neutralizes the impact of productivity growth on prices.

To summarize: There is no intermediate value of the international substitution parameter which is appropriate for reproducing the BS effect and the co-movement of nominal and real exchange rate simultaneously. The remedy is the assumption of pricing to market (PTM). In the presence of PTM the co-movement of the nominal and the real exchange rate remains strong, even if domestic and foreign goods are close substitutes which enables the operation of the BS effect.[23] This result is independent of the assumption on the price setting currency. It remains valid both with local currency pricing (LCP) and producer currency pricing (PCP).

Note, I would rather not take sides in the LCP vs. PCP debate since both approaches can be consistent with the BS effect.[24] As mentioned, PCP can be applied without the assumption of price discrimination. Moreover, in most cases PCP is applied without PTM, which is equivalent to applying version A. The reason for this is that the arguments of the supporters of PCP remain valid without PTM. However, my results points out that if one wants to capture the particularities of emerging markets, then the PCP approach cannot be applied without the assumption of international price discrimination.

[23] The comparison of our study with that of Benigno and Thoenissen (2002) reveals that the analysis of the BS effect is not equivalent to the analysis of the immediate effects of unexpected productivity shocks, as discussed in *section 3.6*. In their NOEM model if a positive shock hits the tradable sector, the real exchange rate depreciates.

[24] LCP vs. PCP is one of the most important undecided debates in the NOEM literature, since the choice of the optimal exchange rate is not independent of this problem. One can read pro LCP arguments in Engel (2002a, 2002b). Obstfeld (2001, 2002) and Obstfeld and Rogoff (2000) presents arguments supporting the PCP approach. Two recent studies on this topic are Bergin (2004), which provides evidences supporting LCP, and Koren et al. (2004) with findings reinforcing PCP.

6 The magnitude of the difference between sectoral inflation rates

In the previous section one aspect of productivity-induced dual inflation and real appreciation was studied: the connection between the dual inflation and real appreciation was the issue. This section focuses on the other part of the mechanism. The relationship between the magnitude of the difference of sectoral inflation rates and that of sectoral productivity growth rates is considered. In *section 4* it was mentioned that usually the difference between sectoral inflation rates is smaller than the difference between productivity growth. This phenomenon can be explained by sectoral asymmetry. However, this section presents another explanation, because some empirical findings cast doubt on the presence of strong sectoral asymmetry.

6.1 Theoretical background

As discussed in *section 4*, some empirical studies show that the difference between sectoral inflation rates is smaller than the difference between productivity growth, see, e.g., Halpern and Wyplosz (2001). This short theoretical survey explains the above phenomenon. For expositional simplicity, in this section let us assume only sectoral productivity shocks affect the economy.

First, the role of sectoral symmetry in the determination of dual inflation is considered. As an illustration, consider the case, when prices and wages are flexible, and goods and capital markets are homogenous. In *Appendix A.2* it is shown that in this case the $\pi_t^R = \pi_t^N - \pi_t^T$ difference of sectoral inflation rates is determined by

$$\pi_t^R = \frac{n_N}{n_T} d\widetilde{A}_t^T - d\widetilde{A}_t^N,$$

where n_T and n_N are the labor utilization parameters in the technological equation (16). If the sectors are symmetric, that is, if $n_T = n_N$, then one percentage point difference between the sectoral productivity growth rates induces one percentage point difference between sectoral inflation rates. If the sectors are asymmetric, the magnitude of these differences will deviate. If prices are sticky, then the asymmetry of other parameters can cause deviation. If one relaxes the assumptions of flexible prices and homogeneity, then the asymmetry of other parameters can also cause deviation.

However, in my model the magnitude of π_t^R can deviate from that of the difference of sectoral productivity growth rates even if the sectors are symmetric. Suppose that $n_T = n_N$ and in equations (39) and (40) the price setting parameters are symmetric, i.e. $\xi_T^l = \xi_N^l$ and $\psi_1^k = \psi_2^k$, $l = 0, 1$,

$k = 1, 2$, and $\vartheta_T = \vartheta_N$. Let us denote the common parameters by ξ^l, ψ^k, and ϑ. Then one can express the $\pi_t^R = \pi_t^N - \pi_t^T$ difference between sectoral inflation rates by using equations (39) and (40) as

$$\bar{\pi}_t^R = \psi^1 \mathrm{E}_t \left[\bar{\pi}_{t+1}^R \right] - \psi^2 \mathrm{E}_t \left[\bar{\pi}_{t+2}^R \right] + \xi^0 \widetilde{mc}_t^R - \xi^1 \mathrm{E}_t \left[\widetilde{mc}_{t+1}^R \right], \tag{48}$$

where $\bar{\pi}^R = \pi_t^R - \vartheta \pi_{t-1}^R$, and $\widetilde{mc}_t^R = \widetilde{mc}^N - \widetilde{mc}^T$ is the relative real marginal cost. Price stickiness reduces the impact of the relative marginal cost on the sectoral relative price: If prices are flexible, i.e. if $1/\xi^0 = 0$,then the relative real marginal cost, and the difference of the productivity factors directly influence the sectoral relative inflation rate, while in the sticky price version this influence becomes indirect. On the other hand, if the degree of price stickiness is increased, then ξ^0 and ξ^1 will be smaller, hence the impact of the relative marginal cost on the relative inflation rate will be weaker.

Price stickiness modifies the impact of differences in productivity across sectors in another way as well. By the combination of equations (44), and (45) one can show that in the presence of sticky prices the relative marginal cost is defined by

$$\widetilde{MC}_t^R = \frac{1}{1-\alpha} \left(\widetilde{A}_t^T - \widetilde{A}_t^N \right) + \frac{\alpha}{1-\alpha} \left(\tilde{y}_t^N - \tilde{y}_t^T \right) - \frac{\alpha}{1-\alpha} \left(\tilde{k}_t^T - \tilde{k}_t^N \right) \tag{49}$$

if $n_T = n_N$. The above equation reveals that the frictions in capital accumulation combined with asynchronized price setting result in short-run scarcity of resources, hence the short-run marginal cost will have decreasing returns-to-scale characteristics. As a consequence, demand factors directly influence this expression represented by the term $\tilde{y}_t^N - \tilde{y}_t^T$.

The presence of the demand factor may weaken the impact of the difference of productivity measures. But this effect is not independent of the size of investments adjustment costs. If adjustment costs are infinitely large, then the capital stock is fixed, that is $\tilde{k}_t^T = \tilde{k}_t^N = 0$. Thus, the effect of $\tilde{y}_t^T$ and $\tilde{y}_t^N$ fully prevails. On the other hand, as adjustment costs decrease, $\tilde{k}_t^T$ and $\tilde{k}_t^N$ become more volatile and this may compensate the effects of $\tilde{y}_t^T$ and $\tilde{y}_t^N$.

So far, it has been shown in an intuitive way that both sectoral asymmetry, and nominal rigidities combined with real frictions can explain the empirical finding that usually the difference between sectoral inflation rates is smaller than the difference between sectoral productivity growth rates. Now I argue that some empirical observations cast doubt on the explanation based on sectoral asymmetry. As mentioned in *section 4*, Engel (1999) showed that in developed countries the internal-real-exchange-rate movements were negligible. This result suggests that asymmetric shock, like asymmetric sectoral productivity shocks are not important in these economies. Moreover, one may conjecture that sectoral symmetry is also a necessary condition of this finding. Otherwise, even symmetric shocks would result in internal-real-exchange rate movements. Let us accept the hypothesis that the sectoral structure of the emerging economies is similar to that of the developed ones, and what makes the difference between them is the different structure of shocks. Then the explanation for the magnitude of the difference between sectoral inflation rates has to be based on nominal rigidity and frictions in capital allocation.

6.2 Simulation results

First, the aforementioned conjecture that sectoral symmetry is necessary to replicate the result of Engel (1999) will be supported by numerical simulations. In the simulation exercise two symmetric shocks are considered, the shock of the foreign business cycle $\tilde{c}_t^*$ as a demand shock, and the nominal-wage shock $\tilde{v}_t^w$ as a cost-push shock.[25] The effects of the shocks are examined separately. The contribution of the external-real-exchange-rate depreciation to the variance of the real depreciation is measured. If it is close to 1, then the role of the internal real exchange rate is negligible.[26] All versions of the model are symmetrically parameterized in the benchmark case. In what follows, the sensitivity of the aforementioned measure to the asymmetry of sectoral parameters is studied. *Table 4* contains the results related to version *B*. Since these findings properly represent the results of the other two versions, for the sake of simplicity, I forego displaying them. I study the effects of price rigidity parameters γ_s, indexation parameters ϑ_s, and adjustment costs parameters ε_s, $s = T, N$. One of the sectoral parameters always takes its benchmark value, the other is changed. The labor utilization parameters n_T and n_N are set such that $1 - n_N = \upsilon(1 - n_T)$, where $\upsilon = 0.75, 0.5, 0.25, 0.1$.

The table reveals that the asymmetry of price rigidity parameters significantly increases the role of the internal real exchange rate. The effect

[25]The importance of demand and cost-push shocks is supported by Ireland (2004) and Smets and Wouters (2003).

[26]Following Engel (1999), the contribution of the external real exchange rate to the variance of the real exchange rate is measured by

$$\left[\operatorname{var}\left(d\tilde{q}^T\right) + a_N \operatorname{cov}\left(d\tilde{q}_t^T, d\widetilde{P}_t^R\right)\right] \operatorname{var}\left(d\tilde{q}_t\right)^{-1}.$$

This index number can be greater than 1 if the covariance in the formula is negative, since $\operatorname{var}\left(d\tilde{q}_t\right) = \operatorname{var}\left(d\tilde{q}_t^T\right) + 2a_N \operatorname{cov}\left(d\tilde{q}_t^T, d\widetilde{P}_t^R\right) + \operatorname{var}\left(d\widetilde{P}_t^R\right)$.

of the price indexation parameters are weaker. The impact of the adjustment costs parameters is especially strong if demand shocks prevail. Finally, the effects of the technological parameters n_T and n_N are significant if $1 - n_N < 0.75(1 - n_T)$. To summarize: a symmetric model structure is required to replicate Engel's finding.

Table 4
Contribution of the external real exchange rate
to the variance of the real exchange rate in version B

	Parameter values of γ_T and γ_N			
Shocks	0.33 0.79	0.79 0.33	0.90 0.79	0.79 0.90
Demand	1.16	0.84	0.84	1.16
Nominal-wage	1.12	0.88	0.87	1.13

	Parameter values of ϑ_T and ϑ_N			
Shocks	0.10 0.37	0.37 0.10	0.90 0.37	0.37 0.90
Demand	0.96	1.04	1.11	0.89
Nominal-wage	0.97	1.03	1.09	0.92

	Parameter values of ε_T and ε_N			
Shocks	0 3	3 0	∞ 3	3 ∞
Demand	0.78	1.31	1.26	0.77
Nominal-wage	0.99	0.98	1.10	0.91

	Parameter values of n_T and n_N			
Shocks	0.49 0.62	0.44 0.72	0.38 0.85	0.34 0.93
Demand	0.95	0.90	0.84	0.80
Nominal-wage	0.94	0.88	0.81	0.76

Now it will be demonstrated that the empirical magnitude of the difference between sectoral inflation rates (recall Halpern and Wyplosz (2001)) can be reproduced without sectoral asymmetry, however, both nominal rigidities and frictions in capital allocation are necessary to achieve this goal.

The simulation results displayed in *Figures 1*, *3*, and *4* demonstrate that the presence of investments adjustment costs and sticky prices reduces the depreciation of the internal real exchange rate. Since non-tradables become more and more expensive, relative demand for tradables increases. Hence, the difference $\tilde{y}_t^N - \tilde{y}_t^T$ decreases. But in this case equations (48) and (49) imply that the appreciation of the internal rate becomes weaker, thus the net effect of productivity factors becomes weaker as well.

Figures 1 shows that the evolution of $d\tilde{k}_t^N - d\tilde{k}_t^T$ fully neutralize the effect of $d\tilde{y}_t^N - d\tilde{y}_t^T$. Thus, the relatively small appreciation of the internal real exchange rate is a consequence of price stickiness only. On the other hand, *Figures 3 - 4* display the results when $d\tilde{k}_t^N - d\tilde{k}_t^T$ amplifies the effect of $d\tilde{y}_t^N - d\tilde{y}_t^T$. As a consequence, in the benchmark economy of version B and C the real appreciation of the internal real exchange rate is smaller than in version A.

Now a sensitivity analysis related to the investments adjustment costs and price rigidity will be performed. *Figure 5* and *6* display the results generated by version B. In *Figure 5* investments adjustment costs are omitted from the benchmark economy. The lack of adjustment costs amplifies the appreciation of the internal real exchange rate since now $d\tilde{k}_t^N - d\tilde{k}_t^T$ do not support the effect of relative sectoral demand, furthermore the coefficients of equation (48) are transformed. In *Figure 6* it is assumed assume that the degree of price rigidity is smaller. Again, the appreciation of the internal real exchange rate is stronger than in the benchmark economy.

Thus, if investments adjustment costs or the degree of price rigidity is reduced, the difference of sectoral inflation rates will be similar to the difference of productivity growth rates. That is, the simultaneous presence of both factors is necessary for the difference of sectoral inflation rates to be significantly smaller than the difference of productivity growth rates. This result is also supported by the observation that in versions B and C if prices are flexible the difference of sectoral inflation rates becomes insensible to adjustment costs.

To summarize: the higher investments adjustment costs are and the stickier the prices are, the smaller the difference of sectoral inflation rates becomes, which is induced by asymmetric productivity growth. In order to replicate the empirically observable size of the difference of sectoral inflation rates the presence of both adjustment costs and sticky prices is required.

7 Other explanations for dual inflation

As was mentioned in *section 4*, some empirical studies stress alternative factors in the determination of dual inflation, and also cast doubt on the relevance of productivity factors. The objective of this section is to check whether it is possible to generate via a NOEM model large and persistent dual inflation, similar to the empirically observable ones in European post-communist countries.

7.1 Theoretical background

In this model beyond productivity a lot of other factor can influence the π_t^R difference of sectoral inflation rates, since frictions in capital allocation yields decreasing-returns-to-scale features in the model. Recall equations (48), which determines the difference between sectoral inflation rates in this model:

$$
\begin{aligned}
\bar{\pi}_t^R &= \psi^1 \mathrm{E}_t\left[\bar{\pi}_{t+1}^R\right] - \psi^2 \mathrm{E}_t\left[\bar{\pi}_{t+2}^R\right] \\
&+ \xi^0\left(\widetilde{mc}_t^R + \frac{\tau^N}{1-\tau^N}\tilde{\tau}_t^N\right) - \xi^1 \mathrm{E}_t\left[\widetilde{mc}_{t+1}^R + \frac{\tau^N}{1-\tau^N}\tilde{\tau}_{t+1}^N\right].
\end{aligned}
$$

Using equations (44) and (45) the relative real marginal cost can be expressed as

$$
\begin{aligned}
\widetilde{mc}_t^R &= \frac{1}{1-\alpha}\left(\widetilde{A}_t^T - \widetilde{A}_t^N\right) + \frac{\alpha}{1-\alpha}\left(\tilde{y}_t^N - \tilde{y}_t^T\right) - \frac{\alpha}{1-\alpha}\left(\tilde{k}_t^T - \tilde{k}_t^N\right) \\
&+ (n_N - n_T)\,\widetilde{w}_t + (n_T - n_N)\left(\widetilde{P}_t^{m*} + \tilde{q}_t\right) - \widetilde{P}_t^R.
\end{aligned}
$$

The above equations reveal that π_t^R is influenced by several non-productivity factors. If sectors are symmetric, then all shocks can influence π_t^R, which have an impact on variables $\tilde{y}_t^T$, $\tilde{y}_t^N$, $\tilde{k}_t^T$, and $\tilde{k}_t^N$. For example, relative demand factors can affect these variables. Furthermore, $\tilde{\tau}_t^N$ representing the tax/transfer policy of the government also influences π_t^R. If sectors are asymmetric, then beyond the aforementioned factors, the import-price shock $\widetilde{P}_t^{m*}$ and all the shocks which influence the real exchange rate can have an impact on π_t^R. Furthermore, $\tilde{\tau}_t^w$, which represents wage shock, also influences it.

Nevertheless, I want to emphasize that the goal of the section is not to enumerate all the possible determinants of the difference of sectoral inflation rates, but rather to select those ones which are able to generate large and persistent dual inflation in a NOEM model.

7.2 Simulation results

First, in the simulation exercise the symmetric benchmark economy is used to study the effects of asymmetric shocks. Let us start with the effects of price liberalization in the non-tradable sector. It is modelled by the *permanent* increase of shock $\tilde{\tau}_t^N$, i.e. the autoregressive parameter of the shock is equal to 1. $\tilde{\tau}_t^N$ is interpreted as a tax/transfer variable, which influences the markup of the non-tradable sector and represents exogenous government interventions. Thus, a permanent increase of $\tilde{\tau}_t^N$ can be seen as a once and for all decrease of state transfer to the sector. *Figure 7* displays the impulse responses belonging to version B, the other two versions provide basically similar results. The size of the shock is chosen such that the markup is changed by 10 percent. The figure reveals that the inflation rate of the non-tradable sector becomes higher than that of the tradable sector. Moreover, at initial dates the difference is quite large, around 5 percentage points.

Then let us consider the effects of the relative-demand shock $\widetilde{\chi}_t^N$. The autoregressive parameter of the shock is 1, i.e. the effect of a permanent 1 percent increase of relative demand for non-tradables is studied. The simulations reveal that the size of the difference between sectoral inflation rates induced by this shock is negligible: in version A, the difference is 0.03, in versions B and C it is 0.04 percentage points.

The exports-demand shock $\tilde{x}_t^*$ is the next one. The magnitude of the difference between sectoral inflation rates induced by a 1 percent shock is the order of 10^{-3} in version A and practically zero in version B and C.

Second, let us study the effects of three symmetric shocks. Now a mild asymmetry of the model is assumed: $1 - n_N = 0.75(1 - n_T)$.

The nominal-wage shock is represented by $\tilde{\tau}_t^w$. The size of the shock is chosen in such a way that it results in 1 percent nominal-wage inflation. Then the generated difference of sectoral inflation rates is only 0.03 percentage point in version A, and in versions B and C it is even smaller.

One may ask whether it enhances the effect of wage shocks if it is assumed that the labor inputs used by the sectors are different. But I neglect this problem because it does not have empirical support. Halpern and Wyplosz (2001) showed that in the examined post-communist countries sectoral wages are equalized, hence sectoral labor inputs must be close substitutes.

The next two symmetric shocks are the foreign-business-cycle shock $\tilde{c}_t^*$, and the imports-price shock $\widetilde{P}_t^{m*}$. A 1 percent rise of $\tilde{c}_t^*$ yields 0.1 percentage point difference in version A, and 0.03 in versions B and C. A 1 percent rise of $\widetilde{P}_t^{m*}$ results in 0.02 percentage point difference in all the versions.

As the above discussion reveals, only one shock can generate considerable dual inflation. I therefore repeat the simulation exercises with asymmetric sectors: the asymmetry of sectoral labor and import utilization, price setting, and investment adjustment costs parameters is increased. But these modifications do not provide significantly different results.

It was mentioned in *section 4* that large nominal-exchange-rate devaluations reduced the size of the difference of sectoral inflation rates since in this case tradable inflation rose more than non-tradable inflation. This reveals that in the considered countries, where the non-tradable inflation rate is significantly higher, nominal-exchange-rate movements cannot cause the observed dual inflation. One could explain it if in the transition periods nominal *revaluations* had been typical. But these events occurred only in the last couple of years, because potential EMU accession countries started a more radical disinflation policy.[27]

To summarize this section: In addition to the BS effect, deregulation can yield large size dual inflation. But large price liberalizations occurred only in the beginning of post-communist transition. Thus, they cannot provide a general explanation. Large nominal devaluations influence the inflationary difference, but just reduce it. Thus, some auxiliary factors can help explaining emerging market dual inflation, but without long run asymmetric productivity growth these factors cannot by themselves cause the empirically observable phenomena.

8 Conclusions

This paper has reviewed how the models of the new open economy macroeconomics (NOEM) can explain the permanent dual inflation and the accompanying real appreciation often observed in emerging markets.

The coexistence of dual inflation and real appreciation is usually explained by the Balassa - Samuelson (BS) effect, i.e. by the faster productivity growth in the tradable sector. Traditionally, the BS effect is derived from models with flexible prices and internationally homogenous tradable goods markets. On the other hand, NOEM models assume sticky prices and/or wages and heterogeneous goods markets. The traditional approach focuses on the determinants of the internal real exchange rate, while NOEM emphasize the importance of the external real exchange rate.

It was shown that a NOEM model can simultaneously guarantee the strong correlation of nominal and real exchange rates and generate the BS effect only if there is pricing to market in the model. This result is independent of the assumption on price setting currency. It remains valid both with local currency pricing (LCP) and producer currency pricing (PCP).

As a consequence of the latter statement, I do not want to take sides in LCP vs. PCP debate since both approaches can be consistent with the BS effect. Although in most cases PCP is applied without PTM, my results point out that if one wants to capture the particularities of emerging markets, then the PCP approach cannot be applied without the assumption of international price discrimination.

[27]The benchmark economies of my model with symmetric sectoral parameters cannot explain the connection between nominal-exchange-rate movements and dual inflation. However, the model of Burnstein et al. (2002) is able to explain this phenomenon. In their argument it is crucial that large devaluations are responses to the tightenings of foreign credit constraints, and tightened credit constraints imply that households increase the consumption of lower quality local tradables.

The study also looks at how the presence of nominal rigidities and investments adjustment costs modify the effects of asymmetric productivity growth on dual inflation. It was demonstrated that the presence of these two factors enable demand to directly influence the price setting behavior of firms, which weakens the link between asymmetric productivity growth and dual inflation.

Alternative explanations for dual inflation were considered as well. Price deregulation in the tradable sector can significantly contribute to differences in sectoral inflation rates. But in general these alternative factors can only provide auxiliary explanations for dual inflation and real appreciation, in addition to asymmetric sectoral productivity growth.

A Appendix

A.1 The steady state

In this section the non-stochastic steady state of the model is described. Variables without time indices refer to their steady-state values.

In the steady state there is no difference between the three model versions since $eP^x = P^{x*}$, the technologies of the tradable and the exports sector are the same, and in the steady state nominal rigidities do not exist. Hence, in this section it is sufficient to discuss version A: thus index T will refer both on local and exported tradables. In the steady state there is no intra-household and intra-sector heterogeneity. Therefore the index j of the households and the index i of the firms are omitted to simplify the notations.

It is assumed that $P = P^T = P^N = 1$. Then equations (6) and (25) imply that

$$c^T = a_T c, \quad c^N = a_N c, \quad \mathcal{I}^{TT} + \mathcal{I}^{TN} = a_T I^T, \quad \mathcal{I}^{NT} + \mathcal{I}^{NN} = a_T I^N. \tag{50}$$

Furthermore, it is assumed that $Px = eP^{m*}m$. Hence,

$$GDP = a^T \left(c^T + \mathcal{I}^{TT} + \mathcal{I}^{TN}\right) + a^N \left(c^N + \mathcal{I}^{NT} + \mathcal{I}^{NN}\right) = c + I.$$

Since $\Phi_s(I^s/k^s) = I^s/k^s$ and $\Phi_s'(I^s/k^s) = 1$, in the steady state investments do not have adjustment costs, and as was mentioned, in the steady state nominal rigidities do not exist. Hence, firms' optimization problem will be the same as in the case when there is a rental market for physical capital and the real rental rate of capital is determined by the real interest and the depreciation rate. Equation (7) implies that the real interest rate is equal to $1/\beta - 1$. If the real rental rate of physical capital, which is uniform in all sectors, is denoted by r, then

$$r = \frac{1}{\beta} - 1 + \delta.$$

This formula is a special case of equation (20). I set the values of τ^T and τ^N such a way that the markups

$$1 = \tau_t^s \frac{\theta}{\theta - 1}, \;\; s = T,\, N.$$

Then it is true for all sectors that the marginal product of capital is equal to r. Thus, equation (15) implies that

$$\varkappa = \left(\frac{r}{\alpha}\right)^{\frac{1}{1-\alpha}},$$

where $\varkappa = z^T/k^T = z^N/k^N$. Furthermore, equations (15), (28), and (30) imply that

$$c^T + I^T + x = k^T \varkappa^{1-\alpha}, \quad c^N + I^N = k^N \varkappa^{1-\alpha}. \tag{51}$$

Beyond this, in the steady-state equation (18) takes the form $I^s = \delta k^s$. Thus, if one defines the $k = k^T + k^N$ aggregate capital stock, then $I = \delta k$.

It is assumed that $w = W = eP^{m*}$. Then equation (17) implies that $w^{z,s} = w$ in both sectors. Since in each sector $w^{z,s}$ is equal to the marginal product of z^s

$$w = (1-\alpha)\varkappa^{-\alpha}.$$

In the benchmark economy $w = 1.212$. Let us denote the exogenous exports/GDP fraction by s_x, and I set $s_x = 0.6$. Since $x = eP^{m*}m$,

$$s_x = \frac{x}{c+I} = \frac{eP^{m*}m}{c+I}. \tag{52}$$

Imports demand equation (23) implies that

$$m = (1-n_T)z^T + (1+n_N)z^N.$$

It is assumed $1 - n_N = \upsilon(1-n_T)$, where υ is an exogenous parameter (In the benchmark economy $\upsilon = 1$). Then one can show that

$$m = (1-n_T)\varkappa\left(k^T + \upsilon k^N\right) = (1-n_T)\mathsf{N}\varkappa k, \tag{53}$$

where

$$\mathsf{N} = \frac{a_T + s_x + \upsilon a_N}{1+s_x}$$

since equations (50), (51), and (52) imply that $k^T/k = (a_T + s_x)/(1+s_x)$ and $k^N/k = a_N/(1+s_x)$.

Using the formula $I = \delta k$, the previous expression for m, and equation (52) yields

$$c = \mathsf{K}k, \tag{54}$$

where

$$\mathsf{K} = eP^{m*}(1-n_T)\mathsf{N}\varkappa s_x^{-1} - \delta.$$

By equation (51) one can similarly show that

$$k\varkappa^{1-\alpha} = c + \delta k + x = \left(eP^{m*}(1-n_T)\mathsf{N}\varkappa s_x^{-1} - \delta\right)k + \delta k + eP^{m*}(1-n_T)\mathsf{N}\varkappa k.$$

This implies that

$$n_T = 1 - \frac{\varkappa^{1-\alpha}}{eP^{m*}\mathsf{N}\varkappa\left(1+s_m^{-1}\right)}, \quad n_N = 1 - \upsilon(1-n_T).$$

In the benchmark economy $n_T = n_N = 0.526$.

In the steady state the labor supply function of the households takes the form

$$\tau^w \frac{\theta_w}{\theta_w - 1} mrs = w.$$

It is assume that τ^w is chosen such that the markup $\tau^w \theta_w / (\theta_w - 1) = 1$. Hence,

$$mrs = [(1-h)c]^\sigma l^\varphi = w. \tag{55}$$

As for imports, one can derive a similar expression for labor:

$$l = \frac{n_T(a_T + s_x) + n_N a_N}{1 + s_x} \varkappa k. \tag{56}$$

Substituting equations (54) and (56) into equation (55) yields and expression for the capital stock:

$$k = \left\{ w \left[(1-h)\mathsf{K}\right]^{-\sigma} \left[\frac{(a_T + s_x)n_T + a_N n_N}{1 + s_x} \varkappa \right]^{-\varphi} \right\}^{\frac{1}{\sigma+\varphi}}.$$

Using this expression one can calculate the steady-state value of the capital stock and investments. In the benchmark economy $k = 21.008$, and $I = \delta k = 0.525$. Then using formula (54) yields the value of consumption, $c = 2.076$, and equation (56) provides the value of labor, $l = 1.43$. Furthermore, one can calculate $rk/(c + I)$, that is, the capital's share in GDP. The value of α is set in such a way that capital's share is equal to 1/3.

A.2 Wage and price setting

Sticky wages

As discussed in *section 3.1*, if household j resets its wage in a rational forward-looking manner at date T, then maximizes

$$\sum_{t=T}^{\infty} (\beta\gamma_w)^{t-T} \mathrm{E}_T \left[\mathcal{U}\left(H_t, l_t(j)\right)\right]$$

with respect to $W_T(j)$ subject to the constraints (4), (11), and (10). It is useful to substitute the constraints (11) and (10) into the budget constraint (4).

To derive the optimal solution it is necessary to utilize the information related to the optimal consumption path. The first-order condition with respect to the aggregate consumption in states of the world relevant for wage setting is

$$(\beta\gamma_w)^{t-T}\frac{u'(c_t)}{P_t} = \kappa_t,$$

where κ_t is the Lagrangian multiplier of the budget constraint. The first-order condition with respect to $W_T(j)$ is

$$\sum_{t=T}^{\infty}(\gamma_w\beta)^{T-t}\mathrm{E}_T\left[b_t(j)\left\{v'(l_t(j))\theta_w W_T(j)^{-1} + \kappa_t\frac{(1-\theta_w)}{\tau_t^w}\bar{P}_{t-1}\right\}\right] = 0,$$

where $\bar{P}_{t-1} = P_{t-1}^{\theta_w}P_{T-1}^{-\theta_w}$ and $b_t(j) = l_t W_t^{\theta_w} W_T(j)^{-\theta_w}\bar{P}_{t-1}^{-\theta_w}$. Substitute the first-order condition for consumption into the previous expression, then

$$\sum_{t=T}^{\infty}(\gamma_w\beta)^{t-T}\mathrm{E}_T\left[b_t(j)\left\{[l_t(j)]^{\varphi}\theta_w W_T(j)^{-1} + \frac{c_t^{-\sigma}}{\tau_t^w P_t}(1-\theta_w)\bar{P}_{t-1}\right\}\right] = 0.$$

Log-linearizing and rearranging it yields

$$\frac{\widetilde{W}_T(j)}{1-\beta\gamma_w} = \sum_{t=T}^{\infty}(\beta\gamma_w)^{t-T}\mathrm{E}_T\left[\widetilde{W}_t + \widetilde{mrs}_t(j) - \widetilde{w}_t + \bar{\tau}^w\tilde{\tau}_t^w - \vartheta_w\left(\widetilde{P}_{t-1} - \widetilde{P}_{T-1}\right)\right], \tag{57}$$

where $\bar{\tau}^w = \tau^w/(1-\tau^w)$, τ^w is the steady-state value of τ_t^w, and the tilde denotes the log-deviation of a variable from its steady-state values.

Since each households' consumption is the same, the individual and the average marginal rate of substitution are related in the following manner:

$$\widetilde{mrs}_t(j) = \widetilde{mrs}_t + \varphi\left[\tilde{l}_t(j) - \tilde{l}_t\right].$$

Substitute the demand function (11) into the previous equation, then

$$\widetilde{mrs}_t(j) = \widetilde{mrs}_t - \theta_w\varphi\left[\widetilde{w}_t(j) - \widetilde{w}_t\right].$$

Substitute the log-linearized version of equation (10) into the above expression, then

$$\widetilde{mrs}_t(j) = \widetilde{mrs}_t - \theta_w\varphi\left[\widetilde{w}_T(j) - \widetilde{w}_t - \vartheta_w\left(\widetilde{P}_{T-1} - \widetilde{P}_{t-1}\right)\right].$$

Combining this equation with the formula (57) yields

$$\frac{\widetilde{W}_T(j)}{1-\beta\gamma_w} = \sum_{t=T}^{\infty}(\beta\gamma_w)^{t-T}\mathrm{E}_T\left[\widetilde{W}_t + \frac{\widetilde{mrs}_t - \widetilde{w}_t + \bar{\tau}^w\tilde{\tau}_t^w}{1+\theta_w\varphi} - \vartheta_w\left(\widetilde{P}_{t-1} - \widetilde{P}_{T-1}\right)\right]. \tag{58}$$

Since on the right hand side there is no term depending on j, $\widetilde{W}_T(j)$ will be uniform, and denote this common value by $\widetilde{W}_T^{new}$. Let us define the following variables:

$$X_t = \widetilde{W}_t - \vartheta_w \widetilde{P}_{t-1}, \qquad \bar{X}_t = \widetilde{W}_t^{new} - \vartheta_w \widetilde{P}_{t-1}.$$

Then using the new variables one can rewrite equation (58) as

$$\bar{X}_T = (1-\beta\gamma_w)\sum_{t=T}^{\infty}(\beta\gamma_w)^{t-T}\mathrm{E}_T\left[X_t + \frac{\widetilde{mrs}_t - \widetilde{w}_t + \bar{\tau}^w\tilde{\tau}_t^w}{1+\theta_w\varphi}\right].$$

This implies that

$$\bar{X}_T - \beta\gamma_w \mathrm{E}_T\left[\bar{X}_{T+1}\right] = (1-\beta\gamma_w)\left(X_T + \frac{\widetilde{mrs}_T - \widetilde{w}_T + \bar{\tau}^w\tilde{\tau}_T^w}{1+\theta_w\varphi}\right). \tag{59}$$

Using formula (12) one can describe the evolution of the aggregate wage index:

$$W_T^{1-\theta_w} = \gamma_w\left[W_{T-1}\left(\frac{P_{T-1}}{P_{T-2}}\right)^{\vartheta_w}\right]^{1-\theta_w} + (1-\gamma_w)(W_T^{new})^{1-\theta_w}.$$

Log-linearizing it yields

$$\widetilde{W}_T = \gamma_w\widetilde{W}_{T-1} + \gamma_w\vartheta_w\left(\widetilde{P}_{T-1} - \widetilde{P}_{T-2}\right) + (1-\gamma_w)\widetilde{W}_T^{new}.$$

This implies that

$$X_T = \gamma_w X_{T-1} + (1-\gamma_w)\bar{X}_T. \tag{60}$$

Express $\bar{X}_T$ and $\mathrm{E}_T\left[\bar{X}_{T+1}\right]$ by (60), and substitute it into equation (59). Some manipulations yield

$$dX_T = \beta\mathrm{E}_t\left[dX_{T+1}\right] + \xi_w\left(\widetilde{mrs}_T - \widetilde{w}_T + \frac{\tau^w}{1-\tau^w}\tilde{\tau}_T^w\right), \tag{61}$$

where $dX_T = \pi_T^W - \vartheta_w\pi_{T-1}$, $\pi_T^w = \widetilde{W}_T - \widetilde{W}_{T-1}$ is wage inflation, π_T is CPI inflation, and

$$\xi_w = \frac{(1-\gamma_w)(1-\gamma_w\beta)}{\gamma_w\left(1+\theta_w\varphi\right)}.$$

Sticky prices

As shown in *section 3.2*, if firm i of sector s resets its price in a rational forward-looking way at date T, then maximizes the expected profit function

$$\mathrm{E}_T\left[\gamma_s^{t-T}D_{T,t}\left\{\frac{P_T^s(i)}{\tau_t^s}\left(\frac{P_{t-1}^s}{P_{T-1}^s}\right)^{\vartheta_s}-MC_t^s(i)\right\}\right]$$

with respect to $P_T^s(i)$ and $y_t^s(i)$ subject to the constraint (14). Let us substitute the constraint into the objective function. Then one can get the following log-linearized first-order condition:

$$\sum_{t=T}^{\infty}(\beta\gamma_s)^{t-T}\mathrm{E}_T\left[(X_T^s(i)-X_t^s)-\widetilde{mc}_t^s(i)+\bar{\tau}^s\tilde{\tau}_t^s\right]=0,$$

again the tilde denotes the log-deviation of a variable from its steady state value, $X_T^s(i)=\widetilde{P}_T^s(i)-\vartheta_s\widetilde{P}_{T-1}^s$, $X_t^s=\widetilde{P}_t^s-\vartheta_s\widetilde{P}_{t-1}^s$ and $\widetilde{mc}_t^s(i)=\widetilde{MC}_t^s(i)-\widetilde{P}_t^s$, which represents the individual real marginal cost Furthermore, $\bar{\tau}^s=\tau^s/(1-\tau^s)$, where τ^s is the steady-state value of τ_t^s. Since the real input price $\widetilde{w}_t^{z,s}$ and the productivity factor is the same for all firms in sector s, the individual and the average real marginal cost is related in the following way:

$$\widetilde{mc}_t^s(i)=\widetilde{mc}_t^s+\hat{\alpha}\left(\tilde{y}_t^s(i)-\tilde{y}_t^s\right)-\hat{\alpha}\hat{k}_t^s(i),$$

where $\hat{k}_t^s(i)=\tilde{k}_t^s(i)-\tilde{k}_t^s$. Substitute the log-linearized version of the demand function (14) into the above expression, then

$$\widetilde{mc}_t^s(i)=\widetilde{mc}_t^s-\theta\hat{\alpha}\left(\widetilde{P}_t^s(i)-\widetilde{P}_t^s\right)-\hat{\alpha}\hat{k}_t^s(i).$$

Implicit price indexation implies that $\widetilde{P}_t^s(i)=\widetilde{P}_T^s(i)+\vartheta_s\left(\widetilde{P}_{t-1}^s-\widetilde{P}_{T-1}^s\right)$. Thus,

$$\widetilde{mc}_t^s(i)=\widetilde{mc}_t^s-\theta\hat{\alpha}\left(X_T^s(i)-X_t^s\right)-\hat{\alpha}\hat{k}_t^s(i).$$

Substituting the above formula into the first-order condition yields

$$\sum_{t=T}^{\infty}(\beta\gamma_s)^{t-T}\mathrm{E}_T\left[(1+\theta\hat{\alpha})\left(X_T^s(i)-X_t^s\right)-\widetilde{mc}_t^s+\bar{\tau}^s\tilde{\tau}_t^s+\hat{\alpha}\hat{k}_t^s(i)\right]=0. \quad (62)$$

Let us eliminate the terms containing $\hat{k}_t^s(i)$ from this expression. In *section 3.2* it was shown that

$$\varepsilon_s\left(\hat{k}_{t+1}^s(i)-\hat{k}_t^s(i)\right)=\beta\varepsilon_s\mathrm{E}_t\left[\hat{k}_{t+2}^s(i)-\hat{k}_{t+1}^s(i)\right]$$
$$+[1-\beta(1-\delta)]\hat{\alpha}\mathrm{E}_t\left[\tilde{y}_{t+1}^s(i)-\tilde{y}_{t+1}^s-\hat{k}_{t+1}^s(i)\right].$$

Substituting equation (14) and the price indexation formula into this equation yields the difference equation

$$\beta^{-1}\Theta\left(X_T^s(i) - X_{t+1}^s\right) =$$
$$\hat{k}_{t+2}^s(i) - \beta^{-1}[1+\beta+(1-\beta(1-\delta)\hat{\alpha}(\varepsilon_s)^{-1})]\hat{k}_{t+1}(i) + \beta^{-1}\hat{k}_t(i),$$

where

$$\Theta = \frac{1-\beta(1-\delta)\hat{\alpha}\theta}{\varepsilon_s} > 0.$$

The roots of the lag polynomial defined by the difference equation is λ_{s1} λ_{s2}. It is easy to show that $0 < \lambda_{s1} < 1 < \lambda_{s2}$. The closed form solution of the difference equation can be expressed by the two roots:

$$\hat{k}_{t+1}^s(i) = \lambda_{s1}\hat{k}_t^s - \beta^{-1}\Theta\sum_{l=1}^{\infty}\lambda_{s2}^{-1}\left(X_T^s(i) - X_{t+l}^s\right).$$

This implies that

$$\sum_{t=T}^{\infty}(\beta\gamma_s)^{t-T}\mathrm{E}_T\left[\hat{k}_t(i)\right] = \frac{1}{1-\gamma_s\beta\lambda_{s1}}\hat{k}_T(i)$$
$$-\Omega\left[\sum_{t=T}^{\infty}\lambda_{s2}^{-(t-T)}\mathrm{E}_T\left[X_T^s(i) - X_t^s\right] - \sum_{t=T}^{\infty}(\beta\gamma_s)\,\mathrm{E}_T\left[X_T^s(i) - X_t^s\right]\right], \quad (63)$$

where

$$\Omega = \frac{\beta\gamma_s}{1-\gamma_s\beta\lambda_{s1}}\frac{\Theta}{\beta(1-\gamma_s\beta\lambda_{s2})}.$$

Let us substitute equation (63) into equation (62). Since $\hat{k}_T(i) = 0$, $X_T^s(i)$ depends only on aggregate variables. Hence, it is the same for all i. Denote this common variable by $\bar{X}_T^s$. Then

$$\begin{aligned}(\mathsf{a}-\mathsf{b})\left(\bar{X}_T^s - X_T^s\right) &= \sum_{t=T}^{\infty}(\beta\gamma_s)^{t-T}\mathrm{E}_T\left[\widetilde{mc}_t^s + \bar{\tau}^s\tilde{\tau}_t^s\right] + \mathsf{a}\sum_{t=T}^{\infty}(\beta\gamma_s)^{t-T}\mathrm{E}_T\left[dX_t^s\right] \\ &- \mathsf{b}\sum_{t=T}^{\infty}(\lambda_{s2})^{-(t-T)}\mathrm{E}_T\left[dX_t^s\right],\end{aligned}$$

where $dX_t^s = X_t^s - X_{t-1}^s$, and

$$\begin{aligned}\mathsf{a} &= \hat{\alpha}\frac{\gamma_s}{1-\beta\gamma_s}\frac{\Theta}{(1-\beta\gamma_s\lambda_{s1})(1-\beta\gamma_s\lambda_{s2})} + \frac{1+\theta\hat{\alpha}}{1-\beta\gamma_s} > 0, \\ \mathsf{b} &= \hat{\alpha}\frac{\gamma_s}{1-\lambda_{s2}^{-1}}\frac{\Theta}{(1-\beta\gamma_s\lambda_{s1})(1-\beta\gamma_s\lambda_{s2})} > 0.\end{aligned}$$

Quasi-difference the above relation, then

$$
\begin{aligned}
&(\mathsf{a}-\mathsf{b})\mathrm{E}_T\left[\left(1-\beta\gamma_s L^{-1}\right)\left(1-\lambda_{s2}^{-1}L^{-1}\right)\left(\bar{X}_T^s - X_T^s\right)\right] = \\
&\mathsf{a}\beta\gamma_s\mathrm{E}_T\left[\left(1-\lambda_{s2}^{-1}L^{-1}\right)dX_{T+1}^s\right] - \mathsf{b}\beta\lambda_{s2}^{-1}\mathrm{E}_T\left[\left(1-\beta\gamma_s L^{-1}\right)dX_{T+1}^s\right] \\
&+\mathrm{E}_T\left[\left(1-\lambda_{s2}^{-1}L^{-1}\right)\widetilde{mc}_T + \bar{\tau}^s\tilde{\tau}_T^s\right], \qquad (64)
\end{aligned}
$$

where L is the lag-operator. Equation (13) implies that the log-linearized sectoral price indices follow the process

$$X_T^s = \gamma_s X_{T-1}^s + (1-\gamma_s)\bar{X}_T^s.$$

This implies that

$$\bar{X}_T^s - X_T^s = \frac{\gamma_s}{1-\gamma_s}dX_T^s.$$

Substitute this expression into equation (64). Then some manipulations yield

$$
\begin{aligned}
dX_T^s &= \psi_s^1\mathrm{E}_T\left[dX_{T+1}^s\right] - \psi_s^2\mathrm{E}_T\left[dX_{T+2}^s\right] \qquad (65)\\
&+ \xi_s^0\left(\widetilde{mc}_T^s + \frac{\tau^s}{1-\tau^s}\tilde{\tau}_T^s\right) - \xi_s^1\mathrm{E}_T\left[\widetilde{mc}_{T+1}^s + \frac{\tau^s}{1-\tau^s}\tilde{\tau}_{T+1}^s\right],
\end{aligned}
$$

where $dX_t^s = \bar{\pi}_t^s = \pi_t^s - \vartheta_s\pi_{t-1}^s$, and $\pi_t^s = \widetilde{P}_t^s - \widetilde{P}_{t-1}^s$ is the sectoral inflation rate. Furthermore,

$$\xi_s^0 = \frac{1-\gamma_s}{\gamma_s}\frac{1}{\mathsf{a}-\mathsf{b}}, \qquad \xi_s^1 = \frac{\xi_s^0}{\lambda_{s2}},$$

and

$$\psi_s^1 = \frac{\mathsf{a}\left(\beta+\lambda_{s2}^{-1}\right) - \mathsf{b}\left(\beta\gamma_s + \gamma_s^{-1}\lambda_{s2}^{-1}\right)}{\mathsf{a}-\mathsf{b}}, \qquad \psi_s^2 = \frac{\beta}{\lambda_{s2}}.$$

Some special cases

Let us consider some special cases. Assume that the adjustment costs for investments are infinitely large, that is, $\varepsilon_s = \infty$. Then capital the stock is fixed, and z_t is the only variable input. Thus, technology exhibits decreasing returns to scale. Then $\lambda_{s1} = 1$, $\lambda_{s2} = 1/\beta$ and $\Theta = 0$. Hence,

$$\mathsf{a} = 1 + \frac{\theta\hat{\alpha}}{1-\beta\gamma_s}, \qquad \mathsf{b} = 0.$$

This implies that

$$\xi_s^0 = \frac{(1-\gamma_s)(1-\beta\gamma_s)}{\gamma_s(1+\theta\hat{\alpha})}, \qquad \xi_s^1 = \beta\xi_s^0,$$

furthermore $\psi_s^1 = 2\beta$ and $\psi_s^2 = \beta^2$. Then the price setting equation (65) takes the form

$$\mathrm{E}_T\left[\left(1-\beta L^{-1}\right)\bar{\pi}_T^s\right] = \\ \xi_s^0 \mathrm{E}_T\left[\left(1-\beta L^{-1}\right)\left(\widetilde{mc}_T + \frac{\tau^s}{1-\tau^s}\tilde{\tau}_T^s\right)\right] + \beta \mathrm{E}_T\left[\left(1-\beta L^{-1}\right)\bar{\pi}_{T+1}^s\right].$$

The bounded solution of the above difference equation is equivalent to that of the following simpler difference equation:

$$\mathrm{E}_T\left[\bar{\pi}_T^s\right] = \xi_s^0\left(\widetilde{mc}_T + \frac{\tau^s}{1-\tau^s}\tilde{\tau}_T^s\right) + \beta \mathrm{E}_T\left[\bar{\pi}_{T+1}^s\right]. \tag{66}$$

Thus, in this special case the pricing formula coincides with the standard Calvo formula.

Let us consider the case when price setting is flexible, that is, $\gamma_s = 0$, but investment is still firm specific. Then the price setting equation and the particular Cobb-Douglas form of the technology defined by equation (15) imply that

$$1 = \tau_t^s \frac{\theta}{\theta-1} mc_t^s = \frac{\mu^s}{1-\alpha_t} w_t^{z,s}\left(\frac{y_t^s}{k_t^s}\right)^{\frac{\alpha}{1-\alpha}} (A_t^s)^{\frac{-1}{1-\alpha}},$$

for all firms in sector s. (For $s = x$, the $e_t = 1$ normalization is applied. If prices are flexible this can be done without loss of generality.) This implies that

$$\tau_t^s \frac{\theta}{\theta-1} w_t^{z,s} = A_t^s(1-\alpha)\left(\frac{k_t^s}{z_t^s}\right)^{\alpha},$$

that is, the real price of z_t^s multiplied by the markup is just equal to the marginal product of z_t^s. Then equation (21) takes the form

$$r_t^s = \frac{\alpha}{1-\alpha} w_t^{z,s} \frac{z_t^s}{k_t^s}.$$

Substitute the expression for $w_t^{z,s}$ into the above formula, then

$$\tau_t^s \frac{\theta}{\theta-1} r_t^s = A_t^s \alpha \left(\frac{z_t^s}{k_t^s}\right)^{1-\alpha},$$

which is just the marginal product of capital. Thus, when prices are flexible the model behaves as if there were a separate rental market for physical capital in each sector, and the sectoral real rental rate were r_t^s (the sectoral rental rates would not necessarily equalized because of the adjustment costs). Furthermore, one can show that expression

$$mc_t^s = \frac{(r_t^s)^\alpha \left(w_t^{z,s}\right)^{1-\alpha}}{A_t^s \alpha^\alpha (1-\alpha)^{1-\alpha}} \tag{67}$$

determines the real marginal cost of all the firms in sector s. Thus, firms follow the standard constant-returns-to-scale price setting practice.

Now let us consider the case, when prices are sticky, and physical capital is not firm specific, but in each sector there exists a separate rental market for physical capital. Then the real marginal cost of firms is described by formula (67). On the other hand, price setting is determined by equation (66), but the coefficient of the real marginal cost will be different:

$$\xi_s^0 = \frac{(1-\gamma_s)(1-\beta\gamma_s)}{\gamma_s}.$$

Assume that in equation (66) $\xi_T^0 = \xi_N^0 = \xi^0$ and $\vartheta_T = \vartheta_N = \vartheta$. Then the difference of sectoral inflation rates is determined by

$$\bar{\pi}_t^R = \xi_s^0 \widetilde{mc}_t^R + \beta \mathrm{E}_t \left[\bar{\pi}_{t+1}^R\right],$$

where $\bar{\pi}^R = \bar{\pi}_t^N - \bar{\pi}_t^T$, and equation (67) and the definition of $\widetilde{w}_t^{z,s}$ implies that the $\widetilde{mc}_t^R = \widetilde{mc}_t^T - \widetilde{mc}_t^N$ relative real marginal cost is

$$\begin{aligned} \widetilde{mc}_t^R &= \widetilde{A}_t^T - \widetilde{A}_t^N + \alpha\left(\tilde{r}_t^T - \tilde{r}_t^N\right) + (1-\alpha)\left(n_N - n_T\right)\widetilde{w}_t \\ &+ (1-\alpha)\left(n_T - n_N\right)\left(\widetilde{P}_t^{m*} + \tilde{q}_t\right) - \widetilde{P}_t^R. \end{aligned}$$

In the models of the traditional approach, there is an economy-wide rental market for physical capital, hence $\tilde{r}_t^T = \tilde{r}^N - t$. Furthermore, prices are flexible, hence the real marginal cost is constant, that is, $\widetilde{mc}_t^s = 0$. This implies that in each sector

$$\widetilde{P}_t^{s*} = \alpha \widetilde{R}_t^* + (1-\alpha) n_s \widetilde{W}_t^* + (1-\alpha) n_s \widetilde{P}_t^{m*} - \widetilde{A}_t^s, \tag{68}$$

where $\widetilde{R}_t^*$ is the nominal rental rate of capital and $\widetilde{W}_t^*$ is the nominal wage, both are denominated in foreign currency terms. Using the above formula one can derive an expression for the sectoral relative price:

$$\begin{aligned} \widetilde{P}_t^R &= \widetilde{A}_t^T - \widetilde{A}_t^N + \alpha\left(\widetilde{R}_t^{T*} - \widetilde{R}_t^{N*}\right) \\ &+ (1-\alpha)\left(n_N - n_T\right)\widetilde{W}_t^* + (1-\alpha)\left(n_T - n_N\right)\widetilde{P}_t^{m*}. \end{aligned} \tag{69}$$

This becomes even simpler if other assumptions of the traditional approach are used: First, sector T is internationally homogenous, thus the PPP is valid. Second, physical capital is formed by the goods of sector T without adjustment costs, and the financial markets are internationally homogenous. These imply that $\widetilde{R}_t^*$ will be determined by the world real interest rate. For simplicity, assume that foreign prices and interest rates are fixed. Then $\widetilde{P}_t^{T*} = \widetilde{R}_t^* = \widetilde{P}_t^{m*} = 0$. Substituting them into equation (68) ($s = T$) yields

$$\widetilde{W}_t^* = \frac{\widetilde{A}_t^T}{(1-\alpha)n_T}.$$

Substitute the above formula into expression (69), then

$$\widetilde{P}_t^R = \frac{n_N}{n_T}\widetilde{A}_t^T - \widetilde{A}_t^N.$$

In this case, the sectoral relative price is not influenced by demand, neither directly, nor indirectly. Only technological factors matter.

A.3 The complete log-linearized model

To solve the model described in *section 3* its log-linear approximation around the steady state is taken. In this section the log-linearized version is described. Variables without time indices refer to their steady-state values, and the tilde denotes the log-deviation of a variable from its steady-state value.

The log-linearization of the price index formula (5) yields

$$\widetilde{P}_t = a_T\widetilde{P}_t^T + a_N\widetilde{P}_t^N, \tag{70}$$

where I used the assumption that $P = P^T = P^N$, and that

$$a_T\widetilde{\chi}_t^T + a_N\widetilde{\chi}_t^N = 0,$$

which is a consequence of equation (3).

The log-linearized versions of the real exchange rate indices in equation (32), and the assumption that P_t^{F*}, P_t^{FT*} and P_t^{FR} are constant are used for the derivation of the following formulas:

$$\pi_t^T = d\tilde{e}_t - \left(\tilde{q}_t^T - \tilde{q}_{t-1}^T\right), \tag{71}$$

$$\pi_t^N = \pi_t^T + \widetilde{P}_t^R - \widetilde{P}_{t-1}^R, \tag{72}$$

$$\pi_t^{x*} = \widetilde{P}_t^{x*} - \widetilde{P}_{t-1}^{x*}, \tag{73}$$

$$\pi_t^x = d\tilde{e}_t + \widetilde{P}_t^{x*} - \widetilde{P}_{t-1}^{x*}, \tag{74}$$

$$\tilde{q}_t = \tilde{q}_t^T - a_N\widetilde{P}_t^R, \tag{75}$$

where $d\tilde{e}_t = \tilde{e}_t - \tilde{e}_{t-1}$ is the depreciation rate of the nominal exchange rate.

Log-linearizing equation (17), and using the assumption that $W = eP^{m*}$ yields

$$\begin{aligned} \widetilde{w}^{z,T} &= n_T \widetilde{w}_t + (1 - n_T) \left(\widetilde{P}_t^{m*} + \tilde{q}_t \right), & (76) \\ \widetilde{w}^{z,N} &= n_N \widetilde{w}_t + (1 - n_N) \left(\widetilde{P}_t^{m*} + \tilde{q}_t \right). & (77) \end{aligned}$$

It is assumed that $n_x = n_T$, hence it is not necessary to have a separate equation for the exports sector.

Using the log-linearized version of equations (3), (6), (25), (32), and using equation (70) one can obtain the following expressions:

$$\begin{aligned} \tilde{c}_t^T &= \eta a_N \widetilde{P}_t^R - \frac{a_T}{a_N} \widetilde{\chi}_t^N + \tilde{c}_t, \\ \widetilde{\mathcal{I}}_t^{Ts}(i) &= \eta a_N \widetilde{P}_t^R - \frac{a_T}{a_N} \widetilde{\chi}_t^N + \tilde{I}_t^s(i). \end{aligned}$$

Let us define $I_t^s = \int_0^1 I_t^s(i)\, di$. Then one can show[28] that

$$\widetilde{\mathcal{I}}_t^{Ts} = \eta a_N \widetilde{P}_t^R - \frac{a_T}{a_N} \widetilde{\chi}_t^N + \tilde{I}_t^s.$$

The above formulas imply that the log-linearized version of the equilibrium condition (28) takes the form

$$\tilde{y}_t^T = \mathsf{x}\tilde{x}_t + \mathsf{c}\tilde{c}_t + \mathsf{I}\tilde{I}_t + (\mathsf{c} + \mathsf{I}) \left(\eta a_N \widetilde{P}_t^R - \frac{a_T}{a_N} \widetilde{\chi}_t^N \right), \tag{78}$$

where $I_t = \sum_s I_t^s$, and equation (50) implies that

$$\begin{aligned} \mathsf{c} &= \frac{c}{GDP + x} = \frac{a_T c (c + I)^{-1}}{a_T + s_x}, \quad \mathsf{I} = \frac{I}{GDP + x} = \frac{a_T I (c + I)^{-1}}{a_T + s_x}, \\ \mathsf{x} &= \frac{x}{GDP + x} = \frac{s_x}{a_T + s_x}. \end{aligned}$$

Similarly, the log-linearized equilibrium condition (29) takes the form

$$\tilde{y}_t^T = \frac{c}{c + I} \tilde{c}_t + \frac{I}{c + I} \tilde{I}_t + \eta a_N \widetilde{P}_t^R - \frac{a_T}{a_N} \widetilde{\chi}_t^N. \tag{79}$$

[28]If a variable is defined in the following manner: $\mathfrak{z} = \int_0^1 \mathfrak{z}(i)\, di$ then its log-linear approximation yields $\tilde{\mathfrak{z}} = \int_0^1 \tilde{\mathfrak{z}}(i)\, di + o^2$, where o^2 denotes those second and higher order errors, which were neglected in the approximation process.

Finally the log-linear approximation of the equilibrium condition (30) is

$$\tilde{y}_t^N = \frac{c}{c+I}\tilde{c}_t + \frac{I}{c+I}\tilde{I}_t - \eta a_T \widetilde{P}_t^R + \widetilde{\chi}_t^N. \quad (80)$$

The log-linearization of equations (9) and (32) yields the expression which determines the trajectory of aggregate consumption:

$$\tilde{c}_t = h\tilde{c}_{t-1} + \frac{1-h}{\sigma}\tilde{q}_t + \tilde{c}_t^*. \quad (81)$$

Here $\tilde{c}_t^* = -(1-h)\widetilde{\Lambda^*}_t/\sigma$, which represents the shock of the foreign business cycle.

In version A of the model $\widetilde{P}_t^T - \tilde{e}_t = \widetilde{P}_t^{x*}$, hence the log-linearized version of the exports demand equation (27) is

$$\tilde{x}_t = \eta^* \tilde{q}_t^T + \tilde{x}_t^*, \quad (82)$$

where equation (32) was used. In versions B and C the log-linearized exports demand becomes

$$\tilde{x}_t = -\eta^* \widetilde{P}_t^{x*} + \tilde{x}_t^*. \quad (83)$$

Define the aggregate stock of physical capital in sector s as $k_t^s = \int_0^1 k_t^s(i)\,di$. Log-linearizing the investment equation (18) yields

$$\tilde{k}_{t+1}^s = (1-\delta)\tilde{k}_t^s + \delta\tilde{I}_t^s,$$

where the steady-state properties of Φ_s is used. As a consequence, the log-linearized equation for the aggregate investment is

$$\delta\tilde{I}_t = \sum_s \frac{I^s}{I}\left[\tilde{k}_{t+1}^s - (1-\delta)\tilde{k}_t^s\right], \quad (84)$$

where in version A $s = T, N$, in versions B and C $s = T, x, N$. Equations (50), (51), and formula $I^s = \delta k^s$ imply that in version A

$$\frac{I^T}{I} = \frac{a_T + s_x}{1+s_x}, \qquad \frac{I^N}{I} = \frac{a_N}{1+s_x}.$$

In versions B and C I^N/I is the same. The expressions for the tradable and the exports sector are

$$\frac{I^T}{I} = \frac{a_T}{1+s_x}, \qquad \frac{I^x}{I} = \frac{s_x}{1+s_x}.$$

Let us combine the log-linearized versions of equations (15), (22), (31), (32), and equations (76), (77). Then aggregating the result yields an expression for aggregate labor demand:

$$\tilde{l}_t = \sum_{s=H,x,N} \frac{l^s}{l} \left[(1-n_s)\rho_s \left(\widetilde{P}_t^{m*} + \tilde{q}_t - \widetilde{w}_t \right) + \bar{\alpha} \left(\tilde{y}_t^s - \widetilde{A}_t^s \right) - \hat{\alpha} \tilde{k}_t^s \right], \quad (85)$$

where, again, in version A $s = T, N$, and in versions B and C $s = T, x, N$. Furthermore, $\bar{\alpha} = 1/(1-\alpha)$ and $\hat{\alpha} = \alpha/(1-\alpha)$. It is assumed that in versions B and C the technology of the local tradable and the exports sector is the same, thus $\rho_T = \rho_x$ and $\widetilde{A}^T = \widetilde{A}^x$. In version A

$$\frac{l^T}{l} = \frac{n_T(a_T + s_x)}{\mathsf{n}}, \qquad \frac{l^N}{l} = \frac{n_N a_N}{\mathsf{n}},$$

where $\mathsf{n} = n_T(a_T + s_x) + n_N a_N$. l^N/l is the same in versions B and C. Finally, one can obtain

$$\frac{l^T}{l} = \frac{n_T a_T}{\mathsf{n}}, \qquad \frac{l^x}{l} = \frac{n_T s_x}{\mathsf{n}}.$$

Log-linearizing and combining equations (7), (19) and (20) results in

$$\begin{aligned} &\mathrm{E}_t \left[\widetilde{\Lambda}_{t+1} \right] - \widetilde{\Lambda}_t + \varepsilon_s \left(\tilde{k}_{t+1}^s(i) - \tilde{k}_t^s(i) \right) = \\ &\mathrm{E}_t \left[[1-\beta(1-\delta)] \tilde{r}_{t+1}^s + \beta \varepsilon_s \left(\tilde{k}_{t+2}^s(i) - \tilde{k}_{t+1}^s(i) \right) \right], \end{aligned}$$

where $\varepsilon_s = -\Phi_s''(\delta)\delta$. Log-linearizing and combining equations (15) and (21) yields

$$\tilde{r}_t^s(i) = \widetilde{w}_t^{z,s} + \bar{\alpha} \left(\tilde{y}_t^s(i) - \widetilde{A}_t^s - \tilde{k}_t^s(i) \right).$$

Combining the above two equations, aggregating the result, and using the definition of Λ_t results in the equation, which determines the evolution of physical capital in the tradable sector:

$$\begin{aligned} &\frac{\sigma h}{1-h} \tilde{c}_{t-1} - \frac{\sigma(1+h)}{1-h} \tilde{c}_t + \frac{\sigma}{1-h} \mathrm{E}_t \left[\tilde{c}_{t+1} \right] + \varepsilon_T \left(\tilde{k}_{t+1}^T - \tilde{k}_t^T \right) \quad (86) \\ &= \Delta \mathrm{E}_t \left[\widetilde{w}_{t+1}^{z,T} + \bar{\alpha} \left(\tilde{y}_{t+1}^T - \widetilde{A}_{t+1}^T - \tilde{k}_{t+1}^T \right) \right] + \beta \varepsilon_T \mathrm{E}_t \left[\tilde{k}_{t+2}^T - \tilde{k}_{t+1}^T \right], \end{aligned}$$

where $\Delta = [1-\beta(1-\delta)]$. The same equation for the non-tradable sector is

$$\begin{aligned} &\frac{\sigma h}{1-h} \tilde{c}_{t-1} - \frac{\sigma(1+h)}{1-h} \tilde{c}_t + \frac{\sigma}{1-h} \mathrm{E}_t \left[\tilde{c}_{t+1} \right] + \varepsilon_N \left(\tilde{k}_{t+1}^N - \tilde{k}_t^N \right) \quad (87) \\ &= \Delta \mathrm{E}_t \left[\widetilde{w}_{t+1}^{z,N} + \bar{\alpha} \left(\tilde{y}_{t+1}^N - \widetilde{A}_{t+1}^N - \tilde{k}_{t+1}^N \right) \right] + \beta \varepsilon_N \mathrm{E}_t \left[\tilde{k}_{t+2}^N - \tilde{k}_{t+1}^N \right]. \end{aligned}$$

Finally, for the exports sector it is

$$
\begin{aligned}
&\frac{\sigma h}{1-h}\tilde{c}_{t-1} - \frac{\sigma(1+h)}{1-h}\tilde{c}_t + \frac{\sigma}{1-h}\mathrm{E}_t\left[\tilde{c}_{t+1}\right] + \varepsilon_T\left(\tilde{k}^x_{t+1} - \tilde{k}^x_t\right) \\
&= \Delta\mathrm{E}_t\left[\widetilde{w}^{z,T}_{t+1} + \bar{\alpha}\left(\tilde{x}_{t+1} - \widetilde{A}^T_{t+1} - \tilde{k}^x_{t+1}\right)\right] + \beta\varepsilon_T\mathrm{E}_t\left[\tilde{k}^x_{t+2} - \tilde{k}^x_{t+1}\right],
\end{aligned} \tag{88}
$$

where the second equilibrium condition of equations (29) is used.

Equation (65) implies that the inflation rate in the tradable sector is determined by

$$
\bar{\pi}^T_t = \psi^1_T\mathrm{E}_t\left[\bar{\pi}^T_{t+1}\right] - \psi^2_T\mathrm{E}_t\left[\bar{\pi}^T_{t+2}\right] + \xi^0_T\widetilde{mc}^T_t - \xi^1_T\mathrm{E}_t\left[\widetilde{mc}^T_{t+1}\right], \tag{89}
$$

where I used the assumption that τ^T_t constant, hence $\tilde{\tau}^T_t = 0$. The log-linearized average real marginal cost is defined by

$$
mc^s_t = \frac{MC^s_t}{P^s_t},
$$

and by using equation (26) it can be expressed as

$$
\widetilde{mc}^T_t = \hat{\alpha}\left(\tilde{y}^T_t - \tilde{k}^T_t\right) - \bar{\alpha}\widetilde{A}^T_t + \widetilde{w}^{z,T}_t + a_N\widetilde{P}^R_t. \tag{90}
$$

Similarly, expression

$$
\begin{aligned}
\bar{\pi}^N_t &= \psi^1_N\mathrm{E}_t\left[\bar{\pi}^N_{t+1}\right] - \psi^2_N\mathrm{E}_t\left[\bar{\pi}^N_{t+2}\right] \\
&+ \xi^0_N\left(\widetilde{mc}^N_t + \frac{\tau^N}{1-\tau^N}\tilde{\tau}^N_t\right) - \xi^1_N\mathrm{E}_t\left[\widetilde{mc}^N_{t+1} + \frac{\tau^N}{1-\tau^N}\tilde{\tau}^N_{t+1}\right]
\end{aligned} \tag{91}
$$

determines the inflation rate in the non-tradable sector, and the formula for the real marginal cost is

$$
\widetilde{mc}^N_t = \hat{\alpha}\left(\tilde{y}^N_t - \tilde{k}^N_t\right) - \bar{\alpha}\widetilde{A}^N_t + \widetilde{w}^{z,N}_t - a_T\widetilde{P}^R_t. \tag{92}
$$

The equation for the inflation rate of the exports sector in version B can be derived as

$$
\bar{\pi}^{x*}_t = \psi^1_T\mathrm{E}_t\left[\bar{\pi}^{x*}_{t+1}\right] - \psi^2_T\mathrm{E}_t\left[\bar{\pi}^{x*}_{t+2}\right] + \xi^0_T\widetilde{mc}^x_t - \xi^1_T\mathrm{E}_t\left[\widetilde{mc}^x_{t+1}\right]. \tag{93}
$$

While in version C as

$$
\bar{\pi}^{x}_t = \psi^1_T\mathrm{E}_t\left[\bar{\pi}^{x}_{t+1}\right] - \psi^2_T\mathrm{E}_t\left[\bar{\pi}^{x}_{t+2}\right] + \xi^0_T\widetilde{mc}^x_t - \xi^1_T\mathrm{E}_t\left[\widetilde{mc}^x_{t+1}\right]. \tag{94}
$$

To derive the above two equations it is assumed that the technology and the price setting parameters of the tradable and the exports sector are the same.

Hence, the coefficients of these two equations are the same as in equation (89). The average real marginal costs are defined as

$$mc_t^x = \frac{MC_t^x}{e_t P_t^{x*}} = \frac{MC_t^x}{P_t^x}.$$

Hence, using (26) provides the log-linearized real marginal cost formula of the exports sector:

$$\widetilde{mc}_t^x = \hat{\alpha}\left(x_t - \tilde{k}_t^x\right) - \bar{\alpha}\widetilde{A}_t^T + n_T\left(\widetilde{w}_t - \tilde{q}_t\right) + (1 - n_T)\widetilde{P}_t^{m*} - \widetilde{P}_t^{x*}. \tag{95}$$

Equation (61) determines the wage-setting process of the model. The marginal substitution between consumption and labor is

$$\widetilde{mrs}_t = \varphi\tilde{l}_t + \frac{\sigma}{1-h}\left(\tilde{c}_t - h\tilde{c}_{t-1}\right).$$

Substituting the above formula into equation (61) and using definition (70) yields

$$\begin{aligned} \pi_t^w - \vartheta_w\left(a_T\pi_{t-1}^T + a_N\pi_{t-1}^N\right) &= \beta\mathrm{E}_t\left[\pi_{t+1}^w - \vartheta_w\left(a_T\pi_t^T + a_N\pi_t^N\right)\right] \\ +\xi_w&\left[\varphi\tilde{l}_t + \frac{\sigma}{1-h}\left(\tilde{c}_t - h\tilde{c}_{t-1}\right) - \widetilde{w}_t + \frac{\tau^w}{1-\tau^w}\tilde{\tau}_t^w\right]. \end{aligned} \tag{96}$$

As mentioned in *section 3.6*, exchange rate policy is represented by the following simple rule:

$$d\tilde{e}_t = -\omega\left(a_T\pi_t^T + a_N\pi_t^N\right) + \mathcal{S}_t^{de}, \tag{97}$$

where $d\tilde{e}_t = \tilde{e}_t - \tilde{e}_{t-1}$ is the nominal depreciation rate, and $\mathcal{S}_t^{de}$ is the shock of an exogenous nominal depreciation.

Finally, the following identities close the system:

$$\bar{\pi}^T = \pi_t^T - \vartheta_T\pi_{t-1}^T, \tag{98}$$
$$\bar{\pi}^N = \pi_t^N - \vartheta_N\pi_{t-1}^N, \tag{99}$$
$$\bar{\pi}^{x*} = \pi_t^{x*} - \vartheta_T\pi_{t-1}^{x*}, \tag{100}$$
$$\bar{\pi}^x = \pi_t^x - \vartheta_T\pi_{t-1}^x, \tag{101}$$
$$\pi_t^w = \widetilde{w}_t - \widetilde{w}_{t-1} + a_t\pi_t^T + a_N\pi_t^N, \tag{102}$$

where it was once again assumed that the price setting parameters of the tradable and the exports sector are the same.

Version A of the model (no PTM) contains 22 equations: (71), (72), (75) – (78), (80) – (82), (84) – (87), (89) – (92),

(96) – (99) and (102). This system determines the trajectories of the following 22 endogenous variables: $\tilde{c}_t$, $\tilde{x}_t$, $\tilde{I}_t$, $\tilde{y}_t^T$, $\tilde{y}_t^N$, $\tilde{l}_t$, $\tilde{k}_t^T$, $\tilde{k}^N$, $\widetilde{mc}_t^T$, $\widetilde{mc}_t^N$, $\tilde{w}_t$, $\tilde{w}_t^{z,T}$, $\tilde{w}_t^{z,N}$, $\tilde{q}_t$, $\tilde{q}_t^T$, $\tilde{P}_t^R$, $d\tilde{e}_t$, π_t^T, π_t^N, π_t^w, $\bar{\pi}_t^T$, $\bar{\pi}_t^N$.

To obtain version B (PTM, LCP) replace equations (78) and (82) by equations (79) and (83). Furthermore, add equations (73), (88), (93), (95) and (100) to the system. This is a system of 27 equations. It determines the paths of the variables belonging to version A, plus the trajectories of $\tilde{k}_t^x$, $\widetilde{mc}_t^x$, $\tilde{P}_t^{x*}$, π_t^{x*}, $\bar{\pi}_t^{x*}$.

To derive version C (PTM, PCP) replace equations (78) and (82) in version A by equations (79) and (83). Beyond this, add equations (74), (88), (94), (95) and (101) to the system. This system of 27 equations determines the paths of the variables belonging to version A and the trajectories of $\tilde{k}_t^x$, $\widetilde{mc}_t^x$, $\tilde{P}_t^{x*}$, π_t^x, $\bar{\pi}_t^x$.

A.4 Second moments of the model

This section provides the formulas for statistics used in *section 5*. First, let us supplement the log-linearized model of *Appendix A.3* with two new variables, $d\tilde{q}_t$, $d\tilde{q}_t^T$, and the equations defining them:

$$d\tilde{q}_t = \tilde{q}_t - \tilde{q}_{t-1}, \quad d\tilde{q}_t^T = \tilde{q}_t^T - \tilde{q}_{t-1}^T.$$

Let us denote by Y_t the vector of endogenous variables of the augmented system. Since it is assumed that the exogenous shocks of the model are uncorrelated, one can study them separately. Let us denote the nth shock by $\mathcal{S}_t^n$. It is determined by a first-order autoregressive process:

$$\mathcal{S}_t^n = \varrho_n \mathcal{S}_{t-1}^n + \epsilon_t^n, \quad |\varrho_n| < 1, \quad \mathrm{E}\left[\epsilon_t^n\right] = 0, \quad \mathrm{E}\left[(\epsilon_t^n)^2\right] = \varsigma_n^2.$$

The undetermined coefficient method, the solution algorithm used, provides matrix Q and R, and the paths of the endogenous variables are determined by[29]

$$Y_t = QY_{t-1} + R\mathcal{S}_t^n.$$

Define the following variables and matrix:

$$\bar{Y}_t = \begin{bmatrix} Y_t \\ \mathcal{S}_{t+1}^n \end{bmatrix}, \quad \mathcal{E}_t = \begin{bmatrix} 0 \\ \epsilon_{t+1}^n \end{bmatrix}, \quad F = \begin{bmatrix} Q & R \\ \varrho_n & 0 \end{bmatrix}.$$

Then the log-linearized model can be represented by the following *first-order vector autoregressive* process:

$$\bar{Y}_t = F\bar{Y}_{t-1} + \mathcal{E}_t.$$

[29]The eigenvalues of matrix Q are smaller than 1 in absolute value.

Let us denote by g the number of the elements of $\bar{Y}$ and $\mathcal{E}_t$. The variance-covariance matrix of $\mathcal{E}_t$ is Σ, which is a $g \times g$ matrix, with elements equal to zero, except the gth diagonal element, which is equal to ς_n^2.

Let us denote by V_0 the *unconditional variance-covariance* matrix of $\bar{Y}_t$, that is,

$$V_0 = \mathrm{E}\left[\bar{Y}_t \bar{Y}_t'\right],$$

and let us denote by $V_0(ij)$ the element in row i and column j. Apply formula (10.2.16) and (10.2.17) of Hamilton (1994), then

$$\mathrm{vec}(V) = \left(I_{g^2 \times g^2} - \mathcal{A}\right)^{-1} \mathrm{vec}(\Sigma),$$

where $I_{g^2 \times g^2}$ is an appropriate identity matrix, $\mathcal{A} = F \otimes F$. Symbol $\otimes$ represents the *Kronecker product*, and operator *vec* transforms a quadratic matrix into a column vector by stacking the columns of the matrix one below the other, with the columns ordered from left to right.

The *lth autocovariance* matrix is defined by

$$V_l = \mathrm{E}\left[\bar{Y}_t \bar{Y}_{t-l}'\right].$$

Formula (10.2.21) provides an expression for it:

$$V_l = F^l V_0.$$

The *variance* of the ith endogenous variable (that is, the ith element of $\bar{Y}_t$) is $V_0(ii)$. The *covariance* of the ith and jth endogenous variable is $V_0(ij)$. Their *correlation coefficient* is $V_0(ij)\left[V_0(ii)V_0(jj)\right]^{-\frac{1}{2}}$. Finally, the *lth autocovariance* of the ith endogenous variable is defined by $V_l(ii)V_0(ii)^{-1}$.

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Figure 1
Balassa-Samuelson effect
No PTM – version *A*
Benchmark economy

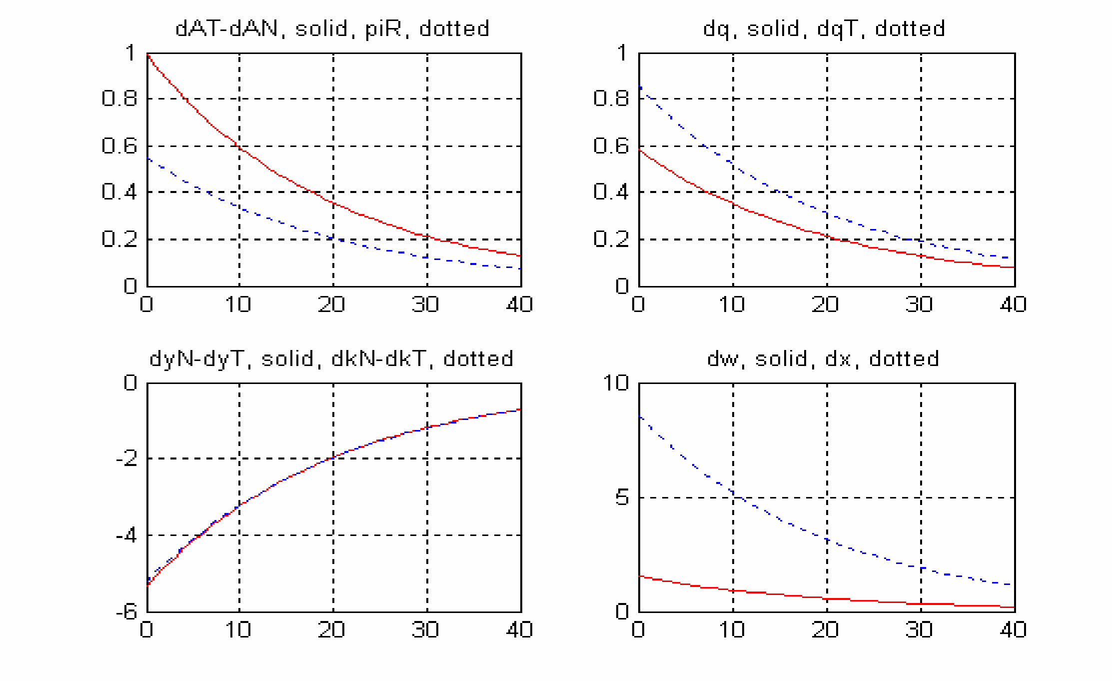
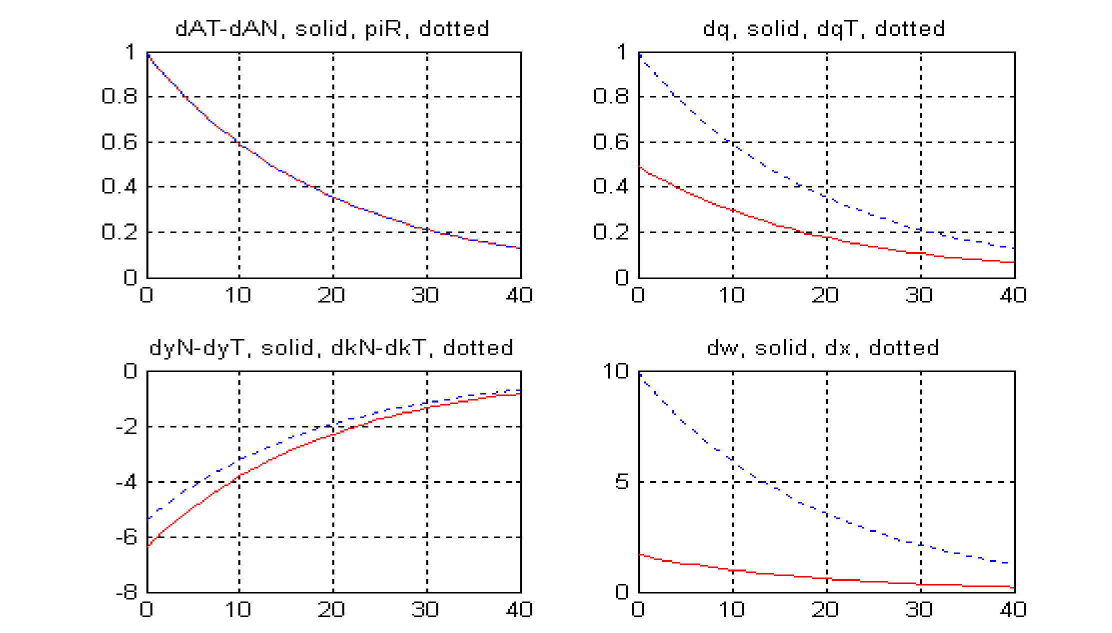


Figure 2
Balassa-Samuelson effect
No PTM – version *A*
Flexible prices and wages, no investment adjustment costs



Units on horizontal axis represent quarters, on vertical axis percentage points.
Growth rates are displayed in annualized terms.

Figure 3
Balassa-Samuelson effect
PTM with LCP – version *B*
Benchmark economy

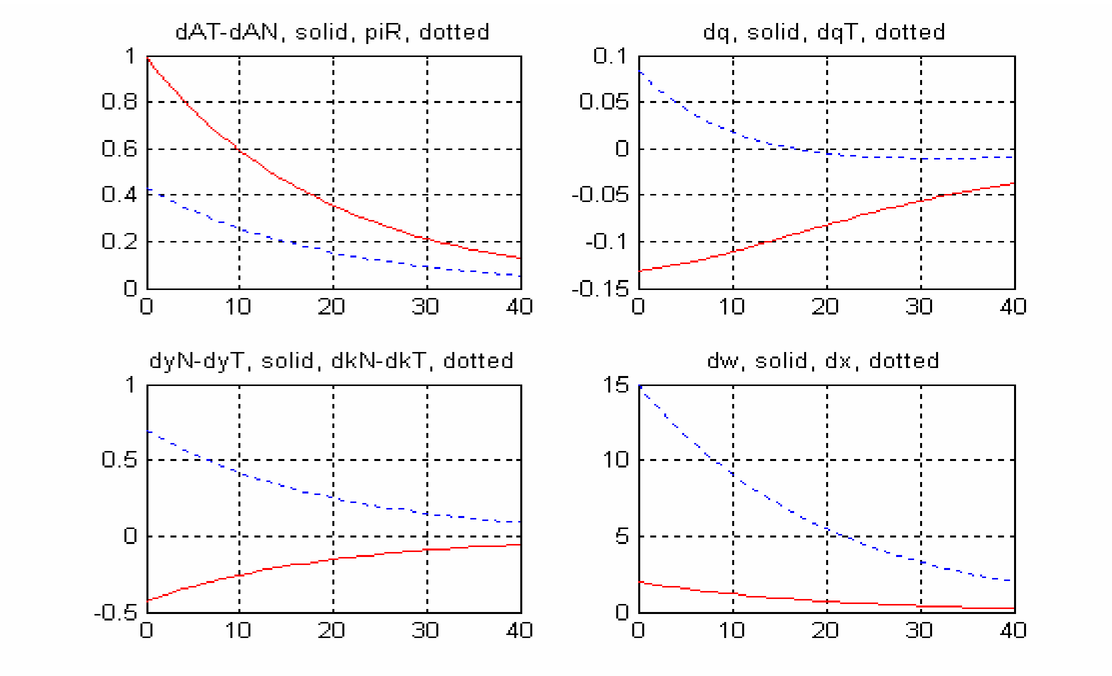
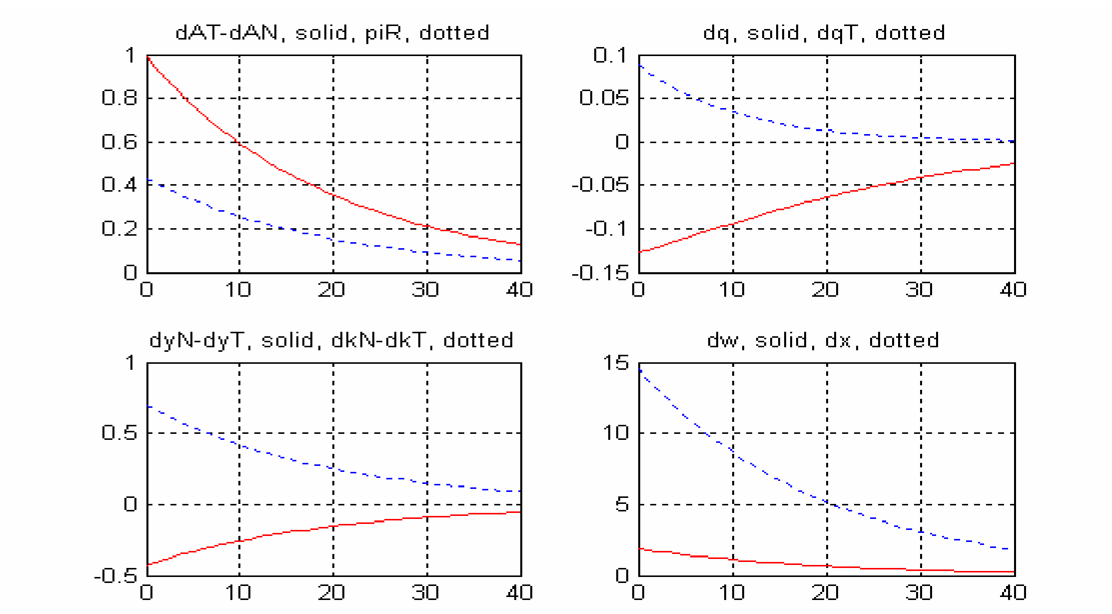


Figure 4
Balassa-Samuelson effect
PTM with PCP –version *C*
Benchmark economy



Units on horizontal axis represent quarters, on vertical axis percentage points.
Growth rates are displayed in annualized terms.

Figure 5
Balassa-Samuelson effect
PTM with LCP – version *B*
No investments adjustment costs

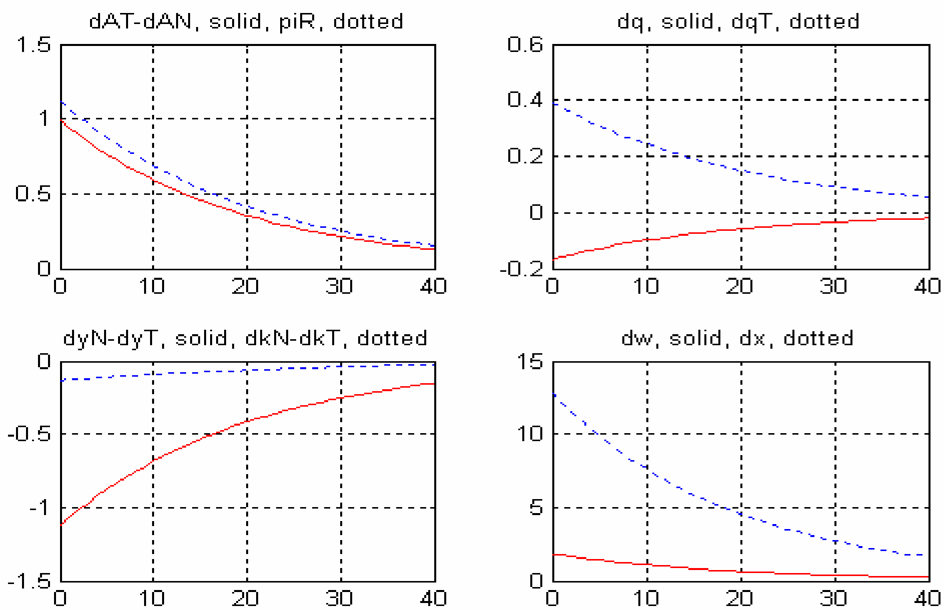
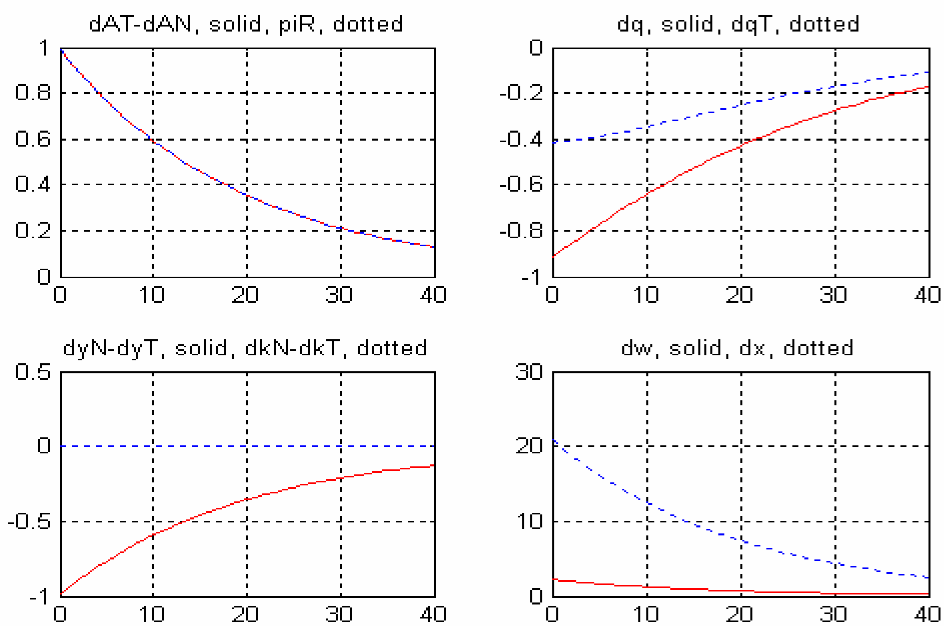
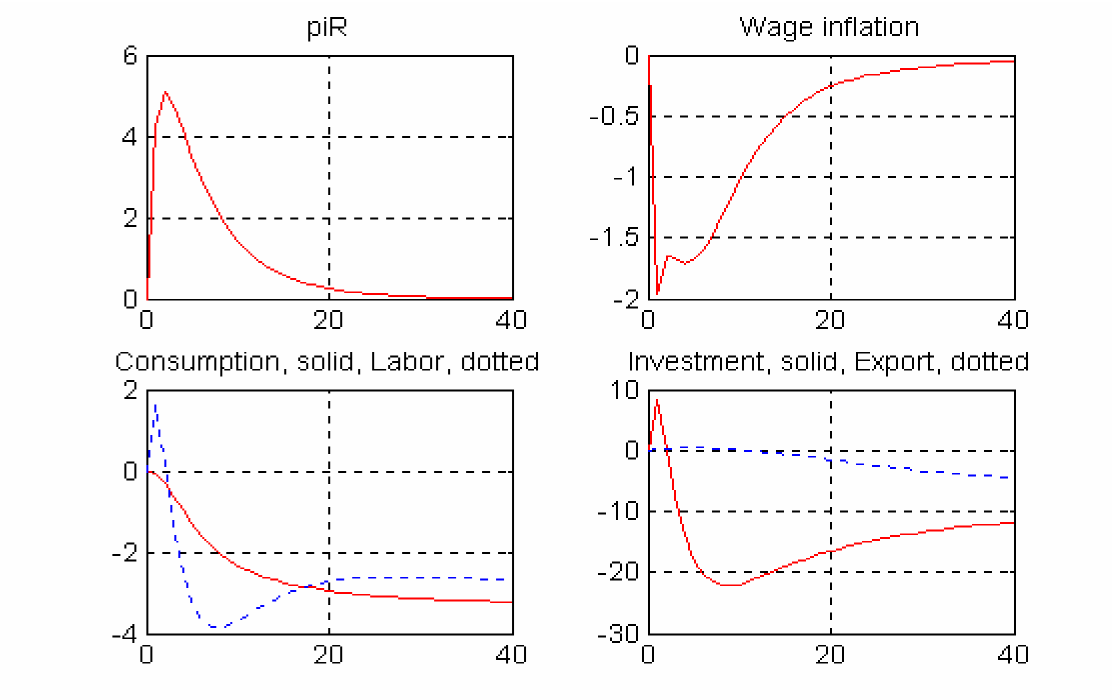


Figure 6
Balassa-Samuelson effect
PTM with LCP – version *B*
Flexible prices



Units on horizontal axis represent quarters, on vertical axis percentage points.
Growth rates are displayed in annualized terms.

Figure 7
Regulation shock in non-tradable sector (υ^N)
PTM with LCP – version *B*
Benchmark economy



Units on horizontal axis represent quarters, on vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

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