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Nonparametric Identification of Auction Models with Non-Separable Unobserved Heterogeneity*

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Abstract

We propose a novel methodology for nonparametric identification of first-price auction models with independent private values, which accommodates auction-specific unobserved heterogeneity and bidder asymmetries, based on recent results from the econometric literature on nonclassical measurement error in Hu and Schennach (2008). Unlike Krasnokutskaya (2009), we do not require that equilibrium bids scale with the unobserved heterogeneity. Our approach accommodates a wide variety of applications, including settings in which there is an unobserved reserve price, an unobserved cost of bidding, or an unobserved number of bidders, as well as those in which the econometrician fails to observe some factor with a non-multiplicative effect on bidder values.

1 Introduction

In this short paper, we propose a methodology for nonparametric identification of first-price auctions with unobserved heterogeneity, in the independent private values framework. By unobserved heterogeneity, we mean auction-specific factors that are observed by the bidders and affect their equilibrium bids. However, these factors are not observed by the econometrician, leading to spurious correlation between the bids for a given auction, even under the assumption of independent private values.

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Krasnokutskaya (2009) proposed an ingenious approach to identify and estimate such a model, as long as the unobserved heterogeneity affects bidders' valuations multiplicatively and is independent of the bidder-specific component of each bidder's valuation. Under these assumptions, Krasnokutskaya shows that results from the classical measurement error literature can be applied, so that two bids per auction are sufficient to identify and estimate the structural components of the model, which consist of the marginal distributions of the unobserved heterogeneity and of bidders' valuations. Subsequently, Krasnokutskaya's results have been used in applied analyses of timber auctions (Athey, Levin, and Seira (2005)), stamp auctions (Asker (2009)), and procurement auctions (Decarolis (2009)).

In this paper, we apply findings from the more recent literature on *nonclassical* measurement error in Hu and Schennach (2008). Using these more powerful tools, we obtain nonparametric identification of bidder values under much weaker assumptions. In particular, we allow the unobserved heterogeneity to affect bidders' valuations in arbitrary nonlinear fashion, and we do not assume that bidders' private signals are independent of the unobserved heterogeneity. We show that, using two bids per auction *plus* a third instrument (which could be a third bid), we can identify the marginal distribution of the unobserved heterogeneity, as well as the distributions of bidder valuations conditional on the unobserved heterogeneity.

Our identification results are very similar in spirit to those in d'Haultfoeille and Fevrier (2008), who focus on conditionally independent common value ("mineral rights") models. Roberts (2009) takes a control-function approach to identify an ascending auction model with unobserved heterogeneity, using two bids and the reserve price as an instrument. Our identification approach, which is based on measurement error results, is quite distinct from these two papers.

The rest of the paper is organized as follows. In Section 2, we present the model, develop our main identification result, discuss an extension to settings with both unobserved heterogeneity and endogenous entry, and compare our approach with that of Krasnokutskaya (2009). Section 3 then discusses various additional examples in which this paper's methods can be applied, including settings with an unobserved reserve price, an unobserved cost of bidding, and an unobserved number of bidders.

2 Model and identification

A fixed set of bidders $i = 1, ..., n$ participates in a first-price auction t, where $n \geq 3$. Bidder i's value is distributed as $V_{it} = v(X_{it}, Y_t)$, where X_{it} is privately observed, Y_t is one-dimensional "unobserved heterogeneity" that is commonly observed by all bidders but not by the econometrician, and $v(\cdot, \cdot)$ is some deterministic function.[1] X_{it} are independent across bidders conditional on Y_t, so that our model is one of asymmetric independent private values (IPV). For all Y_t, $X_{it}|Y_t$ has a well-defined, continuous density over the same support for all i.

Given these assumptions, there exists a unique Bayesian equilibrium of the bidding game in strictly increasing and differentiable strategies that depend on Y_t (Lebrun (1999)). We assume that bidders play these equilibrium strategies, generating random bids $B_t = (B_{it} : i = 1, ..., n)$ observed by the econometrician.

The assumption here that there are at least three bidders is important, as the third bid plays the role of an instrument for the unobserved heterogeneity Y_t. If there are only two bidders, our analysis still applies if an appropriate alternative instrument can be found. Loosely speaking, such an instrument must be correlated with the bids but independent of the bids conditional on Y_t. See Assumptions 2-4 below for precise conditions.

2.1 Identification

In this paper, we provide a novel approach to identify *both* the distribution of the unobserved heterogeneity Y_t and the distribution of bids B_t conditional on Y_t from the distribution of bids B_t. Given these distributions, identification of the distribution of bidder values $V_t = (V_{it} : i = 1, ..., n)$ is straightforward from existing results. To see why, let $G_{it}(b|y) = \Pr(\max_{j \neq i} B_{jt} \leq b | Y_t = y)$ denote the cdf of the highest bid submitted by any of bidder i's competitors conditional on $Y_t = y$, and let $g_{it}(b|y)$ be the associated pdf. The first-order condition of equilibrium bidding implies that bidder i's realized private value v_{it} when he bids b_{it} in equilibrium can be expressed simply as his bid plus a mark-up that depends on

[1] Random variables are capitalized while realizations are in lower case. For simplicity, we assume that the function $v(\cdot, \cdot)$ is identical across bidders. Our approach allows $v(\cdot, \cdot)$ to differ across bidders but, in any case, this distinction is not very important because we will not be able to identify $v(\cdot, \cdot)$.

the elasticity of his probability of winning:

$$v_{it} = b_{it} + \frac{G_{it}(b_{it}|y)}{g_{it}(b_{it}|y)}. \tag{1}$$

(See Guerre, Perrigne, and Vuong (2000) and Athey and Haile (2002) for more details on this standard step.) Finally, given the distributions of both $V_t|Y_t$ and Y_t, one may recover the latent distribution of bidder values V_t.

Fix any three bidders i, j, k. We shall provide conditions given which *both* the distribution of Y_t and the joint distribution of (B_{it}, B_{jt}, B_{kt}) conditional on Y_t are identified from the joint distribution of (B_{it}, B_{jt}, B_{kt}). When these conditions hold for all triplets of bidders, then our methods allow one to identify both the distribution of Y_t and of $B_t|Y_t$ from the distribution of B_t, as desired.

Assumption 1 *The joint density of* $(B_{it}, B_{jt}, B_{kt}, Y_t)$ *exists and is bounded away from zero and infinity.*

Let $f_i(b_{it}|Y_t)$, $f_j(b_{jt}|Y_t)$, $f_k(b_{kt}|Y_t)$, and $f(Y_t)$ denote the marginal densities and $\mathcal{B}_i$, $\mathcal{B}_j$, $\mathcal{B}_k$, and $\mathcal{Y} \subset \mathbb{R}$ the supports of $b_{it}|Y_t$, $b_{jt}|Y_t$, $b_{kt}|Y_t$, and Y_t, respectively.

Assumption 2 *(i)* $f_i(b_{it}|Y_t, b_{jt}, b_{kt}) = f_i(b_{it}|Y_t)$ *and (ii)* $f_j(b_{jt}|Y_t, b_{kt}) = f_j(b_{jt}|Y_t)$.

That is, (b_{it}, b_{jt}, b_{kt}) are independent conditional on Y_t. This assumption is satisfied in the IPV bidding model.

Assumption 3 *For all bounded functions* $h : \mathbb{R} \to \mathbb{R}$, $E[h(b_{kt})|b_{it}] = 0$ *for all* $b_{it} \in \mathcal{B}_i$ *implies that* $h(b_{kt}) = 0$ *for all* $b_{kt} \in \mathcal{B}_k$.

Assumption 4 *For all bounded functions* $h : \mathbb{R} \to \mathbb{R}$, either $E[h(Y_t)|b_{jt}] = 0$ *for all* $b_{jt} \in \mathcal{B}_{jt}$ or $E[h(Y_t)|b_{kt}] = 0$ *for all* $b_{kt} \in \mathcal{B}_{kt}$ *implies that* $h(y) = 0$ *for all* $y \in \mathcal{Y}$.

Assumption 3 implies that the bid b_{it} is correlated with b_{kt} through the unobserved heterogeneity Y_t, while Assumption 4 implies that the bids b_{jt}, b_{kt} are correlated with Y_t. In Krasnokutskaya (2009)'s convolution setting, Assumptions 3-4 are implied by her assumption that Y_t and X_{it} have non-vanishing characteristic functions.

Assumption 5 *There exists a known functional M such that $M[f_i(\cdot|y)] = y$ for all $y \in \mathcal{Y}$.*

Examples of functionals M that could be used to satisfy Assumption 5 include those corresponding to any location of the distribution of $b_{it}|Y_t$, such as the mean, i.e., $M[f] = \int xf(x)dx$, the mode, i.e., $M[f] = \arg\max_x f(x)$, or the τ-th quantile, i.e., $M[f] = \inf\left\{x^* : \int_{-\infty}^{x^*} f(x)dx \geq \tau\right\}$. For instance, suppose that the mean of $b_{it}|Y_t$ is known to be strictly monotone in Y_t. If so, one can normalize Y_t via some monotone transformation so that $E[b_{it}|Y_t = y] = y$ for all $y \in \mathcal{Y}$ and Assumption 5 is satisfied.

As a simple illustration, consider a symmetric example in which bidders' valuations take the form $V_{it} = X_{it} * Y_t$, where $X_{it} \sim U[0,1]$ i.i.d. across bidders. In equilibrium, $b_{it} = Y_t * \frac{n-1}{n} X_{it}$. Moreover, $\text{med}(b_{it}|Y_t) = aY_t$, for the constant $a \equiv \frac{n-1}{2n}$. Hence, the functional $M[f_i(\cdot|y)] \equiv \text{med}(b_{it}|Y_t = y)/a = y$ satisfies Assumption 5. See Section 3 for a variety of additional examples.

Let $f(b_{it}, b_{jt}, b_{kt}) = \int f_i(b_{it}|Y_t) f_j(b_{jt}|Y_t) f_k(b_{kt}|Y_t) f(Y_t) \, \mathrm{d}Y_t$ denote the joint density of (b_{it}, b_{jt}, b_{kt}). The main result of the paper is that, under these assumptions, one may identify the distributions of $b_{it}|Y_t$, $b_{jt}|Y_t$, $b_{kt}|Y_t$, and Y_t from the distribution of (b_{it}, b_{jt}, b_{kt}).

Theorem 1 *Under Assumptions 1-5 above, the density $f(b_{it}, b_{jt}, b_{kt})$ uniquely determines $f_i(b_{it}|Y_t)$, $f_j(b_{jt}|Y_t)$, $f_k(b_{kt}|Y_t)$, and $f(Y_t)$.*

Proof. We prove the identification by showing that all the assumptions in Theorem 1 in Hu and Schennach (2008) are satisfied. Their Assumptions 1, 2, 5 are directly assumed. As discussed in their paper, Assumption 3 and the $(b_{kt}, \mathcal{B}_k)$ part of Assumption 4 here together imply their Assumption 3. Their Assumption 4 requires that for all $y_1, y_2 \in \mathcal{Y}$, the set $\{b : f_j(b|y_1) \neq f_j(b|y_2)\}$ has positive probability whenever $y_1 \neq y_2$. This can be shown as follows. Suppose their Assumption 4 does not hold in our model. Then there exist two $y_1, y_2 \in \mathcal{Y}$ such that $f_j(\cdot|y_1) = f_j(\cdot|y_2)$, i.e., the two distributions are the same. We may then construct a function h such that $\int f_j(b|y) h(y)dy = 0$ for all b and the function $h(\cdot)$ not equal to zero at y_1 and y_2. Therefore, their Assumption 4 is implied by the $(b_{jt}, \mathcal{B}_j)$ part of Assumption 4 here. Finally, their Theorem 1 implies our identification results. ■

2.2 Extension: endogenous participation

Consider an augmented version of our model in which there is a universe $\{1, ..., \overline{n}\}$ of potential bidders, of which only a random *subset* N_t choose to bid. Theorem 1 provides conditions under which to identify the distributions of the unobserved heterogeneity and of bidder values *conditional on* N_t for all $N_t \subset \{1, ..., \overline{n}\}$. In this way, our analysis can accommodate models of endogenous entry, in which the set of bidders N_t is associated with the unobserved heterogeneity Y_t. Recently, there has been interest in such models (e.g. Li and Zheng (2009), Marmer, Shneyerov, and Xu (2009)).

Indeed, across a random sample of auctions in which all bids and bidders are observed, one can estimate directly from the data the probability that N_t is the set of active bidders, for all N_t. Hence, given our estimates of $f(Y_t|N_t)$, one can also form the joint distribution $f(Y_t, N_t) = f(Y_t|N_t) \cdot f(N_t)$, and hence also the conditional distribution $f(N_t|Y_t)$. This suggests that, in the IPV context, a model with non-separable unobserved heterogeneity and endogenous number of bidders can be estimated.

2.3 Comparison with Krasnokutskaya (2009)

In a path-breaking paper, Krasnokutskaya (2009) provided conditions under which results from the literature on classical measurement error can be used to identify IPV first-price auction models in the face of unobserved heterogeneity. Our paper is similar in spirit, but differs in that we bring to bear more powerful results from the recent literature on non-classical measurement error.

These results allow us to weaken most of the restrictive assumptions imposed by Krasnokutskaya (2009). First, Krasnokutskaya assumes that the unobserved heterogeneity has the same multiplicative effect on all bidders' valuations, i.e. $V_{it} = X_{it} * Y_t$, with the implication that equilibrium bids also scale multiplicatively with Y_t. By contrast, we require only that some location of the distribution of equilibrium bids be increasing in Y_t. This condition is automatically satisfied in her setting, since the mean of each bidder's equilibrium bid is not only increasing but linear in Y_t. Second, Krasnokutskaya assumes that the unobserved heterogeneity Y_t is independent of the idiosyncratic components X_{it} of bidders' values, as well as that X_{it} are independent conditional on Y_t. By contrast, we only require that X_{it} are independent conditional on Y_t. Also, as noted earlier in the text, the completeness Assumptions 3-4 are implied by Krasnokutskaya's assumption that Y_t and X_{it} have non-vanishing characteristic functions. On the other hand, Krasnokutskaya makes weaker demands on the data. Whereas we require the observation of three bids, she requires only two.

By insisting only on a monotone relationship between bids and unobserved heterogeneity, rather than a multiplicative one, our approach opens up a wide variety of new applications.

For instance, suppose that bidders are symmetric and each bidder faces a cost C_t of bidding which varies across auctions. While equilibrium bids will certainly not be multiplicative in C_t, the probability that each bidder chooses not to submit a bid is increasing in C_t. Using our approach, one may therefore non-parametrically identify both the distribution of C_t and the distribution of bidder values conditional on C_t. (See Section 3.2 for details.)

3 Examples

This section provides a variety of examples in which this paper's identification approach may be applied. In each example, one object is sold via first-price auction to risk-neutral bidders having independent private values. Except as otherwise specified, the econometrician observes all bids and the set of bidders is fixed and known to the econometrician.

3.1 Unobserved reserve price

Suppose that each auction t has a reserve price R_t that is random and unobserved by the econometrician, common knowledge among the bidders before the bidding, and independent of bidders' private valuations. For example, if the seller lacks the power to commit to a reserve price, then R_t would equal his opportunity cost of selling the good. Or, if the seller has the power to set an optimal reserve price based on the true distribution of bidder values, then R_t is some strictly increasing function of that seller cost. Let $\emptyset$ denote the "null bid" ($\emptyset < b$ for all $b \geq 0$) made in equilibrium by any bidder whose value is less than the reserve.

This paper's approach allows the econometrician to identify the distribution of R_t as well as the distribution of each (asymmetric) bidder's valuation, as long as (i) there are at least three bidders and (ii) the probability that the reserve price R_t binds on bidder i is strictly increasing in R_t over its support.[2] In particular, condition (ii) implies that the functional $M\left[f_i(\cdot|r)\right] = \inf\{b : \Pr(B_{it} > b|R_t = r) < \Pr(B_{it} > \emptyset|R_t = r)\}$ corresponding to the minimal submitted bid equals r. Thus, Assumption 5 is satisfied without any need for a

[2] Condition (ii) is satisfied in a broad set of circumstances. For example, suppose that the seller is able to set an optimal reserve, and that the seller's private cost C_t has support $[0, \overline{c}]$ while each bidder i's value has support $[0, \overline{v}_i]$ for some $\overline{c} < \max\{\overline{v}_1, ..., \overline{v}_N\}$. Condition (ii) fails for bidder i only if his maximal value is low enough that he is sometimes "priced out" of the auction, i.e. $\Pr(R_t > \overline{v}_i) > 0$. An optimally chosen reserve price will never price out all bidders, so (ii) must be satisfied for at least one bidder. Furthermore, should bidders be symmetric, it will be satisfied for all bidders.

normalization of R_t. The other assumptions are trivial to check, given that bidders have independent private values that are uncorrelated with the reserve price. The distribution of R_t and of bidders' values are therefore non-parametrically identified.

3.2 Unobserved cost of bidding

Suppose that bidders are symmetric and the reserve price is zero, but some bidders choose not to participate because of a common cost C_t of submitting a bid that is random and independent of bidders' valuations.

It is straightforward to show that, in the unique (symmetric) equilibrium, the probability that each bidder chooses not to submit a bid is increasing in C_t. Thus, the functional $M\left[f_i(\cdot|c)\right] = \Pr(b_i = \emptyset)$ corresponding to the probability of non-participation is strictly increasing in the unobserved heterogeneity C_t. Hence, Assumption 5 is satisfied after an appropriate normalization, i.e. when $Y_t = \gamma(C_t)$ for some strictly increasing $\gamma(\cdot)$. All other assumptions are again trivial to check. Thus, the distribution of bidder values conditional on Y_t *and conditional on having made a bid* are non-parametrically identified.

Since $\gamma(\cdot)$ is an unknown normalization, however, more work is necessary in order to back out the bidding costs C_t corresponding to each conditional distribution of values. Let $\underline{b}_t(y)$ denote the minimal bid submitted by each bidder when $Y_t = y$, and let $\underline{v}_t(y)$ denote the corresponding bidder value. In equilibrium, a bidder having value $\underline{v}_t(y)$ must be indifferent between bidding $\underline{b}_t(y)$ and not bidding at all:

$$C_t = \left(\underline{v}_t(Y_t) - \underline{b}_t(Y_t)\right) \Pr(\max_{j \neq i} b_{jt} = \emptyset | Y_t). \tag{2}$$

Through this indifference condition, the distribution of C_t is also identified.

3.3 Unobserved number of bidders

Suppose that N_t symmetric bidders choose to participate in an auction with zero reserve price, where N_t is random and common knowledge among the participating bidders before the bidding, and each participating bidder's value is drawn iid from the same distribution regardless of N_t. The econometrician observes detailed bid-data for *three bidders* i, j, k, i.e. whether they chose to participate and what they bid, but does not observe the bids made by other bidders, nor the total number of bidders.[3] Such a scenario could arise if a researcher has acquired data from individual bidders only.

The condition here that the distribution of bidder values does not depend on N_t is naturally satisfied in several sorts of settings. (See Section 2.2 for discussion of an alternative setting in which bidder values are correlated with N_t.) For example, if participation is costly and bidders observe their values before deciding to participate, then in equilibrium each bidder will participate iff his value exceeds a symmetric threshold. In this case, the distribution of participating bidders' values is just a truncation of the original distribution of values. Or, if bidders do not observe their values until they decide to participate, then obviously the distribution of participating bidders' values will just be the original distribution. Or, finally, participation might simply be exogenous, as when participants are comprised of "random passers-by".

The distribution of each bidder's bid in the unique equilibrium is easily shown to be strictly increasing in the number of bidders N_t, in the sense of first-order stochastic dominance. (Since equilibrium strategies are also symmetric across bidders, we will drop reference to bidder i in what follows.) In particular, the mean of $B_t|N_t$ is strictly increasing in N_t. Consequently, Assumption 5 is satisfied for an appropriate normalization $Y_t = \gamma(N_t)$, when we use the functional corresponding to the mean. The other assumptions are again trivial to check, given that bidders have independent private values drawn from the same distribution regardless of the number of bidders. By our results, then, the distribution of Y_t and the distribution of each bidder's bid conditional on Y_t are non-parametrically identified. However, more work is required to identify the distribution of bidder values: The mark-up of values over bids in (1) depends on the number of bidders, and N_t cannot be directly inferred from Y_t since $\gamma(\cdot)$ is unknown.

Since $\gamma(\cdot)$ is strictly increasing, each element $y \in \text{supp}(Y_t)$ corresponds to a different number of bidders $n(y) = \gamma^{-1}(y)$ in the support of N_t. For any quantile $\alpha \in (0, 1)$, let $v_t(\alpha)$ and $b_t(\alpha|y)$ denote the α-th quantiles of the (symmetric) distributions of bidder values and equilibrium bids conditional on $Y_t = y$, respectively. (Recall that, by assumption, the distribution of bidder values does not depend on the number of bidders.) Conditional on $Y_t = y$, the probability that a bidder wins the object with bid $b_t(\alpha|y)$ is simply $G_t(b_t(\alpha|y)|y) = \alpha^{n(y)-1}$.

[3] See Athey and Haile (2007) Section 6.3 for an excellent discussion of scenarios in which bidders do not observe the number of other bidders. Here, only the econometrician fails to observe N_t.

Consequently, for each $\alpha \in (0,1)$, (1) may be re-written as

$$v_t(\alpha) = b_t(\alpha|y) + \frac{\alpha b'(\alpha|y)}{n(y)-1} \text{ for all } y \in \mathcal{Y}. \tag{3}$$

In particular, for any pair of quantiles $\tilde{\alpha}, \alpha$ and pair $\tilde{y}, y \in \text{supp}(Y_t)$,

$$b_t(\tilde{\alpha}|\tilde{y}) + \frac{\tilde{\alpha} b'(\tilde{\alpha}|\tilde{y})}{n(\tilde{y})-1} = b_t(\tilde{\alpha}|y) + \frac{\tilde{\alpha} b'(\tilde{\alpha}|y)}{n(y)-1} \tag{4}$$

$$b_t(\alpha|\tilde{y}) + \frac{\alpha b'(\alpha|\tilde{y})}{n(\tilde{y})-1} = b_t(\alpha|y) + \frac{\alpha b'(\alpha|y)}{n(y)-1}. \tag{5}$$

Note that we have already identified all variables in the system of equations (4-5) except for $(n(\tilde{y}), n(y))$. N_t is therefore identified, as long as (4-5) has a unique solution for all $\tilde{y}, y \in \text{supp}(Y_t)$.[4] Given the distribution of $N_t = n(Y_t)$, one may now identify the distribution of bidder values from the first-order condition (3).

In this way, the distribution of the number of bidders N_t and of bidder values can be non-parametrically identified *as long as the support of N_t has at least two elements*. By contrast, if N_t is unknown but non-random (i.e. the identified distribution of Y_t has singleton support), then bidder values cannot possibly be identified.

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[4] The $\tilde{\alpha}$-th and α-th quantiles of the distribution of bidder values solve (4-5). Thus, this system always has at least one solution under our maintained assumptions about the data generating process. As can be easily shown, it has a unique solution as long as the matrix $A = \begin{bmatrix} \alpha b'(\alpha|y) & -\alpha b'(\alpha|\tilde{y}) \\ \tilde{\alpha} b'(\tilde{\alpha}|y) & -\tilde{\alpha} b'(\tilde{\alpha}|\tilde{y}) \end{bmatrix}$ is invertible.

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