Public Debt and the Macroeconomic Stability of Japan [*]

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Abstract

Recently, the outstanding debt of the Japanese government amounts to 695 trillion yen, which implies 139.5% of GDP. In this paper, we constructed three IS–LM type dynamic models and estimate the eigenvalues of their differential systems. Then we confirm whether or not the huge amount of public debt violates the stability conditions for the Japanese economy.

Our estimation concludes the Japanese economy to be unstable with the existence of a saddle–point equilibrium. Our simulation also shows that severe tax reform would be required to restore the economic stability. Concretely, the government has to raise the consumption tax rate to 15% from 5%, and in addition, allowing the income elasticities of income taxes and inhabitant taxes to increase by 0.033 each, which is equivalent to tax hikes of about 8.3 trillion yen. We assert that structural reform for the government budget including a tax system is essential and emergent.

1.   Introduction

The purpose of this paper is to analyze the influence of public debt on Japan's macroeconomic stability. Recently, the sustainability of the Japanese government's debt is widely perceived as a major issue because of its huge amount of 695 trillion yen, which implies 139.5% of GDP, at the end of fiscal year 2003. High interest rates raised by a recovery in private capital demand could possibly cause a spiral increase in public debt. For example, the yield on newly issued 10-year

government bonds jumped from 0.44% on the 22nd of June to 1.67% about three months later, on the 2nd of September in 2003. In other words, it is questionable whether Japan is at a stable equilibrium as defined by macroeconomics: In that case, there are no grounds for the effectiveness of macroeconomic policy.

In this paper, we consider this issue by investigating the local stability conditions at the steady state theoretically and empirically. Assuming the existence of unemployment, we make use of the traditional IS−LM framework. To begin with, we take the "Fixed Capital Model" in Blinder−Solow (1973), which explicitly includes public debt, and generalize it in several directions, and then construct and estimate an empirical model consistent with this theory. Then we compute a coefficient matrix of a differential system of our model to confirm the stability conditions by using the estimated eigenvalues or the Routh−Hurwitz conditions. [1] [2] To the best of our knowledge, this is the first study that explicitly estimates the coefficient matrix of the differential system.

The stability is classified as follows by the signs of the real part of the eigenvalues of the estimated coefficient matrix. First, the equilibrium state is characterized as "unstable" when the real parts of all eigenvalues are positive. In this case, an increase in public debt caused by any change in exogenous variables, such as policy variables, makes interest rates spiral, and then the economy diverges. Second, the equilibrium state is said to be "stable" when the real parts of all eigenvalues are negative. In this case, even if some shocks boost the public debt permanently, the incremental amount decreases gradually, which means that the economy converges to its new equilibrium. Finally, we can find a situation in which some parts are positive and the rest are negative; this is called the saddle−point equilibrium. This equilibrium state is also stable if economic agents are assumed to be rational and some of the variables are "jumpable". However, it will be taken as unstable in this paper because we do not have any jumpable variables here.

This study is related to two research areas. First, although the stability conditions of the IS−LM type dynamic model should be tested empirically as is pointed out in Blinder−Solow (1973), Turnovsky (1977) and so on, no preceding literature exists at least regarding the Japanese economy. Second, as is mentioned in Blinder−Solow (1973), the stability conditions are sufficient conditions for the sustainability of the government budget, which means our study is also a test for the sustainability using the IS−LM type theory. Many analyses on the government budget sustainability have been executed since the oft−cited study of Hamilton−Flavin (1986); however, some variables such as the yield on bonds are taken as given in such literature. [3] By contrast, our

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[1]   As is well known, this means that we assume the existence of wealth effects of bonds, which Barro (1974) discussed from a negative viewpoint. However, to discuss the sustainability of public debt, we have to assume the existence of wealth effects. Since sustainability is a part of macroeconomic stability, the assumption is inevitable for studies of macroeconomic stability as well.

[2]   This method is equivalent for considering whether the economy diverges or converges based on out−of−sample simulation with all future values of exogenous variables taken as constant. To the best of our knowledge, however, there is no preceding literature in which macroeconometric simulation was conducted to confirm the stability.

[3]   Ihori, T., Doi, T. and Kondo, H. (2001) and Ihori, T., Nakazato, T., and Kawade, M. (2003) are for the Japanese cases.

approach deals with these variables endogenously and should be regarded as complementary to the above-mentioned studies. [4)]

The numerical results of the present study are as follows. Our simulation concludes that the Japanese economy is unstable with the existence of a saddle-point equilibrium at the end of fiscal year 2002, the final year in our estimation. One of the reasons for the economic instability is the huge amount of public debt, as we have demonstrated. In our estimation, we must decrease the BOJ's "Flow of Funds Accounts"-based net debt/GDP ratio from the actual value of about 104% to about 60% at the end of fiscal year 2002 to re-stabilize the economy because of a remarkable increase in the budget deficit derived from a slight rise in the yield on bonds. It is expected that this cumbersome situation would be revealed with economic recovery; therefore, reduction in public debt could be the emergent policy issue for Japan's economic stability. [5)]

Our estimation shows that we need a tax reform to raise the consumption tax rate to 15% from 5%, and in addition, allowing the income elasticities of income taxes and inhabitant taxes to increase by 0.033 each. This 0.033 rise in the income elasticities is equivalent to tax hikes of about 8.3 trillion yen. Given that revenues from income taxes and inhabitant taxes totaled about 23 trillion yen in fiscal year 2002, this tax reform must be severe.

The plan of this paper is as follows. In the next section, the benchmark model in Blinder-Solow (1973) is modified to fit our estimation using the macroeconomic data as the SNA. Then, two derivatives of this model are developed as dynamic variables are increased. In Section 3, macroeconometric models are constructed, and the local stability conditions are tested in each model. The policies that aim to restore the stability are discussed in Section 4. Section 5 is the summary of this study.

2. Theoretical Analysis on the Stability [6)]

In this section, we develop three theoretical models increasing the dynamic variables one at a time. This may seem redundant in view of the enormous amount of studies conducted over the decades; however, we believe it useful to clarify the relation between public debt and macroeconomic stability.

[4)] Hamilton-Flavin (1986) resolved the solution of the first-order differential equation (which means the government budget constraint), and then empirically analyzed its No-Ponzi game conditions, whereas we focus on its convergence conditions.

[5)] The Ministry of Finance declared its apprehension that rising interest rates might cause a further increase in public debt, due in part to the fact that interest on Japanese government bonds briefly exceeded 1.9% in June 2004. (June 17, 2004, Nihon Keizai Shimbun)

[6)] We are grateful to Hisao Nagata (Niigata University) for his suggestion.

2–1. Fixed Capital Model

In order to be consistent with our empirical analyses, the Fixed Capital model in Blinder–Solow (1973) (referred to as the original model below) is modified as follows. First, the original continuous model is changed into a discrete one. Second, "beginning of period" equilibrium in the original model is changed to "end of period" equilibrium. As Tobin (1980) stressed, the "end of period" equilibrium model is essential when using the "Flow of Funds Accounts" data. Finally, the bonds dealt with here are floating rate bonds with a fixed face value rather than perpetual bonds paying 1 unit of goods per period, which reflects that the data of the government's interest payment is correlated to market interest rates. Furthermore, it is assumed that the entire public debt is financed by government bonds, not by money creation.

Taking into account these three modifications and Turnovsky (1977), the original model is changed as follows:

$$Y_t = C(\overset{(+)}{(1-t)(Y_t + R_{t-1}B_{t-1})}, \overset{(+)}{B_{t-1} + M_{t-1}}) + I(\overset{(-)}{R_t}) + G_t \tag{1}$$

$$M_t = l(\overset{(+)}{Y_t}, \overset{(-)}{R_t}, \overset{(+)}{B_{t-1} + M_{t-1}}) \tag{2}$$

$$B_t = (1 + (1-t)R_{t-1})B_{t-1} + G_t - tY_t \tag{3}$$

With no loss of generality, we can assume $P_t = 1$ and $M_t = M_{t-1} = \overline{M}$. Notations are as in textbooks except for interest rate, R_t, respectively. The most important change from the original model is that the interest rate becomes a dynamic variable. By using a linear approximation at the steady state, we can express a short–term equilibrium in reduced form assuming that wealth effects on private consumption surpass those on money demand.

$$Y_t = f(\overset{+}{R_{t-1}}, \overset{+}{B_{t-1}}) , R_t = g(\overset{+}{R_{t-1}}, \overset{+}{B_{t-1}}) \tag{4}$$

Using these functions, we can obtain the dynamic system for this economy as follows:

$$\begin{pmatrix} b_t \\ r_t \end{pmatrix} = \begin{pmatrix} 1 + (1-t)\overline{R} - tf_{\overline{B}} & (1-t)\overline{B} - tf_{\overline{R}} \\ g_{\overline{B}} & g_{\overline{R}} \end{pmatrix} \begin{pmatrix} b_{t-1} \\ r_{t-1} \end{pmatrix} \quad \text{where} \quad x_t = X_t - \overline{X}$$

Rewriting this differential system, it becomes:

$$\begin{pmatrix} \Delta b_t \\ \Delta r_t \end{pmatrix} = \begin{pmatrix} (1-t)\overline{R} - tf_{\overline{B}} & (1-t)\overline{B} - tf_{\overline{R}} \\ g_{\overline{B}} & g_{\overline{R}} - 1 \end{pmatrix} \begin{pmatrix} b_{t-1} \\ r_{t-1} \end{pmatrix} \tag{5}$$

Trace = $[(1-t)\overline{R} - tf_{\overline{B}}] + [g_{\overline{R}} - 1]$

Determinant = Δ_F = $[(1-t)\overline{R} - tf_{\overline{B}}][g_{\overline{R}} - 1] - g_{\overline{B}}[(1-t)\overline{B} - tf_{\overline{R}}]$

The necessary and sufficient conditions for the stability of this system are Trace<0 and Determinant>0 (Routh–Hurwitz conditions). The steady state will be the saddle point with Trace<0 and Determinant<0, and unstable with Trace>0 and Determinant>0. However, the saddle–point equilibrium is not attainable because we do not have any jumpable variables here, so hereafter we consider this saddle–point equilibrium to be unstable in the following. [7]

The first term on the right–hand side in the trace definition implies the stability condition for the original model. New issues of bonds at time t(ΔB) raise both the government's interest payment at time t+1($(1-t)\overline{R}\Delta B$) and tax revenues by wealth effects ($tf_{\overline{B}}\Delta B$). If the former surpasses the latter, the government has to issue new bonds at time t+1. However, if the amount is less than that at time t, which makes that at time t+2 much smaller, then the amount converges to 0 as the time tends to infinity. Hence, in the original model with a non–dynamic interest rate, it is a necessary and sufficient condition that the first term is negative.

In the model developed here, however, the interest rate is dynamic. Therefore, that the first term is negative is not sufficient to create stability. It can be seen that the sign of the second term is ambiguous in general, but it is negative when the amount of an increase in the interest rate at time t+1 derived from that at time t – a high interest rate at time t gives high interest income, high consumption, and then a high interest rate at time t+1 – is lower than the latter. In this case, the trace is negative when the original stability condition is satisfied.

Next, let us consider whether the determinant condition can be satisfied. The first term on the right–hand side in the determinant is positive when both the first and the second term in the trace are negative. The second term in the determinant ($g_{\overline{B}}[(1-t)\overline{B} - tf_{\overline{R}}]$) is also positive due to the signs of the short–term equilibrium equation (4). Thus, the sign of the determinant depends on which term surpasses the other. However, the determinant will be negative when there is a sufficient amount of issued bonds, since the effect of the second term overwhelms that of the first term. New issues of bonds at time t+1 in the present model are the sum of an amount of $(1-t)\overline{B}\Delta R$ and $(1-t)\overline{R}\Delta B$, the correspondence in the original model. Thus, when the amount of outstanding bonds is large, the economy diverges as the issuing of new bonds tends to infinity.

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[7]   The interest rate is not jumpable since it is determined in the equilibrium condition for the money market every period.

It should be noticed that the [2,2] element of the coefficient matrix can be positive. In this case, the interest rate diverges because of a greater increase in interest rate at time t+1 than at time t. If this effect satisfies Trace>0 and Determinant>0, then the steady state is unstable since the amount of bonds tends to infinity due to this spiral increase in the interest rate.

2−2. *Variable Capital Model*

In this subsection, we extend the model to include private capital as in Blinder−Solow (1973) (Variable Capital Model).

$$Y_t = C(\overset{(+)}{(1-t)(Y_t + R_{t-1}B_{t-1})}, \overset{(+)}{B_{t-1} + M_{t-1} + K_{t-1}}) + I(\overset{(-)}{R_t}, \overset{(-)}{K_{t-1}}) + G_t \tag{6}$$

$$M_t = l(\overset{(+)}{Y_t}, \overset{(-)}{R_t}, \overset{(+)}{B_{t-1} + M_{t-1} + K_{t-1}}) \tag{7}$$

$$B_t = (1 + (1-t)R_{t-1})B_{t-1} + G_t - tY_t \tag{8}$$

$$K_t = K_{t-1} + I(\overset{(-)}{R_t}, \overset{(-)}{K_{t-1}}) \tag{9}$$

For simplicity, we ignore capital depreciation and add two assumptions as shown below. When private capital increases, first, a decrease in private investment is assumed to surpass an increase in consumption through wealth effects ($C_W + I_K < 0$), and second, the LM curve shifts more to the left than the IS curve does. [8] Under these assumptions, we obtain the reduced forms,

$$Y_t = f(\overset{+}{R_{t-1}}, \overset{+}{B_{t-1}}, \overset{-}{K_{t-1}}), \quad R_t = g(\overset{+}{R_{t-1}}, \overset{+}{B_{t-1}}, \overset{+}{K_{t-1}}) \tag{10}$$

and the dynamic system:

$$\begin{pmatrix} \Delta b_t \\ \Delta r_t \\ \Delta k_t \end{pmatrix} = \begin{pmatrix} (1-t)\overline{R} - tf_{\overline{B}} & (1-t)\overline{B} - tf_{\overline{R}} & -tf_{\overline{K}} \\ g_{\overline{B}} & g_{\overline{R}} - 1 & g_{\overline{K}} \\ I_{\overline{R}}g_{\overline{B}} & I_{\overline{R}}g_{\overline{R}} & I_{\overline{K}} + I_{\overline{R}}g_{\overline{K}} \end{pmatrix} \begin{pmatrix} b_{t-1} \\ r_{t-1} \\ k_{t-1} \end{pmatrix} \tag{11}$$

where $x_t = X_t - \overline{X}$

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[8]   An increase in the private capital shifts both the IS curve and the LM curve; therefore, GDP decreases but the effect on interest rates is ambiguous.

The necessary and sufficient conditions for stability are as follows (Routh–Hurwitz conditions):

・Trace<0

・Determinant<0

・−Trace×(the sum of all 2×2 principal minors of the coefficient matrix)+ Determinant>0

Let us consider these conditions in comparison with the Fixed Capital model in Subsection II-1. An investment function of (6) and (9) with $I_{\bar{K}} < 0$, possibly derived from the stock adjustment principle, makes the [3,3] element in (11) negative. [9] This means that the trace condition can be satisfied more easily than in the Fixed Capital model. The determinant is defined as

$$
\text{Determinant} = \ I_{\bar{K}}\begin{vmatrix} (1-t)\overline{R}-tf_{\bar{B}} & (1-t)\overline{B}-tf_{\bar{R}} \\ g_{\bar{B}} & g_{\bar{R}}-1 \end{vmatrix} - I_{\bar{R}}\begin{vmatrix} (1-t)\overline{R}-tf_{\bar{B}} & -tf_{\bar{K}} \\ g_{\bar{B}} & g_{\bar{K}} \end{vmatrix} \tag{12}
$$

and it can be seen that the first term on the right–hand side in (12) besides $I_{\bar{K}}$ is equivalent to the determinant of the Fixed Capital model, Δ_F. Hence, it is negative when the determinant condition in the Fixed Capital model is satisfied, and vice versa. The second term in (12) can be negative if an increase in interest payments ($(1-t)\overline{R}\Delta B$) is less than that in tax revenues due to wealth effects ($tf_{\bar{B}}\Delta B$), which implies that the determinant can be negative even when the first term is positive due to the debt outstanding. Therefore, the determinant condition can also be satisfied more easily than in the Fixed Capital model.

However, the third condition above is complex. Even under the assumptions that $[(1-t)\overline{R}-tf_{\bar{B}}]<0, [g_{\bar{R}}-1]<0, \Delta_F>0$ and that the steady state in the Fixed Capital model is stable,

(The left–hand side in the third condition) $=$

$$
-\{[(1-t)\overline{R}-tf_{\bar{B}}]+[g_{\bar{R}}-1]+[\overset{-}{I_{\bar{K}}}+\overset{-}{I_{\bar{R}}}\overset{+}{g_{\bar{K}}}]\}(\overset{+}{\Delta_F}+\overset{+}{\Delta_{KB}}+\overset{+}{\Delta_{RK}})
$$

$$
+\overset{-}{I_{\bar{K}}}\overset{+}{\Delta_F}-\overset{-}{I_{\bar{R}}}\begin{vmatrix} (1-t)\overline{R}-tf_{\bar{B}} & -tf_{\bar{K}} \\ g_{\bar{B}} & g_{\bar{K}} \end{vmatrix} \tag{13}
$$

[9] $I_{\bar{K}} + I_{\bar{R}}g_{\bar{K}} = I_{\bar{K}} - I_{\bar{R}}[(1-C_{\bar{Y}}(1-t))l_{\bar{W}} + (C_{\bar{W}}+I_{\bar{K}})l_{\bar{Y}}]/[(1-C_{\bar{Y}}(1-t))l_{\bar{R}} + I_{\bar{R}}l_{\bar{Y}})] < 0$

Where $\Delta_{KB} = \begin{vmatrix} (1-t)\overline{R} - tf_{\overline{B}} & -tf_{\overline{K}} \\ I_{\overline{R}} g_{\overline{B}} & I_{\overline{K}} + I_{\overline{R}} g_{\overline{K}} \end{vmatrix} > 0$、 $\Delta_{RK} = \begin{vmatrix} g_{\overline{R}} - 1 & g_{\overline{K}} \\ I_{\overline{R}} g_{\overline{R}} & I_{\overline{K}} + I_{\overline{R}} g_{\overline{K}} \end{vmatrix} > 0$

then, the sign of the determinant is ambiguous due to the third term on the right-hand side in (13).

The intuition is as follows. Suppose, in the Fixed Capital model, that new bonds are issued when its stability conditions are satisfied. The economy tends to its new steady state as both the bonds and the interest rate increase. In the absence of the one possibility that we will discuss soon, it is also true in the Variable Capital model since the private capital works as a "cushion" to suppress the upward pressure of the interest rate. The exception that makes the economy diverge in the Variable Capital model is when the interest rate has a strong effect on private investment. In this case, tax revenues decrease since the increasing interest rate reduces private investment and GDP, so that the amount of issued bonds cannot converge. Therefore, the steady state cannot be stable in this case.

2-3. Price Adjustment Model

In the previous subsections, economic stability is attained mainly through the increase in tax revenues derived from the wealth effects of public bonds. However, the stability conditions will change in the case of introducing price adjustment that can absorb an increase in GDP through wealth effects. In this subsection, we consider the price level as the fourth dynamic variable using the Phillips Curve and Okun's Law.

$$Y_t = C(\overset{(+)}{(1-t)(Y_t + \frac{R_{t-1}B_{t-1}}{P_t})}, \overset{(+)}{\frac{B_{t-1} + M_{t-1} + K_{t-1}}{P_t}}) + I(\overset{(-)}{R_t - (\frac{P_{t+1}^e - P_t}{P_t})}, \overset{(-)}{K_{t-1}}) + G_t \quad (14)$$

$$\frac{M_t}{P_t} = l(\overset{(+)(-)}{Y_t, R_t}, \overset{(+)}{\frac{B_{t-1} + M_{t-1} + K_{t-1}}{P_t}}) \quad (15)$$

$$\frac{B_t}{P_t} = (1 + (1-t)R_{t-1})\frac{B_{t-1}}{P_t} + G_t - tY_t \quad (16)$$

$$K_t = K_{t-1} + I(\overset{(-)}{R_t - (\frac{P_{t+1}^e - P_t}{P_t})}), \overset{(-)}{K_{t-1}}) \quad (17)$$

$$\frac{P_{t+1} - P_t}{P_t} = \gamma(Y_t - \overset{(+)}{F(K_{t-1})}) \quad (18)$$

$F(K_{t-1})$ in (18) is a production function, so (18) implies that the inflation rate is determined by the gap between actual and potential GDP. [10] The expectation of the price level (P_{t+1}^{e}) is assumed to be perfectly foresighted. In addition to the assumptions in the previous subsections, and assuming a greater shift in the LM curve than in the IS curve when the price rises, we obtain the short–term equilibrium equations below.

$$Y_t = f(\overset{+}{R}_{t-1}, \overset{+}{B}_{t-1}, \overset{-}{K}_{t-1}, \overset{-}{P}_t) \quad R_t = g(\overset{+}{R}_{t-1}, \overset{+}{B}_{t-1}, \overset{+}{K}_{t-1}, \overset{+}{P}_t) \tag{19}$$

With no loss of generality, we can set the initial price level equal to unity. By using a linear approximation at the steady state, the dynamics for this model are represented in the following differential equation system:

$$\begin{pmatrix} \Delta b_t \\ \Delta r_t \\ \Delta k_t \\ \Delta p_{t+1} \end{pmatrix} = \begin{pmatrix} (1-t)\overline{R}-tf_{\overline{B}} & (1-t)\overline{B}-tf_{\overline{R}} & -tf_{\overline{K}} & \overline{G}-tf_{\overline{P}} \\ g_{\overline{B}} & g_{\overline{R}}-1 & g_{\overline{K}} & g_{\overline{P}} \\ I_{\overline{R}}(g_{\overline{B}}-\gamma f_{\overline{B}}) & I_{\overline{R}}(g_{\overline{R}}-\gamma f_{\overline{R}}) & I_{\overline{K}}+I_{\overline{R}}(g_{\overline{K}}-\gamma(f_{\overline{K}}-F')) & I_{\overline{P}}(g_{\overline{P}}-\gamma f_{\overline{P}}) \\ \gamma f_{\overline{B}} & \gamma f_{\overline{R}} & \gamma(f_{\overline{K}}-F') & \gamma f_{\overline{P}} \end{pmatrix} \begin{pmatrix} b_{t-1} \\ r_{t-1} \\ k_{t-1} \\ p_t \end{pmatrix} \tag{20}$$

As before, the necessary and sufficient conditions for the local stability are that the real parts of all eigenvalues of the coefficient matrix are negative. Although we would like to consider the stability as we did in the previous two subsections, it is quite difficult to do so in this extended model. [11] However, since what we would like to demonstrate is that the stability conditions here differ from those in the previous subsections, it is enough to explain that an unstable steady state possibly exists in this model when the stability conditions for the Variable Capital model are satisfied.

Suppose new bonds are issued in the Variable Capital model and its stability conditions are satisfied. The economy converges to its new steady state as both the bonds and the interest rate increase and as private capital decreases. Since the stability conditions are satisfied here, the increase in bonds raises tax revenues through wealth effects that are greater than interest payments. If the price level is introduced, however, the story changes because the increase in tax revenues depends on the increase in GDP. The growth of GDP raises the price level as well, which shrinks the growth of GDP itself. If this effect is large, the tax increase cannot cover the interest payments, and the amount of debt may diverge.

As shown in this section, the stability conditions differ among the models used. Even though it is possible to confirm the stability in one model, it does not necessarily hold true for another. In

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[10]   We consider constant labor supply for potential GDP, but do not deal with it explicitly.
[11]   We do not express them in matrix form due to their complexity, although it is possible.

other words, even when the data of public debt seems to diverge, other dynamic variables can work to make the economy converge.

As mentioned in the previous section, policies based on the macroeconomic theories do not work when the steady state is unstable. Therefore, checking the stability conditions for the Japanese economy is quite important not only for academic interests but also for practical policy debates. In the next section, we compute the conditions and check whether or not they are satisfied in the Japanese economy.

3. Empirical Analysis of the Stability Conditions

In this section, we estimate the local stability conditions for the Japanese macroeconomy and verify whether or not it is satisfied. Our methodology is as follows. First, we estimate a macroeconometric model consistent with our theoretical analyses. Next, for each of our three theoretical models, we compute the coefficient matrix of a differential system on the basis of the estimated parameters. Then we calculate the trace, the determinant and so on, and check the Routh–Hurwitz conditions. At the same time, we can obtain the eigenvalues of the coefficient matrix as solutions of the characteristic equations so that we also investigate the stability conditions by checking the signs of the real part of the eigenvalues.

3-1. Relation to Previous Literature

If the outstanding public debt is less than the amount that violates the stability conditions in macroeconomics, sustainable fiscal management is possible. In this sense, our empirical analysis is related to the literature on sustainability tests of the Japanese government's debts. Ihori, Nakazato and Kawade (2003) tested the fiscal sustainability of Japan's central and local governments using the methodology of Hamilton and Flavin (1986). Ihori, Doi and Kondo (2001) employed the approach suggested by Bohn (1998) and tested the sustainability of the General Account of the national government. These researches presented some empirical results that gave us a negative prospects for fiscal sustainability of the Japanese government. Both Hamilton and Flavin (1986) and Bohn (1998), however, deal with some variables such as the yield on bonds as exogenous. By contrast, our approach is based on the IS–LM type macro model and we handle the aggregate demand and the yield on bonds endogenously, making it possible to take into account the interdependence between these variables and the government's debts.

3-2. The Macroeconometric Model

Data for the goods and services markets are mainly obtained from the "Annual Report on National Accounts" (from fiscal year 1980 to 2002), compiled by the Economic and Social Research

Institute (Cabinet Office, Government of Japan). The financial asset market data is obtained from the "Financial and Economic Statistics (from fiscal year 1980 to 2002)" and "Flow of Funds Accounts (from fiscal year 1989 to 2002)", compiled by the Bank of Japan. All the macro data is based on 93SNA and the data frequency is annual. Under the restriction of an small sample, we simply employ OLS to estimate each behavioral equation. [12)]

We constructed the macroeconometric model on the basis of the Price Adjustment model introduced in Section 2−3. However, we added a considerable number of structural equations to the original theoretical model to enhance goodness of fit for the actual Japanese economy. Figure 1 shows an outline of our macroeconometric model. (See the appendix for all the structural equations and details of the data.) The major modifications are as follows:

1) The private sector is divided into households and firms in the goods markets.

2) The government sector is divided into central and local government. This means that we separate the budget balance or the outstanding debts between the central and the local first, and then we reconstruct "government debt" as the total of them.

3) Tax revenues of the government are classified by tax items: income tax (included in the national tax), inhabitant tax on individuals (included in the local tax), corporate tax revenue (included in the national tax) and inhabitant tax on corporations (included in the national tax). In addition, we estimate the elasticity with respect to the corresponding income (or profits), instead of the marginal/average tax rate used in the theoretical models.

4) The government bonds market represents financial asset markets in the econometric model, while the theoretical model employs the money market.
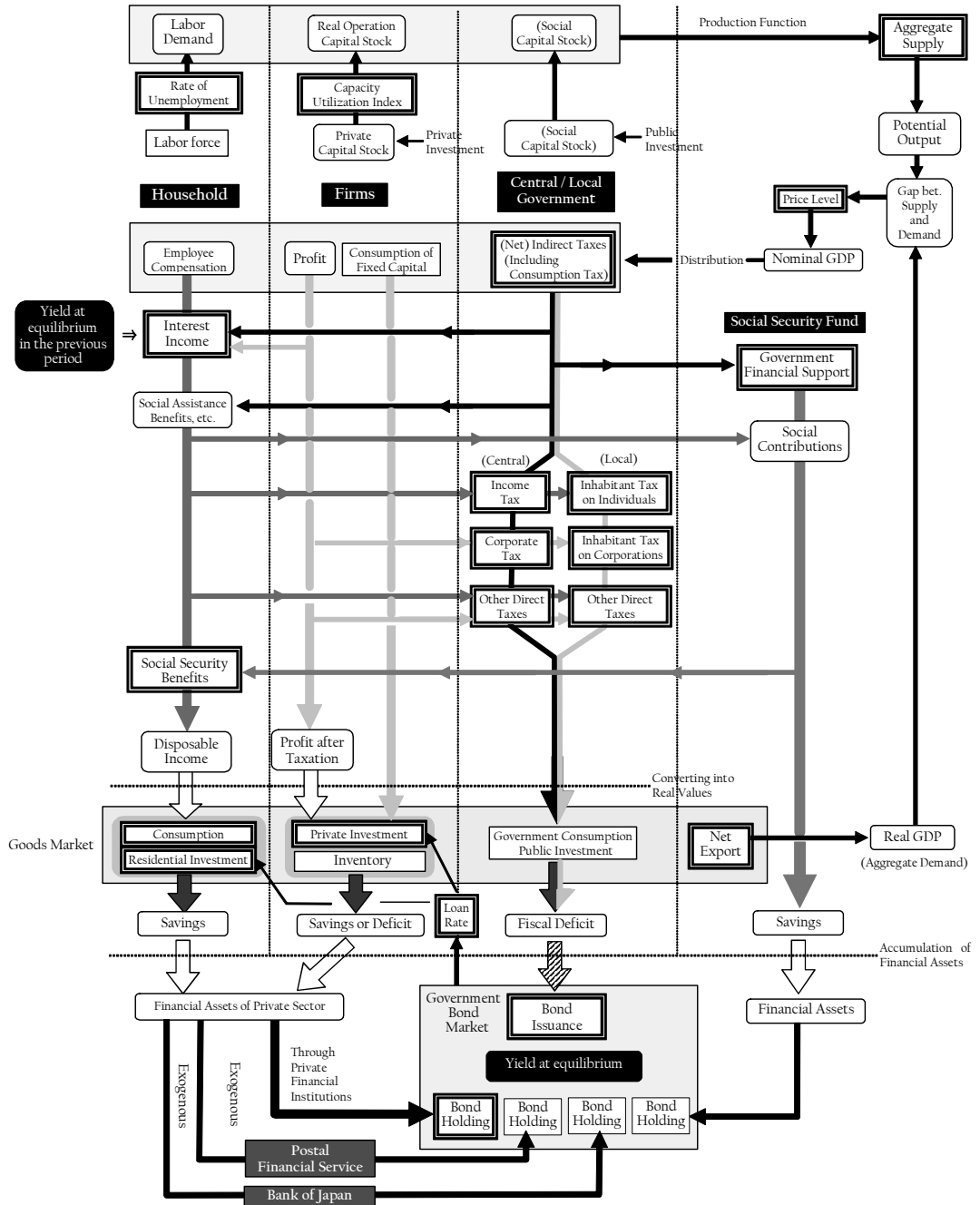
In our macroeconometric model, the structure of the actual Japanese financial markets is simplified as follows. First, the balance sheets of the private financial institutions are consolidated into that of the private non−financial sector. That is, we treat the portfolio selection of the private financial institutions exactly the same as that of the depositors or the insured for them. [13)] Second, the funding transactions between the private non−financial sector and the private financial institutions are offset by consolidating their balance sheets. Then, private financial instruments (bank loan, industrial debenture, etc.) are not explicitly considered in this model. Third, we treat postal savings and postal life insurance as an independent financial asset since, at present, the funds of public financial sectors are not completely operated in accordance with the market mechanism. [14)]

[12)]   The sample size available is quite small because of insufficient retroaction of 93SNA in Japan. This restriction makes the two−stage least squares estimator or the three−stage least squares estimator unreliable. Therefore, we simply employ OLS, although we recognize the problem of the simultaneous−equations bias.

[13)]   Actually, a private non−financial sector has only about 3% of the national bonds outstanding, while the private financial institutions own about 36% (at the end of fiscal year 2003). However, the private financial institutions demand deposits or life insurance mainly from the private non−financial sector. Then the private non−financial sector can be regarded as the ultimate holder of the government bonds held by the private financial institutions.

[14)]   The funds of postal savings and postal life insurance should be invested in safer assets because they are protected by government guarantee. They mainly purchase national and local government bonds. For example, their share in relation to all assets (excluding Fiscal Loan Deposits) is about 83% in postal savings at the end of fiscal year 2003.

Figure 1. Outline of the Macroeconometric Model

Labor Demand — Rate of Unemployment — Labor force

Real Operation Capital Stock — Capacity Utilization Index — Private Capital Stock ← Private Investment

(Social Capital Stock) — (Social Capital Stock) ← Public Investment

Production Function → Aggregate Supply → Potential Output

Household — **Firms** — Central / Local Government

Employee Compensation — Profit — Consumption of Fixed Capital — (Net) Indirect Taxes (Including Consumption Tax) ← Distribution ← Nominal GDP

Gap bet. Supply and Demand → Price Level

Yield at equilibrium in the previous period ⇒ Interest Income

Social Security Fund — Government Financial Support

Social Assistance Benefits, etc. — Social Contributions

(Central) — (Local)

Income Tax — Inhabitant Tax on Individuals

Corporate Tax — Inhabitant Tax on Corporations

Other Direct Taxes — Other Direct Taxes

Social Security Benefits

Disposable Income — Profit after Taxation

Converting into Real Values

Goods Market: Consumption / Residential Investment — Private Investment / Inventory — Government Consumption Public Investment — Net Export → Real GDP (Aggregate Demand)

Savings — Savings or Deficit — Loan Rate — Fiscal Deficit — Savings — Financial Assets

Accumulation of Financial Assets

Financial Assets of Private Sector — Exogenous — Exogenous — Through Private Financial Institutions

Government Bond Market — Bond Issuance — Yield at equilibrium — Bond Holding — Bond Holding — Bond Holding — Bond Holding

Financial Assets

Postal Financial Service

Bank of Japan

1) We regard ☐ as a behavioral equation, ☐ as an identity and ☐ as an exogenous variable.

2) In the actual macroeconometric model, budget balance (furthermore, outstanding debt) of the central government and the local government is separated once, and then we define "government debt" as the total of them.

3) The funding transactions between the non-financial sector and the private financial institutions are offset by consolidating their balance sheets. Then, private financial instruments (bank loans, industrial debentures, etc.) are not explicitly considered in this model.

4) The estimated coefficient of the social capital stock in the production function is not significant in this macroeconometric model.

So, we separate the balance sheets for them from those of the private sector. We assume that the amount of postal savings and postal life insurance deposited by the private non-financial sector is given. In addition, we also deal with the asset portfolio of the public financial sector as exogenous. Fourth, financial transactions with foreign countries are omitted.

By applying these simplifications, only three assets remain to be considered: money, government debts and postal financial services (postal savings and postal life insurance). However, because we are assuming the supply of postal financial services as exogenous, it is only the money market and the government bonds market that must be treated endogenously, which is consistent with our theoretical framework.

Now we add one more assumption regarding the government bond market. In Japan, the Japanese government bond (JGB) market is overwhelmingly larger than the market for local government bonds. Besides, the rate of return on local government bonds is determined in accordance with that on JGBs. We therefore suppose that the government debt market (shown in the lowest part of Figure 1) is represented by the JGB market in our macro econometric model.

To affirm the forecast performance of our macroeconometric model, we executed a simulation within the sample period of fiscal year 1998–2002. Table 1 presents the calculated Theil's U statistics for some of the important endogenous variables. [15]

Table 1. Calculated Theil's U statistics of major endogenous variables
: In the case of simulation within the sample period FY1998–2002

	Theil's U
Real gross domestic product (as aggregate demand)	0. 006
Real private consumption	0. 005
Real investment	0. 026
Real capital stock	0. 003
Nominal gross domestic product	0. 006
GDP deflator (1 period ahead)	0. 014
Nominal tax revenue of the central government	0. 012
Nominal tax revenue of the local government	0. 016
Real yields on Japanese government bonds	0. 015
Fiscal deficit / Nominal GDP ratio of the central government	0. 041
Fiscal deficit / Nominal GDP ratio of the local government	0. 095
Nominal outstanding public debt ※	0. 012

※ Net outstanding public debt: The sum of the net financial liability of the central and
   local governments (based on 93SNA).
   Net financial liability is calculated by subtracting the gross financial assets from the
   gross financial liabilities.

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[15] Theil's U statistics $\left(= \sqrt{\dfrac{(1/n)\sum_i(y_i - \hat{y}_i)^2}{(1/n)\sum_i y_i^2}} \right)$ is interpreted as a standardized version of a root mean squared error. A small value indicates a rich forecasting performance.

On the whole, the forecast accuracy of our macro model is high, although Theil's U statistics with respect to the deficit/GDP ratio of both the central and the local government is slightly large. Therefore we consider it legitimate to use this macroeconometric model for our estimation of the coefficient matrix.

3–3. *Estimating the Coefficient Matrix: Basic Methodology*

The methodology of estimating the coefficient matrix of the dynamic system is as follows. For example, we illustrate the two–variable dynamic system.

$$\begin{pmatrix} \Delta b_t \\ \Delta r_t \end{pmatrix} = \begin{pmatrix} b_t - b_{t-1} \\ r_t - r_{t-1} \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}\begin{pmatrix} b_{t-1} \\ r_{t-1} \end{pmatrix} \quad \text{where,} \quad b_t = B_t - \overline{B} \quad r_t = R_t - \overline{R} \tag{21}$$

First, we solve the macroeconometric model for the period of fiscal year 2002 without any restriction. Then, we obtain the solutions for the net public debt and the yield on government bonds and regard them as the stationary equilibrium values $(\overline{B}, \overline{R})$.

Second, we assume the situation in which an increase of 1000 units of the net public debt occurs in fiscal year 2001 while the yield on government bonds does not change (b_{t-1} = 1000, r_{t-1} = 0). Under this situation, we solve the macroeconometric model again for the period of fiscal year 2002 and then obtain the new solutions for the net public debt and the yield on government bonds $(\tilde{B}, \tilde{R})$.

By substituting $(\overline{B}, \overline{R})$ and $(\tilde{B}, \tilde{R})$ into (21), we can estimate both the [1,1] element (α) and the [2,1] element (γ) of the coefficient matrix.

$$b_t - b_{t-1} = (\tilde{B}_t - \overline{B}) - 1000 = \alpha \times 1000 \quad \Rightarrow \quad \alpha = (\tilde{B}_t - \overline{B} - 1000)/1000$$

$$r_t - r_{t-1} = (\tilde{r}_t - \overline{r}) - 0 = \gamma \times 1000 \quad \quad \Rightarrow \quad \gamma = (\tilde{r}_t - \overline{r})/1000$$

Next, we assume the new situation in which a rise of 1 units in the yield on government bonds occurs in fiscal year 2001 while the net outstanding public debt does not change (b_{t-1} = 0, r_{t-1} = 1 basis point). Under this new situation, we solve the macroeconometric model again for the period of fiscal year 2002 and then obtain the solutions for the net public debt and the yield on government bonds $(\overline{\overline{B}}, \overline{\overline{R}})$. We substitute $(\tilde{B}, \tilde{R})$ and $(\overline{\overline{B}}, \overline{\overline{R}})$ into (21), and calculate the [1,2] element (β) and [2,2] element (δ) of the coefficient matrix as shown below.

$$\beta = \overline{\overline{B}}_t - \overline{B} \quad , \quad \delta = \overline{\overline{r}}_t - \overline{r} - 1$$

By repeatedly using this methodology, we estimate the coefficient matrix of the dynamic system with three or four dynamic variables.

3–4. *The Stability Conditions of the Fixed Capital Model*

The dynamic system of the Fixed Capital model shown in Section 2–2 is as follows:

$$\begin{pmatrix} \Delta b_t \\ \Delta r_t \end{pmatrix} = \begin{pmatrix} b_t - b_{t-1} \\ r_t - r_{t-1} \end{pmatrix} = \begin{pmatrix} (1-t)\overline{R} - tf_{\overline{B}} & (1-t)\overline{B} - tf_{\overline{R}} \\ g_{\overline{B}} & g_{\overline{R}} - 1 \end{pmatrix} \begin{pmatrix} b_{t-1} \\ r_{t-1} \end{pmatrix} \tag{5´}$$

The left–hand side of the equation is the difference between the current and the previous fluctuation of the dynamic variable concerned. In this regard, the sign of the corresponding diagonal element indicates the dynamic stability of its own variable when we disregard the interaction with another dynamic variable.

If the sign is a minus, the current fluctuation of the dynamic variable concerned always falls below that in the previous period and this dynamic variable is in the process of converging to the steady state. However, when the dynamics are described as a simultaneous equations system, the dynamics of one variable are influenced by those of the other variables. For that reason, the variable concerned does not always converge only by the fact that the sign of a corresponding diagonal element is negative. The off–diagonal element of the coefficient matrix indicates the magnitude of the interaction between dynamic variables.

In this empirical analysis, we totaled the real net financial liability of the central and the local government from the "Flow of Funds Account" (based on 93SNA) and regard this total value as the real net outstanding public debt. [16] And, we use the real interest rate of the Japanese government 10–year bonds as the yield on government bonds. Equation (22) shows the estimated coefficient matrix of the dynamic system (5´). [17]

$$\begin{pmatrix} \Delta b_t \\ \Delta r_t \end{pmatrix} = \begin{pmatrix} -0.10 & 25.8 \\ 0.0057 & -0.84 \end{pmatrix} \begin{pmatrix} b_{t-1} \\ r_{t-1} \end{pmatrix} \tag{22}$$

   * The unit of b (fluctuation of real net outstanding public debt) is billion yen, while the unit of r (shift in the yield on bonds) is 1 basis point (0.01%).

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[16] Net financial liability is calculated by subtracting the gross financial assets from the gross financial liabilities.
[17] This macroeconometric model was constructed corresponding to the Price Adjustment model. Then the private capital and the price level are viewed as exogenous variables if we use this model as the Fixed Capital model. Similarly, when we use this model as the Variable Capital model, we execute the simulation handling the price level as an exogenous variable.

Here, we examine the estimated coefficient of the diagonal elements. The estimated value of the [1,1] element is −0.1, which implies that an increase in the tax revenue slightly exceeds an increase in the interest payment burden. Therefore, as long as we disregard the influence from fluctuation of the yield on government bonds, the real net outstanding public debt gradually converges to a steady state. On the other hand, the estimated value of the [2,2] element is −0.84. That is, if the yield on government bonds in the previous period rose by 1 basis point, the yield on current government bonds also rises, but the increment is 0.84 basis points smaller than that in the previous period. Therefore, the yield on government bonds converges as long as we disregard its dynamic interaction with the real net outstanding public debt.

Next, we examine the estimated coefficient values of the off−diagonal elements. The [1,2] element of the coefficient matrix shows the amount of fluctuation of the real net outstanding public debt for this period when the yield on government bonds in the previous period rose by 1 basis point. The estimated value is 25.8, and an increase in the interest payment burden greatly exceeds an increase in the tax revenue by the wealth effects. On the other hand, the [2,1] element shows the increment of the yield on government bonds for this period when the net outstanding public debt in the previous period increased by one unit (billion yen). In the IS−LM system, the yield on government bonds rises when the wealth effects work. The estimated value of this coefficient is 0.0057. Thus, in the dynamic system estimated as (22), there exists the interaction from which the stability of dynamic variables is mutually ruined.

The Routh−Hurwitz conditions of the coefficient matrix were calculated as follows:

Trace $= −0.10 −0.84 = −0.94 < 0$

Determinant $= −0.10 \times (−0.84) − 25.8 \times 0.0057 = −0.06 < 0$

The determinant does not satisfy the stability conditions, whereas the trace does satisfy them. Therefore, when considering the Japanese macroeconomy at the end of fiscal year 2002 to be in a state of stationary equilibrium, these values do not satisfy the local stability. We can also confirm the local stability conditions from the signs of the eigenvalues of the coefficient matrix. When the real part of each obtained eigenvalue takes a negative value, the local stability of stationary equilibrium is secured. The obtained eigenvalues are −1.00 and 0.06; then the necessary and sufficient conditions of the local stability are still not satisfied.

To obtain an intuitive understanding, we executed simple simulation using (22) only in the case where the net outstanding public debt at the end of the preceding period increases by 1000 units (1 trillion yen) while the yield on government bonds in the previous period is invariable (Table 2).

Table 2. Fluctuation of dynamic variables of the Fixed Capital model

	Fluctuation of dynamic variables	
	Net outstanding public debt (Real, trillion yen)	Yield of Japanese government bonds (Real, basis points)
Initial value	1. 00	0. 0
1st period	0. 90	5. 7
2nd period	0. 96	6. 1
3rd period	1. 02	6. 5
4th period	1. 08	6. 9
5th period	1. 15	7. 3
6th period	1. 23	7. 8

When the net public debt in the previous period increases by one trillion yen, the net increase of the net public debt in the 1st period remains at 900 billion yen. At the same time, however, the yield on government bonds rises by 5.7 basis points. As a result, the public debt newly increases to 960 billion yen in the 2nd period and the yield on government bonds rises more than the previous period by 6.1 basis points. After the 2nd period, the gains of the net outstanding public debt and the yield on government bonds always exceed those of the previous period. This is because the rise of the yield on government bonds expands the interest payment, causing further flotation of the government bonds. Hence, the net outstanding public debt and the yield on government bonds increase (rise) divergently.

The main factor in the collapse of the local stability conditions is in the [1,2] element of the coefficient matrix. When the amount of public debt is already large enough, the interest payment burden increases by a slight rise in the yield on government bonds. The net outstanding public debt diverges from the stationary state as a consequence and an increase in flotation of government bonds causes the yield on government bonds to rise further.

3–5. *The Stability Conditions of the Variable Capital Model*

The actual macroeconomy is not as simple as that described with the Fixed Capital model. In this subsection, we test the local stability of the Japanese macroeconomy using the Variable Capital model that includes real private capital as a dynamic variable. We estimated the dynamic system expressed in equation (11) on the basis of our macroeconometric model, and the result is as follows:

$$\begin{pmatrix} \Delta b_t \\ \Delta r_t \\ \Delta k_t \end{pmatrix} = \begin{pmatrix} -0.10 & 25.6 & 0.01 \\ 0.0057 & -0.84 & 0.0001 \\ -0.17 & -26.1 & -0.04 \end{pmatrix} \begin{pmatrix} b_{t-1} \\ r_{t-1} \\ k_{t-1} \end{pmatrix} \tag{23}$$

* The unit of b (fluctuation of the real net outstanding public debt) and k (fluctuation of the real private capital) is billion yen, while the unit of r (shift in the yield on bonds) is 1 basis point (0.01%).

The 2×2 principal submatrix is almost the same as the coefficient matrix of the Fixed Capital model; therefore, we focus only on the other elements here. The estimated value of the [3,3] element (−0.04) shows the stock adjustment speed of real private capital, and the [3,1] and [3,2] elements indicate the impact on real private capital for this period caused by fluctuation of a corresponding dynamic variable in the previous period. According to the estimated value of the [3,1] element, an increase of one unit in the net public debt in the previous period decreases real private capital by 0.17 units through the rise in interest rate. On the other hand, the estimated value of the [3,2] element (−26.1) is the decrement of private investment caused by a rise of one basis point in the interest rate in the previous period. These two negative elements imply that private capital decreases when the net public debt and/or the yield on government bonds increases.

The [1,3] and [2,3] elements show the influence that the dynamics of private capital exerts on the net public debt and the yield on government bonds. If private capital increases in the previous period, current private investment decreases because of the stock adjustment principle. Since this decreases the aggregate demand, tax revenues also decrease. The estimated value of the [1,3] element (0.01) shows a change in flotation of government bonds in accordance with such a decrease in tax revenues. And, the estimated value of the [2,3] element indicates that the yield on government bonds rises by 0.0001 basis points through an increase of one unit in private capital in the previous period. Of course, this influence acts in the reverse direction when private capital decreases in the previous period. Considering the effects of these off−diagonal elements, it seems that the dynamics of private capital has improved the stability of the net public debt.

The Routh−Hurwitz conditions of the coefficient matrix were calculated as follows:

Trace = −0.981 < 0

Determinant = −0.0006 < 0

−Trace×(the sum of all 2×2 principal minors of the coefficient matrix)+ Determinant =−0.018

cf. The eigenvalues of the coefficient matrix   (1.00, 0.01±0.03i)

Only the conditions of the trace and the determinant are satisfied from among the three

necessary and sufficient conditions concerning the local stability. Similarly, the real parts of all eigenvalues are not negative, so the local stability of the Japanese macroeconomy is not secured.

As in the case of the Fixed Capital model, we executed simple simulation using (23). We assumed the case in which the net outstanding public debt at the end of the preceding period increases by 1000 units (1 trillion yen) while the yield on government bonds and the real private capital in the previous period did not change. Table 3 presents the simulation results.

Table 3. Fluctuation of dynamic variables of the Variable Capital model

	Fluctuation of dynamic variables		
	Net outstanding public debt (Real, trillion yen)	Yield of Japanese government bonds (Real, basis points)	Private capital stock (Real, trillion yen)
Initial value	1. 00	0. 0	0. 0
1st period	0. 90	5. 7	−0. 17
2nd period	0. 95	6. 1	−0. 46
3rd period	1. 01	6. 4	−0. 76
4th period	1. 07	6. 7	−1. 06
5th period	1. 12	7. 1	−1. 37
6th period	1. 18	7. 4	−1. 69

Certainly, the amount of both the fluctuation of the net public debt and the yield on government bonds is smaller than that in the case of the Fixed Capital model shown in Table 2. However, the results here are the same as in the Fixed Capital model, to the extent that the fluctuations of dynamic variables increase as time passes. The Japanese macroeconomy diverges even under the Variable Capital model when judging from this result. Accordingly, although the dynamics of private capital slightly suppresses the macroeconomic divergence, this effect is not as large as the local stability is recovered. In other words, the interest payment burden of a large sum of government bonds makes the Japanese macroeconomy unstable even in the Variable Capital model.

3−6. *The Stability Conditions of the Price Adjustment Model*

In the Fixed Capital and Variable Capital models, it was assumed that the price level was invariable. In this section, we take more of a mid/long−term view. Concretely, we take price variation into consideration, and then affirm the dynamic stability of the Japanese macroeconomy. The estimated coefficient matrix of the dynamic system with respect to the Price Adjustment model shown in (20) is as follows:

$$
\begin{pmatrix} \Delta b_t^N \\ \Delta r_t \\ \Delta k_t \\ \Delta p_{t+1} \end{pmatrix} = \begin{pmatrix} -0.10 & 23.8 & 0.009 & -2482.9 \\ 0.0057 & -0.84 & 0.0001 & -10.2 \\ -0.17 & -26.0 & -0.04 & -2684.9 \\ 0.0000002 & 0.00003 & -0.00000005 & 0.0022 \end{pmatrix} \begin{pmatrix} b_{t-1}^N \\ r_{t-1} \\ k_{t-1} \\ p_t \end{pmatrix} \qquad (24)
$$

* The unit of b_N (fluctuation of the nominal net outstanding public debt) and k (fluctuation of the real private capital) is billion yen, while the unit of r (shift in the yield on bonds) is 1 basis point (0.01%). And, the unit of p (fluctuation of the price level) is 0.01 when the price level in 1995 is standardized to be unity.

The net public debt has been changed to nominal value based, so that we may consider the price level to be an endogenous variable in the Price Adjustment model. However, the estimated coefficients of the 3×3 principal submatrix are almost the same as those of the coefficient matrix in the Variable Capital model. Therefore, we will focus on the other elements of the matrix in the following.

It can be seen from the signs of the off-diagonal elements in the fourth row that an increase in the net public debt, a rise of the yield on government bonds, and a decrease in private capital are all factors of inflation. On the other hand, the estimated [1,4] and [2,4] elements are negative. Hence, the net public debt decreases and the yield on government bonds falls when the price level goes up, because tax revenues increase as nominal GDP increases. If these effects are sufficiently large, the dynamics of the net public debt and the yield on government bonds become stable as a result of fluctuations in price. The estimated value of the [3,4] element is negative, which means that the inflation of prices decreases the private capital divergently. In addition, the estimated value of the [4,4] diagonal element shows that the present inflation (or deflation) raises (lowers) the price level further in the succeeding periods. [18] If these influences are large, the dynamics of private capital and the price level become unstable as a result of fluctuations in price. We confirmed whether or not the necessary and sufficient conditions concerning the local stability were satisfied, by checking the sign of the real part of each eigenvalue of the coefficient matrix in (24). [19] The set of calculated eigenvalues are ($\lambda_1 = -0.99$, $\lambda_2 = -0.03$, λ_3, $\lambda_4 = 0.02 \pm 0.05i$); then the necessary and sufficient conditions were still not satisfied.

4.   Policies to Restore Macroeconomic Stability

In the previous section, we obtained the result that the steady state is locally unstable in each

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[18]   The sign of the estimated [4, 4] diagonal element is thought to reflect the so-called "deflation spiral" that the Japanese economy has experienced since the 1990s.
[19]   We only checked the signs of the eigenvalues of the coefficient matrix, because the expression of the 4×4 matrix version of the Routh-Hurwitz condition is very complex.

model, mainly because of a huge amount of issued public bonds. In this case, a slight rise in interest rates boosts interest payments, which causes the government to issue additional bonds so that the outstanding amount diverges. At the same time, market interest rates rise divergently and private capital is crowded out, then the economy diverges completely. In this section, we consider stability−restoring policies to avoid this catastrophe

To begin with, we compute the level of net outstanding public debt that can be maintained in the current economic situation in Japan using the Price Adjustment model. The terms of the current economy are defined such that 1) structural parameters of the economy are equal to the estimated parameters and they are invariable, and 2) all exogenous variables of the economy are evaluated at the fiscal year 2002 level.

In our model, interest on public debt at time t−1 is paid at time t by the government, so it is public debt in fiscal year 2001 that affects the economy in fiscal year 2002. As previously mentioned, the BOJ's "Flow of Funds Accounts"−based net debt / GDP ratio is about 104% at the end of fiscal year 2002. (The gross debt / GDP ratio is about 146%.) In the following, we suppose that the ratio in fiscal year 2001 is equal to 100% at first and confirm the stability using the same method as in the previous section. If the stability conditions are not satisfied, we reduce the ratio by 5% and check it again. Repeating this procedure until the condition is fulfilled, we obtain the level of public debt that allows for a stable Japanese economy.

The purpose here is only to estimate the level numerically, so we assume that the initial reduction in the net public debt is absorbed by the Bank of Japan. In addition, it is assumed that a change in public debt derived by dynamic mechanism is purchased by the private sector. The results are shown in Table 4.

Table 4. The level of net public debt needed to satisfy the stability conditions: Tax rates and government spending fixed

Outstanding amount of nominal net public debt (March 31, 2002;Ratio to nominal GDP)	Actual value (104%)	100%	95%	90%	85%	80%	75%	70%	65%	60%
Eigenvalues of the coefficient matrix	−0. 992	−0. 988	−0. 993	−0. 989	−0. 994	−0. 999	−0. 995	−1. 000	−0. 996	−1. 001
	−0. 032	−0. 031	−0. 031	−0. 030	−0. 030	−0. 029	−0. 028	−0. 028	−0. 027	−0. 026
	+0. 021 $\pm$0. 050i	+0. 019 $\pm$0. 051i	+0. 016 $\pm$0. 052i	+0. 014 $\pm$0. 052i	+0. 011 $\pm$0. 053i	+0. 008 $\pm$0. 054i	+0. 006 $\pm$0. 054i	+0. 003 $\pm$0. 055i	+0. 001 $\pm$0. 055i	−0. 002 $\pm$0. 055i
Number of eigenvalues with a negative real part	2/4	2/4	2/4	2/4	2/4	2/4	2/4	2/4	2/4	4/4
Satisfaction of the stability conditions	×	×	×	×	×	×	×	×	×	○

Satisfaction of the stability conditions: "○" if satisfied, "×" otherwise.

Assuming that government spending is fixed at the fiscal year 2002 level, a level of only 60% of the net public debt / GDP ratio could be accepted for the stability. [20] However, the assumption that the Bank of Japan absorbs such amount of public debt is not realistic. Considering that social security expenditures must increase due to aging, a reduction in government expenditures would be difficult as well. Hereafter, we compute the level of public debt that could restore the stability by a tax increase only.

First, we supposed a raise in consumption tax rate to 10% from 5%. Then, we continued to reduce the net public debt / GDP ratio by decrements of 5% until the conditions were satisfied. [21] The results are summarized in Table 5.

Table 5. The level of public debt needed to satisfy the stability conditions: In the case of 10% consumption tax rate

Outstanding amount of nominal net public debt (March 31, 2002; Ratio to nominal GDP)	Actual value (104%)	100%	95%	90%	85%	80%	75%
Eigenvalues of the coefficient matrix	−0.997	−0.993	−0.998	−0.994	−0.999	−0.995	−1.000
	−0.031	−0.030	−0.030	−0.028	−0.029	−0.028	−0.026
	+0.016 ±0.053i	+0.014 ±0.053i	+0.011 ±0.053i	+0.008 ±0.052i	+0.006 ±0.055i	+0.004 ±0.055i	−0.00001 ±0.054i
The number of eigenvalues with a negative real part	2/4	2/4	2/4	2/4	2/4	2/4	4/4
Satisfaction of the stability conditions	×	×	×	×	×	×	○

Satisfaction of the stability conditions: "○" if satisfied, "×" otherwise.

Even in the case of the 10% consumption tax rate, we need to decrease the net public debt / GDP ratio to 75% or less to restore the stability. This means that a 5% rise in the consumption tax is not sufficient to absorb the current interest payments. Hence, we perform the same simulation under the 15% consumption tax, the results of which are shown in Table 6.

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[20]   When we reduced the ratio of the net outstanding public debt to nominal GDP from 65% in 1% decrements, the necessary and sufficient conditions of the local stability were satisfied at 63%.
[21]   When we estimate the coefficient matrix of the differential system using the methodology introduced in Section 3-3, we are assuming an economy in which the consumption tax rate is always 10%. That is, we are not deriving the coefficient matrix from a comparison of the state before and after the hike in the consumption tax rate. What we want to analyze is not the temporary impact of the tax increase but the permanent effects in the steady state.

Table 6. The level of public debt needed to satisfy the stability conditions: In the case of 15% consumption tax rate

Outstanding amount of nominal net public debt (March 31, 2002; Ratio to nominal GDP)	Actual value (104%)	100%	95%	90%	85%	80%
Eigenvalues of the coefficient matrix	−0. 993	−0. 999	−0. 995	−1. 000	−1. 005	−1. 001
	−0. 034	−0. 029	−0. 031	−0. 029	−0. 028	−0. 026
	+0. 013 ±0. 050i	+0. 009 ±0. 052i	+0. 008 ±0. 053i	+0. 004 ±0. 055i	+0. 001 ±0. 055i	−0. 002 ±0. 054i
The number of eigenvalues with a negative real part	2/4	2/4	2/4	2/4	2/4	4/4
Satisfaction of the stability conditions	×	×	×	×	×	○

Satisfaction of the stability conditions: "○" if satisfied, "×" otherwise.

As is hardly different from the last case, it is necessary to reduce the ratio to 80% or less to restore the stability even under the 15% consumption tax rate.

We can continue these procedures but it would be infeasible to raise the consumption tax rate more than 15% due to political factors. Therefore, next we consider income taxes to increase government revenues. As discussed in Section 3−2, income taxes and inhabitant taxes can be estimated through an income tax function and an inhabitant tax function in our macroeconometric model. In these equations, the parameters are interpreted as the income elasticity of these taxes and are estimated as 1.17 and 1.29 each in fiscal year 2002 (see Appendix). Assuming that we can control the elasticity through, for example, abolition of the so−called permanent tax reduction in income taxes, taxation on public pension and so forth, we simulate these effects on the stability. In this simulation, we increase the elasticities of both taxes by increments of 0.001 each until the conditions are satisfied when the economy is unstable with the 15% consumption tax rate. Table 7 shows the results.

Table 7. The effect of the increase in income taxes and inhabitant taxes under the 15% consumption tax rate

Outstanding amount of nominal net public debt (March 31, 2002; Ratio to nominal GDP)	Actual value (104%)	100%	95%	90%	85%
Increase in the elasticity of both income taxes (national tax) and inhabitant taxes on individuals (local tax)	+0. 033	+0. 028	+0. 020	+0. 015	+0. 010
Estimated amount of tax increase (trillion yen)	Approx. ¥8. 3	Approx. ¥7. 0	Approx. ¥4. 7	Approx. ¥3. 5	Approx. ¥2. 2
Eigenvalues of the coefficient matrix	−0. 996	−0. 998	−0. 998	−0. 999	−1. 001
	−0. 026	−0. 026	−0. 027	−0. 027	−0. 029
	−0. 0008 ±0. 0520i	−0. 0012 ±0. 0521i	−0. 0002 ±0. 0539i	−0. 0008 ±0. 0537i	−0. 0009 ±0. 0544i
The number of eigenvalues with a negative real part	4/4	4/4	4/4	4/4	4/4

In the actual situation at the end of fiscal year 2002, such that the net public debt / GDP ratio is equal to approximately 104%, we need a tax reform allowing the income elasticity of income taxes and inhabitant taxes on individuals to increase by 0.033 each, in addition to raising the consumption tax rate to 15%. With this tax reform, the estimated amount of tax income increase is 8.3 trillion yen. This amount is much larger than each year reduction of 4.1 trillion yen in so−called permanent tax reduction that was initiated in fiscal year 1999, which means that we need a drastic fiscal reform including a tax increase to restore dynamic stability for the Japanese macroeconomy.

In the present study, we used the net debt / GDP ratio based on the BOJ's "Flow of Funds Accounts" in fiscal year 2002. However, the value amounts to about 118% at the end of fiscal year 2003 (166% for the gross debt / GDP ratio). Taking this figure into account, significantly more severe fiscal reform including tax increase would be essential for the economic stability of Japan.

5. Summary

We constructed three IS−LM type dynamic models to estimate the eigenvalues of their differential systems in order to confirm whether or not the huge amount of public debt violates the stability conditions for the Japanese economy. When the real parts of all eigenvalues are positive, the steady state is unstable and there are no grounds for the effectiveness of any policies based on the theory. To the best of our knowledge, this is the first study that explicitly estimates the coefficient matrix of the differential system, which is the distinguishing feature of this paper.

Under the assumption that the Japanese economy is at the steady state in fiscal year 2002, our estimation shows that the equilibrium is the saddle point, which is equivalent to unstable in our

framework with no jumpable variables. Unless we decrease the BOJ's "Flow of Funds Accounts"−based net debt / GDP ratio from about 104% to about 60% at the end of fiscal year 2002, we could not restore economic stability. Otherwise, a slight rise in the interest rates boosts public debt by increased interest payments, which swells public debt even further. It is expected that this cumbersome situation would be revealed with economic recovery; therefore, reduction in public debt could be the emergent policy issue for Japan's economic stability.

We also consider tax policies to restore the economic stability. As explained, we could not decrease government spending for this purpose in the simulation since government expenditure is exogenous in the IS−LM type framework used here. Our estimation shows that we need a tax reform to raise the consumption tax rate to 15%, and in addition, allowing the income elasticities of income taxes and inhabitant taxes to increase by 0.033 each. This 0.033 rise in the income elasticities is equivalent to tax hikes of about 8.3 trillion yen. Given that revenues from income taxes and inhabitant taxes totaled about 23 trillion yen in fiscal year 2002, this tax reform must be severe.

Finally, we summarize the relationships with the relevant studies introduced in the first section. First, although some literature such as Blinder−Solow (1973) and Turnovsky (1977) stresses that the stability conditions of the IS−LM type dynamic model should be tested empirically, no such study has been conducted so far. Our first trial shows that the Japanese economy is unstable. Second, as is mentioned in Blinder−Solow (1973), the stability conditions are sufficient conditions for the sustainability of the government budget. Since the Japanese economy is unstable as we estimated, it would be quite difficult to sustain the budget deficit of the government. Therefore, we assert again that structural reform for the government budget including a tax system is essential and emergent.

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Appendix. All Structural Equations of the Macroeconometric Model

This macroeconometric model is estimated mainly on the basis of three official statistical sources. Data for the goods and services markets is mainly from the "Annual Report on National Accounts" (from fiscal year 1980 to 2002) by the Economic and Social Research Institute (Cabinet Office, Government of Japan). We obtained the financial data from the "Financial and Economic Statistics (from fiscal year 1980 to 2002)" and "Flow of Funds Accounts (from fiscal year 1989 to 2002)", which are compiled by the Bank of Japan. All of this macro data is based on 93SNA and the data frequency is annual. The sources for the other data that we used are as follows:

"Annual Report on Population Estimates",
　(Statistics Bureau, Ministry of Internal Affairs and Communications)
"Fiscal Statistics" (Ministry of Finance)
"Gross Capital Stock of Private Enterprises",
(Economic and Social Research Institute, Cabinet Office)
"Labour Force Survey" (Statistics Bureau, Ministry of Internal Affairs and Communications)
　"Monthly Finance Review" (Policy Research Institute, Ministry of Finance)
"Monthly Labour Survey" (Ministry of Health, Labour and Welfare)
"Social Capital of Japan" (Cabinet Office)

It was necessary to estimate the equations using a small sample because of insufficient retroaction of 93SNA in Japan. Under this restriction, the two–stage least squares estimators and the three–stage least squares estimators are unreliable. Therefore, we simply employ OLS, although we recognize the problem of the simultaneous–equations bias. The only exception is the production function, which we estimated by the maximum likelihood method considering the first–order autocorrelation of the error term.

[Note]

1) "D_-_" in the equation denotes a dummy variable that takes '1' in the corresponding period. For instance, "D91_94" is a dummy variable in which '1' is taken from fiscal year 1991 to 1994. We express the fiscal year 2000 as "00"; similarly, fiscal year 2001 (2002) as "01(02)".

2) "・" (dot) on the variable denotes the rate of change from the previous year, while "Δ"(delta) ahead of the variable denotes the amount of the change from the previous year.

3) The t–statistics are shown in parentheses. '**' denotes coefficients significantly different from zero at the 5% level, and '*' at the 10% level.

4) The real value of each variable is standardized by the price level in 1995.

5) The unit of the amount is "billion yen", the unit of the interest rate and that of the rate of change is "%", and the unit of the population is "10,000 persons". The price deflators are standardized so that the value in 1995 might become 1.

6) Series that existed only in the calendar–year–base were converted into fiscal–year–based series using the following equation:

Pseudo fiscal–year–based value = 0.75 × the value of the present calendar year

+ 0.25 × the value of the next calendar year

Structural Equations

1. Real Aggregate Demand

・Real gross domestic expenditure

$$RY = RCP + RIH + RIP + RCG + RIG + RIF + RIN + (REX - RIM)$$

・Real private consumption expenditure

FY1981–2002

$$RCP = 23500 + 5150 \times D95\_96$$
$$[4.14^{**}]\ [4.70^{**}]$$
$$- 5450 \times D99\_00$$
$$[-4.72^{**}]$$
$$+ (0.67 - 0.02 \times D81\_85) \times DI\_H / P_Y$$
$$[20.7^{**}][-4.46^{**}]$$
$$+ 0.04 \times A\_H_{-1} / P_Y$$
$$[10.5^{**}]$$

$$R^2(adj) = 0.99 \quad DW = 2.03$$

・Real private residential investment

FY1981–2002

$$RIH = -3770$$
$$[-0.54]$$
$$- 2620 \times D84\_85 - 2720 \times D01\_02$$
$$[-3.13^{**}] \qquad [-2.92^{**}]$$
$$+ (0.28 - 0.02 \times D97\_02) \times DI\_H/P_Y$$
$$[4.71^{**}]\ [-6.25^{**}]$$
$$- (14.1 + 1110 \times D91\_02) \times (RL - \dot{P}_Y)$$
$$[-0.03][-4.34^{**}]$$
$$- 2530 \times KH_{-1} / POP$$
$$[-2.32^{**}]$$

$$R^2(adj) = 0.89 \quad DW = 2.07$$

・Real private equipment investment
(Based on "Gross Capital Stock of Private Enterprises")

FY1981–2002

$$RIP\# - \delta_P \times KP_{-1}$$
$$= 68900 + 6010 \times D94 - 7420 \times D02$$
$$[8.85^{**}]\ [2.33^{**}] \qquad [-2.59^{**}]$$
$$+ (0.87 + 0.08 \times D99\_00) \times (DI\_F + CF\_F) / P_Y$$
$$[5.99^{**}][2.84^{**}]$$
$$+ (10800 + 3380 \times D90\_01) \times P_{L-1} / P_Y$$
$$[8.25^{**}]\ [3.75^{**}]$$
$$- 4910 \times (RL + \delta_P - \dot{P}_I) - 0.06 \times KP_{-1}$$
$$[-7.85^{**}] \qquad\qquad [-7.20^{**}]$$

$$R^2(adj) = 0.99 \quad DW = 1.97$$

・Real private equipment investment
(Based on "Annual Report on National ccounts")

$$RIP = ADJ \times RIP\#$$

・Real general government consumption

$$RCG = MED / P_{CG} + (RC\_CG + RC\_LG)$$

・Real general government investment

$$RIG = RI\_CG + RI\_LG$$

・Real exports of goods and services

FY1981–2002

$$\log(REX) = 4.38 + 0.04 \times D00 - 0.08 \times D01$$
$$[15.8^{**}]\ [2.00^{**}] \qquad [-4.33^{**}]$$
$$+ (1.41 - 0.03 \times D98\_02) \times \log(USRY)$$
$$[22.6^{**}][-6.18^{**}]$$
$$+ (0.002 + 0.0007 \times D91\_02) \times ER \times P_U / P_Y$$
$$[8.42^{**}] \quad [5.71^{**}]$$

$$R^2(adj) = 0.99 \quad DW = 1.63$$

・Real imports of goods and services

FY1980–2002

$$\log(RIM) = -13.4 \quad +0.10 \times D89\_90$$
$$[-13.3^{**}]\ [3.84^{**}]$$

$$+1.83 \times \log(RY)$$
$$[24.8^{**}]$$

$$- (0.001 +0.0010 \times D85\_89$$
$$[-2.45^{**}][-6.54^{**}]$$

$$+0.0013 \times D91\_94 - 0.0005 \times D00\_02)$$
$$[-7.19^{**}] \qquad [2.47^{**}]$$

$$\times ER \times (P_U / P_Y)$$

$$R^2(adj) = 0.99 \quad DW = 2.02$$

2. Real Aggregate Supply

・Private sector, Real domestic product

FY1982–2002

$$\log(RYP / LDP) = -0.685 +0.01 \times TIME$$
$$[-0.90]\ [2.89^{**}]$$

$$+(0.22 +0.16 \times D82\_89)$$
$$[2.93^{**}][3.88^{**}]$$

$$\times \log((KP_{-1} \times CU) / LDP)$$
$$- 0.02 \times \log(KG_{-1}) +0.32 \times e_{-1}$$
$$[-0.29] \qquad\qquad [1.65^{*}]$$

$$R^2(adj) = 0.99 \quad DW = 1.84$$

・General government, Real domestic product

FY1983–2002

$$RYG = -16700 +1.03 \times RYG_{-1}$$
$$[-1.92^{*}]\ [64.3^{**}]$$

$$+(39.8 \quad +1.17 \times D98\_02) \times LDG$$
$$[2.14^{**}]\ [2.34^{**}]$$

$$R^2(adj) = 0.99 \quad DW = 1.69$$

・Private sector, Labor demand

[10000 persons * Working hours]

$$LDP = ((1-RU) \times LS - LDG) \times LT$$

・Rate of unemployment in labor force

FY1985–2002

$$\log(RU) \quad = -3.25 \quad +0.06 \times D95\_99$$
$$[-27.6^{**}]\ [3.70^{**}]$$

$$- 0.05 \times D97 - 0.05 \times D00$$
$$[-1.86^{*}] \qquad [-1.71^{*}]$$

$$- 5.25 \quad \times \log(RY / SY95)$$
$$[-12.5^{**}]$$

$$+0.78 \times \log(RU_{-1})$$
$$[27.1^{**}]$$

$$R^2(adj) = 0.99 \quad DW = 1.71$$

・Capital utilization index

FY1986–2002

$$\dot{CU} = -0.04$$
$$[-5.81^{**}]$$

$$+0.05 \times (D89 +D94\_95$$
$$[8.86^{**}] \qquad +D97 +D99 +D02)$$

$$+(0.94 \quad -0.46 \times D(\text{if } \Delta YKE/P_Y <0))$$
$$[8.19^{**}][-2.32^{**}]$$

$$\times YKE / P_Y$$

$$R^2(adj) = 0.96 \quad DW = 1.66$$

* D(if __) denotes the dummy variable that takes '1' if the condition in parentheses is satisfied.

・Potential Gross Domestic Product (Real)

$$SY = e^{-0.685 +0.01 \times TREND}$$
$$\times (KP_{-1} \times CU^{*})^{(0.22+0.16 \times D82\_89)}$$
$$\times ((1-RU^{*}) \times LS)^{(1- (0.22+0.16 \times D82\_89))}$$
$$+ RYG$$

3. Real Capital Stock

· Real private capital stock (equipment)
$$KP = (1 - \delta_P) \times KP_{-1} + RIP$$

· Real housing stock
$$KH = (1 - \delta_H) \times KH_{-1} + RIH$$

· Real social capital stock
$$KG = (1 - \delta_G) \times KG_{-1} + (RIG + RIF)$$

4. GDP Deflator and Nominal GDP

· GDP deflator
$$P_Y = (1 + RMP) \times PT_Y$$

· GDP deflator excluding consumption tax factor
FY1990–2002

$$\log(PT_Y / PT_{Y-1})$$
$$= 0.017 - 0.005 \times D98$$
$$[13.8^{**}] \; [-2.91^{**}]$$
$$- 0.01 \times (D95\_97 + D00\_01)$$
$$[-10.1^{**}]$$
$$+ 0.68 \times \log(1 + (RY_{-1} - SY_{-1})/RY_{-1})$$
$$[23.5^{**}]$$

$$R^2(adj) = 0.99 \quad DW = 2.02$$

· Nominal GDP
$$NY = P_Y \times RY$$

5. Nominal Aggregate Demand

(Limited to the ones that are required when we consider the nominal balance of savings and investments of household, firm and government.)

· Nominal private consumption expenditure
$$NCP = RCP \times P_{CP}$$

· Nominal private residential investment
$$NIH = RIH \times P_{IH}$$

· Nominal private equipment investment
$$NIP = RIP \times P_{IP}$$

· Nominal inventory investment
$$NIN = RIN \times P_{IN}$$

· Nominal general government consumption
$$NCG = (RC\_CG + RC\_LG) \times P_{CG} + MED$$

· Nominal general government investment
$$NIG = (RI\_CG + RI\_LG) \times P_{IG}$$

6. Deflator of each demand component

(Limited to the ones that are required when we consider the nominal balance of savings and investments of household, firm and government.)

· Deflator of private consumption expenditure
$$P_{CP} = (1 + RMP) \times PT_{CP}$$

· Deflator of private consumption expenditure excluding consumption tax factor
FY1981–2002

$$\log(PT_{CP}) = -0.01 \times D81\_86$$
$$[-3.26^{**}]$$
$$+ 0.04 \times \log(P_{IM}) + 0.40 \times \log(PT_Y)$$
$$[5.42^{**}] \qquad\qquad [11.5^{**}]$$
$$+ 0.62 \times \log(PT_{CP-1})$$
$$[17.8^{**}]$$

$$R^2(adj) = 0.99 \quad DW = 1.77$$

· Deflator of private residential investment
$$P_{IH} = (1 + RMP) \times PT_{IH}$$

· Deflator of private residential investment
 excluding consumption tax factor

$$\text{FY1982–2002}$$

$$\log(PT_{IH}) = 0.008$$
$$[2.26^{**}]$$

$$- 0.03 \times D85\_86 - 0.02 \times D98$$
$$[-4.01^{**}] \qquad [-2.03^{**}]$$

$$+0.39 \times \log(PT_Y)$$
$$[4.75^{**}]$$

$$+0.65 \times \log(PT_{IH-1})$$
$$[11.1^{**}]$$

$$R^2(\text{adj}) = 0.98 \quad DW = 2.36$$

· Deflator of private equipment investment:
 Exogenous

· Deflator of general government consumption
$$P_{CG} = (1+RMP) \times PT_{CG}$$

· Deflator of general government consumption
 excluding consumption tax factor

$$\text{FY1981–2002}$$

$$\log(PT_{CG}) = 0.008 \quad - 0.01 \times D99$$
$$[3.27^{**}] \; [-2.07^{**}]$$

$$+0.54 \times \log(PT_{CP})$$
$$[6.47^{**}]$$

$$+ 0.58 \times \log(PT_{CG-1})$$
$$[10.5^{**}]$$

$$R^2(\text{adj}) = 0.99 \quad DW = 1.70$$

· Deflator of inventory investment: Exogenous

· Deflator of public investment: Exogenous

7. Distribution of Nominal GDP

· National income
$$YD = NY - CF - (CTX + OTX - SUB)$$

· Consumption tax

$$\text{FY1989–2002}$$

$$CTX = -2840$$
$$[-18.8^{**}]$$

$$-2050 \times D97 + 487 \times D98 + 725 \times D99$$
$$[-21.8^{**}] \qquad [5.18^{**}] \qquad [7.69^{**}]$$

$$+ (0.77 \quad +0.12 \times D90\_96)$$
$$[90.2^{**}] \, [17.9^{**}]$$

$$\times RCTX \times (NCP + NIH + NCG + NIG)$$

$$R^2(\text{adj}) = 0.99 \quad DW = 2.00$$

· Indirect taxes excluding consumption tax

$$\text{FY1981–2002}$$

$$OTX = 5910$$
$$[5.45^{**}]$$

$$+4020 \times D87\_88 + 1290 \times D95\_96$$
$$[7.03^{**}] \qquad [2.33^{**}]$$

$$+0.06 \times YLE \quad +0.08 \times YKE$$
$$[13.2^{**}] \qquad [5.12^{**}]$$

$$R^2(\text{adj}) = 0.97 \quad DW = 1.56$$

· Subsidies
$$SUB = SB\_CG + SB\_LG$$

· Consumption of fixed capital
$$CF = CF\_H + CF\_F + CF\_CG + CF\_LG + CF\_O$$

· Employee compensation
$$YLE = \alpha \times YD$$

· Operating surplus of firm sector
$$YKE = YD - YLE$$

・Operating surplus
   (Share of the private unincorporated enterprises)
   $YKE\_H = \beta_H \times YKE$

・Operating surplus
   (Share of the private corporations)
   $YKE\_F = \beta_F \times YKE$

8. Tax Revenue of the Central Government

・Central government, Tax revenue
   $TAX\_CG = HTX + FTX + DTX\_CG + CPTX$
   $\qquad\qquad\qquad + CTX\_CG + OTX\_CG$

・Income tax (National taxes)
                              FY1981–2002
   $\log(HTX) = -5.16 + 0.12 \times D89\_91$
   $\qquad\qquad [-7.45^{**}][4.92^{**}]$

   $\qquad -0.06 \times D99 + 0.14 \times D00\_01$
   $\qquad [-1.57] \qquad [4.25^{**}]$

   $\qquad +(1.21 -0.02 \times D94\_97 -0.03 \times D98\_02)$
   $\qquad [21.4^{**}][-7.73^{**}] \qquad\quad [-11.2^{**}]$

   $\qquad \times \log(YLE + YKE\_H + RI\_H - PI\_H - PEN\_C)$

   $\qquad\qquad R^2(adj) = 0.98 \qquad DW = 1.79$

・Corporate tax (National taxes)
                              FY1982–2002
   $\log(FTX) = -1.53$
   $\qquad\qquad [-1.67^{*}]$

   $\qquad +0.31 \times D89\_91 -0.07 \times D02$
   $\qquad [8.76^{**}] \qquad\quad [-2.02^{**}]$

   $\qquad +(1.07 +0.02 \times D91\_94 -0.02 \times D00\_02)$
   $\qquad [12.0^{**}][6.83^{**}] \qquad\quad [-2.96^{**}]$

   $\qquad \times \log(YKE\_F + RI\_F - PI\_F)$

   $\qquad\qquad R^2(adj) = 0.94 \qquad DW = 1.92$

・Inheritance tax, donation tax (National taxes)
   (Including Land−value tax, FY1992–1997)
   $\qquad\qquad\qquad\qquad\qquad\qquad$ FY1981–2002

   $CPTX = -849$
   $\qquad\quad [-4.56^{**}]$

   $\qquad -578 \times D90 -791 \times D91 -258 \times D02$
   $\qquad [-2.89^{**}] \quad [-4.19^{**}] \qquad [-1.60]$

   $\qquad +0.004 \times NY$
   $\qquad [10.3^{**}]$

   $\qquad +(4.97 +3.58 \times D91\_97) \times L_P$
   $\qquad [6.25^{**}][6.19^{**}]$

   $\qquad\qquad R^2(adj) = 0.96 \qquad DW = 1.78$

・Central government, Other direct taxes
   $DTX\_CG = \nu_{CG} \times (ETX\_H + ETX\_F)$

・Consumption tax
   (Share of the central government)
   $CTX\_CG = \gamma_{CG} \times CTX$

・Indirect taxes excluding consumption tax
   (Share of the central government)
   $OTX\_CG = \theta_{CG} \times OTX$

9. Tax Revenue of the Local Government

・Local government, Tax revenue
   $TAX\_LG = LHTX + LFTX + DTX\_LG$
   $\qquad\qquad\qquad + CTX\_LG + OTX\_LG$

・Inhabitant tax on individuals (Local taxes)
                              FY1982–2002
   $\log(LHTX) = -8.12$
   $\qquad\qquad [-15.6^{**}]$

   $\qquad -0.06 \times D96 -0.06 \times D98\_99$
   $\qquad [-2.08^{**}] \quad [-2.69^{**}]$

$+(1.38 \quad -0.09 \times D94\_02) \quad \times \log(YLE$
$[32.8^{**}][-6.74^{**}]$

$+YKE\_H +RI\_H -PI\_H -PEN\_C\,)$

$R^2(adj) = 0.98 \qquad DW = 1.63$

· Inhabitant tax on corporations (Local taxes)
$\qquad\qquad\qquad FY1981-2002$

$\log(LFTX) \quad =-2.66$
$\qquad\qquad [-2.73^{**}]$

$+0.42 \times D89\_91 \quad -0.21 \times D02$
$[9.65^{**}] \qquad\qquad [-3.08^{**}]$

$+ (1.04+0.04 \times D92\_95 +0.01 \times D00\_02)$
$[10.9^{**}][8.70^{**}] \qquad\qquad [4.02^{**}]$

$\times \log(YKE\_F +RI\_F -PI\_F)$

$R^2(adj) = 0.93 \qquad DW = 2.60$

· Local government, Other direct taxes
$DTX\_LG = (1 - \nu_{CG}) \times (ETX\_H +ETX\_F)$

· Consumption tax
(Share of the local government)
$CTX\_LG = (1 - \gamma_{CG}) \times CTX$

· Indirect taxes excluding consumption tax
(Share of the local government)
$OTX\_LG = (1 - \theta_{CG}) \times OTX$

10. Household: Disposable Income,
**   Balance of Savings and Investment (Nominal)**
**   (Including Private Unincorporated Enterprises)**

· Household, Disposable income
$DI\_H \;=\; (RI\_H -PI\_H) + (YLE +YKE\_H)$
$\qquad +(SGA\_CG +SGA\_LG) +PEN\_S$
$\qquad - (HTX +LHTX +ETX\_H)$
$\qquad - PEN\_C - OFT\_H$

· Household, Property (interest) income
$\qquad\qquad\qquad\qquad FY1989-2002$

$RI\_H /A\_H_{-1} = 0.006 \quad -0.002 \times D95\_96$
$\qquad\qquad [6.31^{**}]\;[-2.24^{**}]$

$+0.003 \times D98\_99 -0.003 \times D01$
$[3.13^{**}] \qquad\qquad [-2.04^{**}]$

$+0.005 \times RB +0.004 \times RB_{-1}$
$[10.7^{**}] \qquad\quad [7.52^{**}]$

$R^2(adj) = 0.99 \qquad DW = 1.96$

· Household, Debt interest payment
$\qquad\qquad\qquad\qquad FY1992-2002$

$PI\_H / L\_H_{-1} \quad = 0.037 -0.003 \times D95$
$\qquad\qquad [20.2^{**}]\;[-1.86^{*}]$

$+(0.005 \quad -0.002 \times D99\_02) \times RL_{-1}$
$[14.1^{**}] \quad [-2.71^{**}]$

$R^2(adj) = 0.98 \qquad DW = 1.80$

· Household, Social security contribution
$PEN\_C = \quad \eta \quad \times YD$

· Household, Social security benefits
$\qquad\qquad\qquad\qquad FY1982-2002$

$PEN\_S = \quad -26650 \quad -885 \times D01$
$\qquad\qquad [-19.4^{**}]\;[-2.25^{**}]$

$+ (798 +29.5 \times D81\_89) \quad \times YLE / (LDP +LDG)$
$[11.1^{**}][2.65^{**}]$

$+10.8 \quad \times POP65 +2270 \times RU$
$[6.52^{**}] \qquad\qquad [7.03^{**}]$

$R^2(adj) = 0.99 \qquad DW = 1.41$

· Household, Balance of savings and investment
$ISB\_H = (DI\_H +CF\_H)$
$\qquad - (NCP +NIH +NIP\_H +OCT\_H) -CPTX$

・Consumption of fixed capital
(Share of the household sector)
$$CF\_H / P_{IH} = RC\_H \times KH95_{-1}$$

・Equipment investment by private
unincorporated enterprises (Nominal)
$$NIP\_H = \chi_H \times NIP$$

11. Private Corporations: Profit after Taxation, Balance of Savings and Investment (Nominal)

・Private corporations, Profit after taxation
$$DI\_F = (RI\_F - PI\_F) + YKE\_F$$
$$- (FTX + LFTX + ETX\_F) - OFT\_F$$

・Private corporations, Property income
$$FY1992-2002$$
$$RI\_F / A\_F_{-1} = 0.02 \quad -0.003 \times D95\_96$$
$$[23.8^{**}] \; [-3.41^{**}]$$

$$-0.007 \times D99 + 0.003 \times D02$$
$$[-6.07^{**}] \qquad [2.24^{**}]$$

$$+0.007 \times RB \quad +0.001 \times RB_{-1}$$
$$[8.61^{**}] \qquad [2.11^{**}]$$

$$R^2(adj) = 0.99 \qquad DW = 1.75$$

・Private corporations, Debt interest payment
$$FY1990-2002$$
$$PI\_F / L\_F_{-1} = 0.021 \quad -0.006 \times D99\_01$$
$$[7.40^{**}] \quad [-2.29]$$

$$+0.007 \times RL_{-1}$$
$$[12.5^{**}]$$

$$R^2(adj) = 0.96 \qquad DW = 2.04$$

・Private corporations, Balance of savings and investment
$$ISB\_F = (DI\_F + CF\_F)$$
$$- (NIP\_F + NIN\_F + OCT\_F)$$

・Private corporations, Consumption of fixed capital
(Share of the household sector)
$$CF\_F/P_{IP} = RC\_F \times KP95_{-1}$$

・Private corporations, Equipment investment
(Nominal)
$$NIP\_F = (1 - \chi_H) \times NIP$$

・Private corporations, Inventory investment
(Nominal)
$$NIN\_F = \xi_F \times NIN$$

12. Fiscal Balance of the Central Government

・Central government, Fiscal balance
$$ISB\_CG = (RI\_CG - PI\_CG) + TAX\_CG$$
$$- (LATG\_G + SS\_CG + SGA\_CG + SB\_CG)$$
$$- (RC\_CG \times P_{CG} + RI\_CG \times P_{IG} + NIN\_CG)$$
$$+ CF\_CG - OTH\_CG$$

・Central government, Debt interest payment
$$FY1991-2002$$
$$PI\_CG / L\_CG_{-1} = 0.02 - 0.004 \times D00$$
$$[12.6^{**}][-2.82^{**}]$$

$$-0.008 \times D02 + 0.001 \times RCA$$
$$[-7.31^{**}] \qquad [3.17^{**}]$$

$$+0.004 \times RB_{-1} + 0.001 \times RB_{-2}$$
$$[4.90^{**}] \qquad [2.25^{**}]$$

$$R^2(adj) = 0.99 \qquad DW = 2.03$$

・Central government,
Financial Support toward Social Security Fund
$$FY1981-2002$$
$$SS\_CG = 5020 \quad +972 \times D88\_89$$
$$[23.1^{**}] \; [4.19^{**}]$$

$$+2580 \times D99$$
$$[7.62^{**}]$$

$+(0.28 + 0.04 \times D01\_02) \times PEN\_B$
$[35.7^{**}][7.23^{**}]$

$$R^2(adj) = 0.99 \qquad DW = 1.41$$

・Central government,
　Social assistance benefits toward household
　　$SGA\_CG = \phi_{CG} \times YLE$

・Central government, Inventory investment
　(Nominal)
　　$NIN\_CG = \xi_{CG} \times NIN$

13. Fiscal Balance of the Local Government

・Local government, Fiscal balance
　　$ISB\_LG = (RI\_LG - PI\_LG) + TAX\_LG$
　　　　　　$+ LATG\_G - (SGA\_LG + SB\_LG)$
　　　　　　$- (RC\_LG \times P_{CG} + RI\_LG \times P_{IG}$
　　　　　　$+ NIN\_LG) + CF\_LG - OTH\_LG$

・Local government, Debt interest payment
　　　　　　　　　　FY1993−2002
　　$PI\_LG / LL\_LG_{-1} = 0.03 \quad + 0.003 \times D94$
　　　　　　　　$[21.0^{**}]\ [2.00^{**}]$

　　　　　　　　$-0.005 \times D01\_02$
　　　　　　　　$[-3.92^{**}]$

　　　　　　　　$+0.004 \times RB_{-1} + 0.002 \times RB_{-2}$
　　　　　　　　$[3.60^{**}] \qquad [2.58^{**}]$

$$R^2(adj) = 0.98 \qquad DW = 1.85$$

・Local government,
　Social assistance benefits toward household
　　$SGA\_LG = \phi_{LG} \times YLE$

・Local government, Inventory investment (Nominal)
　　$NIN\_LG = \xi_{LG} \times NIN$

14. Financial Markets

・Household, Gross financial liabilities (Nominal)
　　$L\_H = LP\_H + LG\_H$

・Household, Debts from private sector (Nominal)
　　　　　　　　　　　　FY1990−2002
　　$LP\_H = 221000 - 4530 \times RL$
　　　　$[22.5^{**}][-15.9^{**}]$

　　　　$+0.07 \times YLE \quad +0.28 \times YKE$
　　　　$[1.50] \qquad\quad [2.68^{**}]$

$$R^2(adj) = 0.99 \quad DW = 1.92$$

・Household, Net financial liabilities (Nominal)
　　　　　　　　　　　FY1993−2002

　　$\Delta DAL\_H = 10100$
　　　　　　$[4.47^{**}]$

　　　　　　$-24600 \times D99 - 8380 \times D00\_01$
　　　　　　$[-3.21^{**}] \qquad [-1.71^{*}]$

　　　　　　$+532 \times \dot{PS} + ISB\_H$
　　　　　　$[3.73^{**}]$

$$R^2(adj) = 0.99 \quad DW = 2.29$$

・Household, Gross financial assets (Nominal)
　　$A\_H = (LP\_H + LG\_H) + DAL\_H$

・Household, Government bond holding (Nominal)
　　　　　　　　　　　FY1990−2002
　　$\Delta (B\_P / (A\_H_{-1} + A\_F_{-1} - POS\_H_{-1}))$
　　　　$= 0.007 \times D97 - 0.026 \times D01$
　　　　$[2.66^{**}] \qquad [-7.97^{**}]$

　　　　　$+ (0.006 + 0.004 \times D99\_02) \times RRB$
　　　　　$[6.21^{**}]\ [9.57^{**}]$

　　　　　$+0.23 \times RISK_{-1} - 0.49 \times 10^{-7} \times RY$
　　　　　$[4.26^{**}] \qquad [-4.95^{**}]$

$$R^2(adj) = 0.99 \quad DW = 1.72$$

・Government bond issuance (Nominal)

FY1990–2002

$$\Delta \text{B\_CG} = +7790 \times \text{D99\_00}$$
$$[3.65^{**}]$$

$$+0.77 \times (-\text{ISB\_CG}) + \text{AJBG}$$
$$[11.5^{**}]$$

$$R^2(\text{adj}) = 0.99 \quad \text{DW} = 1.55$$

・Real government bond yield (Market equilibrium)

$$(\text{B\_CG} + \text{B\_FIL}) / P_Y = (\text{B\_P} + \text{B\_OTH}) / P_Y$$

・Nominal government bond yield

$$\text{RB} = \text{RRB} + \dot{P}_Y$$

・Nominal interest rate of bank lending

FY1981–2002

$$\text{RL} = 0.73 \quad -0.36 \times (\text{D80\_82} + \text{D95\_96})$$
$$[7.31^{**}] \ [-4.78^{**}]$$

$$+0.46 \times \text{D98} + 0.19 \times \text{RCA} + 0.83 \times \text{RB}$$
$$[3.06^{**}] \qquad [5.87^{**}] \qquad [19.0^{**}]$$

$$R^2(\text{adj}) = 0.99 \quad \text{DW} = 1.67$$

・Outstanding government bonds (Real)

$$\text{RB\_CG} = \text{B\_CG} / P_Y$$

・Central government, Net financial liabilities (Nominal)

$$\text{ND\_CG} = \text{B\_CG} + \text{D\_CG}$$

・Central government, Gross financial liabilities (Nominal)

$$\text{GD\_CG} = \text{ND\_CG} + \text{A\_CG}$$

・Local government, Net financial liabilities (Nominal)

FY1991–2002

$$\Delta \text{ND\_LG} = -962 \quad -6220 \times \text{D97} - 5230 \times \text{D00}$$
$$[-2.34^{**}] \ [-10.0^{**}] \qquad [-8.21^{**}]$$

$$+0.83 \times \ (-\text{ISB\_LG}) + \text{AJLG}_{-1}$$
$$[11.5^{**}]$$

$$R^2(\text{adj}) = 0.99 \quad \text{DW} = 2.23$$

・Local government, Gross financial liabilities (Nominal)

$$\text{GD\_LG} = \text{ND\_LG} + \text{A\_LG}$$

・Net public debt (Nominal)

$$\text{ND\_G} = \text{ND\_CG} + \text{ND\_LG}$$

Endogenous Variables

1. Real Aggregate Demand

RY   Real gross domestic expenditure

RCP   Real private consumption expenditure

RIH   Real private residential investment

RIP#   Real private equipment investment
        (Based on "Gross Capital Stock of Private
        Enterprises")

RIP   Real private equipment investment
        (Based on the "Annual Report on National
        Accounts")

RCG   Real general government consumption

RIG   Real general government investment

REX   Real exports of goods and services

RIM   Real imports of goods and services

2. Real Aggregate Supply

RYP   Private sector, Real domestic product

RYG   General government, Real domestic product

LDP   Private sector, Labor demand
        [10000 persons * Working hours]

RU   Rate of unemployment in labor force

CU   Capital utilization index

SY   Potential gross domestic product (real)

3. Real Capital Stock

KP   Real private capital stock (equipment)

KH   Real housing stock

KG   Real social capital stock

4. GDP Deflator and Nominal GDP

PY   GDP deflator

PT_Y GDP deflator excluding consumption tax
        factor

NY   Nominal GDP

5. Nominal Aggregate Demand

NCP   Nominal private consumption expenditure

NIH   Nominal private residential investment

NIP   Nominal private equipment investment

NIN   Nominal inventory investment

NCG   Nominal general government consumption

NIG   Nominal general government investment

6. Deflator of Each Demand Component

P_{CP} Deflator of private consumption expenditure

PT_{CP} Deflator of private consumption expenditure
           excluding consumption tax factor

P_{IH} Deflator of private residential investment

PT_{IH} Deflator of private residential investment
           excluding consumption tax factor

P_{CG} Deflator of general government consumption

PT_{CG} Deflator of general government consumption
           excluding consumption tax factor

7. Distribution of Nominal GDP

YD   National income

CTX   Consumption tax

OTX   Indirect taxes excluding consumption tax

SUB   Subsidies

CF   Consumption of fixed capital

YLE   Employee compensation

YKE   Operating surplus of firm sector

YKE_H   Operating surplus distributed to private
        unincorporated enterprises

YKE_F   Operating surplus distributed to private
        corporations

**8. Tax Revenue of the Central (National)
   Government**

TAX_CG Central government, Tax revenue

HTX   Income tax (National taxes)

FTX   Corporate tax (National taxes)

CPTX   Inheritance tax, donation tax
           (National taxes)
           (Including land–value tax, FY1992–1997)

DTX_CG   Central government, Other direct taxes

CTX_CG   Consumption tax
           (Share of the central government)

OTX_CG   Indirect taxes excluding consumption tax
           (Share of the central government)

9. Tax Revenue of the Local Government

TAX_LG   Local government, Tax revenue

LHTX   Inhabitant tax on individuals (Local
           taxes)

LFTX   Inhabitant tax on corporations (Local
           taxes)

DTX_LG   Local government, Other direct taxes

CTX_LG   Consumption tax
           (Share of the local government)

OTX_LG   Indirect taxes excluding consumption tax
           (Share of the local government)

10. Household: Disposable Income,
   Balance of Savings and Investment (Nominal)
   (Including Private Unincorporated Enterprises)

DI_H   Household, Disposable income

RI_H   Household, Property (interest) income

PI_H   Household, Debt interest payment

PEN_C   Household, Social security contribution

PEN_S   Household, Social security benefits

ISB_H   Household, Balance of savings and
           investment

CF_H   Consumption of fixed capital
           (Share of the household sector)

NIP_H   Equipment investment by private
           unincorporated enterprises (Nominal)

11. Private Corporations: Profit after Taxation,
   Balance of Savings and Investment (Nominal)

DI_F   Private corporations, Profit after taxation

RI_F   Private corporations, Property income

PI_F   Private corporations, Debt interest payment

ISB_F   Private corporations, Balance of Savings
           and Investment

CF_F   Private corporations, Consumption of fixed
           capital (Share of the household sector)

NIP_F   Private corporations, Equipment
           investment (Nominal)

NIN_F   Private corporations, Inventory
           investment (Nominal)

12. Fiscal Balance of the Central Government

ISB_CG   Central government, Fiscal balance

PI_CG   Central government, Debt interest
           payment

SS_CG   Central government, Financial
           support toward Social Security Fund

SGA_CG   Central government, Social assistance
           benefits toward household

NIN_CG   Central government, Inventory
           investment (Nominal)

13. Fiscal Balance of the Local Government

ISB_LG   Local government, Fiscal balance

PI_LG   Local government, Debt interest payment

SGA_LG   Local government, Social assistance
           benefits toward household

NIN_LG   Local government, Inventory
           investment (Nominal)

14. Financial Markets

L_H   Household, Gross financial liabilities
           (Nominal)

LP_H	Household, Debts from private sector (Nominal)	RL	Nominal interest rate of bank lending
DAL_H	Household, Net financial liabilities (Nominal)	RB_CG	Outstanding government bond (Real)
A_H	Household, Gross financial assets (Nominal)	ND_CG	Central government, Net financial liabilities (Nominal)
B_P	Household, Government bond holding (Nominal)	GD_CG	Central government, Gross financial liabilities (Nominal)
B_CG	Outstanding government bonds (Nominal)	ND_LG	Local government, Net financial liabilities (Nominal)
RRB	Real government bond yield	GD_LG	Local government, Gross financial liabilities (Nominal)
RB	Nominal government bond yield	ND_G	Net public debt (Nominal)

Exogenous Variables

A. Domestic Macroeconomy

LDG   Government Sector, Employed persons

LS   Labor force

LT   Working hours (unit: hours/month)

POP   Nationwide population

POP65   Population 65 years old and older

MED   Nationwide medical expenses (Nominal)

LP   Index of urban land prices (FY2000 = 100)

PS   Nikkei 225 Stock Average (Unit: yen)

RISK   Loan loss risk index

(Liabilities of bankruptcy enterprises / liabilities of corporate businesses)

B. Foreign Macroeconomy

PU   United States, GDP deflator

ER   Nominal Exchange Rate ($/¥)

USRY   United States, Real GDP (Unit: $billion)

C. Potential Output

CU*   Mean value of capacity utilization index in the 11th and the 12th business cycle

RU*   Structural unemployment rate (assumed as 2.5%)

D. Capital Depletion Rate

δ_P Depletion rate of private capital stock

δ_H Depletion rate of private housing stock

δ_G Depletion rate of social capital stock

E. Consumption of Fixed Capital

RC_H   Household, Consumption rate of fixed capital

RC_F   Private corporations, Consumption rate of fixed capital

CF_CG   Central government, Consumption of fixed capital

CF_LG   Local government, Consumption of fixed capital

F. Aggregate Demand

RIN   Real inventory investment

RIF   Real equipment investment by public corporations

ADJ   Adjustment factor that we use to convert private equipment investment from "Gross Capital Stock of Private Enterprises" basis to "Annual Report on National Accounts" basis

G. Deflators Regarded as Exogenous

P_{IP} Deflator of private equipment investment

P_{IN} Deflator of inventory investment

P_{IG} Deflator of public investment

P_{IM} Deflator of imports of goods and services

RMP   Rate of price shift caused by consumption tax

H. Ratio Related to Income Distribution

α Labor share

β_H Share of operating surplus distributed to private unincorporated enterprises

β_F Share of operating surplus distributed to private corporations

I. Tax Revenue: Distribution Between the Central and Local Government

ν_{CG} Share of other direct taxes distributed to the central government

γ_{CG} Share of consumption tax distributed to the central government

θ_{CG} Share of other indirect taxes excluding

consumption tax distributed to the central government

J. Policy Variables

RCTX   Consumption tax rate

LATG_G Distribution of local allocation tax

RCA   Call rate

RI_CG   Central government, Investment (Real)

RI_LG   Local government, investment (Real)

RC_CG Central government, consumption (Real)

RC_LG Local government, consumption (Real)

SB_CG   Central government, subsidies (Nominal)

SB_LG   Local government, subsidies (Nominal)

K. Private Sector: IS−Balance

ETX_H   Household, Other direct taxes (Payable)

OFT_H   Household, Other net current transfers (Payable)

OCT_H   Household, Other net capital transfers (Payable)

η Household, Social security contribution as a percentage of national income

χ_H Share of private equipment investment occupied by private unincorporated enterprises

ETX_F   Private corporations, Other direct taxes (Payable)

OFT_F   Private corporations, Other net current transfers (Payable)

OCT_F   Private corporations, Other net capital transfers (Payable)

ξ_F Share of inventory investment occupied by private corporations

L. Governments: IS−Balance

OTH_CG Central government, Other net current and capital transfers (Payable)

ϕ_{CG} Central government, Ratio of social assistance benefit to employee compensation

ξ_{CG} Share of inventory investment occupied by the central government

RI_LG   Local government, Interest payment

OTH_LG Local government, Other net current and capital transfers (Payable)

ϕ_{LG} Local government, Ratio of social assistance benefits to employee compensation

ξ_{LG} Share of inventory investment occupied by the local government

M. Financial Market

LG_H   Household, Debt from public sector

POS_H   Outstanding amount of postal savings and postal life insurance

B_FIL   Fiscal Loan Fund Special Account Bonds (A type of government bond)

B_OTH Other sectors, Government bond holding

D_CG   Central government, Net debt other than government bonds

A_F   Private corporations, Gross financial assets (Nominal)

L_F   Private corporations, Gross financial liabilities (Nominal)

A_CG   Central government, Gross financial assets (Nominal)

A_LG   Central government, Gross financial assets (Nominal)

N. Fluctuation of Present Market Value

AJLP   Household, Fluctuation of present market value of debt from private sector

AJBG   Central government, Fluctuation of present market value of government bonds

AJLG   Local government, Fluctuation of present market value of government bonds