SMU ECONOMICS & STATISTICS
WORKING PAPER SERIES

SMU
SINGAPORE MANAGEMENT
UNIVERSITY

Subsides for FDI:
Implications from a Model with Heterogeneous Firms

Davin Chor

August 2007

Subsidies for FDI:
Implications from a Model with Heterogeneous Firms[*]

Davin Chor[†]

Singapore Management University

This version: 15 August 2007

Comments welcome

Abstract

This paper analyzes the welfare effects of subsidies to attract multinational corporations, in a setting where firms are heterogeneous in their productivity levels. I show that the use of a small subsidy raises welfare in the FDI host country, with the consumption gains from attracting more multinationals exceeding the direct costs of funding the subsidy program through a tax on labor income. This welfare gain stems from a selection effect, whereby the subsidy induces only the most productive exporters to switch to servicing the host's market via FDI. I further show that the welfare gain from a subsidy to variable costs is larger than from a subsidy to the fixed cost of conducting FDI, since a variable cost subsidy also raises the inefficiently low output levels stemming from each firm's mark-up pricing power.

Keywords: FDI subsidies; heterogeneous firms; fixed versus variable cost subsidies; import subsidies.

JEL Classification: F12, F13, F23, L23

1 Introduction

This paper presents an application of the trade models with heterogeneous firms advanced by Melitz (2003) and Helpman et al. (2004) to an analysis of policy interventions related to foreign direct investment (FDI). It examines in particular the use of cost subsidies to attract multinational corporations (MNCs) – an increasingly popular practice among prospective FDI host countries – to determine the scope for a welfare improvement from subsidizing FDI in a setting where firms within the target industry are heterogeneous in their productivity levels.

As an economic phenomenon, FDI has expanded considerably over the past two decades.[1] Many countries are in fact now keen to attract multinationals to their shores for a variety of reasons. The consumption gains are perhaps the most direct benefit: The relocation of production lowers the prices that MNCs charge in their host country's market, due to the savings on cross-border transport costs and possibly also labor costs (if the host is a developing country where wages are lower). In addition, host countries often value the increased demand for labor or the injection of foreign capital that MNCs bring.[2] The policy arguments in favor of FDI have also stressed other potential long-term benefits for economic growth, such as industrial spillovers and transfers of technological expertise, although such effects have been more difficult to identify and quantify empirically.[3]

Not surprisingly, countries that adopt these positive views towards FDI have used an array of incentive measures to try to attract a larger share of the FDI pie. Such incentives range from generous tax holidays, to subsidies for employment creation, and even the construction of large-scale infrastructure and industrial facilities for MNCs. A recent edition of the *World Investment Report* (UNCTAD 2003) surmised that "[t]he use of locational incentives to attract FDI has considerably expanded in frequency and value" (p. 124), resulting in an intensifying competition among countries for FDI projects. The *Report* cited how BMW was reportedly wooed by more than 250 European sites before finally locating a plant in Leipzig in 2001. There has also been brewing unease among some Western European countries over the aggressive use of corporate tax cuts by several Central and Eastern European countries, such as Poland and Slovakia, to attract foreign corporations, prompting

---

[1]Inward stocks of FDI into developing countries increased almost threefold as a percentage of GDP, from 12.6% in 1980 to 36.0% in 2002 (UNCTAD 2003).

[2]On the positive labor market effects of FDI, Rama (2001) reports suggestive evidence that PPP-adjusted real wages are positively correlated with the FDI-to-GDP ratio across countries.

[3]For example, Aitken and Harrison (1999) find relatively small net effects of foreign investment on domestic firms in Venezuela. Javorcik (2004), however, presents evidence of positive spillovers in Lithuania arising from backward linkages, as a greater multinational presence helped to improve the productivity levels of local suppliers. Likewise, Haskel et al. (2007) find positive effects at the industry level of a greater foreign presence on domestic plant-level productivity in the UK, although they argue that the per-job value of these spillovers tends to be smaller than the quantum of subsidies recently extended to several MNCs by the UK and US government. Turning to the broader cross-country literature on the growth effects of FDI, Nunnenkamp's (2004) overview concludes that whether these benefits materialize depends crucially on host country conditions such as the quality of the workforce and the strength of local institutions.

France and Germany to propose harmonizing the basic tax rate within the European Union (*The Economist*, July 24th, 2004).[4]

Although FDI subsidies have become a common feature of the international economic landscape, it is not clear *a priori* that such policies are necessarily welfare-improving for the host country even in the absence of strategic competition for FDI. On net, the direct fiscal costs of financing the subsidy schemes have to be weighed against the benefits of an increased multinational presence.

In this paper, I assess this tradeoff formally in the context of a two-country model with heterogeneous firms. I consider the interaction between a Home country where multinationals are headquartered and a Foreign country seeking to attract FDI. Firms differ in their innate productivity levels, as determined by a draw from a pre-existing distribution of technological possibilities. The initial industry equilibrium sees only the most productive Home firms conducting horizontal FDI in Foreign to service that market, since only these firms are sufficiently productive to compensate for the higher fixed costs of operating an overseas plant. I then ask whether welfare in the foreign host country can in fact be improved by a subsidy that draws even more Home firms to undertake FDI. Focusing in particular on the consumption gains that accrue from attracting more MNCs, do these gains to the host economy outweigh the direct cost of financing the subsidy scheme through an income tax on its workers? Of note, the model that I formulate admits a closed-form expression for consumer welfare, which makes the analysis of these policy interventions analytically tractable.

Previewing the main result in Section 3, I establish that a small subsidy for FDI does indeed improve welfare in the host country within this industry setting with heterogeneous firms. This result holds both for a subsidy that reduces MNCs' fixed costs of operation (such as the construction of industrial parks and infrastructure) and when the subsidy is applied to their variable costs of production (such as through corporate tax rate cuts or job-creation subsidies). Importantly, this welfare gain stems from a selection effect that comes into play when firms are heterogeneous in their productivity levels: The subsidy attracts only the most productive Home firms that were initially servicing the Foreign market via exports to switch to horizontal FDI instead. Given their relatively low unit production costs, this margin of firms already sets low prices in the initial equilibrium, with consumers demanding these products in relatively high quantities. The resulting consumption gains in Foreign following the savings on cross-border transport costs from the switch to FDI are thus large, since the subsequent price reduction is applied over a large volume of consumption. At the same

---

[4] *The Economist* reported that "Poland reduced its basic rate this year (2004) from 27% to 19%, and Slovakia from 25% to 19%. Hungary has a 16% rate, while Estonia does not even levy corporate tax on reinvested earnings. By contrast, Germany levies a 38.3% rate ... and France 34.3%." Hines (1996) and Devereux and Griffith (1998) provide empirical evidence on the importance of differences in corporate tax rates in explaining cross-state or cross-country variation in volumes of multinational activity. See however Wells et al. (2001) for a much more skeptical view of the use of tax incentives to attract FDI, where the authors contend that such schemes did not deliver net gains in Indonesia.

time, to ensure that these consumption gains indeed exceed the direct costs of funding the policy, one intuitively also requires that the mass of MNCs in the Foreign host country be relatively small, to cap the size of the total subsidy bill. This translates neatly in our model set-up into an analytic condition regarding the degree of firm heterogeneity, specifically that the distribution of firm productivities not display too thick a right-tail; reassuringly, this is a condition found to be readily satisfied in the estimates of firm productivity distributions in Helpman et al. (2004).

The key role played by firm heterogeneity and the underlying selection effect for these welfare results is made clear in Section 3.3, which contrasts how the scope for net gains from a FDI subsidy is theoretically ambiguous when all firms are identical, as in the antecedent model of Krugman (1980). When all firms share the same productivity level, any subsidy that induces one firm to switch from exporting to FDI necessarily induces all Home firms to make the same decision. This generates a large fiscal cost of funding the subsidy scheme, which can more than negate the consumption gains if, for example, the fixed cost of conducting FDI is high or the productivity level of Home firms is low, so that a large per-firm subsidy is needed to induce the switch to FDI.

Apart from this selection effect, there is an additional varieties effect that emerges when we take into account how the subsidy scheme raises the *ex ante* profitability of potential entrant firms to the Home industry. Section 3.4 shows how this increases the measure of Home firms in the full industry equilibrium, which amplifies the consumption gains to Foreign.

Are there any substantive differences then between the use of fixed versus variable cost subsidies? A simple calibration exercise (Section 3.5) suggests that variable cost subsidies have a much larger impact on the host country's welfare than fixed cost subsidies. Indeed, I show formally that a variable cost subsidy to MNCs delivers a greater welfare gain than a fixed cost subsidy that incurs the same total subsidy bill, subject to a mild sufficient condition. Intuitively, a variable cost subsidy alters both the entry and production decisions of MNCs, whereas a fixed cost subsidy affects only the former. The reduction in variable costs prompts each MNC to raise output levels, which delivers an additional kick to consumption, in effect counteracting the inefficiency arising from firms' mark-up pricing power. Nevertheless, this apparent favorable comparison of variable over fixed cost subsidies warrants qualification should the production facility also serve as a platform for sales to third-country markets (Eckholm et al. 2003, Grossman et al. 2006). If this export-platform motive for FDI is large, this could raise the subsidy bill substantially even while most of the consumption gains accrue to third-country consumers. The net result could ultimately be a welfare loss for the host country, unless the subsidy takes the form of a domestic sales credit (a rebate to local consumption) as opposed to a subsidy to production costs.

This paper contributes to an extensive literature on the welfare effects of subsidies for FDI, pre-

senting a first attempt (to the best of my knowledge) at applying a framework with heterogeneous firms to these policy issues. Moreover, the formal comparison of fixed versus variable cost subsidies is a natural exercise that emerges under this modelling framework, yet this is a question that has been under-explored despite the recognition that FDI subsidies can assume diverse forms. This paper also follows a growing body of research (exemplified by Baldwin et al. (2003)), that takes an industry equilibrium approach towards analyzing the repercussions of trade policy interventions.[5] The model developed here will in fact allow us to be very precise in describing the behavior of individual firms in the industry equilibrium, specifically how each firm's productivity draw and the size of the FDI subsidy jointly pin down whether it can profitably enter the Foreign market, and if so, its optimal mode for servicing that market (via exports or FDI).

Separately, this paper also speaks to a broader literature on optimal policy towards foreign investment. The early theoretical contributions here, by MacDougall (1960), Kemp (1966) and Jones (1967), focused on analyzing the jointly optimal levels of commodity tariffs and capital flow taxes in a two-factor, two-country world where one factor (capital) is allowed to be mobile across borders. In this strand of work however, foreign investment is viewed as international capital movements, in contrast to the more recent literature on MNCs which treats FDI more concretely as the production activities of overseas affiliates. Along these latter lines, there has been work exploring various economic settings under which FDI subsidies might lead to welfare improvements. For example, Haaland and Wooten (1999) examine how FDI subsidies can foster agglomeration effects, while Pennings (2005) shows that a positive subsidy is optimal when foreign investors face uncertainty over demand conditions in the host economy.[6] Others have argued that FDI subsidies can help to alleviate the under-provision of public services (Black and Hoyt 1989) or improve the allocation of firms' production facilities to countries from the standpoint of aggregate efficiency (Fumagalli 2003).[7]

In this paper, the welfare improvement stems instead from the reduction of barriers to entry into the host country market, specifically for the most productive Home firms that would have serviced

---

[5]See also Haufler and Wooten (1999) and Ottaviano and van Ypersele (2005), who use a general equilibrium framework to study how market size can confer countries with an advantage in the competition to attract firms or mobile capital. Of note, Baldwin and Okubo (2006) introduce firm heterogeneity in a two-country new economic geography model, where footloose firms choose which country to base their headquarter operations in after observing their productivity draw. In their setting, the most productive firms locate in the larger Northern country in the long-run equilibrium, since these firms benefit the most from agglomeration effects. A subsidy from the South might then attract some relocation of firms, although the question of the net welfare effect on the South remains to be explored.

[6]Janeba (2002) models uncertainty arising from an inability by the host government to make credible long-term commitments to maintain their announced tax or subsidy policies. His analysis also takes into account both sides of this credibility issue, namely that MNCs themselves may not be able to credibly commit to invest in only one country.

[7]Black and Hoyt (1989) consider how a subsidy to firms can reduce the distortion caused by the average cost pricing of public services, when the marginal cost of providing these services is lower than the tax revenue that they generate. In Fumagalli (2003), firms prefer to be located in a region that is a more developed market, but locating the MNC in a less developed region may yield greater gains from technological spillovers.

the Foreign market via exporting in the absence of the FDI subsidy. I focus in this present analysis solely on the consumption gains accruing to the host country from attracting more MNCs, namely the benefit from accessing MNC's products more cheaply due to the savings on transport costs. While this puts aside other potential benefits such as technological spillovers, agglomeration effects, or an increased demand for local labor, the model nevertheless serves as an important benchmark, since such additional effects would intuitively reinforce the gains from attracting FDI. The results in this paper are closely related to recent work by Demidova and Rodriguez-Clare (2007), who demonstrate how a simple consumption subsidy, import tariff, or export tax can be used to offset the inefficiency due to mark-up pricing in the Melitz (2003) model with heterogeneous firms, to achieve the first-best welfare level. While their paper addresses interventions targeted at domestic consumers or the domestic industry, my results here relate instead to policy incentives that influence the behavior (specifically the production location decisions) of firms headquartered in other countries.

The paper proceeds as follows. Section 2 lays out the building blocks of the model. Section 3 establishes the main propositions on the scope for a welfare gain from either a fixed or variable cost subsidy to FDI. Section 4 explores several extensions. A parallel analysis shows that there is a similar scope for improving welfare in Foreign through an import subsidy (Section 4.1). I also confirm the robustness of the results under an alternative utility function specification that incorporates richer income effects (Section 4.2). Section 5 concludes. Detailed proofs are relegated to the Appendix.

2 A Two-Country Model with Heterogeneous Firms

I proceed to set up the baseline model. There are two countries, called Home and Foreign, indexed by H and F respectively. Each economy is made up of two sectors: (i) a homogenous good sector, and (ii) a (country-specific) differentiated goods sector. Labor is the sole factor of production.

Utility: The utility of the representative consumer in country i is given by:

$$U_i = x_i^0 + \sum_{c=H,F} \frac{1}{\mu}(X_i^c)^\mu \tag{1}$$

Here, x_i^0 denotes consumption of the homogenous good (the domestic numeraire). X_i^c is the familiar Dixit-Stiglitz aggregator of consumption over products, x_i^c, from country c's differentiated goods sector, given by: $X_i^c = \left[\int_{\Omega_i^c} x_i^c(a)^\alpha dG^c(a)\right]^{\frac{1}{\alpha}}$, where Ω_i^c is the set of products from country-c firms available to consumers in country i. For example, when $c = H$ and $i = F$, this set is the union of goods exported from Home to Foreign and goods produced in Foreign by Home MNCs. I shall assume that $0 < \mu < \alpha < 1$, so the differentiated products are pair-wise substitutes, and moreover, products from the same country are closer substitutes than products from different countries.

Differentiated products are indexed by a, which is the amount of labor required to produce one unit of output. $1/a$ is thus a measure of a firm's labor productivity. Upon paying the fixed cost of entry into the industry, each firm draws its a from a pre-determined technological distribution, $G^c(a)$, and the resulting productivity differences are the key dimension along which firms in the differentiated goods sector are heterogeneous.

The utility function in (1) is maximized with respect to the budget constraint:

$$x_i^0 + \sum_{c=H,F} \int_{\Omega_i^c} p_i^c(a) x_i^c(a) dG^c(a) = w_i \tag{2}$$

where w_i is the wage income of a representative consumer in country i, and p_i^c is the unit price of good x_i^c. (The homogenous good price is normalized to 1.) Solving the consumer's maximization problem yields a demand function for each product with constant elasticity, $\varepsilon = \frac{1}{1-\alpha} > 1$, given by: $x_i^c(a) = (X_i^c)^{\frac{\mu-\alpha}{1-\alpha}} p_i^c(a)^{-\varepsilon}$. Substituting this into the definition of X_i^c delivers the following expression for the CES consumption aggregates in terms of goods prices that is useful for future computations:

$$X_i^c = \left[\int_{\Omega_i^c} p_i^c(a)^{1-\varepsilon} dG^c(a) \right]^{\frac{1}{\varepsilon-1} \frac{1}{1-\mu}} \tag{3}$$

Intuitively, the overall consumption of differentiated goods decreases as individual goods prices rise.[8]

Welfare measure: As a measure of welfare for the subsequent analysis, I also derive the indirect utility function, W_i, for a representative consumer. The demand function for differentiated products, $x_i^c(a)$, and the budget constraint (2) together imply a level of demand for the homogenous good, x_i^0. Substituting this expression for x_i^0 into the utility function (1) and simplifying, one obtains:

$$W_i = w_i + \left(\frac{1-\mu}{\mu} \right) \sum_{c=H,F} (X_i^c)^\mu \tag{4}$$

Note the natural property that welfare is increasing in the consumer's labor income, as well as in the consumption aggregates purchased.

The analysis which follows focuses on the industry equilibrium for the Home differentiated goods sector, namely $c = H$, and the effects of a Foreign subsidy on FDI from this sector. For brevity, I henceforth suppress the c superscript unless there is cause for ambiguity.

Nominal wages: The homogenous good is produced under a constant returns to scale technology. I assume that the labor force in each country is sufficiently large, so that output in this numeraire sector is strictly positive in equilibrium. The nominal wage in each economy is then pinned down by

---

[8]An alternative way to see this is to recognize that X_i^c is equal to the ideal price index for Home goods raised to the power of $-1/(1-\mu)$.

the marginal product of labor in this sector.[9] While a more general model might allow the wage in the host country to respond to the increased demand for labor, this intuitively raises welfare in Foreign if wages rise as a consequence. The results below would then continue to hold, as long as the rise in w_F were not so large as to erode the original incentives for MNCs to locate production in Foreign.

Production and profits: The structure of production in the differentiated goods sector follows closely that in Helpman et al. (2004). Upon entering the industry, each Home firm takes a productivity draw, a, from the distribution $G^H(a)$. Production for the Home domestic economy requires a fixed cost of f_D units of Home labor in each period, while the marginal cost of each unit of output is aw_H. Under profit-maximization, prices are set equal to a constant mark-up, $\frac{1}{\alpha}$, over marginal costs.

Home firms may service the Foreign market through one of two means, namely exporting or horizontal FDI. Firms that export incur two additional costs: (i) a per-period fixed cost of exporting, equal to f_X units of Home labor; and (ii) the conventional iceberg transport costs, which raise unit production costs by a factor $\tau > 1$. Alternatively, Home firms which are sufficiently productive may opt to operate an additional manufacturing plant in Foreign, employing Foreign labor at wage cost w_F, while saving on transport costs. However, FDI entails a higher per-period fixed cost, $f_I > f_X$, than exporting.

Denote the number of workers in country i by M_i ($i = H, F$). For a Home firm with productivity level $1/a$, the per-period profits from sales to the domestic economy, from exporting, and from FDI in Foreign are given respectively by $\pi_D(a)$, $\pi_X(a)$ and $\pi_I(a)$:

$$\pi_D(a) = \left(\frac{aw_H}{\alpha}\right) M_H x_H(a) - aw_H M_H x_H(a) - f_D w_H = (1-\alpha) A_H \left(\frac{aw_H}{\alpha}\right)^{1-\varepsilon} - f_D w_H \quad (5)$$

$$\pi_X(a) = \left(\frac{\tau aw_H}{\alpha}\right) M_F x_F(a) - \tau aw_H M_F x_F(a) - f_X w_H = (1-\alpha) A_F \left(\frac{\tau aw_H}{\alpha}\right)^{1-\varepsilon} - f_X w_H \quad (6)$$

$$\pi_I(a) = \left(\frac{aw_F}{\alpha}\right) M_F x_F(a) - aw_F M_F x_F(a) - f_I w_H = (1-\alpha) A_F \left(\frac{aw_F}{\alpha}\right)^{1-\varepsilon} - f_I w_H \quad (7)$$

where $A_i = M_i(X_i)^{\frac{\mu-\alpha}{1-\alpha}}$ ($i = H, F$) is the level of demand in country i. Since there is a continuum of firms, individual firms take these aggregate demand levels as given.[10]

Productivity cut-offs: Firms engage in production for the domestic market if profits from (5) are positive. Solving $\pi_D(a) = 0$, this establishes a cut-off value, a_D, which is the maximum labor

---

input coefficient at which production for the Home market is profitable. In addition, firms for which $\pi_X(a) \geq 0$ export to Foreign. This implies a cut-off value, a_X, such that exporting is profitable for all firms with $a < a_X$. However, Home firms service the Foreign market via FDI instead if $\pi_I(a) \geq \pi_X(a)$; solving for the value of a that equates (6) and (7) yields a third cut-off, a_I, such that the Home firm opts for FDI over exporting if $a < a_I$. The explicit expressions for these three cut-offs are:

$$(a_D)^{1-\varepsilon} = \frac{f_D w_H}{(1-\alpha)A_H(w_H/\alpha)^{1-\varepsilon}} \tag{8}$$

$$(a_X)^{1-\varepsilon} = \frac{f_X w_H}{(1-\alpha)A_F(\tau w_H/\alpha)^{1-\varepsilon}} \tag{9}$$

$$(a_I)^{1-\varepsilon} = \frac{(f_I - f_X)w_H}{(1-\alpha)A_F[(w_F/\alpha)^{1-\varepsilon} - (\tau w_H/\alpha)^{1-\varepsilon}]} \tag{10}$$

Following Helpman et al. (2004), I introduce several restrictions on parameter values that ensure a natural sorting pattern of firms to the various modes of servicing the two markets. In particular, I stipulate that $a_D > a_X > a_I$, so that only the most productive firms (with $a < a_I$) are able to conduct FDI, while firms with an intermediate level of productivity (with $a_X > a > a_I$) export to Foreign. Firms with $a_D > a > a_X$ serve only the Home market, while firms that draw an a larger than a_D have labor input requirements that are too high and thus exit the industry immediately. The condition $a_D > a_X$ simplifies to: $\tau^{\varepsilon-1}(\frac{f_X}{A_F}) > \frac{f_D}{A_H}$. Intuitively, the fixed cost of exporting must be large relative to that for domestic production, so that only sufficiently productive firms are able to overcome this cost barrier to exporting.[11] Similarly, the condition $a_X > a_I$ boils down to: $f_I > (\frac{\tau w_H}{w_F})^{\varepsilon-1}f_X$. In other words, the fixed cost of FDI must be large, so that FDI is more profitable than exporting only for the most productive Home firms. Figure 1 illustrates this sorting pattern of firms according to $a^{1-\varepsilon}$, which proxies for firms' productivity levels (since $1-\varepsilon < 0$). Note also from (10) that we require $w_F < \tau w_H$, in order for the FDI cut-off to be well-defined ($a_I > 0$). Thus, Foreign wages must be lower than the marginal cost of the exporting option, in order to make horizontal FDI a profitable operation for some positive productivity levels.

Technology: Firm productivity levels, $1/a$, are independent draws taken from a Pareto distribution with shape parameter k and support $[1/a_H, \infty)$, a parametrization commonly adopted in the industrial organization literature. The shape parameter conveniently summarizes several key features of this distribution, with both the mean and the variance decreasing in k. Thus, a larger k corresponds to a distribution that places less weight on its right-tail, or equivalently, on obtaining high productivity draws, as illustrated in Figure 2.[12] It is useful to define $V^H(a) = \int_0^a \tilde{a}^{1-\varepsilon}dG^H(\tilde{a})$, as this

---

[11]For the full restriction, one must solve for the equilibrium values of A_H and A_F to substitute back into the inequality.
[12]The mean of this Pareto distribution is $\frac{k}{k-1}(\frac{1}{a_H})$ for $k > 1$; when $k \leq 1$, the mean is infinite as a lot of weight is placed on the right-tail of the distribution. Likewise, the variance of this distribution is $\frac{k}{(k-1)^2(k-2)}(\frac{1}{a_H})^2$ for $k > 2$;

expression will show up repeatedly. The Pareto distribution facilitates an analytical solution, with $G^H(a)$ and $V^H(a)$ both polynomials in a, given by:

$$G^H(a) = \left(\frac{a}{a_H}\right)^k \tag{11}$$

$$V^H(a) = \frac{k}{k-\varepsilon+1}\left(\frac{a}{a_H}\right)^k a^{1-\varepsilon} \tag{12}$$

for $a \in (0, a_H]$, where $(0, a_H]$ is the support of the distribution.[13]

Helpman et al. (2004) show that if the underlying productivity distribution is Pareto with shape parameter k, then the distribution of observed firm sales will be Pareto with shape parameter $k-\varepsilon+1$. (The cumulative distribution function of this sales distribution is in fact equal to $V^H(a)$ up to a multiplicative constant.) Their estimation based on European firm-level data establishes the goodness of fit of the Pareto distribution for firm sales, while yielding estimates for $k - \varepsilon + 1$ that are always significantly greater than 0 across manufacturing industries. This empirical evidence motivates a key assumption: $k > \varepsilon - 1$. In essence, this condition ensures that the distribution of productivity levels does not place too much weight on obtaining very high productivity draws.[14]

Equilibrium consumption of differentiated goods: We can now solve for the equilibrium levels of X_H and X_F, which are the CES aggregates for Home differentiated goods consumed in the Home and Foreign markets respectively. The expression for X_F, in particular, will be important for evaluating welfare based on the indirect utility function, (4). From (3) and the sorting pattern within the Home industry, these CES aggregates can be re-written as:

$$X_H = \left[N \int_0^{a_D} \left(\frac{a w_H}{\alpha}\right)^{1-\varepsilon} dG^H(a)\right]^{\frac{1}{\varepsilon-1}\frac{1}{1-\mu}} = \left[N \frac{k}{\alpha}\Lambda_H(A_H)^{\frac{k-\varepsilon+1}{\varepsilon-1}}\right]^{\frac{1}{\varepsilon-1}\frac{1}{1-\mu}} \tag{13}$$

$$X_F = \left[N \left(\int_{a_I}^{a_X} \left(\frac{\tau a w_H}{\alpha}\right)^{1-\varepsilon} dG^H(a) + \int_0^{a_I} \left(\frac{a w_F}{\alpha}\right)^{1-\varepsilon} dG^H(a)\right)\right]^{\frac{1}{\varepsilon-1}\frac{1}{1-\mu}}$$

$$= \left[N \frac{k}{\alpha}\Lambda_F(A_F)^{\frac{k-\varepsilon+1}{\varepsilon-1}}\right]^{\frac{1}{\varepsilon-1}\frac{1}{1-\mu}} \tag{14}$$

where N is the measure of Home firms. Note that I have substituted the expressions for the productivity cut-offs from (8), (9) and (10), as well as for the Pareto distribution from (11) in evaluating the

when $k \leq 2$, this variance is instead infinite. Note that both the mean and variance are decreasing functions in k for $k > 2$, so that a larger shape parameter corresponds to a Pareto distribution with a thinner right-tail.

[13]We assume that $a_H > a_D$, so that some prospective entrants to the industry fail to obtain a good enough productivity draw to engage in production for the domestic Home market, and thus exit the industry immediately.

[14]It should be stressed that the condition $k - \varepsilon + 1 > 0$ is a mild assumption regarding the extent of firm heterogeneity: For the variance of firm sales to be finite, we would in fact require the even more stringent condition that $k - \varepsilon + 1 > 2$.

integrals above. The terms Λ_H and Λ_F are given explicitly by:

$$\Lambda_H = \frac{\varepsilon-1}{k-\varepsilon+1}\left(\frac{\alpha}{a_H}\right)^k\left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}}\frac{(f_D)^{\frac{k}{1-\varepsilon}+1}w_H}{(w_H)^k}$$

$$\Lambda_F = \frac{\varepsilon-1}{k-\varepsilon+1}\left(\frac{\alpha}{a_H}\right)^k\left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}}\left[\frac{(f_X)^{\frac{k}{1-\varepsilon}+1}w_H}{(\tau w_H)^k}+\frac{(f_I-f_X)^{\frac{k}{1-\varepsilon}+1}w_H}{((w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}\right]$$

In words, Λ_H and Λ_F are equal to the *ex ante* profits that a prospective entrant firm in the Home sector would obtain in expectation (before observing its productivity draw a) from sales in Home and Foreign respectively, if the market demand levels A_H and A_F were normalized to 1. I therefore refer to Λ_H and Λ_F as the normalized profit levels in these respective markets.

Recall that $A_i = M_i(X_i)^{\frac{\mu-\alpha}{1-\alpha}}$ (for $i=H,F$), which is a definition I now substitute into (13) and (14). Some algebra then leads to the following compact expression for the consumption aggregates:

$$(X_i)^\mu = \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha-k(\mu-\alpha)}}\frac{\tilde{M}_i}{M_i}\tilde{\Lambda}_i, \quad i=H,F \tag{15}$$

where $\tilde{\Lambda}_i = (\Lambda_i)^{\frac{\mu\alpha}{\mu\alpha-k(\mu-\alpha)}}$ and $\tilde{M}_i = (M_i)^{\frac{k\alpha(1-\mu)}{\mu\alpha-k(\mu-\alpha)}}$. (15) is a very useful expression for computing country welfare levels; in particular, observe from (4) that $(X_F)^\mu$ is proportional to the quantum of utility derived by Foreign from the consumption of Home differentiated goods. Not surprisingly, $(X_F)^\mu$ and hence Foreign welfare are both increasing in N, the measure of Home firms, so that there is the usual "love of varieties" effect. In addition, $(X_F)^\mu$ rises with Λ_F ($\frac{\mu\alpha}{\mu\alpha-k(\mu-\alpha)}>0$, since $\mu<\alpha$); a higher normalized profit level from sales in Foreign implies that a larger subset of Home firms will be able to service the Foreign market profitably, thus raising consumer welfare in Foreign. Finally, we have a market size effect, whereby $(X_F)^\mu$ increases with M_F ($\frac{k\alpha(1-\mu)}{\mu\alpha-k(\mu-\alpha)}-1=\frac{\mu(k-k\alpha-\alpha)}{\mu\alpha-k(\mu-\alpha)}>0$, which follows from $k>\frac{\alpha}{1-\alpha}=\varepsilon-1$).[15]

To close the model fully, one needs to pin down the measure of Home firms, N, with a free-entry condition for the Home sector. I defer this step to Section 3.4, since (15) already facilitates a closed-form expression for Foreign welfare that corresponds to the case of a "small" host country, when the Foreign market is too small to affect the entry decisions of prospective entrant firms in Home. Analyzing this case where N is exogenous helps to isolate and highlight the selection effect of FDI subsidies, by focusing attention on the set of Home firms that switch their mode of servicing Foreign from exporting to FDI. It will turn out later in the endogenous N case that the additional entry or varieties effect works to reinforce this selection effect, so that the welfare implications are qualitatively identical. I therefore turn first to the policy analysis when the measure of Home firms is fixed.

---

[15]It is important for the derivation of the closed-forms in (13), (14) and (15) that f_D and f_X both be strictly positive. For instance, if $f_X = 0$, the expression for the cut-off a_D would then involve both A_H and A_F, since all firms that engage in domestic production also export when the fixed cost of exporting is 0. A_H (in addition to A_F) then enters on the right-hand side of (14) when computing that CES consumption aggregate, and it is subsequently not possible to isolate a closed-form for X_F or A_F.

3 The Welfare Implications of FDI Subsidies

We now consider the effects of FDI subsidies to attract more Home firms to locate in Foreign, focusing on the welfare implications for the host economy. I first establish that the use of a small subsidy increases welfare levels when the subsidy is applied either to the fixed costs of FDI (Section 3.1) or to the variable component of MNCs' production costs (Section 3.2). Section 3.3 highlights the key role played by firm heterogeneity in the analysis, by contrasting the results against a model where all firms have identical productivity levels. As promised, Section 3.4 shows how to endogenize N for the full industry equilibrium, a step that leaves the qualitative welfare implications unchanged. Section 3.5 compares the relative efficacy of fixed versus variable cost subsidies.

3.1 FDI subsidy to fixed costs

Consider first the use of a subsidy by the Foreign government that reduces the per-period fixed costs of FDI for Home multinationals by the amount: $s_f(f_I - f_X)w_H$, where $s_f < 1$. For example, this subsidy could come in the form of the provision of basic infrastructure such as roads or factory space, which the MNC would otherwise have to bear the cost for. Alternatively, the subsidies could remove lump-sum regulatory fees that need to be paid on a recurrent basis. For subsequent notational convenience, the subsidy is applied to the difference between the fixed costs of investment and exporting, to capture how it closes the gap between the upfront costs of these two modes of servicing the Foreign market.

Throughout the analysis, I restrict myself to subsidy policies that are "budget-neutral", in that the subsidy bill is exactly paid for by revenues raised from a tax on Foreign labor income. In the case of a fixed cost subsidy, the income tax rate, t_f, must therefore satisfy the following equation to balance Foreign's state budget:

$$t_f w_F M_F = s_f(f_I - f_X)w_H N G^H(a_I) \tag{16}$$

The expression on the right-hand side of this balanced-budget constraint is the total fiscal bill from subsidizing each Home firm with $a < a_I$ by the amount $s_f(f_I - f_X)w_H$. Firms continue to pay w_F for each unit of Foreign labor they employ, but Foreign workers now maximize utility subject to the budget constraint (2) with w_F replaced by $(1 - t_f)w_F$. (t_f is thus the minimum tax rate that needs to be levied on consumers in order to cover the costs of funding the subsidy to MNCs.)

To evaluate the net impact on welfare, it suffices to examine what happens to the terms, $W_{Ff} \equiv (1 - t_f)w_F + \frac{1-\mu}{\mu}(X_F^H)^\mu$, in the formula for indirect utility in (4). This expression for W_{Ff} highlights the nature of the tradeoff facing Foreign: The FDI subsidy lowers the productivity cut-off, $a_I^{1-\varepsilon}$, allowing some Home firms that were previously exporting to turn to FDI as their mode of servicing

12

Foreign. There is thus a margin of goods that were previously exported to Foreign at price $\frac{\tau a w_H}{\alpha}$ that are now priced more cheaply at $\frac{a w_F}{\alpha}$ by the MNC's Foreign facility (by assumption, $\tau w_H > w_F$). While this boosts the consumption aggregate, X_F^μ, the consumption gain must be weighed against the direct cost of the income tax, $-t_f w_F$.

The following proposition formally establishes that the net effect from a "small" positive subsidy to the fixed costs of FDI is indeed a welfare improvement:

Proposition 1 [Fixed Cost Subsidy]: *Consider the family of fixed cost FDI subsidies characterized by s_f that: (i) satisfy the balanced-budget constraint (16); and (ii) preserve the sorting pattern of organizational forms within the industry, namely $a_D > a_X > a_I$. Then the optimal policy, s_f^*, that maximizes welfare in Foreign is a strictly positive subsidy, namely $s_f^* \in (0,1)$.*

Proof. Substituting from (10) and (11) into (16), one can express t_f in terms of s_f and the underlying model parameters. Together with the expression for $(X_F)^\mu$ from (15), this yields:

$$W_{Ff} = w_F + \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu - \alpha)}} \frac{\tilde{M}_F}{M_F}\left[\frac{1-\mu}{\mu}\tilde{\Lambda}_{Ff} - s_f\left(\frac{\alpha}{k}\right)\frac{\tilde{\Lambda}_{Ff}}{\Lambda_{Ff}}\frac{\partial\Lambda_{Ff}}{\partial s_f}\right]$$

where Λ_{Ff} and $\tilde{\Lambda}_{Ff}$ are Λ_F and $\tilde{\Lambda}_F$ respectively with $f_I - f_X$ replaced by $(1-s_f)(f_I - f_X)$, while:

$$\frac{\partial\Lambda_{Ff}}{\partial s_f} = \left(\frac{\alpha}{a_H}\right)^k \left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}} \frac{(f_I - f_X)^{\frac{k}{1-\varepsilon}+1} w_H}{((w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}(1-s_f)^{\frac{k}{1-\varepsilon}}$$

(The term $\frac{\partial\Lambda_{Ff}}{\partial s_f}$ appears in W_{Ff} purely through an algebraic substitution; no first-order conditions have been taken yet.)

Now, differentiating W_{Ff} with respect to s_f and simplifying gives:

$$\frac{\partial W_{Ff}}{\partial s_f} = \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu - \alpha)}} \frac{\tilde{M}_F}{M_F}\frac{\tilde{\Lambda}_{Ff}}{\Lambda_{Ff}}\left[\frac{\alpha(1-\mu)}{\mu\alpha - k(\mu - \alpha)}\frac{\partial\Lambda_{Ff}}{\partial s_f} - \frac{\alpha}{k}\frac{\partial\Lambda_{Ff}}{\partial s_f}\cdots\right.$$

$$\left.\cdots - s_f\frac{\alpha(\mu - \alpha)}{\mu\alpha - k(\mu - \alpha)}\frac{1}{\Lambda_{Ff}}\left(\frac{\partial\Lambda_{Ff}}{\partial s_f}\right)^2 - s_f\frac{\alpha}{k}\frac{\partial^2\Lambda_{Ff}}{\partial s_f^2}\right]$$

It suffices to examine the terms in square brackets to deduce how Foreign welfare varies with the level of the fixed cost subsidy, since $\tilde{\Lambda}_F > 0$ whenever $s_f < 1$.

To examine the scope for a welfare improvement from a small subsidy, set $s_f = 0$. The key trade-off is now the following: Does the consumption gain from a marginal increase in s_f (captured by the $\frac{\alpha(1-\mu)}{\mu\alpha - k(\mu - \alpha)}\frac{\partial\Lambda_{Ff}}{\partial s_f}$ term) exceed the increased fiscal cost of the subsidy scheme? At $s_f = 0$, this increased fiscal cost (captured by the $-\frac{\alpha}{k}\frac{\partial\Lambda_{Ff}}{\partial s_f}$ term) is simply equal to the cost of transferring a marginal dollar to each of the multinationals already operating in the host economy (the infra-marginal MNCs). Since

$\frac{\partial \Lambda_{Ff}}{\partial s_f} > 0$ for $s_f < 1$, a sufficient condition for $\frac{\partial W_{Ff}}{\partial s_f} > 0$ at $s_f = 0$ is:

$$\frac{\alpha(1-\mu)}{\mu\alpha - k(\mu - \alpha)} - \frac{\alpha}{k} = \frac{\alpha}{k}\left(\frac{k(1-\alpha) - \mu\alpha}{\mu\alpha - k(\mu - \alpha)}\right) = \frac{\alpha}{k}\frac{(1-\alpha)(k - \mu(\varepsilon - 1))}{\mu\alpha - k(\mu - \alpha)} > 0$$

where this last inequality holds because $k - \mu(\varepsilon - 1) > k - (\varepsilon - 1) > 0$ and $\mu < \alpha$. Thus, $\frac{\partial W_{Ff}}{\partial s_f} > 0$ in a neighborhood of $s_f = 0$.

One can show moreover that $\frac{\partial W_{Ff}}{\partial s_f} > 0$ for all $s_f < 0$, so that a tax on FDI is never beneficial. Finally, $W_{Ff} \to -\infty$ as $s_f \to 1^-$. (The details of these proofs are not particularly illuminating, and are relegated to Appendix 7.1.) W_{Ff} therefore increases with s_f for small values of the subsidy, but has at least one turning point in the interval $(0, 1)$. In practice though, not all subsidies in the interval $(0, 1)$ will be feasible: As s_f is raised towards 1, the FDI cut-off $a_I^{1-\varepsilon}$ decreases monotonically and eventually overshoots $a_X^{1-\varepsilon}$ ($a_I \to \infty$ as $s_f \to 1^-$). Thus, to retain the ordering of the productivity cut-offs, we must have $s_f < \tilde{s}_f$, where $\tilde{s}_f \in (0, 1)$ is the subsidy level that equates the FDI and exporting cut-offs ($a_I = a_X$). Notwithstanding this, a small fixed cost subsidy unambiguously improves welfare for Foreign's workers, and the optimal policy for the FDI host country is thus a positive subsidy. ∎

It is useful at this point to dissect the intuition behind this welfare improvement. The fixed cost, f_I, prevents a segment of Home firms from servicing the Foreign market directly through FDI, and so the subsidy helps to alleviate the inefficiency caused by this fixed cost barrier. Specifically, the subsidy enables the most productive Home exporters – those firms just to the left of the original $a_I^{1-\varepsilon}$ cut-off – to switch into FDI, as illustrated in Figure 1. What then ensures that the subsequent consumption gains to Foreign outweigh the direct costs of the subsidy scheme? First, the consumption gains to host country consumers are large because the firms newly attracted to undertake FDI form a sufficiently productive margin: The more productive a firm, the lower the unit price it sets, and hence the larger the increase in consumption when the price of this good falls from $\frac{\tau a w_H}{\alpha}$ (under exporting) to $\frac{a w_F}{\alpha}$ (under FDI). Firm heterogeneity is a key ingredient for this selection effect, since it is the underlying productivity differences that allows the FDI subsidy to attract precisely just this productive margin of Home firms.

Second, the cost of funding the subsidy scheme must also be kept reasonably small. For this to be the case, the measure of MNCs cannot be too large, or equivalently, the distribution of Home firm productivities cannot have too thick a right-tail. This is why the condition $k - \mu(\varepsilon - 1) > 0$ comes into play in the above proof: Higher values of k correspond to Pareto distributions with thinner right-tails (see Figure 2), and hence a smaller mass of Home firms productive enough to undertake FDI. k must therefore be sufficiently large, to ensure that the funding cost of the subsidy scheme remains reasonably small. Reassuringly, the condition on the extent of firm heterogeneity required

for the result to hold is a mild one, consistent with the estimates from Helpman et al. (2004) showing that $k > \varepsilon - 1$. Naturally, if the Foreign government were able to discriminate between new and existing FDI projects, and extend the subsidy only to new multinational activity (without subsidizing the infra-marginal MNCs), the welfare gains would be even larger.

It is once again worth emphasizing that the welfare improvement in this result arises simply from the consumption gains to the host economy, putting aside other considerations that have been highlighted in the literature as potential gains from attracting FDI. Such effects as technological spillovers (Fumagalli 2003, Javorcik 2004), agglomeration economies (Haaland and Wooten 1999), and an increase in the demand for Foreign labor would intuitively reinforce and amplify the welfare improvement from a subsidy to FDI (so long as wages in the host country do not rise too much to undermine MNCs' incentives to undertake production there).

A natural question to ask is whether the optimal fixed cost subsidy is unique. Note first that one cannot obtain a closed-form expression for s_f^*. Moreover, the welfare function is in general not globally concave, and there are parameter values, albeit extreme ones, for which W_{Ff} exhibits more than one turning point in the interval $(0,1)$.[16] For all practical purposes though, the calibration exercise in Section 3.5 will confirm that for the empirically relevant parts of the parameter space, the welfare function does exhibit a unique turning point. For those that desire a formal treatment of this issue, Appendix 7.1 derives a sufficient condition for the uniqueness of the optimal subsidy: $2(\varepsilon - 1) > k$. This restriction is typically satisfied by the parameter choices of prior calibrations in the literature, in particular Ghironi and Melitz (2005) where the baseline values are $k = 3.4$ and $\varepsilon = 3.8$.[17]

A final remark is that the model can rationalize a situation in which two countries offer FDI subsidies to attract each others' multinationals. To see this, suppose that the structure of the Foreign differentiated goods sector is identical to Home's. Even in the absence of any subsidy, the most productive firms in Foreign would choose to service the Home market via horizontal FDI so long as $\tau w_F > w_H$, namely that the variable cost of exporting from Foreign exceeds the unit cost of locating production in Home itself.[18] Proposition 1 then implies that it would also be optimal for Home to extend a positive FDI subsidy to attract even more Foreign MNCs.

---

[16]Experimenting with the calibration parameters in Section 3.5 shows that when both k is increased (to about $k = 50$) and μ is raised closer to the value of α (for example, $\mu = 0.6$ and $\alpha = 0.74$), then W_{Ff} has 3 turning points in $(0,1)$.

[17]This optimal subsidy discussion puts aside normative issues with regards to whether the subsidy can in practice be implemented. A more thorough treatment of the determinants of FDI subsidy levels would likely need to incorporate political economy considerations. See for example Janeba (2004), where FDI subsidies generate a re-distributive effect from workers to firms, which inherently limits the scope for large subsidies if workers have sufficient political clout.

[18]Together with the earlier condition $\tau w_H > w_F$, this implies the necessary condition $\frac{1}{\tau} < \frac{w_H}{w_F} < \tau$. Intuitively, wages cannot differ too much between the two countries if horizontal FDI is to take place in both directions.

3.2 FDI subsidy to variable costs

What happens if the financial incentives to multinationals are targeted towards their variable costs of production instead? Many of the incentive schemes offered in practice, such as job-creation subsidies or corporate tax rate cuts, fall into this category. I show below that the welfare implications of such variable cost subsidies are qualitatively similar to what we have seen for a fixed cost subsidy.

Consider then a subsidy to the variable costs of Home MNCs' production in Foreign that reduces their effective unit wage costs from w_F to $(1 - s_v)w_F$, where $s_v < 1$. As before, suppose that the state pays for these subsidies by levying an income tax on its citizens, t_v. If this incentive scheme is to be budget-neutral, then:

$$t_v w_F M_F = s_v w_F N A_F \left(\frac{(1 - s_v)w_F}{\alpha} \right)^{-\varepsilon} V^H(a_I) \tag{17}$$

where the right-hand side of (17) is the total amount paid out as production subsidies to the multinationals. Note that a higher demand for Home final goods, A_F, will now raise the total subsidy bill directly under a variable cost subsidy scheme.

We have the following parallel result concerning the welfare improvement from a subsidy to MNCs' variable costs of production:

Proposition 2 [Variable Cost Subsidy]: *Consider the family of variable cost FDI subsidies characterized by s_v that: (i) satisfy the balanced-budget constraint (17); and (ii) preserve the sorting pattern of organizational forms within the industry, namely $a_D > a_X > a_I$. Then the optimal policy, s_v^*, that maximizes welfare in Foreign is a strictly positive subsidy level, namely $s_v^* \in (0, 1)$.*

Proof. Once again, it suffices to examine the terms $W_{Fv} \equiv (1 - t_v)w_F + \frac{1-\mu}{\mu}(X_F^H)^\mu$ in the indirect utility function. The state's budget constraint (17) allows us to re-write t_v in terms of s_v; making this substitution and using the expression for $(X_F)^\mu$ from (15), one obtains:

$$W_{Fv} = w_F + \left(N \frac{k}{\alpha} \right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu - \alpha)}} \frac{\tilde{M}_F}{M_F} \left[\frac{1 - \mu}{\mu} \tilde{\Lambda}_{Fv} - s_v \left(\frac{\alpha}{k} \right) \frac{\tilde{\Lambda}_{Fv}}{\Lambda_{Fv}} \frac{\partial \Lambda_{Fv}}{\partial s_v} \right]$$

where Λ_{Fv} and $\tilde{\Lambda}_{Fv}$ are given by Λ_F and $\tilde{\Lambda}_F$ respectively except with $(w_F)^{1-\varepsilon}$ replaced by $((1 - s_v)w_F)^{1-\varepsilon}$ in the denominator, while:

$$\frac{\partial \Lambda_{Fv}}{\partial s_v} = \frac{\varepsilon - 1}{k - \varepsilon + 1} \left(\frac{\alpha}{a_H} \right)^k \left(\frac{1 - \alpha}{w_H} \right)^{\frac{k}{\varepsilon - 1}} \frac{(f_I - f_X)^{\frac{k}{1-\varepsilon}+1} w_H}{(((1 - s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}+1}} k w_F^{1-\varepsilon}(1 - s_v)^{-\varepsilon}$$

The welfare function to be maximized, W_{Fv}, is thus completely analogous to that from the fixed cost subsidy case (W_{Ff}). It is straightforward to check that the proof that $\frac{\partial W_{Fv}}{\partial s_v} > 0$ at $s_v = 0$ follows the same steps as in Proposition 1. For this, we will once again require that the distribution of firm

16

productivity levels not be too thick-tailed, namely $k > \mu(\varepsilon - 1)$, which ensures a reasonably small mass of infra-marginal multinationals and hence a small total subsidy bill in the neighborhood of $s_v = 0$. In addition, one can show that $\frac{\partial W_{Fv}}{\partial s_v} > 0$ for all $s_v < 0$, while $W_{Fv} \rightarrow -\infty$ when s_v approaches 1 (see Appendix 7.2).

The welfare function in the variable cost subsidy case thus has a positive slope at $s_v = 0$, a turning point in the interior of $(0, 1)$ and an asymptote to $-\infty$ at $s_v \rightarrow 1^-$. To retain the ordering of productivity cut-offs, s_v cannot be increased all the way towards 1, as this would eventually reduce the FDI cut-off, $a_I^{1-\varepsilon}$, below the cut-off for exporting, $a_X^{1-\varepsilon}$. Nevertheless, this does not detract from the conclusion that the optimal policy is a strictly positive subsidy level, $s_v^* \in (0, 1)$. Once again, while the equation $\frac{\partial W_{Fv}}{\partial s_v} = 0$ may in principle have more than one zero in $(0, 1)$, this is the case only for extreme parameter values, so that the optimal subsidy is in general unique. ∎

As with a fixed cost subsidy, the variable cost subsidy exhibits the same selection effect of drawing in the most productive Home exporters who were just shy of the $a_I^{1-\varepsilon}$ cut-off for FDI. In addition, there is now also a production effect at play here, in that the variable cost subsidy raises output levels at all overseas assembly plants, even those belonging to MNCs that would have conducted FDI without the subsidy. This production effect raises further the consumption gains from a variable cost subsidy. Put otherwise, the variable cost subsidy helps to (partially) counteract the inefficiency stemming from the firms' monopoly pricing power, by reducing the effective price mark-up of MNCs from $\frac{1}{\alpha}$ to $\frac{1-s_v}{\alpha}$.

Section 3.5 shall turn to a more careful comparison of the relative efficacy of fixed versus variable cost subsidies. But first, it is useful to isolate the key role played by firm heterogeneity for the welfare results, by explicitly contrasting what happens with the use of FDI subsidies when all Home firms have identical productivity levels.

3.3 Comparison to a model with homogenous firms

Consider now the case where all firms in Home's differentiated goods sector share the same unit labor input coefficient, $\bar{a}$, as in Krugman (1980). Within the framework set up in Section 2, this corresponds to a situation where $G^H(a)$ has its entire mass concentrated at a single point. For simplicity, I shall continue to treat N as exogenous.

Suppose that all the Home firms initially service Foreign via exports instead of via FDI, namely $\bar{a}$ satisfies $\pi_D(\bar{a}), \pi_X(\bar{a}) > 0$, but $\pi_X(\bar{a}) < \pi_I(\bar{a})$, where the profit functions come from (5), (6) and (7). The question of interest would then be whether a subsidy to the Home firms inducing a switch from exporting to FDI improves welfare in Foreign. It turns out that in this setting of identical firm productivity levels, the scope for welfare improvement from a subsidy to FDI is **not** guaranteed.

For the purpose of illustration, let us focus on the case of a fixed cost subsidy. Consider a subsidy, s_f, that would make the Home firms exactly indifferent between exporting and FDI as their preferred mode of servicing the Foreign market. This is the smallest subsidy level that would make the policy effective in inducing Home firms to switch to FDI. Setting $\pi_X(\bar{a}) = \pi_I(\bar{a})$, s_f therefore satisfies:

$$(1-\alpha)A_{F,I}\left(\frac{\bar{a}w_F}{\alpha}\right)^{1-\varepsilon} - (1-\alpha)A_{F,X}\left(\frac{\tau\bar{a}w_H}{\alpha}\right)^{1-\varepsilon} = (1-s_f)(f_I - f_X)w_H \qquad (18)$$

where $A_{F,X}$ and $A_{F,I}$ are respectively the levels of Foreign market demand in the old equilibrium where all Home firms export and in the new equilibrium where all Home firms conduct FDI. Now, the expression for X_i^c in (3) implies that $(X_{F,X})^\mu = N^{\frac{1-\alpha}{\alpha}\frac{\mu}{1-\mu}}\left(\frac{\tau\bar{a}w_H}{\alpha}\right)^{-\frac{\mu}{1-\mu}}$ when Home firms export, and $(X_{F,I})^\mu = N^{\frac{1-\alpha}{\alpha}\frac{\mu}{1-\mu}}\left(\frac{\bar{a}w_F}{\alpha}\right)^{-\frac{\mu}{1-\mu}}$ when Home firms undertake FDI (the X and I subscripts distinguish between the consumption aggregates in these two scenarios). From this, one can compute the market demand levels, $A_{F,X}$ and $A_{F,I}$, using the definition: $A_F = M_F(X_F)^{\frac{\mu-\alpha}{1-\alpha}}$, and substitute the subsequent expressions obtained back into (18).

As before, let the fixed cost subsidy be paid for by revenues from a tax, t_f, on workers:

$$t_f w_F M_F = s_f(f_I - f_X)w_H N$$

Recall that the relevant welfare measure is: $(1-t_f)w_F + \frac{1-\mu}{\mu}(X_F^H)^\mu$. The minimum effective FDI subsidy level given by (18) can be substituted into the above balanced-budget constraint to obtain an expression for t_f. One can now derive an expression for the change in welfare for Foreign from the use of this fixed cost subsidy to Home firms to induce a switch to FDI:

$$\Delta W_F = N^{\frac{1-\alpha}{\alpha}\frac{\mu}{1-\mu}}\left(\frac{1}{\mu} - \alpha\right)\left[\left(\frac{\bar{a}w_F}{\alpha}\right)^{-\frac{\mu}{1-\mu}} - \left(\frac{\tau\bar{a}w_H}{\alpha}\right)^{-\frac{\mu}{1-\mu}}\right] - \frac{(f_I - f_X)w_H N}{M_F} \qquad (19)$$

The first summand in (19) represents the consumption gains to Foreign from a lower price of Home final goods; this is strictly positive since $\frac{1}{\mu} > 1 > \alpha$ and $\tau w_H > w_F$. Meanwhile, the second term is equal to the funding cost of the FDI subsidy program. Observe now that ΔW_F can in fact be negative: The higher the fixed cost of FDI (the larger is $(f_I - f_X)$), the larger the total subsidy bill required to attract multinationals, and this could potentially overwhelm the consumption gains from this policy action. Likewise, the higher is $\bar{a}$ (the less productive the Home firms are), the smaller the price decrease for Foreign consumers when Home firms switch from exporting to FDI, and the resulting consumption gains may end up being too small relative to the subsidy bill to generate a welfare increase. (The set-up of the model does not constrain how large $f_I - f_X$ or $\bar{a}$ can be: The initial condition $\pi_X(\bar{a}) > \pi_I(\bar{a})$ delivers a positive lower bound on the magnitude of $(f_I - f_X)$, and not an upper bound, as would be needed to limit the size of the subsidy bill. Likewise, this inequality implies a lower bound on $\bar{a}$.)

18

This exercise highlights the key role played by firm heterogeneity and the selection effect for the results of Sections 3.1 and 3.2. When firms are instead homogenous with respect to their productivity levels, any subsidy that is effective in attracting Home multinationals necessarily also induces the full measure of Home exporters to switch to FDI. This can imply a large subsidy burden if the fixed cost of conducting FDI is very high. Moreover, the consumption gain can be too small to justify attracting all Home firms to undertake FDI if the Home firms are not particularly productive to begin with. In essence then, the industry equilibrium with heterogeneous firms moderates the amount of FDI induced by the subsidy by sieving out the most productive Home exporters, a selection mechanism that is critical for delivering a welfare improvement.

3.4 Endogenizing the measure of varieties (N)

Up till now, the measure of Home firms, N, has been treated as exogenous in order to focus on the shift in the FDI cut-off, $a_I^{1-\varepsilon}$, that an FDI subsidy induces. However, when the host country is a large market, this policy action by Foreign also increases the *ex ante* profitability of potential entrant firms to the Home differentiated goods sector. I show in this subsection how to incorporate this additional varieties effect by endogenizing N in the full industry equilibrium. The model remains highly tractable, with the subsequent increase in the measure of Home firms reinforcing the gains that accrue to Foreign consumers.

Free-entry: N is pinned down by a free-entry condition for the Home sector, which closes the industry equilibrium in Section 2. Potential entrants do not observe their productivity draw $1/a$ until after they have started paying a per-period fixed cost of entry equal to f_E units of Home labor. These prospective firms therefore weigh their expected profits after entry against this fixed cost, with zero *ex ante* profits prevailing in equilibrium. This free-entry condition for the Home sector is:

$$
\begin{aligned}
f_E w_H &= (1-\alpha)A_H \left(\frac{w_H}{\alpha}\right)^{1-\varepsilon} V^H(a_D) + (1-\alpha)A_F \left(\frac{\tau w_H}{\alpha}\right)^{1-\varepsilon} (V^H(a_X) - V^H(a_I)) \ldots \\
&\quad \ldots + (1-\alpha)A_F \left(\frac{w_F}{\alpha}\right)^{1-\varepsilon} V^H(a_I) - f_D w_H G^H(a_D) \ldots \\
&\quad \ldots - f_X w_H (G^H(a_X) - G^H(a_I)) - f_I w_H G^H(a_I)
\end{aligned}
\tag{20}
$$

where the right-hand side comes from integrating over the productivity distribution, $G^H(a)$, to compute expected profits prior to entry. By substituting the productivity cut-offs in (8)-(10) and the expressions from the Pareto distribution in (11)-(12) into (20), this free-entry condition simplifies to:

$$
f_E w_H = \Lambda_H (A_H)^{\frac{k}{\varepsilon-1}} + \Lambda_F (A_F)^{\frac{k}{\varepsilon-1}}
\tag{21}
$$

This last equation has an intuitive interpretation: $\Lambda_H (A_H)^{\frac{k}{\varepsilon-1}}$ captures the normalized profits from sales in Home weighted by a measure of the level of demand in the Home market, with $\Lambda_F (A_F)^{\frac{k}{\varepsilon-1}}$

being the analogous terms for profits from the Foreign market. The free-entry condition (21) thus equates the fixed cost of entry with the total expected profits for the firm from both markets.

Substituting now from (13) and (14) into the definition: $A_i = M_i(X_i)^{\frac{\mu-\alpha}{1-\alpha}}$, yields:

$$(A_i)^{\frac{k}{\varepsilon-1}} = \left(N\frac{k}{\alpha}\right)^{\frac{k(\mu-\alpha)}{\mu\alpha-k(\mu-\alpha)}} \frac{\tilde{\Lambda}_i}{\Lambda_i}\tilde{M}_i, \quad i = H, F \tag{22}$$

Bearing in mind that $\mu < \alpha$, (22) implies that the level of demand facing each firm in market i is increasing in the size of that market, M_i, while decreasing in the measure of competing varieties, N.

The free-entry condition (21) and the two equations in (22) comprise a system of three equations in three unknowns, N, A_H and A_F, that can be solved for the equilibrium in the Home sector. Specifically, substituting from (22) into (21) and re-arranging delivers the measure of firms, N, as a function of the model parameters:

$$N = \frac{\alpha}{k}\left[\frac{\tilde{M}_H\tilde{\Lambda}_H + \tilde{M}_F\tilde{\Lambda}_F}{f_E w_H}\right]^{\frac{\mu\alpha-k(\mu-\alpha)}{k(\alpha-\mu)}} \tag{23}$$

Not surprisingly, (23) states that an increase in entry costs, f_E, decreases the measure of Home firms in equilibrium. Similarly, a rise in Home wages, w_H, decreases the expected normalized profits from both markets (both Λ_H and Λ_F), while also raising entry costs, $f_E w_H$, and hence reduces N. This expression for N can be substituted into (15) to evaluate the consumption aggregate $(X_F)^\mu$, and hence welfare in Foreign, as a function of exogenous parameters only. It is easy to observe that both N and $(X_F)^\mu$ exhibit similar comparative statics with respect to most of the structural parameters of the model; in particular, both variables rise as the market size, M_i, or normalized profit levels, Λ_i, of either country increase.

Allowing N to be endogenous introduces an additional varieties effect from the use of an FDI subsidy. Denoting N_{Ff} (respectively N_{Fv}) to be the measure of Home firms in the equilibrium with a fixed cost subsidy s_f (respectively, a variable cost subsidy s_v), we have:

Lemma 1: $\frac{\partial N_{Ff}}{\partial s_f}, \frac{\partial N_{Fv}}{\partial s_v} > 0$ for all $s_f, s_v < 1$.

Intuitively, the subsidy to FDI raises the profitability of potential Home entrants. In equilibrium, the thickness of the supply side of this sector has to increase in response to ensure that firms continue to earn zero *ex ante* profits. Due to the "love of varieties" exhibited by the utility function, this increase in N amplifies the consumption gains arising from the use of a subsidy. For the net effect on Foreign welfare, one must weigh this against the higher subsidy bill to be paid to the larger mass of Home firms. It turns out nevertheless that when N is endogenous, the welfare functions W_{Ff} or W_{Fv} continue to inherit the same shape as in the baseline case where N is fixed: Welfare is an increasing

function of s_f (or s_v) when the subsidy level is negative; exhibits a positive slope when the subsidy level is zero; but hits a negative asymptote as the subsidy level approaches 1. I summarize this result in the following proposition (see Appendix 7.3 for a sketch of the proof):

Proposition 3 [N **endogenous**]: *For both fixed and variable cost subsidy schemes, the optimal subsidy policy that maximizes welfare in Foreign when N is endogenous continues to be a strictly positive subsidy level that lies in the interior of the interval $(0,1)$.*

In sum, the varieties effect introduced when N is endogenous is an additional effect that does not alter the welfare implications from a subsidy to FDI. In fact, Appendix 7.3 shows that the slope of the welfare function when the subsidy level is 0 is larger when N is endogenous compared to the baseline case where N is fixed (for both the fixed and variable cost cases). Thus, for small subsidy levels, the increase induced in the measure of Home varieties amplifies the welfare gains accruing to Foreign. Since Home consumers also benefit from the expansion of varieties, the FDI subsidy in fact generates an improvement for both countries.

3.5 Fixed versus variable cost subsidies

I turn now to a comparison of the impact and efficacy of the two types of subsidy schemes considered. In particular, how does the welfare level at the optimal fixed cost subsidy, s_f^*, compare to that at the optimal variable cost subsidy, s_v^*?

The answer to this question is best illustrated graphically. Figure 3 plots the welfare functions W_{Ff} and W_{Fv} from a simple calibration of the model. As in Ghironi and Melitz (2005), I set the elasticity of consumer demand to $\varepsilon = 3.8$ (which implies $\alpha = 0.74$), and the key productivity spread parameter to $k = 3.4$. (The qualitative nature of Figure 3 is unchanged if the higher value of $\varepsilon = 6$ more commonly seen in the macroeconomics literature is used instead to imply a smaller price mark-up.) Following their lead, I also fix $f_X = 0.23$, $f_E = 1$, and $\tau = 1.3$.[19] There is less precedent in the empirical literature for the remaining model parameters, although the conditions $\mu < \alpha$, $\tau w_H > w_F$, and $a_D > a_X > a_I$ impose some discipline on the values that can be chosen. While the baseline calibration in Figure 3 adopts the values: $\mu = 0.3$, $f_D = 0.1$, $f_I = 2$, $w_H = w_F = 1$, $a_H = 1$, and $M_H = M_F = 1$, the general shape of the welfare functions is nevertheless very similar under a wide

---

[19]Ghironi and Melitz (2005) adopt the value $\varepsilon = 3.8$ from Bernard et al.'s (2003) empirical estimation based on US plant and trade data. Bernard et al. (2003) also estimate the log standard deviation of plant sales to be 1.67; since this moment is equal to $1/(k - \varepsilon + 1)$ in the model, this implies a value of $k = 3.4$. Ghironi and Melitz (2005) set $f_X = 0.23$, in order to match the share of exporting plants reported in Bernard et al. (2003). Last but not least, they set $\tau = 1.3$ following Obstfeld and Rogoff's (2001) deductions regarding the magnitude of this iceberg transport cost parameter needed to reconcile several empirical puzzles in international macroeconomics.

range of calibrations that continue to respect the structural assumptions of the model, in particular the ordering of the productivity cut-offs.[20]

Several observations emerge from Figure 3. First, the shape of the welfare functions confirms the existence of a unique optimal subsidy for this parametrization, both in the fixed and variable cost cases.[21] Second, the variable cost subsidy appears to generate much higher levels of welfare than the fixed cost subsidy for small positive subsidy rates. This confirms the intuition articulated in Section 3.2 that a variable cost subsidy has the potential to generate a greater kick to welfare by reducing the distortion arising from firms' monopoly pricing power: The effective mark-ups that consumers pay is lowered, while output at each firm is raised from their inefficiently low levels. The resulting increase in consumption appears here to generate greater utility gains for Foreign. This production effect is absent with a fixed cost subsidy, which only affects a Home firm's decision on exporting versus FDI, but not its output levels. Third, allowing the measure of firms N to respond to the introduction of the subsidy accentuates the welfare functions without altering their general shape, as was asserted in Proposition 3. For the small to moderate positive subsidy levels graphed in Figure 3, the case with endogenous N (dashed-line graphs) has welfare levels raised above that when the measure of Home firms is fixed (solid-line graphs).[22]

Figure 3 suggests that there is a *prima facie* case in favor of variable cost subsidies from the perspective of Foreign welfare levels. We can in fact derive the following result for the case where N is exogenous (see Appendix 7.4 for a proof):

Proposition 4 [Fixed versus variable cost subsidy]: *Suppose that $\varepsilon > 2$ and that the measure of Home firms is fixed. Consider only those subsidies with $s_f, s_v \in (0,1)$ that: (i) satisfy a balanced-budget constraint; and (ii) preserve the sorting pattern of organizational forms within the industry, namely $a_D > a_X > a_I$. Then, a variable cost subsidy that incurs the same total subsidy bill as a fixed cost subsidy always delivers greater consumption gains to Foreign.*

A variable cost subsidy therefore has more bang for the buck, delivering a greater increase in utility than a corresponding fixed cost subsidy that incurs the same amount of public spending. It follows that the welfare level achieved by the optimal variable cost subsidy, s_v^*, will in general be higher than that reached by the optimal fixed cost subsidy, s_f^*. The requirement that $\varepsilon > 2$ also has an intuitive

---

[20]For the parameters used in Figure 3, it takes relatively large values of s_f and s_v to lower the FDI cut-off, $a_I^{1-\varepsilon}$, down to the level of the exporting cut-off, $a_X^{1-\varepsilon}$. Specifically, a value of either $s_f = 0.86$ or $s_v = 0.40$ is required to equate the two cut-offs (namely, $a_X = a_I$). The range of subsidy values plotted on the horizontal axis in Figure 3 is therefore consistent with the desired ordering of the productivity cut-offs (in particular, $a_X > a_I$), with the welfare functions reaching their maximum turning points within a reasonably low range of subsidy values.

[21]This is not surprising for the case of the fixed cost subsidy: As noted before, the calibration parameters satisfy the sufficient condition $2(\varepsilon - 1) > k$ for a unique turning point.

[22]For the exogenous N graphs, the value of N used is given by (23) with the subsidy level set to 0.

interpretation: Consumer demand needs to be sufficiently elastic, so that a given price decrease with the introduction of a variable cost subsidy will generate a large increase in consumption.[23]

It is important however to raise a key caveat that will qualify the above result placing variable cost subsidies in a more favorable light. As it stands, the only motive in the model for a Home firm to open a plant in Foreign is to take advantage of the proximity-concentration tradeoff in servicing the Foreign market, so that FDI is of a purely horizontal nature. However, much of the foreign affiliate activity that takes place in the real world is intended to service more than just the local market, with some output from the foreign assembly plant being shipped back to the home or third-country markets. It turns out that the scope for welfare improvement from a variable cost subsidy to production is potentially fragile to incorporating such an export-platform motive, since re-exports represent subsidized output for which the consumption gains accrue purely to foreigners. If the third-country market that the affiliate is servicing is large, re-exports would raise the total subsidy bill without generating corresponding gains to domestic consumers, potentially negating the scope for a welfare improvement (the slope of the welfare function at $s_v = 0$ could even be negative). To avoid this outcome, the subsidy would then have to be administered as a domestic sales or retail credit, instead of as a rebate to production. In contrast, this criticism does not apply to a fixed cost subsidy, since it alters only firms' mode of servicing the Foreign market, but does not affect firms' choice of output levels.

4   Some Extensions

I discuss briefly two extensions of the baseline model. The first of these examines a similar policy intervention, namely import subsidies to encourage more Home firms to service the Foreign market. The second extension explores the robustness of the welfare results to alternative utility specifications that build in richer income effects, albeit at the expense of some analytical tractability.

4.1   Import subsidies

The model presented above lends itself naturally to an analysis of trade policy. It is well-established that the optimal policy for a country with a downward-sloping demand curve is to levy a positive import tariff, since the gains in tariff revenue outstrip the loss in consumer surplus when the tariff is small (see Helpman and Krugman (1989)). In view of this result, previous arguments advanced in favor of the opposite policy of import subsidies have relied on the existence of dynamic gains: Dasgupta and Stiglitz (1988) for example posit that when learning-by-doing effects are large, it may

---

[23]The proof in Appendix 7.4 makes it evident that a more precise lower bound for ε can be derived, namely $\varepsilon > \frac{2k+1}{k+1}$. This last inequality clearly implies $\varepsilon > 2$.

be optimal to subsidize imports to promote learning by foreign producers, thus lowering prices in the long run.

The framework with heterogeneous firms suggests one additional mechanism through which import subsidies might be beneficial, by increasing the set of products that is exported to the Foreign market. In this case, however, the intuition is a little less neat: Such a subsidy will induce some Home firms to start exporting to Foreign, delivering positive consumption gains, but it will also prompt some Home MNCs to switch from horizontal FDI back to exporting, raising prices for Foreign consumers for this margin of goods. There is thus a positive selection effect from the leftward shift of the $a_X^{1-\varepsilon}$ cut-off in Figure 1, but a detrimental effect as the $a_I^{1-\varepsilon}$ cut-off moves to the right. *A priori* at least, it is not clear if the net effect will be a positive welfare gain accruing to Foreign.

It is nevertheless straightforward to compute the welfare impact of either a fixed or variable cost subsidy to imports, following the steps laid out in Sections 3.1 and 3.2. It turns out that for this alternative policy intervention, a small subsidy is once again welfare-improving:

Proposition 5 [Import Subsidies]: *Consider the family of fixed cost (respectively variable cost) import subsidies that: (i) satisfy a balanced-budget constraint; and (ii) preserve the sorting pattern of organizational forms within the industry, namely $a_D > a_X > a_I$. Then the optimal policy that maximizes welfare in Foreign is a strictly positive subsidy level.*

The proof of this proposition when N is exogenous is essentially identical to that for Propositions 1 and 2, hinging on the fact that the welfare function has a positive slope in the neighborhood where the subsidy level is zero. (See Appendix 7.5 for details.) Thus, it turns out that the consumption gains from drawing in more firms at the $a_X^{1-\varepsilon}$ cut-off outweigh the loss of MNCs at the $a_I^{1-\varepsilon}$ margin, with the underlying intuition being that there is a greater density of firms at the cut-off for exporting than at the cut-off for FDI. Once again, this result relies on the inequality: $k > \mu(\varepsilon - 1)$, or equivalently that the distribution $G^H(a)$ not have too thick a right tail, so that the measure of firms that needs to be subsidized is not too large. Naturally, when N is allowed to be endogenous, the entry of more firms in the Home differentiated sector generates a positive varieties effect that reinforces the welfare gains from a small subsidy, akin to Proposition 3.

4.2 Robustness to specification of utility function

To this point, the quasilinear utility function (1) has facilitated the analysis by providing a closed-form expression for country welfare that is a function only of the model's structural parameters. One relevant concern, however, is that this quasilinearity carries with it the restriction that any exogenous change in labor income, w_F, affects only the consumption level of the homogenous good. If instead

the demand for differentiated products were also to have a strictly positive income elasticity, then the decrease in disposable income from the labor tax could potentially dampen the welfare gains from an FDI subsidy.

A natural way to incorporate these more general income effects is to consider the following country i utility function:

$$U_i = \left((x_i^0)^\rho + \left(\sum_{c=H,F} \frac{1}{\mu} (X_i^c)^\mu \right)^{\frac{\rho}{\mu}} \right)^{\frac{1}{\rho}} \tag{24}$$

This specification introduces a constant elasticity of substitution between the demand for homogenous goods, x_i^0, and the differentiated varieties aggregate, $\sum_{c=H,F} \frac{1}{\mu}(X_i^c)^\mu$, with the substitution parameter equal to $\frac{1}{1-\rho}$. We stipulate that $0 < \rho < \mu < \alpha < 1$, so that differentiated products are always better substitutes for each other than the homogenous good. Since utility is no longer quasilinear in x_i^0, the demand for differentiated products, $x_i^c(a)$, now depends directly on disposable income, $(1 - t_f)w_F$. This does however come at the expense of some analytical tractability, as welfare can no longer be written down in a closed-form, so the subsidy analysis will require us to perform comparative statics on the system of equations that defines the industry equilibrium.

It turns out nevertheless that the welfare results pertaining to the impact of fixed and variable cost subsidies continue to hold with this alternative utility function (24), so that the optimal policy for Foreign remains a small subsidy to multinationals. The proof of this proposition (sketched out in Appendix 7.6 for the fixed cost subsidy case) hinges once again on the condition $k > \varepsilon - 1$, which ensures a relatively small mass of infra-marginal firms, and hence a small total direct funding cost for the subsidy scheme. Intuitively too, this implies a small per capita income tax burden, t_f, so that the negative income effect of this tax on the consumption of differentiated products fails to dampen out the net welfare gains. In sum, it is reassuring that the main welfare implications for FDI subsidies carry through when some allowance is made for the demand for differentiated products to be subject to income effects.

5 Conclusion

There has been much recent work in international economics on models of firm heterogeneity aimed at understanding the interaction between globalization and industry structure. This paper builds upon this work by extending it to a policy analysis of FDI subsidies, which have been used with increasing frequency by countries that perceive economic gains from attracting MNCs to their shores.

To this end, this paper has developed a two-country version of the Helpman et al. (2004) model that admits a tractable closed-form expression for consumer welfare in each country, greatly facilitating a

welfare analysis of the effects of FDI subsidies. Focusing only on the consumption gains to attracting FDI (from the lowered prices at which MNCs' products are sold in the host country market), I showed that an FDI subsidy to either MNCs' fixed or variable costs of operation leads unambiguously to a rise in consumer welfare in the host country, after netting out the cost of financing this policy through a tax on workers' income. Of note, this result does not require us to appeal to other potential benefits of FDI, such as technological spillovers or agglomeration economies, that have received a fair amount of attention in the related policy debates.

This scope for a welfare improvement is driven by a selection effect, highlighting the key role played by firm heterogeneity in these results. The FDI subsidy enables the host country to attract those Home country exporters that previously fell just shy of the FDI cut-off to now switch to servicing the Foreign market via horizontal FDI. Being from a relatively productive margin, these firms set lower prices and are demanded in relatively high quantities, so that the resulting savings on cross-border transport costs from the switch to FDI delivers substantial consumption gains to Foreign. Nevertheless, for these resulting consumption gains to exceed the direct cost of the subsidy scheme, one also needs the mass of multinationals to be subsidized to be relatively small, which translates in our setting into a requirement that the firm distribution not place too large a mass on high productivity levels. What is particularly neat is that the implied formal condition on the heterogeneity parameter of firm productivities is one that prior empirical research on industry distributions (by Helpman et al. (2004)) has shown is readily satisfied in practice. The model also suggests that a variable cost subsidy generates much greater gains to the host country than a fixed cost subsidy, given that a reduction in MNCs' variable costs of operation has an additional production effect that partially corrects the inefficiently low output levels stemming from firms' monopoly pricing power.

As a cautionary concluding remark, it has to be stressed that this paper should not be seen as a *carte blanche* endorsement of generous subsidies to FDI. Naturally, the actual implementation and governance of any such subsidy schemes will have to be carried out very carefully, in order to determine what the appropriate subsidy level should be. Moreover, while the model does establish a set of sharp benchmark results, it does so by abstracting from such issues as strategic competition among countries for FDI or possible political economy forces (lobbying by parties with vested interests) that could qualify and alter the conclusions concerning the efficacy of such FDI subsidies in practice. These are all relevant issues for future work to consider within this setting with heterogeneous firms.

6 References

Aitken, B., Harrison, A., 1999. Do domestic firms benefit from direct foreign investment? Evidence from Venezuela. American Economic Review 89, 605-618.

Baldwin, R., Forslid, R., Martin, P., Ottaviano, G.I.P., Robert-Nicoud, F., 2003. Economic Geography and Public Policy. Princeton University Press, USA.

Baldwin, R., Okubo, T., 2006. Heterogeneous firms, agglomeration and economic geography: Spatial selection and sorting. Journal of Economic Geography 6, 323-346.

Bernard, A., Eaton, J., Jensen, J.B., Kortum, S., 2003. Plants and productivity in international trade. American Economic Review 93, 1268-1290.

Black, D.A., Hoyt, W.H., 1989. Bidding for firms. American Economic Review 79, 1249-1256.

Dasgupta, P., Stiglitz, J., 1988. Learning-by-doing, market structure and industrial and trade policies. Oxford Economic Papers 40, 246-268.

Demidova, S., Rodriguez-Clare, A., 2007. Export subsidies, productivity and welfare under firm-level heterogeneity. Pennsylvania State University mimeo.

Devereux, M.P., Griffith, R., 1998. Taxes and the location of production: Evidence from a panel of US multinationals. Journal of Public Economics 68, 335-367.

Ekholm, K., Forslid, R., Markusen, J.R., 2003. Export-platform foreign direct investment. NBER Working Paper 9517.

Fumagalli, C., 2003. On the welfare effects of competition for foreign direct investments. European Economic Review 47, 963-983.

Ghironi, F., Melitz, M.J., 2005. International trade and macroeconomic dynamics with heterogeneous firms. Quarterly Journal of Economics 120, 865-915.

Grossman, G., Helpman, E., Szeidl, A., 2006. Optimal integration strategies for the multinational firm. Journal of International Economics 70, 216-238.

Haaland, J., Wooton, I., 1999. International competition for multinational investments. Scandinavian Journal of Economics 101, 631-649.

Haskel, J., Pereira, S.C., Slaughter, M.J., 2007. Does inward foreign direct investment boost the productivity of domestic firms? Review of Economics and Statistics 89, 482-496.

Haufler, A., Wooten, I., 1999. Country size and tax competition for foreign direct investment. Journal of Public Economics 71, 121-139.

Helpman, E., Melitz, M.J., Yeaple, S.R., 2004. Exports versus FDI with heterogeneous firms. American Economic Review 94, 300-316.

Helpman, E., Krugman, P., 1989. Trade Policy and Market Structure. MIT Press, USA.

Hines, J., 1996. Altered states: Taxes and the location of FDI in America. American Economic Review 86, 1076-1094.

Janeba, E., 2002. Attracting FDI in a politically risky world. International Economic Review 43, 1127-1155.

Janeba, E., 2004. Global corporations and local politics: Income redistribution vs FDI subsidies. Journal of Development Economics 74, 367-391.

Javorcik, B.S., 2004. Does foreign direct investment increase the productivity of domestic firms? In search of spillovers through backward linkages. American Economic Review 94, 605-627.

Jones, R.W., 1967. International capital movements and the theory of tariffs and trade. Quarterly Journal of Economics 81, 1-38.

Kemp, M.C., 1966. The gain from international trade and investment: A neo-Heckscher-Ohlin approach. American Economic Review 56, 788-809.

Krugman, P., 1980. Scale economies, product differentiation, and the pattern of trade. American Economic Review 70, 950-959.

Levy, S., Nolan, S., 1992. Trade and foreign investment policies under imperfect competition. Journal of Development Economics 37, 31-62.

MacDougall, G.D.A., 1960. The benefits and costs of private investment from abroad: A theoretical approach. Economic Record 36, 13-35. Reprinted in 1960, Bulletin of the Oxford University Institute of Economics and Statistics 22, 187-212.

Melitz, M.J., 2003. The impact of trade on intra-industry reallocation and aggregate industry productivity. Econometrica 71, 1695-1725.

Nunnenkamp, P., 2004. To what extent can foreign direct investment help achieve international development goals? World Economy 27, 657-677.

Obstfeld, M., Rogoff, K.S., 2001. The six major puzzles in international macroeconomics: Is there a common cause?" in: Bernanke, B.S., Rogoff K.S. (Eds.), NBER Macroeconomics Annual 2000, MIT Press, Cambridge, pp. 339-390.

Ottaviano, G.I.P., van Ypersele, T., 2005. Market size and tax competition. Journal of International Economics 67, 25-46.

Pennings, E., 2005. How to maximize domestic benefits from foreign investments: The effect of irreversibility and uncertainty. Journal of Economic Dynamics and Control 29, 873-889.

Rama, M., 2001. Globalization, inequality and labor market policies. World Bank manuscript.

The Economist, 2004. Tax wars: Corporate tax in the EU. July 24th issue.

UNCTAD, 2003. World Investment Report 2003. FDI Policies for Development: National and International Perspectives. United Nations, New York.

Wells, L.T., Allen, N.J., Morisset, J., Pirnia, N., 2001. Using Tax Incentives to Compete for Foreign Investment: Are They Worth the Costs? International Finance Corporation and the World Bank, USA.

7 Appendix (Details of Proofs)

7.1 Details of Proof of Proposition 1

Proof that $W_{Ff} \to -\infty$ when $s_f \to 1^-$. Recall that:

$$W_{Ff} = w_F + \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}} \frac{\tilde{M}_F}{M_F} \tilde{\Lambda}_{Ff} \left[\frac{1-\mu}{\mu} - s_f \left(\frac{\alpha}{k}\right) \frac{1}{\Lambda_{Ff}} \frac{\partial \Lambda_{Ff}}{\partial s_f}\right] \tag{25}$$

where, from the definition of Λ_F, we have:

$$\Lambda_{Ff} = \frac{\varepsilon-1}{k-\varepsilon+1} \left(\frac{\alpha}{a_H}\right)^k \left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}} \left[\frac{(f_X)^{\frac{k}{1-\varepsilon}+1} w_H}{(\tau w_H)^k} + \frac{((f_I - f_X)(1-s_f))^{\frac{k}{1-\varepsilon}+1} w_H}{((w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}\right]$$

Note that $\lim_{s_f \to 1^-} \Lambda_{Ff} = +\infty$, since $1 - \varepsilon < 0$. It follows that $\lim_{s_f \to 1^-} \tilde{\Lambda}_{Ff} = +\infty$, since $\tilde{\Lambda}_{Ff}$ is Λ_{Ff} raised to a positive power ($\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)} > 0$ as $\mu < \alpha$).

Moreover, some algebraic manipulation shows that $\frac{1}{\Lambda_{Ff}} \frac{\partial \Lambda_{Ff}}{\partial s_f} = \frac{k-\varepsilon+1}{\varepsilon-1} \frac{1}{1-s_f} g(s_f)$, where:

$$g(s_f) = \frac{\frac{((f_I - f_X)(1-s_f))^{\frac{k}{1-\varepsilon}+1}}{((w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}}{\frac{(f_X)^{\frac{k}{1-\varepsilon}+1}}{(\tau w_H)^k} + \frac{((f_I - f_X)(1-s_f))^{\frac{k}{1-\varepsilon}+1}}{((w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}}$$

Clearly, $\lim_{s_f \to 1^-} g(s_f) = 1$, which implies that $\lim_{s_f \to 1^-} \frac{1}{\Lambda_{Ff}} \frac{\partial \Lambda_{Ff}}{\partial s_f} = +\infty$. Hence, the limit of the term in the square brackets in (25) as s_f approaches 1 is $-\infty$. Together with the fact that $\lim_{s_f \to 1^-} \tilde{\Lambda}_{Ff} = +\infty$, we have $W_{Ff} \to -\infty$ when $s_f \to 1^-$ as claimed. $\blacksquare$

Proof that $\frac{\partial W_{Ff}}{\partial s_f} > 0$ for all $s_f < 0$. Since $\frac{\partial^2 \Lambda_{Ff}}{\partial s_f^2} / \frac{\partial \Lambda_{Ff}}{\partial s_f} = \frac{k}{\varepsilon-1} \frac{1}{1-s_f}$, the derivative of (25) can be re-written as:

$$\frac{\partial W_{Ff}}{\partial s_f} = \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}} \frac{\tilde{M}_F}{M_F} \frac{\tilde{\Lambda}_{Ff}}{\Lambda_{Ff}} \frac{\partial \Lambda_{Ff}}{\partial s_f} \frac{\alpha}{k} \left[\frac{k(1-\alpha) - \mu\alpha}{\mu\alpha - k(\mu-\alpha)} \cdots \right.$$
$$\left. \cdots + s_f \frac{k(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} \frac{1}{\Lambda_{Ff}} \frac{\partial \Lambda_{Ff}}{\partial s_f} - \frac{s_f}{1-s_f} \frac{k}{\varepsilon-1}\right] \tag{26}$$

The first summand on the right-hand side is positive, since $k(1-\alpha) - \mu\alpha > 0$ follows from $k > \mu(\varepsilon-1)$. Now observe that for $s_f < 0$, the last two summands are:

$$s_f \frac{k(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} \frac{1}{\Lambda_{Ff}} \frac{\partial \Lambda_{Ff}}{\partial s_f} - \frac{s_f}{1-s_f} \frac{k}{\varepsilon-1} = \frac{s_f}{1-s_f} \frac{k}{\varepsilon-1} \left[\frac{(k-\varepsilon+1)(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} g(s_f) - 1\right]$$
$$> \frac{s_f}{1-s_f} \frac{k}{\varepsilon-1} \left[\frac{(k-\varepsilon+1)(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} - 1\right]$$
$$= \frac{s_f}{1-s_f} \frac{k}{\varepsilon-1} \frac{(\alpha-\mu)(1-\varepsilon) - \mu\alpha}{\mu\alpha - k(\mu-\alpha)}$$
$$> 0$$

where we rely here in particular on the fact that $\varepsilon > 1$ and $\mu < \alpha$. Hence, the expression in the square brackets in (26) is positive, from which it follows that $\frac{\partial W_{Ff}}{\partial s_f} > 0$ whenever $s_f < 0$. ∎

Proof that $2(\varepsilon - 1) > k$ is a sufficient condition for W_{Ff} to have a unique turning point.
Setting the derivative in (26) equal to zero, any turning point of the welfare function must satisfy:

$$\tilde{g}(s_f) \equiv \frac{k(1-\alpha) - \mu\alpha}{\mu\alpha - k(\mu-\alpha)} \frac{1-s_f}{s_f} + \frac{k}{\varepsilon-1}\left(\frac{(k-\varepsilon+1)(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)}g(s_f) - 1\right) = 0$$

Note first that $\frac{1-s_f}{s_f}$ is a decreasing function of s_f, while $g(s_f)$ is increasing in s_f, so it is not possible to conclude that $\tilde{g}(s_f)$ is a monotonic function. Nevertheless, $\lim_{s_f \to 0^+} \tilde{g}(s_f) = +\infty$, while $\lim_{s_f \to 1^-} \tilde{g}(s_f) = \frac{k}{\varepsilon-1}\left(\frac{(k-\varepsilon+1)(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} - 1\right) < 0$. For $\tilde{g}(s_f)$ to have a unique zero in $(0,1)$, it therefore suffices that $\tilde{g}(s_f)$ be a strictly convex function in this interval. It is easy to check that $\frac{1-s_f}{s_f}$ is indeed strictly convex. Since the sum of two convex functions is also convex, I explore a sufficient condition for $g(s_f)$ to be convex. Some differentiation and algebraic substitution yields:

$$g''(s_f) = \frac{k-\varepsilon+1}{\varepsilon-1} \frac{g(s_f)(1-g(s_f))}{(1-s_f)^2}\left[\frac{k}{\varepsilon-1} - 2\frac{k-\varepsilon+1}{\varepsilon-1}g(s)\right]$$

Since $g(s_f) \in (0,1)$ for $s_f \in (0,1)$, we have strict convexity if and only if $g(s_f) < \frac{k}{2(k-\varepsilon+1)}$. A sufficient condition is therefore: $1 < \frac{k}{2(k-\varepsilon+1)}$, or equivalently $2(\varepsilon - 1) > k$. ∎

7.2   Details of Proof of Proposition 2

Proof that $W_{Fv} \to -\infty$ when $s_v \to 1^-$. Recall that:

$$W_{Fv} = w_F + \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}} \frac{\tilde{M}_F}{M_F}\tilde{\Lambda}_{Fv}\left[\frac{1-\mu}{\mu} - s_v\left(\frac{\alpha}{k}\right)\frac{1}{\Lambda_{Fv}}\frac{\partial\Lambda_{Fv}}{\partial s_v}\right] \tag{27}$$

where:

$$\Lambda_{Fv} = \frac{\varepsilon-1}{k-\varepsilon+1}\left(\frac{\alpha}{a_H}\right)^k\left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}}\left[\frac{(f_X)^{\frac{k}{1-\varepsilon}+1}w_H}{(\tau w_H)^k} + \frac{(f_I - f_X)^{\frac{k}{1-\varepsilon}+1}w_H}{(((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}\right]$$

Analogous to the proof in Appendix 7.1, we have $\lim_{s_v \to 1^-}\Lambda_{Fv} = \lim_{s_v \to 1^-}\tilde{\Lambda}_{Fv} = +\infty$.

Some algebraic work shows that $\frac{1}{\Lambda_{Fv}}\frac{\partial\Lambda_{Fv}}{\partial s_v} = \frac{k}{1-s_v}\frac{((1-s_v)w_F)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}h(s_v)$, where:

$$h(s_v) = \frac{\dfrac{(f_I-f_X)^{\frac{k}{1-\varepsilon}+1}}{(((1-s_v)w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}}{\dfrac{(f_X)^{\frac{k}{1-\varepsilon}+1}}{(\tau w_H)^k} + \dfrac{(f_I-f_X)^{\frac{k}{1-\varepsilon}+1}}{(((1-s_v)w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}}$$

Now, $\lim_{s_v \to 1^-}h(s_v) = 1$, while $\lim_{s_v \to 1^-}\frac{((1-s_v)w_F)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon}} = 1 > 0$. Together, these imply that $\lim_{s_v \to 1^-}\frac{1}{\Lambda_{Fv}}\frac{\partial\Lambda_{Fv}}{\partial s_v} = +\infty$. The limit of the term in the square brackets in (27) as $s_v \to 1^-$ is therefore $-\infty$, so that we have $\lim_{s_v \to 1^-}W_{Ff} = -\infty$ as desired. ∎

Proof that $\frac{\partial W_{Fv}}{\partial s_v} > 0$ for all $s_v < 0$. The derivative of (27) is:

$$\frac{\partial W_{Fv}}{\partial s_v} = \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}} \frac{\tilde{M}_F}{M_F} \frac{\tilde{\Lambda}_{Fv}}{\Lambda_{Fv}} \frac{\partial \Lambda_{Fv}}{\partial s_v} \frac{\alpha}{k} \left[\frac{k(1-\alpha) - \mu\alpha}{\mu\alpha - k(\mu-\alpha)} + s_v \frac{k(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} \frac{1}{\Lambda_{Fv}} \frac{\partial \Lambda_{Fv}}{\partial s_v} \cdots \right.$$
$$\left. \cdots - \frac{s_v}{1 - s_v} \frac{(k+1)((1-s_v)w_F)^{1-\varepsilon} - \varepsilon(\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} \right]$$

Since we know that $\frac{k(1-\alpha)-\mu\alpha}{\mu\alpha - k(\mu-\alpha)} > 0$, it suffices to show that the last two summands on the right-hand side add up to a positive quantity. Observe first that $(a_X)^{1-\varepsilon} < (a_I)^{1-\varepsilon}$ implies $\frac{f_X}{(\tau w_H)^{1-\varepsilon}} < \frac{f_I - f_X}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}$. Using this inequality to replace f_X in the denominator of $h(s_v)$ and simplifying, one obtains: $h(s_v) < \frac{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon}}$. For $s_v < 0$, we then have:

$$s_v \frac{k(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} \frac{1}{\Lambda_{Fv}} \frac{\partial \Lambda_{Fv}}{\partial s_v} - \frac{s_v}{1-s_v} \frac{(k+1)((1-s_v)w_F)^{1-\varepsilon} - \varepsilon(\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}$$
$$= \frac{s_v}{1-s_v} \left[\frac{k^2(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} \frac{((1-s_v)w_F)^{1-\varepsilon} h(s_v)}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} - \frac{(k+1)((1-s_v)w_F)^{1-\varepsilon} - \varepsilon(\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}\right]$$
$$> \frac{s_v}{1-s_v} \left[\frac{k^2(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} - \frac{(k+1)((1-s_v)w_F)^{1-\varepsilon} - \varepsilon(\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}\right]$$

For this last expression to be positive whenever $s_v < 0$, it suffices to show that:

$$\frac{(k+1)((1-s_v)w_F)^{1-\varepsilon} - \varepsilon(\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} > k \tag{28}$$

since $\frac{k^2(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} - k = \frac{-k\mu\alpha}{\mu\alpha - k(\mu-\alpha)} < 0$. This will then ensure that $\frac{\partial W_{Fv}}{\partial s_v} > 0$ whenever $s_v < 0$.

Bearing in mind that $((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon} > 0$, a re-arrangement of (28) yields:

$$((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon} + (k - \varepsilon + 1)(\tau w_H)^{1-\varepsilon} > 0$$

which holds, since $k - \varepsilon + 1 > 0$. ∎

7.3 Proofs from Section 3.4 (N endogenous)

Proof of Lemma 1. Log-differentiating (23), one has:

$$\frac{\partial N_{Ff}}{\partial s_f} = N_{Ff} \frac{\mu\alpha}{k(\alpha-\mu)} \frac{\tilde{M}_F \tilde{\Lambda}_{Ff}}{\tilde{M}_H \tilde{\Lambda}_H + \tilde{M}_F \tilde{\Lambda}_{Ff}} \frac{1}{\Lambda_{Ff}} \frac{\partial \Lambda_{Ff}}{\partial s_f} > 0$$

for all $s_f < 1$. An analogous expression holds for $\frac{\partial N_{Fv}}{\partial s_v}$ with s_f replaced by s_v and the subscript f replaced by v. ∎

Sketch of proof of Proposition 3. I illustrate this proof for the case of a fixed cost subsidy, since the argument for the case of a variable cost subsidy is virtually identical. Using the expression for W_{Ff} from (25), we have shown in Appendix 7.1 that $\tilde{\Lambda}_{Ff}\left[\frac{1-\mu}{\mu} - s_f\left(\frac{\alpha}{k}\right) \frac{1}{\Lambda_{Ff}} \frac{\partial \Lambda_{Ff}}{\partial s_f}\right]$ is a positive

increasing function in s_f when $s_f < 0$. Since N_{Ff} is also a positive increasing function in s_f for all $s_f < 1$, this implies that W_{Ff} must be increasing in s_f when s_f is negative.

Also, observe that $\lim_{s_f \to 1^-} N_{Ff} = +\infty$. Since $\tilde{\Lambda}_{Ff} \left[\frac{1-\mu}{\mu} - s_f \left(\frac{\alpha}{k} \right) \frac{1}{\Lambda_{Ff}} \frac{\partial \Lambda_{Ff}}{\partial s_f} \right]$ tends to $-\infty$ when s_f approaches 1, this implies that $\lim_{s_f \to 1^-} W_{Ff} = -\infty$.

Finally, the expression for $\frac{\partial W_{Ff}}{\partial s_f}$ when N is endogenous is given by (26) plus an extra term (from the product rule) to reflect the effect of s_f on N. This extra term is:

$$\frac{\mu\alpha}{\mu\alpha - k(\mu - \alpha)} \left(\frac{k}{\alpha} \right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}} (N_{Ff})^{\frac{k(\mu-\alpha)}{\mu\alpha - k(\mu-\alpha)}} \frac{\partial N_{Ff}}{\partial s_f} \frac{\tilde{M}_F}{M_F} \tilde{\Lambda}_{Ff} \left[\frac{1-\mu}{\mu} - s_f \left(\frac{\alpha}{k} \right) \frac{1}{\Lambda_{Ff}} \frac{\partial \Lambda_{Ff}}{\partial s_f} \right]$$

This is clearly positive when evaluated at $s_f = 0$, and hence the slope of W_{Ff} at $s_f = 0$ is larger when N is endogenous when compared to the baseline case where N is fixed. ■

7.4   Proof of Proposition 4

Proof. We establish this using a proof by contradiction. Suppose that the total subsidy bills from s_f and s_v are equal, where $s_f, s_v \in (0,1)$. From (25) and (27), this implies that: $s_f \frac{\tilde{\Lambda}_{Ff}}{\Lambda_{Ff}} \frac{\partial \Lambda_{Ff}}{\partial s_f} = s_v \frac{\tilde{\Lambda}_{Fv}}{\Lambda_{Fv}} \frac{\partial \Lambda_{Fv}}{\partial s_v}$, or equivalently that:

$$\frac{s_f}{1 - s_f} \tilde{\Lambda}_{Ff} \frac{k - \varepsilon + 1}{\varepsilon - 1} g(s_f) = \frac{s_v}{1 - s_v} \tilde{\Lambda}_{Fv} \frac{k((1 - s_v)w_F)^{1-\varepsilon}}{((1 - s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} h(s_v) \qquad (29)$$

However, suppose to the contrary that the consumption gains from the fixed cost subsidy are larger; pulling out the relevant terms from the welfare functions, this means that $\tilde{\Lambda}_{Ff} \geq \tilde{\Lambda}_{Fv}$. This inequality simplifies to:

$$\frac{((f_I - f_X)(1 - s_f))^{\frac{k}{1-\varepsilon}+1}}{((w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}} \geq \frac{(f_I - f_X)^{\frac{k}{1-\varepsilon}+1}}{(((1 - s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}} \qquad (30)$$

Observe though that (30) implies that $g(s_f) \geq h(s_v)$, and hence that $\tilde{\Lambda}_{Ff} g(s_f) \geq \tilde{\Lambda}_{Fv} h(s_v)$. Looking back at (29), we must therefore have:

$$\frac{s_f}{1 - s_f} \frac{k - \varepsilon + 1}{\varepsilon - 1} \leq \frac{s_v}{1 - s_v} \frac{k((1 - s_v)w_F)^{1-\varepsilon}}{((1 - s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} \qquad (31)$$

Now (30) simplifies directly to $s_f \geq 1 - \left[\frac{(w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} \right]^{\frac{k}{k-\varepsilon+1}}$, so that:

$$\frac{s_f}{1 - s_f} \geq \left[\frac{((1 - s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}{(w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} \right]^{\frac{k}{k-\varepsilon+1}} - 1 \qquad (32)$$

Combining (31) and (32) to eliminate $\frac{s_f}{1-s_f}$, the following inequality needs to be satisfied:

$$\left[\frac{((1 - s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}{(w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} \right]^{\frac{k}{k-\varepsilon+1}} - 1 - \frac{s_v}{1 - s_v} \frac{k(\varepsilon - 1)}{k - \varepsilon + 1} \frac{((1 - s_v)w_F)^{1-\varepsilon}}{((1 - s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} \leq 0 \quad (33)$$

32

Let us define the function in s_v on the left-hand side of this last inequality as $\psi(s_v)$. Observe that $\psi(0) = 0$. I shall now show that if $\varepsilon > 2$, then $\psi'(s_v) > 0$ for all $s_v \in (0,1)$, so that in fact $\psi(s_v) > 0$ for all positive subsidy levels. This will yield the desired contradiction to (33). Some algebra shows that $\psi'(s_v)$ is equal up to a positive multiplicative constant to:

$$
\left[\frac{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}{(w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} \right]^{\frac{k}{k-\varepsilon+1}} - \frac{1}{1-s_v} + \frac{s_v}{1-s_v} \frac{(\varepsilon-1)(\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}
$$

$$
> \left[1 + \frac{((1-s_v)^{1-\varepsilon} - 1)(w_F)^{1-\varepsilon}}{(w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} \right]^{\frac{k}{k-\varepsilon+1}} - \frac{1}{1-s_v}
$$

$$
> (1 + ((1-s_v)^{1-\varepsilon} - 1))^{\frac{k(1-\varepsilon)}{k-\varepsilon+1}} - \frac{1}{1-s_v}
$$

$$
= \frac{1}{1-s_v} \left[(1-s_v)^{\frac{k(1-\varepsilon)}{k-\varepsilon+1}+1} - 1 \right]
$$

This last expression is positive for all $s_v \in (0,1)$ if and only if: $\frac{k(1-\varepsilon)}{k-\varepsilon+1} + 1 < 0$. This holds when $\varepsilon > \frac{2k+1}{k+1}$, and in particular when $\varepsilon > 2$. ∎

7.5 Sketch of proof of Proposition 5

The proofs concerning the welfare implications of an import subsidy mirror closely those for an FDI subsidy. The exposition below is therefore brief, showing that the welfare function under an import subsidy has a positive slope when the subsidy level is less than or equal to zero, but asymptotes towards $-\infty$ as the subsidy level tends towards its maximum value. The optimal policy is therefore a strictly positive subsidy. We focus on the case of a subsidy to the fixed cost, f_X; the proof for a variable cost subsidy is similar and is available upon request.

Fixed cost subsidy to Home exporters.   Consider a subsidy that reduces the fixed cost of exporting for each Home firm by the amount $s_f f_X w_H$, with $s_f < 1$. Suppose as before that this is financed by a tax on labor income equal to $t_f w_H$. The balanced budget constraint for Foreign is:

$$
t_f w_F M_F = s_f f_X w_H N(G^H(a_X) - G^H(a_I))
$$

Substituting the implied value of t_f from this budget constraint into $W_{Ff} = (1-t_f)w_F + \frac{1-\mu}{\mu}(X_F^H)^\mu$, one obtains the following expression for welfare in Foreign:

$$
W_{Ff} = w_F + \left(N\frac{k}{\alpha} \right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}} \frac{\tilde{M}_F}{M_F} \left[\frac{1-\mu}{\mu} \tilde{\Phi}_{Ff} - s_f \left(\frac{\alpha}{k} \right) \frac{\tilde{\Phi}_{Ff}}{\Phi_{Ff}} \frac{\partial \Phi_{Ff}}{\partial s_f} \right] \tag{34}
$$

where:

$$\Phi_{Ff} = \frac{\varepsilon - 1}{k - \varepsilon + 1} \left(\frac{\alpha}{a_H}\right)^k \left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}} \left[\frac{((1-s_f)f_X)^{\frac{k}{1-\varepsilon}+1} w_H}{(\tau w_H)^k} + \frac{(f_I - (1-s_f)f_X)^{\frac{k}{1-\varepsilon}+1} w_H}{((w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}\right]$$

$$\frac{\partial \Phi_{Ff}}{\partial s_f} = \left(\frac{\alpha}{a_H}\right)^k \left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}} \left[\frac{(1-s_f)^{\frac{k}{1-\varepsilon}}(f_X)^{\frac{k}{1-\varepsilon}+1} w_H}{(\tau w_H)^k} - \frac{(f_I - (1-s_f)f_X)^{\frac{k}{1-\varepsilon}} f_X w_H}{((w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}\right]$$

and $\tilde{\Phi}_{Ff} = (\Phi_{Ff})^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}}$. Note that Φ_{Ff} is precisely equal to Λ_F with f_X replaced by $(1-s_f)f_X$. (The switch of notation to Φ is intended to avoid a clash with Λ, which has been used for the analysis of FDI subsidies.) The welfare function in (34) clearly parallels that in (25) for the case of a fixed cost FDI subsidy, except that the *ex ante* profits from sales in Foreign are now given by Φ_{Ff} instead of Λ_{Ff}. The expression for Φ_{Ff} also makes apparent the two opposing effects that an import subsidy has: The first summand in the square brackets captures how s_f lowers the $a_X^{1-\varepsilon}$ threshold for exporting, which results in consumption gains for Foreign, but the second summand captures how s_f raises the $a_I^{1-\varepsilon}$ cut-off for FDI, which cuts into these consumption gains.

Differentiating (34) with respect to s_f yields:

$$\frac{\partial W_{Ff}}{\partial s_f} = \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}} \frac{\tilde{M}_F}{M_F} \frac{\tilde{\Phi}_{Ff}}{\Phi_{Ff}} \frac{\partial \Phi_{Ff}}{\partial s_f} \frac{\alpha}{k} \left[\frac{k(1-\alpha) - \mu\alpha}{\mu\alpha - k(\mu-\alpha)} \cdots \right.$$
$$\left. \cdots + s_f \frac{k(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} \frac{1}{\Phi_{Ff}} \frac{\partial \Phi_{Ff}}{\partial s_f} - s_f \left(\frac{\partial^2 \Phi_{Ff}}{\partial s_f^2} \Big/ \frac{\partial \Phi_{Ff}}{\partial s_f}\right)\right]$$

Evaluating s_f at 0, it is straightforward to check once again that $\frac{\partial W_{Ff}}{\partial s_f} > 0$, given that $k > \mu(\varepsilon - 1)$. Thus, a small subsidy to exporting firms from Home raises indirect utility in Foreign.

Moreover, as $s_f \longrightarrow 1^-$, we have $\Phi_{Ff}, \tilde{\Phi}_{Ff}, \frac{1}{\Phi_{Ff}}\frac{\partial \Phi_{Ff}}{\partial s_f} \longrightarrow +\infty$. This implies from (34) that W_{Ff} asymptotes to $-\infty$ as s_f tends towards its maximum value of 1. Last but not least, $\frac{\partial \Phi_{Ff}}{\partial s_f} > 0$ so long as $a_X^{1-\varepsilon} < a_I^{1-\varepsilon}$. One can also verify that: $\frac{1}{\Phi_{Ff}}\frac{\partial \Phi_{Ff}}{\partial s_f} < \frac{k-\varepsilon+1}{\varepsilon-1}\frac{1}{1-s_f}$ and $\frac{\partial^2 \Phi_{Ff}}{\partial s_f^2}\Big/\frac{\partial \Phi_{Ff}}{\partial s_f} > \frac{k}{\varepsilon-1}\frac{1}{1-s_f}$. Substituting these two inequalities into the above expression for $\frac{\partial W_{Ff}}{\partial s_f}$, one can then show that when $s_f < 0$, we have $\frac{\partial W_{Ff}}{\partial s_f} > 0$.

It is straightforward to see that these properties of W_{Ff} continue to hold when N is endogenized, as the argument in Appendix 7.3 can be adapted with Φ_{Ff} replacing Λ_{Ff} throughout. ∎

7.6   Robustness to alternative utility specification

I illustrate the robustness of the welfare results on FDI subsidies under the alternative utility function (24), under which the demand for differentiated products is also subject to income effects. The exposition focuses on a fixed cost subsidy; the proof for a variable cost subsidy is similar and details are available on request. I focus also on the small-country case, wherein the policy action by Foreign

does not affect the measure of varieties, N. The more general setting that endogenizes this variable is more algebraically involved, without adding substantially to the underlying intuition.

Income effects. Suppose that the utility of a representative consumer from country i is given instead by (24). Maximizing this utility function subject to the budget constraint (2), one obtains a demand function for differentiated products of the familiar iso-elastic form, $x_i^c(a) = A_i^c p_i^c(a)^{-\varepsilon}$, $\varepsilon = \frac{1}{1-\alpha}$, where the level of aggregate demand is now given by:

$$A_i^c = M_i w_i \times (P_i^c)^{\frac{1}{1-\alpha}\frac{\alpha-\mu}{1-\mu}} \times \frac{\mu^{-\frac{1}{\mu}\frac{\rho}{1-\rho}}(\sum_{c=H,F}(P_i^c)^{-\frac{\mu}{1-\mu}})^{\frac{1}{\mu}\frac{\rho-\mu}{1-\rho}}}{1+\mu^{-\frac{1}{\mu}\frac{\rho}{1-\rho}}(\sum_{c=H,F}(P_i^c)^{-\frac{\mu}{1-\mu}})^{\frac{1-\mu}{\mu}\frac{\rho}{1-\rho}}}$$

and $P_i^c = (\int_{\Omega_i^c} p_i^c(a)^{-\frac{\alpha}{1-\alpha}} dG^c(a))^{\frac{1-\alpha}{\alpha}}$ is the ideal price index of country c differentiated varieties in the country i market. Substituting this demand function into the budget constraint, it is straightforward to derive the residual demand for the homogenous good, and from there obtain an expression for the indirect utility function to serve as our welfare metric:

$$W_i = w_i \left[1 + \mu^{-\frac{1}{\mu}\frac{\rho}{1-\rho}} \left(\sum_{c=H,F} (P_i^c)^{-\frac{\mu}{1-\mu}} \right)^{\frac{1-\mu}{\mu}\frac{\rho}{1-\rho}} \right]^{\frac{1-\rho}{\rho}} \tag{35}$$

Our key exercise focuses on $i = F$, namely whether Foreign can raise W_F through an FDI subsidy.

Turning from the demand side to the industry equilibrium, the structure of the Home differentiated goods sector is identical to that in the baseline model with quasilinear utility in Section 2. In particular, we maintain the Pareto parametrization for the distribution of productivity draws from each country's differentiated goods sector. Therefore, from (13) and (14), the ideal price indices for differentiated goods as purchased by Foreign are equal to:

$$P_F^c = \left[N^c \frac{k}{\alpha} \Lambda_{Ff}^c (A_F^c)^{\frac{k-\varepsilon+1}{\varepsilon-1}} \right]^{-\frac{1-\alpha}{\alpha}} , \qquad c = H, F \tag{36}$$

where N^c is the measure of firms in the country-c differentiated goods sector, and Λ_{Ff}^c are the expected profits which firms from this country-c differentiated goods sector can earn from their sales in the Foreign market when the demand level in Foreign is normalized to 1. (The additional f subscript is a reminder that these expected profits are computed taking into account a fixed cost FDI subsidy.)

Consider now a subsidy by Foreign that reduces the fixed cost of FDI for Home firms by $s_f(f_I - f_X)w_H$. The balanced budget constraint (16) still applies, and can be re-written (after some algebraic work) to obtain the following expression for the tax rate levied on Foreign workers to fund the subsidy program:

$$t_f = \frac{s_f N^H}{w_F M_F} \frac{\partial \Lambda_{Ff}^H}{\partial s_f} (A_F^H)^{\frac{k}{\varepsilon-1}} \tag{37}$$

In the presence of this income tax, the expression for the level of Foreign demand for differentiated goods needs to be modified to:

$$A_F^c = M_F(1 - t_f)w_F \times (P_F^c)^{\frac{1}{1-\alpha}\frac{\alpha-\mu}{1-\mu}} \times \frac{\mu^{-\frac{1}{\mu}\frac{\rho}{1-\rho}}(\sum_{c=H,F}(P_i^c)^{-\frac{\mu}{1-\mu}})^{\frac{1}{\mu}\frac{\rho-\mu}{1-\rho}}}{1 + \mu^{-\frac{1}{\mu}\frac{\rho}{1-\rho}}(\sum_{c=H,F}(P_i^c)^{-\frac{\mu}{1-\mu}})^{\frac{1-\mu}{\mu}\frac{\rho}{1-\rho}}} , \qquad c = H, F \quad (38)$$

Observe that (36), (37), and (38) define a system of five equations in the five variables, A_F^H, A_F^F, P_F^H, P_F^F, and t_f, in which s_f acts as a state variable. A complication that arises relative to the quasilinear utility case is that we cannot solve for P_F^c as a closed-form function of the model parameters only, so the behavior of W_F with respect to the subsidy rate, s_f, will have to be deduced by performing comparative statics on the above system of five equations.

Replacing w_F by $(1 - t_f)w_F$ in (35) and log-differentiating with respect to s_f yields:

$$\frac{1}{W_{Ff}}\frac{dW_{Ff}}{ds_f} = -\frac{1}{1-t_f}\frac{dt_f}{ds_f} - \phi_1\left(\phi_2\frac{1}{P_F^H}\frac{dP_F^H}{\partial s_f} + (1-\phi_2)\frac{1}{P_F^F}\frac{dP_F^F}{\partial s_f}\right) \quad (39)$$

where we have defined for notational ease:

$$\phi_1 = \frac{\mu^{-\frac{1}{\mu}\frac{\rho}{1-\rho}}(\sum_{c=H,F}(P_i^c)^{-\frac{\mu}{1-\mu}})^{\frac{1-\mu}{\mu}\frac{\rho}{1-\rho}}}{1 + \mu^{-\frac{1}{\mu}\frac{\rho}{1-\rho}}(\sum_{c=H,F}(P_i^c)^{-\frac{\mu}{1-\mu}})^{\frac{1-\mu}{\mu}\frac{\rho}{1-\rho}}} \in (0,1)$$

$$\phi_2 = \frac{(P_F^H)^{-\frac{\mu}{1-\mu}}}{\sum_{c=H,F}(P_i^c)^{-\frac{\mu}{1-\mu}}} \in (0,1)$$

To obtain an expression for $\frac{dt_f}{ds_f}$, totally differentiating (37) yields:

$$\frac{dt_f}{ds_f} = \frac{t_f}{s_f} + t_f\left[\frac{\partial^2\Lambda_{Ff}^H}{\partial s_f^2}/\frac{\partial\Lambda_{Ff}^H}{\partial s_f} + \frac{k}{\varepsilon-1}\frac{1}{A_F^H}\frac{dA_F^H}{ds_f}\right]$$

$$= \frac{\alpha}{k}\frac{1}{\Lambda_{Ff}^H}\frac{\partial\Lambda_{Ff}^H}{\partial s_f}(1-t_f)\phi_1\phi_2 + t_f\left[\frac{\partial^2\Lambda_{Ff}^H}{\partial s_f^2}/\frac{\partial\Lambda_{Ff}^H}{\partial s_f} + \frac{k}{\varepsilon-1}\frac{1}{A_F^H}\frac{dA_F^H}{ds_f}\right] \quad (40)$$

Note that the expression $\frac{t_f}{s_f} = \frac{\alpha}{k}\frac{1}{\Lambda_{Ff}^H}\frac{\partial\Lambda_{Ff}^H}{\partial s_f}(1-t_f)\phi_1\phi_2$ used in this last step is derived from (37).

Similarly, (36) implies:

$$\frac{1}{P_F^H}\frac{dP_F^H}{ds_f} = -\frac{1-\alpha}{\alpha}\frac{1}{\Lambda_{Ff}^H}\frac{\partial\Lambda_{Ff}^H}{\partial s_f} - \frac{1-\alpha}{\alpha}\frac{k-\varepsilon+1}{\varepsilon-1}\frac{1}{A_F^H}\frac{dA_F^H}{ds_f} \quad (41)$$

$$\frac{1}{P_F^F}\frac{dP_F^F}{ds_f} = \frac{1}{P_F^H}\frac{dP_F^H}{ds_f} + \frac{\alpha(1-\mu)}{\mu\alpha - k(\mu-\alpha)}\frac{1}{\Lambda_{Ff}^H}\frac{\partial\Lambda_{Ff}^H}{\partial s_f} \quad (42)$$

Totally differentiating (38) and using (42) to replace the terms involving $\frac{1}{P_F^F}\frac{dP_F^F}{ds_f}$ yields:

$$\frac{1}{A_F^H}\frac{dA_F^H}{ds_f} = -\frac{1}{1-t_f}\frac{dt_f}{ds_f} + \left(\frac{\alpha-\rho}{(1-\alpha)(1-\rho)} + \frac{\rho}{1-\rho}\phi_1\right)\frac{1}{P_F^H}\frac{dP_F^H}{ds_f}\dots$$

$$\dots + \left(\frac{\mu-\rho}{(1-\mu)(1-\rho)} + \frac{\rho}{1-\rho}\phi_1\right)(1-\phi_2)\frac{\alpha(1-\mu)}{\mu\alpha-k(\mu-\alpha)}\frac{1}{\Lambda_{Ff}^H}\frac{\partial\Lambda_{Ff}^H}{\partial s_f} \quad (43)$$

36

Now, (40), (41) and (43) can be solved simultaneously to obtain expressions for $-\frac{1}{1-t_f}\frac{dt_f}{ds_f}$ and $\frac{1}{P_F^H}\frac{dP_F^H}{ds_f}$ in terms of model parameters only. These can then be substituted into (39) to obtain:

$$\left(K_0 + \frac{t_f}{1-t_f}\frac{k}{\varepsilon-1}\right)\frac{1}{W_{Ff}}\frac{dW_{Ff}}{ds_f} = -K_1\frac{t_f}{1-t_f}\left(\frac{\partial^2\Lambda_{Ff}^H}{\partial s_f^2}\Big/\frac{\partial\Lambda_{Ff}^H}{\partial s_f}\right) + K_2\frac{t_f}{1-t_f}\frac{1}{\Lambda_{Ff}^H}\frac{\partial\Lambda_{Ff}^H}{\partial s_f} + K_3\frac{1}{\Lambda_{Ff}^H}\frac{\partial\Lambda_{Ff}^H}{\partial s_f} \quad (44)$$

where the various constants are given by:

$$K_0 = 1 + \frac{1-\alpha}{\alpha}\frac{k-\varepsilon+1}{\varepsilon-1}\left(\frac{\alpha-\rho}{(1-\alpha)(1-\rho)} + \frac{\rho}{1-\rho}\phi_1\right)$$

$$K_1 = 1 + \frac{1-\alpha}{\alpha}\frac{k-\varepsilon+1}{\varepsilon-1}\left(\frac{\alpha-\rho}{(1-\alpha)(1-\rho)} + \frac{1}{1-\rho}\phi_1\right)$$

$$K_2 = \frac{k}{\varepsilon-1}\frac{1}{1-\rho}\left[\frac{1-\alpha}{\alpha}\left(\phi_1 + \frac{\alpha-\rho}{1-\alpha}\right) - (1-\phi_2)\frac{\alpha(1-\mu)}{\mu\alpha-k(\mu-\alpha)}\left(\phi_1 + \frac{\mu-\rho}{1-\mu}\right)\right]$$

$$K_3 = \frac{\phi_1\phi_2}{k}\frac{1-\alpha}{1-\rho}\left[\frac{k}{\varepsilon-1} - \rho - \left(\frac{k}{\varepsilon-1}-1\right)\phi_1\right]$$

(44) readily allows us to analyze the behavior of the Foreign welfare function. At $s_f = 0$, the tax t_f is also zero. Since $\frac{\partial\Lambda_{Ff}^H}{\partial s_f} > 0$, this implies that $sign(\frac{dW_{Ff}}{ds_f}) = sign(\frac{K_3}{K_0})$. Clearly, $K_0 > 0$ given that $0 < \rho < \alpha < 1$, $\varepsilon > 1$, and $k > \varepsilon - 1$. Moreover, $K_3 > 0$ since $\frac{k}{\varepsilon-1} - \rho > \frac{k}{\varepsilon-1} - 1 > (\frac{k}{\varepsilon-1}-1)\phi_1$. Thus, $\frac{dW_{Ff}}{ds_f} > 0$ in the neighborhood of $s_f = 0$, and so a small subsidy improves welfare under this more general utility specification.

As $s_f \longrightarrow 1^-$, observe that $\frac{t_f}{1-t_f} = \frac{\alpha}{k}\frac{1}{\Lambda_{Ff}^H}\frac{\partial\Lambda_{Ff}^H}{\partial s_f}\phi_1\phi_2 \longrightarrow +\infty$, where we recall from Appendix 7.1 that $\frac{1}{\Lambda_{Ff}^H}\frac{\partial\Lambda_{Ff}^H}{\partial s_f} \longrightarrow +\infty$. Recall also that $\frac{1}{\Lambda_{Ff}^H}\frac{\partial\Lambda_{Ff}^H}{\partial s_f} = \frac{k-\varepsilon+1}{\varepsilon-1}\frac{1}{1-s_f}g(s_f)$ and $\frac{\partial^2\Lambda_{Ff}^H}{\partial s_f^2}\Big/\frac{\partial\Lambda_{Ff}^H}{\partial s_f} = \frac{k}{\varepsilon-1}\frac{1}{1-s_f}$. Some algebraic simplification leads to:

$$\frac{1}{W_{Ff}}\frac{dW_{Ff}}{ds_f} \longrightarrow \frac{1}{1-s_f}\left\{-\frac{k}{\varepsilon-1}K_1 + \frac{k-\varepsilon+1}{\varepsilon-1}g(s_f)K_2\right\}\Big/\frac{k}{\varepsilon-1}$$

$$= \frac{1}{1-s_f}\left\{-1 - \frac{k-\varepsilon+1}{\varepsilon-1}\frac{1}{1-\rho}\left[\frac{1-\alpha}{\alpha}\left(\phi_1 + \frac{\alpha-\rho}{1-\alpha}\right)(1-g(s_f))\ldots\right.\right.$$

$$\left.\left.\ldots + (1-\phi_2)\frac{\alpha(1-\mu)}{\mu\alpha-k(\mu-\alpha)}\left(\phi_1 + \frac{\mu-\rho}{1-\mu}\right)g(s_f)\right]\right\}$$
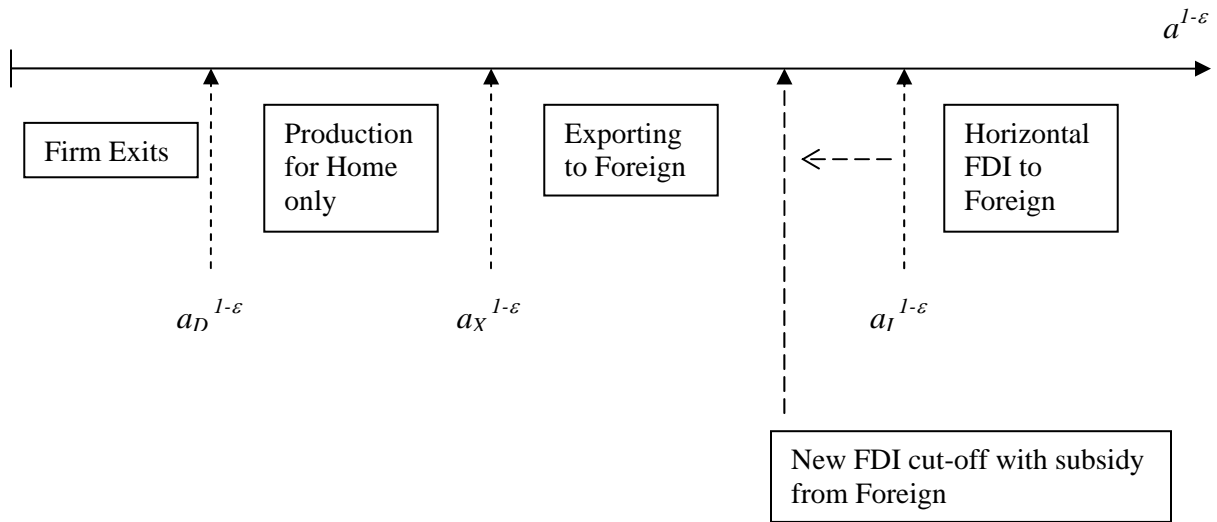
$$\longrightarrow -\infty$$

since the expression in the large curly braces in the penultimate step is negative (bearing in mind that $g(s_f) \in (0,1)$). The welfare function therefore asymptotes to negative infinity as the subsidy is raised to its maximum possible level.

Finally, it remains to show that W_{Ff} is an increasing function in s_f when s_f and hence t_f are negative. Observe that the right-hand side of (44) is equal to: $\frac{1}{1-s_f}\frac{t_f}{1-t_f}\left\{-\frac{k}{\varepsilon-1}K_1 + \frac{k-\varepsilon+1}{\varepsilon-1}g(s_f)K_2\right\} + K_3\frac{1}{\Lambda_{Ff}^H}\frac{\partial\Lambda_{Ff}^H}{\partial s_f}$. The second summand is clearly positive since $K_3 > 0$. Moreover, we have just seen that $\left\{-\frac{k}{\varepsilon-1}K_1 + \frac{k-\varepsilon+1}{\varepsilon-1}g(s_f)K_2\right\} < 0$, from which it follows that $\frac{1}{1-s_f}\frac{t_f}{1-t_f}\left\{-\frac{k}{\varepsilon-1}K_1 + \frac{k-\varepsilon+1}{\varepsilon-1}g(s_f)K_2\right\} > 0$ when $t_f < 0$. Thus, the right-hand side of (44) is positive when $s_f < 0$. As for the coefficient

of $\frac{1}{W_{Ff}}\frac{dW_{Ff}}{ds_f}$ on the left-hand side, note that $\frac{t_f}{1-t_f} = \frac{\alpha}{k}\frac{s_f}{1-s_f}\frac{k-\varepsilon+1}{\varepsilon-1}g(s_f)\phi_1\phi_2 > -\frac{\alpha}{k}\frac{k-\varepsilon+1}{\varepsilon-1}\phi_1\phi_2$, since $\frac{s_f}{1-s_f} \in (-1,0)$ when $s_f < 0$. Substituting this lower bound for $\frac{t_f}{1-t_f}$ and simplifying, we have:
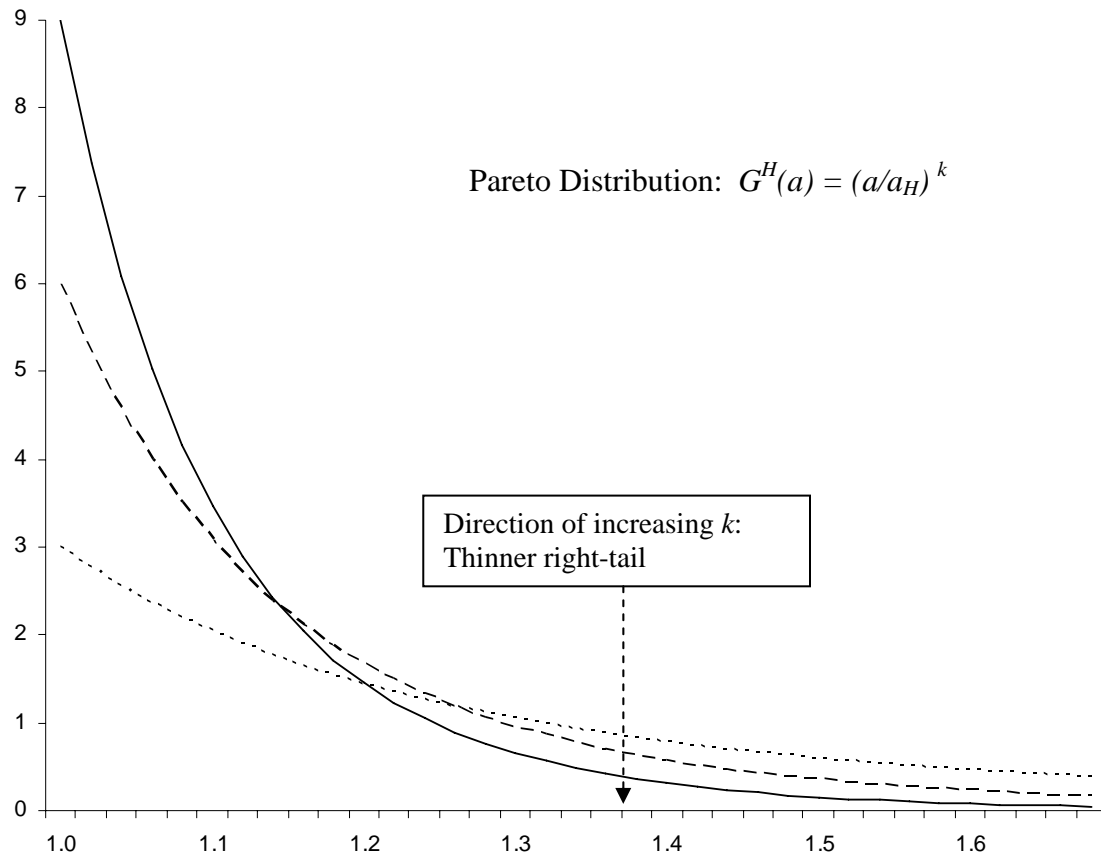
$$
\begin{aligned}
K_0 + \frac{t_f}{1-t_f}\frac{k}{\varepsilon-1} \; &> \; 1 - \frac{\alpha}{\varepsilon-1}\frac{k-\varepsilon+1}{\varepsilon-1}\phi_1\phi_2 + \frac{1-\alpha}{\alpha}\frac{k-\varepsilon+1}{\varepsilon-1}\left(\frac{\alpha-\rho}{(1-\alpha)(1-\rho)} + \frac{\rho}{1-\rho}\phi_1\right) \\
&> \; 1 - \frac{\alpha}{\varepsilon-1}\frac{k-\varepsilon+1}{\varepsilon-1}\phi_1 + \frac{1-\alpha}{\alpha}\frac{k-\varepsilon+1}{\varepsilon-1}\left(\frac{\alpha-\rho}{1-\alpha} + \rho\phi_1\right) \\
&= \; 1 + \frac{k-\varepsilon+1}{(\varepsilon-1)^2}(\alpha-\rho)\left(\frac{1}{1-\alpha} - \phi_1\right) \\
&> \; 0
\end{aligned}
$$

Thus, $\frac{dW_{Ff}}{ds_f} > 0$ for all $s_f < 0$. ∎

$a^{1-\varepsilon}$

| Firm Exits | | Production for Home only | | Exporting to Foreign | | $\leftarrow\; ---$ | | Horizontal FDI to Foreign |

$a_D^{\,1-\varepsilon}$ $\qquad\qquad$ $a_X^{\,1-\varepsilon}$ $\qquad\qquad$ $a_I^{\,1-\varepsilon}$

New FDI cut-off with subsidy from Foreign

Figure 1: The Sorting Pattern within Home's Differentiated Goods Sector

Pareto Distribution: $G^H(a) = (a/a_H)^k$

Direction of increasing k:
Thinner right-tail

**Figure 2: Effect of a Larger Shape Parameter k on
the Pareto Distribution of Productivity Draws**

Notes: Illustrated with a_H set equal to 1. The three Pareto distributions are for $k = 3$ (dotted-line graph), $k = 6$ (dashed-line graph), and $k = 9$ (solid-line graph).
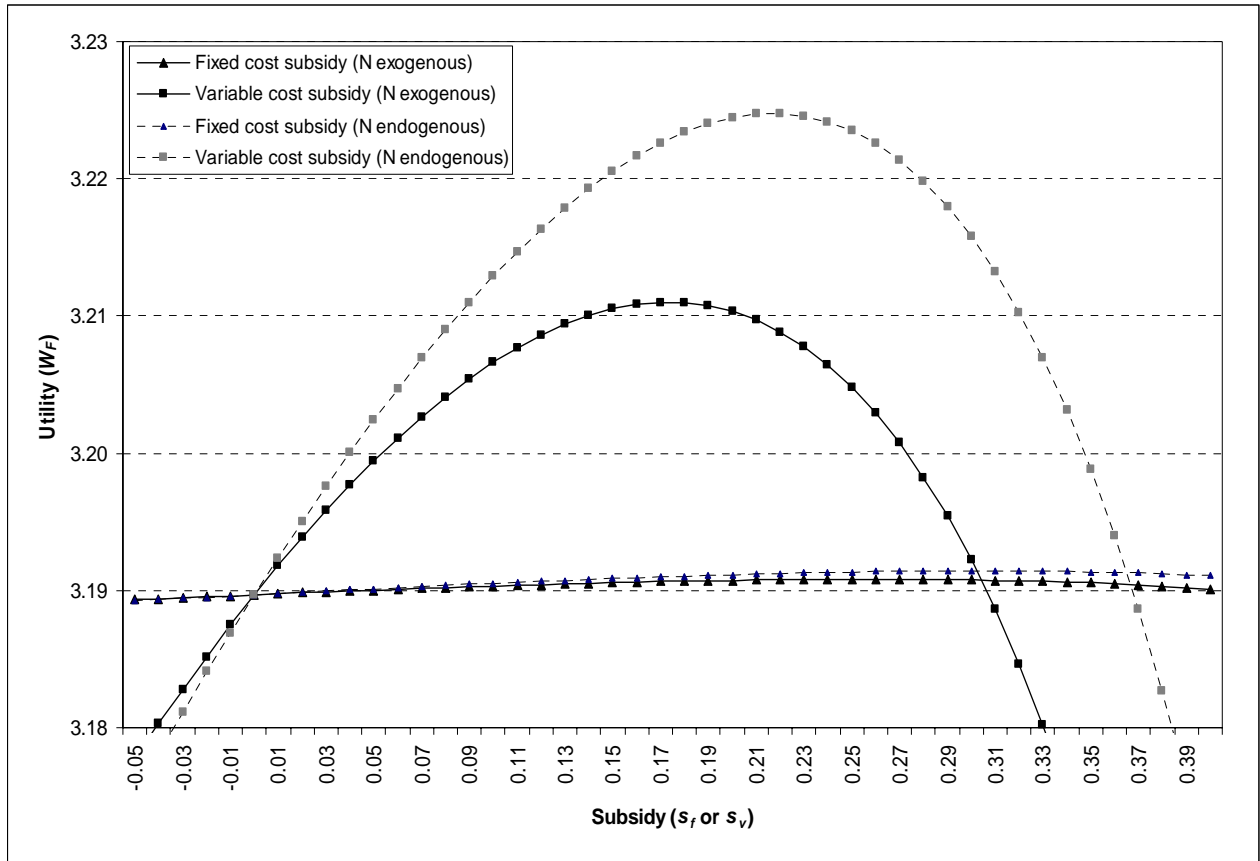
Figure 3: Some Sample Calibrated Welfare Functions

Notes: Calibration parameters are: $k = 3.4$, $\varepsilon = 3.8$ (which implies $\alpha = 0.74$), $\mu = 0.3$, $f_D = 0.1$, $f_X = 0.23$, $f_I = 2$, $w_H = w_F = 1$, $\tau = 1.3$, $a_H = 1$, $f_E = 1$, $M_H = M_F = 1$. These parameter choices imply the order of productivity cut-offs imposed in the model, namely $a_D > a_X > a_I$. For the graphs where the measure of Home firms is exogenous, the value of N used is that obtained when setting the subsidy level to 0 in equation (23).