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Smooth is Smart:
Bullwhip, Inventory, Fill-Rates and the Golden Ratio

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ABSTRACT

A major cause of supply chain deficiencies is the bullwhip effect. This effect refers to the tendency of the variance of the replenishment orders to increase as one moves up a supply chain. Supply chain managers experience this variance amplification in both inventory levels and replenishment orders. As a result, companies face shortages or bloated inventories, run-away transportation and warehousing costs and major production adjustment costs. In this article we analyze a major cause of the bullwhip effect and suggest a remedy. We focus on a unique replenishment rule that is able to reduce the bullwhip effect. In general, bullwhip reduction may have a negative impact on customer service due to inventory variance increases. Our analysis shows that bullwhip can be satisfactorily managed without unduly increasing stock levels to maintain target fill rates.

I. INTRODUCTION

There is ample anecdotic evidence that many companies experience significant extra costs due to supply chain problems. Konicki (2002) reports on a major retailer's inability to master supply chain logistical problems. The company faced sharp spikes and drops in demand for products and sales merchandise was often out of stock when customers got to the store. Furthermore, bloated stocks sat alongside these empty racks and display shelves, but they were no guarantee of high customer service levels. It is a formidable job for logistics managers to design order management systems that optimally match pipelines to the marketplace (see Looman, Ruttins and de Boer (2002), Childerhouse, Aitken and Towill (2002) and Christopher and Towill (2002)).

What is causing all this trouble? How come that the material flow is so hard to predict in supply networks? There are for sure many causes of these deficiencies. In this paper, however, we will focus on the bullwhip problem. The bullwhip problem refers to the tendency of replenishment orders to increase in variability as one moves up a supply chain. As smooth final customer demand patterns are transformed into highly erratic demand patterns for suppliers; the information in the chain gets distorted. The bullwhip is characterised by oscillations of orders at each level of the supply chain and an amplification of these oscillations as one moves farther up the chain (Croson and Donohue (2003)). Jay Forrester (1961) was among the first researchers to describe this phenomenon, then called "Demand Amplification". The Beer Game developed at MIT is one of the most popular games in many business schools and executive seminars and is very useful for illustrating the bullwhip problem, Sterman (1989).

Procter and Gamble first coined the phrase "bullwhip effect" to describe the ordering behaviour witnessed between customers and suppliers of *Pampers* diapers. While diapers enjoy a fairly constant consumption rate, P&G found that wholesale orders tended to fluctuate considerably over time. They observed a further amplification of the oscillations of orders placed to their suppliers of raw material. The bullwhip problem has been given a lot of academic attention after the important contribution of Lee et al. (1997).

There is also a lot of empirical evidence of bullwhip. Our own data shows that the coefficient of variation (the ratio of the standard deviation over the mean) of retail sales typically range between 0.15 and 0.50 whereas the coefficient of variation of production orders (even in small batch driven environments) is typically in the range of 2 to 3. Moreover, the bullwhip effect is multiplicative in traditional supply chains. Incredible though this may seem, there is ample evidence in many business environments to verify this and mathematical models to prove it (Dejonckheere, Disney, Lambrecht and Towill (2004). One of the principal reasons used to justify investments in inventories is its role as a buffer as it is believed that inventories have a stabilising effect on material replenishment. Clearly, however inventory management policies can have a destabilising effect by increasing the volatility of demand in the supply chain.

We will now review causes of the bullwhip effect as mentioned in the literature, and investigate ways to alleviate and to overcome the problem. We distinguish operational and behavioural causes. The behavioural causes are rather straightforward. Supply chain managers may not always be completely rational. Managers over-react (or under-react) to demand changes. People often try to read "too much signal" into a series of demand history as it changes over time. Decision makers sometimes over-react to customer complaints and anecdotes of negative customer reactions. Moreover, there are cognitive limitations as supply chain networks are often very complicated, operating in a highly uncertain environment with limited access to data. Croson and Donohue (2002) and Sterman (1989) found that decision makers consistently under-weight the supply chain. This means that they don't have a clear idea of what is available in the pipeline. This induces some form of decision bias. Strategies to alleviate this problem include: sharing Point-Of-Sales data, sharing inventory and demand information, centralizing ordering decisions and using formal forecasting techniques correctly (we will come back on this issue later on in this paper)

Lee et al (1997) identify five major operational causes of the bullwhip: demand signal processing, lead-time, order batching, price fluctuations and rationing and shortage gaming. We understand demand signal processing as the practice of decision makers adjusting the parameters of the inventory replenishment rule. Target stock levels, safety stocks and demand forecasts are updated in face of new information or deviations from targets. These "rational" adjustments create erratic responses. We will also show that it is possible to design replenishment rules that have a stabilizing, smoothing effect on orders. It is important to realize that most players in supply chains do not respond directly to the market but respond to replenishment demand from downstream echelons. This is why local optimisation often results in global disharmony. It is therefore claimed that centralized control (e.g. Distribution Requirements Planning, Vendor Managed Inventories) is superior to decentralized control (disconnected supply chains).

A second major cause of the bullwhip problem is the lead-time. Lead-times are made of two components; the physical delays as well as the information delays. The lead-time is a key parameter for calculating safety stock, reorder points and order-up-to levels. The increase in variability is magnified with increasing lead-time. A way to alleviate this problem is lead-time compression. The information delay can be reduced by better communication technologies (web-enabled communication, EDI, e-procurement etc) and the order fulfilment lead-time (the physical lead-time) can be reduced by investment in production technology and process, strategic supplier partnerships (supplier hubs, logistics integrators etc) or by eliminating channel intermediaries (direct channels, 'the Dell model').

A third well-known bullwhip creator is the practice of order batching. Economies of scale in ordering, production set-ups or transportation will quite clearly increase order variability. Reduction of set-up, ordering and handling costs is of course a way to alleviate this problem.

The fourth major cause of bullwhip as highlighted by Lee et al (1997) has to do with price fluctuations. Retailers often offer price discounts, quantity discounts, coupons or in-store promotions. This results in forward buying where retailers (as well as consumers) buy in advance and in quantities that do not reflect their immediate needs. Pricing strategies (ranging from deep promotions to Every Day Low Price) should clearly be connected to supply and replenishment policies. However, it is not sure from a marketing perspective whether the positive supply chain effect (e.g. higher efficiencies) outweigh the potential negative marketing effect (e.g. demand-depressing side effects). We refer to Ortmeyer et al.(1991) and Budman (2002) for more details on issues in the operations management, marketing interface.

In general, it is important to transmit into the supply chain the correct demand information. An accurate forecast (see Chen, Drezner, Ryan and Simchi-Levi (2000)) will assist the upstream suppliers' capacity- and material planning. We may want to stimulate forecast accuracy and to penalise forecast errors. We may want to limit the ability to revise forecasts over time, or we may negotiate flexibility contracts with customers (based on risk sharing). These are all ways to have demand better under control and to view forecasting as more than just a courtesy.

A further cause of the bullwhip has to do with rationing and shortage gaming. Inflated orders placed by supply chain members during shortage periods tend to magnify the bullwhip effect. Such orders are common when retailers and distributors suspect that a product will be in short supply. Exaggerated customers orders make it hard for manufacturers to forecast the real demand level. A very simple countermeasure is to allocate products proportional to sales in previous periods and not proportional to what has been ordered.

This short overview of the causes of the bullwhip effect (and a short summary of potential remedies) highlights that the bullwhip effect is a very complex issue. It touches on all aspects of supply chain management. In this article, we will limit ourselves to one specific cause, the (ab)use of replenishment rules. In section II we introduce the order-up-to replenishment rule and demonstrate that it creates bullwhip. In section III we introduce a new smoothing replenishment rule and in section IV we focus on the link between replenishment rules and customer service. Section V highlights implications for management. Section VI concludes.

II. THE ORDER-UP-TO REPLENISHMENT RULE

There are many different types of replenishment policies (for example see Zipkin (2000) and Silver, Pyke and Peterson (1998)), of which two are commonly used: the periodic review, replenishment interval, Order-Up-To (OUT) policy and the continuous review, reorder point, order quantity model.

Given the common practice in retailing to replenish inventories frequently (daily, weekly, monthly) and the tendency of manufacturers to produce to demand, we will focus our analysis on the replenishment strategies known as Order-Up-To (OUT) policies. In such a system we track the inventory position (= amount on-hand + inventory on-order – backlog). The inventory position is reviewed every period (e.g. daily, weekly) and an order is placed to raise the inventory position up to an order-up-to or base stock level that determines order quantities. This policy is sometimes preferred due to qualitative benefits of following a regular repeating schedule of inventory replenishment. Both the review period and the order-up-to level are decision variables but in order to simplify the analysis we set the review period equal to one base period (day, week or month).

The OUT level equals the expected demand during the risk period and a safety stock to cover higher than expected demands during the same risk period. The risk period equals the physical lead-time (*Tp* periods) and the review period (1 period). Consequently,

$$S_t = \hat{D}^{Tp+1} + k.\sigma^{Tp+1}.$$

S_t is the OUT level used in period t and $\hat{D}^{Tp+1}$ is an estimate of mean demand over Tp+1 periods (we assume $\hat{D}^{Tp+1} = (Tp+1)\hat{D}^{\alpha}{}_t$, where $\hat{D}^{\alpha}{}_t$ is the estimate of demand in the next period calculated e.g. with exponential smoothing, with a smoothing constant α). σ^{Tp+1} is an estimate of the standard deviation of the forecast error over Tp+1 periods. k is a constant chosen to meet a desired Customer Service Level (CSL). To simplify the analysis we replace the safety stock term by $a.\hat{D}^{\alpha}{}_t$ (this can always be done and it makes the analysis somewhat easier). After this substitution we obtain,

$$S_t = (Tp+1+a)\hat{D}^{\alpha}{}_t \qquad (1)$$

This more general form of the OUT policy defines the risk period as $Tp+1+a$ and consequently immediately includes the safety stock.

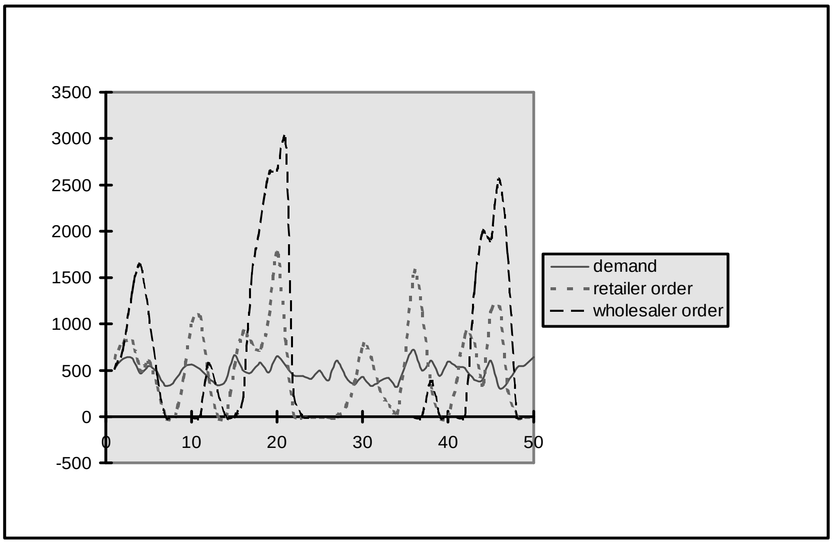
Suppose that the demand process is normally, independently and identically distributed (iid) over time, then it is quite clear that the best demand estimate of next period demand is simply the long-term average demand, $\overline{D}$. Formula (1) then becomes,

$$S = (Tp+1+a)\overline{D}. \qquad (2)$$

What happens now if we apply the above replenishment rule (2) (using $\overline{D}$ as an estimator). The answer to that question is simple and known to most inventory managers (see Dejonckheere et al.(2003)). The OUT policy is generating replenishment orders that are the same as the last periods observed demand. We simply order what the demand was in the base period (sounds very much like a Just-In-Time strategy), that's why this policy is also called "passing on orders" or "lot for lot" or even sometimes "continuous replenishment" when the length of the planning period has been shortened. Either way, the variability of the replenishment orders is exactly the same as the variability of the original demand. So how is it that we observe variance amplification in the real world? The answer to that key question is that decision makers don't know the demand (over the lead-time) and consequently they have to forecast demand and constantly adjust the OUT levels. Unfortunately these adjustments create bullwhip. This observation was already well described by Forrester (1961) and was very elegantly proved by Chen et al (2000).

Let's illustrate one possible adjustment strategy. Assume that the decision-maker follows an OUT policy, that means the retailer orders what the demand was, but we adjust this quantity by the difference between the target safety stock ($a.\overline{D}_t$) and the actual physical inventory at the end of the period. This is a quite logical adjustment, if the physical inventory at the end of a period is less than the safety stock, order more and vice versa. This rather logical, and at first sight innocent adjustment rule, has a very devastating effect on the bullwhip as is illustrated in Figure 1. The example is introduced in the next paragraph.

FIGURE 1
The impact of adjustments



Assume a supply chain consisting of customers, a retailer and a wholesaler. The retailer physical lead-time equals two periods and the wholesaler lead-time also equals two periods. Further, assume a normally distributed demand process with an expected value of 500 ($\overline{D} = 500$) units per period and a standard deviation of 100 units (the coefficient of variation equals 0.2). Furthermore set the safety stock, a = 0.5. The OUT level for the retailer equals (2+1+0.5)500=1750 and the OUT level for the wholesaler equals (2+1+0.5)500=1750. The safety stock equals 500(0.5)=250 units. An OUT policy results in a replenishment pattern with a variance equal to the variance of the demand pattern. The adjustment policy explained in the previous paragraph, however, results in the replenishment patterns shown in Figure 1. No need to say that there is a very significant variance amplification effect. The so-called “adaptive” inventory policies may have a devastating effect on the amplification of oscillations.

The use of forecasting tools has exactly the same impact. Suppose we use exponential smoothing as a forecasting tool:

$$\hat{D}_t = \hat{D}_{t-1} + \alpha(D_t - \hat{D}_{t-1}) \qquad (3)$$

Suppose that the demand was 100 units for the last ten periods, and that in period eleven the demand increases to 150 units. Assume 4 stages in the supply chain each with a four period lead-time (*Tp*=2, review period = 1 and a 1)). The first member of the chain will forecast a demand of 110 units (use α=0.2) and its OUT level equals 440. Given that the demand increased to 150 units, the inventory position will decrease (250 units). The order quantity will consequently increase, in this example from 100 to 190 units. This order quantity is now transferred to the next link in the chain and exactly the same will happen there, that means, the OUT level increases and the inventory position decreases. After 4 stages the order quantity equals 626 units whereas it was only 100 units in the first ten periods. That’s how the bullwhip works, the key driver is the “full adjustment policy” used to recover inventory errors.

The bullwhip problem is akin to a common situation we face every morning. As we stand underneath a cold shower and turn the hot tap too quickly, the water, a few moments later (lead-time), becomes too hot and we respond by reaching for the cold tap or turning back the hot tap. These “full” adjustments are undesirable. We all know that, when in the shower, we should turn the taps very slowly in order to get the temperature “just right”. Well, the same issue is prevalent in a supply chain, we must turn the taps very slowly also. The key word here is “fractional adjustment” that are well known to control engineers (see Deziel and Eilon (1967) and Magee (1956)). This smoothing replenishment strategy is the subject of the next section.

III. A SMOOTHING REPLENISHMENT RULE: SMOOTH IS SMART

Let’s go back to formula (1) and decompose it as follows,

$$O_t = S_t - \text{inventory position}$$

Where O_t is the ordering decision made at the end of period *t*. The inventory position equals the net stock (*NS*) plus inventory on order (Work In Progress or WIP). The net stock equals inventory at hand minus backlog.

$$O_t = (Tp + 1 + a)\hat{D}_t^{\alpha} - NS_t - WIP_t$$

or

$$O_t = \hat{D}^{\alpha}{}_t + (a\hat{D}_t^{\alpha} - NS_t) + (Tp.\hat{D}_t^{\alpha} - WIP_t) \qquad (4)$$

where $a\hat{D}^{\alpha}{}_t$ can be viewed as a target net stock (safety stock) and $Tp.\hat{D}^{\alpha}{}_t$ as a target pipeline stock (on order inventory). Expression (4) is the same as expression (1), but we decomposed the original formula into three components: a demand forecast, a net stock discrepancy term and a WIP or pipeline discrepancy term, see Dejonckheere, Disney, Lambrecht and Towill (2003). Moreover, if we now want to turn the taps slowly, we can give a weight to the discrepancies as in expression (5)

$$O_t = \hat{D}^{\alpha}{}_t + \beta(a\hat{D}_t^{\alpha} - NS_t) + \gamma(Tp.\hat{D}_t^{\alpha} - WIP_t) \qquad (5)$$

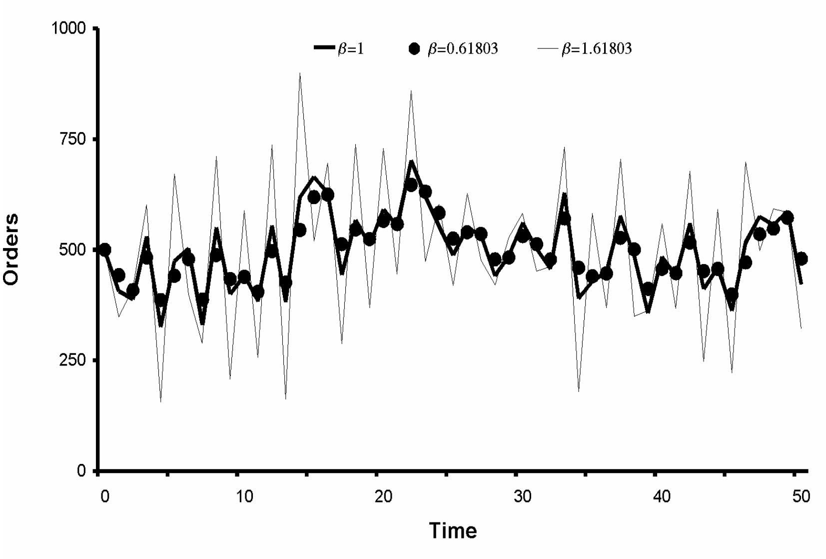
We now have three controllers α, β and γ that will enable us to tune the dynamic behaviour of the supply chain. For $\beta = \gamma > 1$ we will create bullwhip (amplification) and for $\beta = \gamma < 1$ we will create a smooth replenishment pattern (dampening).

This is illustrated in Figure 2. We take the same example as before (i.i.d. normal distribution with $\overline{D}$ =500 and $\sigma_D = 100$ and *Tp*=2) and we use $\overline{D}$ as an estimator; furthermore we assume in this paper that $\beta = \gamma$ throughout.

The first controller α is simply the smoothing constant in the exponential smoothing forecasting rule. Smaller values will produce smoother responses and larger values create more bullwhip. Note that here we have set α = 0, to match the assumption that demand is a stationary iid random variable, simplifying drastically the equations presented. When demand is not stationary, that is, there is a genuine change in mean demand or some auto-correlation in the demand signal exists, then 0>α<2 may be better suited (or indeed a different forecasting mechanism). We refer readers to Dejonckheere et al ((2003), (2004)) and Disney, Farasyn, Lambrecht, Towill and van de Velde (2003) for more details on this aspect. The major advantage of this modification to the OUT policy is that it filters out “noise” in the marketplace sales (through the dampened feedback), whilst tracking genuine changes in demand (admittedly with a lag). By doing this, companies can avoid excess costs due to unnecessary ramping up and down production or ordering levels. The optimal values of the three controllers are obviously sensitive to the economics of the supply chain in question.

FIGURE 2

A smoothing replenishment rule



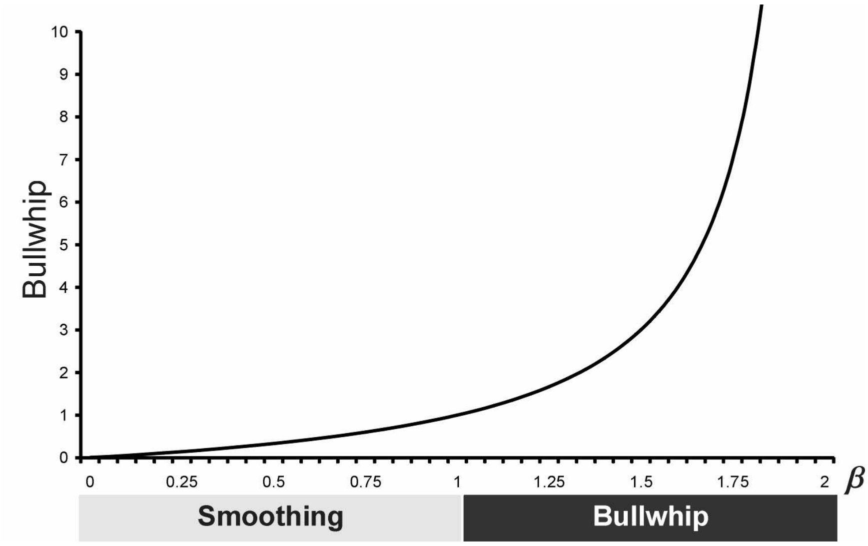
We refer to Dejonckheere et al. (2003) for more details concerning the derivation of bullwhip expressions. For illustration purposes: when $\hat{D}^{\alpha}{}_t = \overline{D}$, bullwhip (defined as the ratio of the variance of the orders over the variance of demand) is equal to given by,

$$Bullwhip = \frac{\sigma_O^2}{\sigma_D^2} = \frac{\beta}{2-\beta} \qquad (6)$$

or graphically.

FIGURE 3

Bullwhip with stationary demand and matched feedback controllers



We observe that for the case of stationary demand and $\beta = \gamma$, bullwhip is independent of lead-time.

IV. BULLWHIP AND CUSTOMER SERVICE

So far we have been concentrating on the variance of orders placed. This is, however, only one side of the coin. We should also study the variance of inventory, because that variance will have an immediate effect on customer service: the higher the variance, the more stock will be needed to maintain customer service at the target level. Recall 'net stock' refers to NS_t in (4).

Remember that $\beta = \gamma = 1$ (and $\hat{D}^{\alpha}{}_t = \overline{D}$) results in a bullwhip measure of 1 as we have a pure chase policy. In such a case the inventory fluctuations will be minimal.

Intuitively, we expect smooth ordering patterns ($\beta = \gamma < 1$) will result in higher inventory fluctuations and consequently in a poorer fill rate, and this is indeed the case. Defining a measure of net stock variance amplification as,

$$NSAmp = \frac{\sigma_{NS}^2}{\sigma_D^2} \qquad (7)$$

Our control systems engineering methodology results in the following interesting expressions for *NSAmp* for $\hat{D}^{\alpha}{}_{t}$ = $\overline{D}$ when $\gamma = \beta$,

$$NSAm = 1 + Tp + \frac{(1-\beta)^2}{(2-\beta)\beta} \quad (8)$$

NSAmp clearly has a 'review' component a 'lead time' component and a 'smoothing' component. Figure 4 shows NSAmp as a function of $\gamma = \beta$ for *Tp* = 2. For a pure OUT ($\beta = 1$) strategy, the smoothing component equals zero. Note that even then, inventory variance exceeds demand variance by a factor 3 (= 1+*Tp*). Otherwise, for 0<$\beta = \gamma < 1$, the smoothing component is always positive. As expected, smooth replenishments increase the variance of inventory.

FIGURE 4
NSAmp as a function of $\beta = \gamma$ *; Tp = 2*

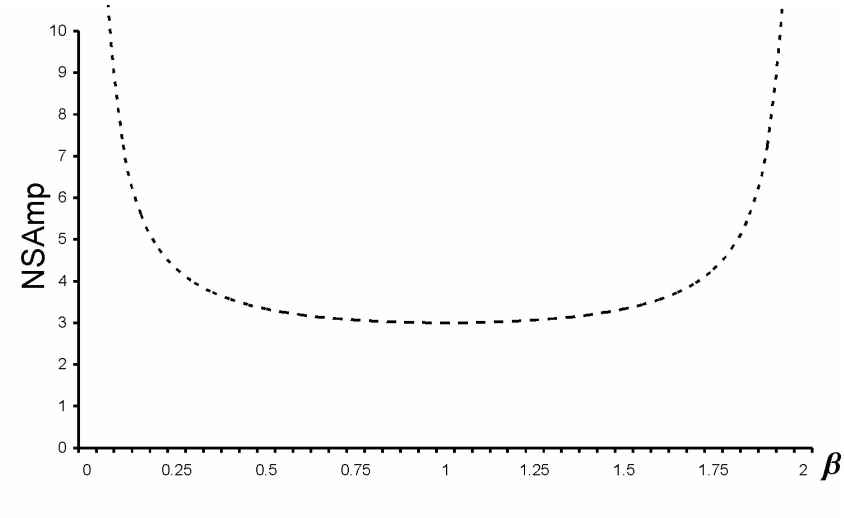


Figure 4 shows that *NSAmp*,

- is minimal at $\beta = 1$,

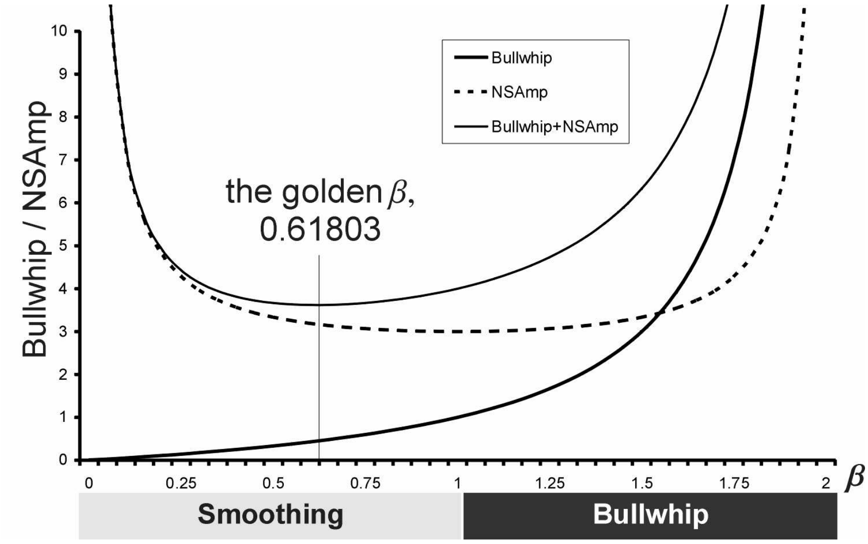
- increases with decreasing β, but also with increasing β. This means, that from an inventory point of view, smoothing ($\beta < 1$) and bullwhip ($\beta > 1$) are as equally 'bad'.

It is interesting to know that the shape of the NS*Amp* depends on the demand pattern. For ARMA demands the characteristic U-shaped inventory variance curve flexes to the right or the left depending on the parameters of the demand pattern. These extensions will not be handled in this paper. In this paper we focus on independently indentically distributed demand patterns. That means that the results of this paper has to be interpreted with care. The shape of the curves in Figures 3 and 4 are slightly different for ARMA demand patterns.

These observations lead to an interesting trade-off between bullwhip and customer service.

FIGURE 5

The variability trade-off (using the same data as Figures 3 and 4)



It can be shown that the sum of NSAmp and Bullwhip is minimised at β = 0.618, irrespective of lead-time. As a side note: 0.618, and its inverse, 1.618, is known since ancient history as the Golden Ratio, often found in many forms of the arts and nature. For example it describes the optimal placement of seeds and leaves in growing plants, the optimal ratio of female and male bees and geometric patterns in architecture.

Net Stock variance (let alone variance amplification) is not a common supply chain measure. Fortunately, the fill rate, defined as the fraction of volume delivered from inventory, is a popular customer service measure is closely related. Here we will give the basic insights, with the use of a minimum amount of mathematics.

First we will express Target Net Stock as follows,

$$TNS = z \times \sigma_{NS} \quad , \tag{9}$$

where,

z = safety factor

σ_{NS} = standard deviation of the net stock, which can be derived from the NSAmp expression (7) as

$$\sigma_{NS} = \sigma_D \sqrt{1 + Tp + \frac{(1-\beta)^2}{(2-\beta)\beta}} \tag{10}$$

The fill rate is a popular metric used to measure customer service (Zipkin, 2000).

$$\text{Fill Rate} = 1 - \frac{\text{expected number of backorders}}{\text{expected demand}} \tag{11}$$

Expression (12) can be rewritten as (the proof is omitted):

$$\text{Fill Rate} = 1 - \frac{\sigma_{\text{NS}} \times E(z)}{\overline{\text{D}}} \tag{12}$$

where, $E(z)$ = expected number of units backordered per period for a safety factor z

We can now determine E(z) from (13). Once E(z) is known, we can easily determine z using standard tables. This in turn will determine the Target Net Stock (TNS) to be used in (4).

$$TNS = z \times \sigma_{NS}$$

or equivalently, expressing TNS as a number of periods coverage, *a*:

$$TNS = a \times \overline{D} \tag{13}$$

While the safety factor z is related to σ_{NS}, a , represents how many periods of average demand $\overline{D}$ are covered by the Target Net Stock (TNS). The resulting ‘smoothing’ replenishment rule, guaranteeing a specified fill rate equals (for $\hat{D}^{\alpha}{}_{t} = \overline{D}$):

$$O_t = \overline{D} + \beta\,((Tp+a)\,\overline{D} - NS_t - WIP_t) \qquad (15)$$

In order to quantify the trade-off between the degree of ‘smoothing’ and the associated investment in safety stock we have to know the costs involved. Our experience is that a lot of ‘smoothing’ can be obtained with a small investment in extra safety stock. This is exposed by our numerical example ($\overline{D} = 500, \sigma_D = 100, Tp = 2$) by calculating the *TNS* for eight different values of β (see Table 1)

TABLE 1
Sample results highlighting the link between bullwhip, inventory and service levels

β	*Bullwhip*	*NSAmp*	*Bullwhip + NSAmp*	*a, number of periods coverage required to achieve a 99,5% fill rate*	*Fill rate at constant TNS*
1.667	5.000	3.800	8.800	0.717	99.1%
1.000	1.000	3.000	4.000	*0.622*	*99.5%*
0.618	0.447	3.171	*3.618*	0.643	99.4%
0.500	0.333	3.333	*3.666*	0.662	99.3%
0.333	0.200	3.800	*4.000*	0.717	99.1%
0.250	0.143	4.286	*4.429*	0.773	98.8%
0.167	0.091	5.273	5.364	*I0.875*	98.1%
0.100	0.053	7.263	7.316	1.060	96.7%
0.050	0.026	12.256	12.282	1.446	92.8%

It is obvious now that we can remove 90% of the order rate variance (i.e. by setting β = 0.17 rather than β = 1) with a quarter of a period extra inventory (0.875 - 0.622 = 0.253), whilst still maintaining a 99.5% fill rate.
The last column of the table shows the fill rate that would result from adopting the smoothing replenishment rule, but maintaining the Target Net Stock at the level required for β = 1. Depending on the profitability of the product (and/or the customer) and the cost of holding inventory, one may elect to 'pay' for smooth replenishments through slightly lower customer service rather than increasing inventory.

Note also that the safety stock required for 99.5% fill-rate at β= 0.333 is the same as for β=1.667, whereas the bullwhip differs by a factor of 25. The "Golden β", 0.61803, minimises the sum of bullwhip and inventory variance, which is then equal to $Tp+1+\beta$. The simple formulas above (for iid demand) can be extended to cope with different types of demand, Disney, Farasyn, Lambrecht, Towill and Van de Velde (2003).

In the discussion above, we have presented the bullwhip and customer service as a trade-off situation, in other words as a win-lose situation where one can win on bullwhip and lose on inventory investment (more inventory needed to guarantee the same fill rate). Fortunately, this is not a general conclusion. For certain stochastic demand patterns with Auto Regressive and Moving Average components (ARMA, see Box and Jenkins (1970)) it can be shown that win-win situations exist. That is, we may win on bullwhip and simultaneously win on inventory levels. Both bullwhip and inventory variability can be reduced simultaneously. We refer the reader to Disney et al. (2003)) for a detailed discussion on the win/win opportunities.

V. MANAGERIAL IMPLICATIONS

In a production environment with a wide product range, it is worth emphasizing that the bullwhip effect, measured at item level, may be dampened overall (a portfolio effect), but the inventory and fill rate considerations continue to hold. However, shortages can generally not be compensated by excess inventories of a different item.

We have shown that the longer the replenishment lead-time, the relative negative impact of smoothing is reduced. This insight is particularly relevant for global supply chains, typically designed to exploit low-cost manufacturing opportunities. Such long supply chains are by definition very sensitive to demand variability. An aspect all too often ignored in myopic optimisations of unit cost. It is this situation that the modified OUT policy may be particularly useful.

Finally, multi-echelon inventory policy research has shown that upstream inventories offer only indirect protection to customer service. In a distribution network, it is generally claimed that upstream fill rates are not as critical and can be set significantly lower than the target for the final customer, such as the 99.5% in our example. In Table 1 above, we have computed the increase in net stocks required to maintain the fill rate at the pre-specified level, or to let the customer service decrease if inventory investment is to remain constant. Companies upstream in a supply chain (where bullwhip hurts most) may well accept a small deterioration in fill rates as an alternative to increasing inventory, without compromising on the performance of the supply chain as a whole. This has obviously commercial implications, and calls for performance measures spanning the supply chain (a topic well beyond this paper). This last conclusion typically holds for i.i.d. demand patterns.

VI. CONCLUSION

A worst-case scenario occurs for supply chain partners experiencing outspoken bullwhip. Their production costs increase due to the bullwhip effect, along with the investment in additional safety stocks or conversely, missed sales resulting from insufficient inventories.

We have demonstrated that a smoothing replenishment rule is very effective at taming the bullwhip effect. Smoothing generally comes at the expense of increased investment in inventories in order to guarantee a given fill rate. We have shown that this extra investment can be relatively small or alternatively that the smoothing objective could be achieved with a lower customer service level. This conclusion holds for stationary identically and independently distributed demand patterns. For ARMA demand patterns, careful tuning of the replenishment rule can result in simultaneous reductions in bullwhip and inventory variability.

VIII. ACKNOWLEDGEMENTS

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REFERENCES

Anderson, E.G. Jr, Fine, C.H. and Parker, G.G., 2000, Upstream Volatility in the Supply Chain: the Machine Tool Industry as a Case Study, *POMS Series in Technology and Operations Management* 9, 239-261.

Box, G.E., Jenkins, G.M., 1970, Time Series Analysis Forecasting and Control, (Holden-Day, San Francisco).

Butman, J., 2002, A Pain in the (Supply) Chain, *Harvard Business Review* 80, 5, 31-44.

Chen, F., Drezner, Z., Ryan J.K., Simchi-Levi, D., 2000, Quantifying the Bullwhip Effect in a Simple Supply Chain: the Impact of Forecasting, Lead-Times and Information, *Management Science* 46, 436-443.

Childerhouse, P, Aitken, J. and Towill, D.R., 2002, Analysis and Design of Focused Demand Chains, *Journal of Operations Management* 20, 675-689.

Christopher, M., and Towill, D. R., 2002, Developing Market Specific Supply Chain Strategies, *The International Journal of Logistics Management* 13, 1, 1-14.

Craig, M. 2000, Thinking Visually, (Continuum Press, London.).

Croson, R., Donohue, K., 2002, Experimental Economics and Supply-Chain Management, *Interfaces* 32, 5, September-October, 74-82.

Croson, R., Donohue, K., 2003, Impact of POS Data Sharing on Supply Chain Management: an Experimental Study, *Production and Operations Management* 12, 1, 1-11.

Dejonckheere, J., Disney, S.M., Farasyn, I., Janssen, F., Lambrecht, M., Towill, D.R. and Van de Velde, W., 2002, Production and Inventory Control; the Variability Trade-Off, Proceedings of the 9th EUROMA, June 2-4, (Copenhagen, Denmark).

Dejonckheere, J., Disney, S.M., Lambrecht, M.R. and Towill, D.R., 2004, The Impact of Information Enrichment on the Bullwhip Effect in Supply Chains: a Control Engineering Perspective, *European Journal of Operational Research*, 152, 727-750.

Dejonckheere, J., Disney, S.M., Lambrecht, M.R., Towill, D.R., 2003, Measuring the Bullwhip Effect: a Control Theoretic Approach to Analyse Forecasting Induced Bullwhip in Order-Up-to Policies, *European Journal of Operations Research* 147, 3, 567-590.

Deziel, D.P. and Eilon, S., 1967, A Linear Production – Inventory Control Rule, *The Production Engineer*, 43, 93-104.

Disney, S. M., Farasyn, I., Lambrecht, M., Towill, D.R. and Van de Velde, W., 2003, Taming the Bullwhip Effect whilst Watching Customer Service, Working Paper, (Cardiff University).

Disney, S.M. and Towill, D.R., 2003, On the Bullwhip and Inventory Variance Produced by an Ordering Policy, *Omega*, 31, 157-167.

Forrester, J., 1958, Industrial Dynamics – a Major Break Though for Decision-Makers, *Harvard Business Review* 36, 4, 37-66.

Forrester, J., 1961, Industrial Dynamics, (Cambridge MA, MIT Press).

Geary, S., Disney, S.M. and Towill, D.R., 2003, Bullwhip in Supply Chains ~ Past, Present and Future, 17th International Conference on Production Research, (Virginia, USA), 3-7 August.

Hayes, R.H. and Wheelwright, S.C., 1984, Restoring our Competitive Edge: Competing through Manufacturing, (John Wiley & Sons).

Holmström, J. 1997, Product Range Management: a Case Study of Supply Chain Operations in the European Grocery Industry, *International Journal of Supply Chain Management* 2, 8, 107-115.

Houlihan, J.B., 1987, International Supply Chain Management, *International Journal of Physical Distribution and Materials Management* 17, 2, 51-66.

Konicki,S. (2002), Now in Bankruptcy, Kmart Struggled with Supply Chain, Information week, Url: http://www.informationweek.com/story/IWK20020125S0020, [28/01/2002].

Lee, H.L., Padmanabhan, P., Whang, S., 1997, Information Distortion in a Supply Chain: the Bullwhip Effect, *Management Science*, 43, 543-558.

Looman, A., Ruttins, F.A.J. and de Boer, L, .2002, Designing Ordering and Inventory Management Methodologies for Purchased Parts, *Journal of Supply Chain Management*, Spring, 38, 12, 22-29.

Magee, J.F.,1956, Guides to Inventory Control, part II, *Harvard Business Review*, Mar-April, 106-116.

Mason-Jones R., and Towill D.R., 1997, Information Enrichment: Designing the Supply Chain for Competitive Advantage, *Supply Chain Management* 2, 4, 37-149.

McCullen, P. and Towill, D.R., 2002, Diagnosis and Reduction of Bullwhip in Supply Chains, *Supply Chain Management, an International Journal* 7, 3-4, 164-179.

Mitchell, T., 1923, Competitive Illusion as a Cause of Business Cycles, *Quarterly Journal of Economics* 38, 631-652.

Ortmeyer, G., Quelch, J. and Salmon, W., 1991, Restoring Credibility to Retail Pricing, *Sloan Management Review Fall*, 55-66

Silver, E.A., Pyke, D.F. and Peterson, R., 1998, Inventory Management and Production Planning and Scheduling, (John Wiley and Sons, New York).

Sterman, J., 1989, Modelling Managerial Behaviour : Misperceptions of Feedback in a Dynamic Decision Making Experiment, *Management Science* 35, 3, 321-339.

Towill D.R. and McCullen, P., 1999, The Impact of an Agile Manufacturing Programme on Supply Chain Dynamics, *International Journal Logistics Management* 10, 1, 83-96.

Towill, D.R., 1997, FORRIDGE – Principles of Good Practice in Material Flow, *International Journal of Production Planning and Control* 8, 7, 622-632.

Wikner, J., Towill, D.R., and Naim, M.M., 1991, Smoothing Supply Chain Dynamics, *International Journal of Production Economics* 22, 231-248.

Zipkin, P.H., 2000, Foundations of Inventory Management, (McGraw-Hill, New York).

TABLE 1.
Sample results highlighting the link between bullwhip, inventory and service levels

β	***Bullwhip***	***NSAmp***	***Bullwhip* + *NSAmp***	***a*, number of periods coverage required to achieve a 99.5% fill rate**	**Fill rate at constant TNS**
1.667	5.000	3.800	8.800	0.717	99.1%
1.000	1.000	3.000	4.000	**0.622**	**99.5%**
0.618	0.447	3.171	**3.618**	0.643	99.4%
0.500	0.333	3.333	3.666	0.662	99.3%
0.333	0.200	3.800	4.000	0.717	99.1%
0.250	0.143	4.286	4.429	0.773	98.8%
0.167	0.091	5.273	5.364	**0.875**	98.1%
0.100	0.053	7.263	7.316	1.060	96.7%
0.050	0.026	12.256	12.282	1.446	92.8%

FIGURE 1

The impact of adjustments

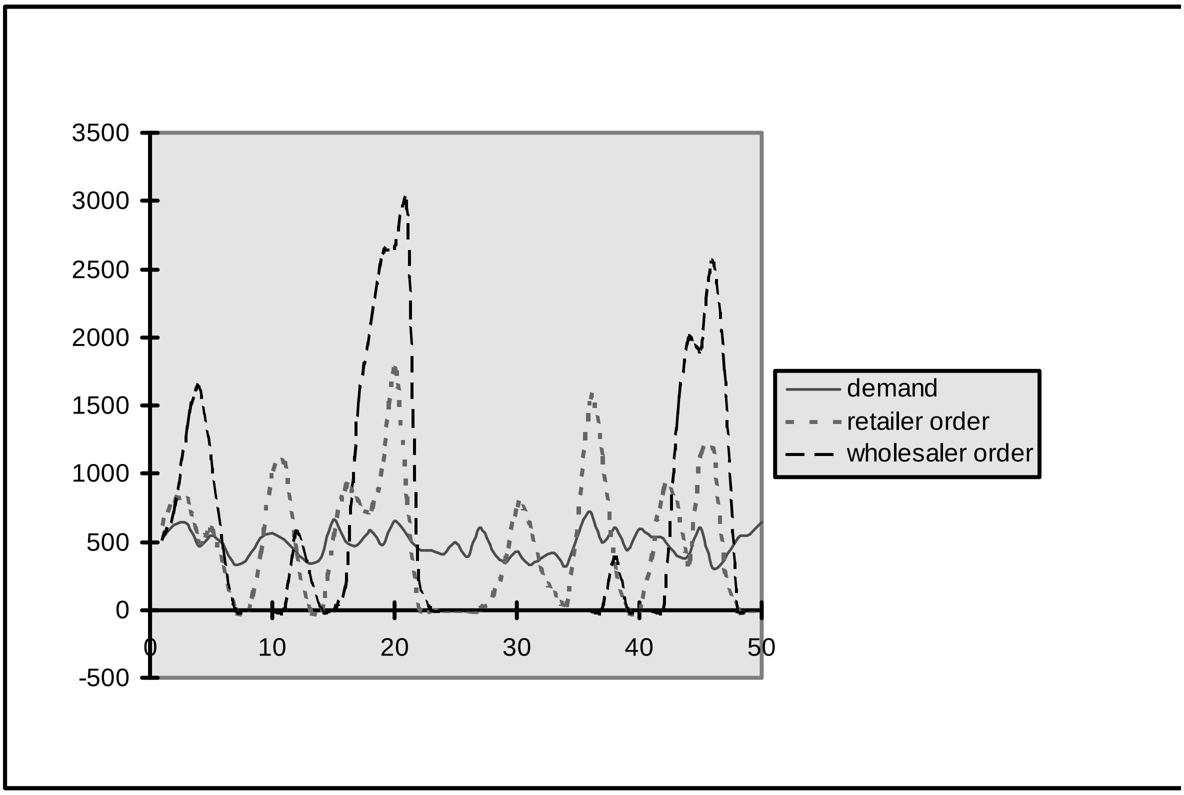

FIGURE 2

A smoothing replenishment rule

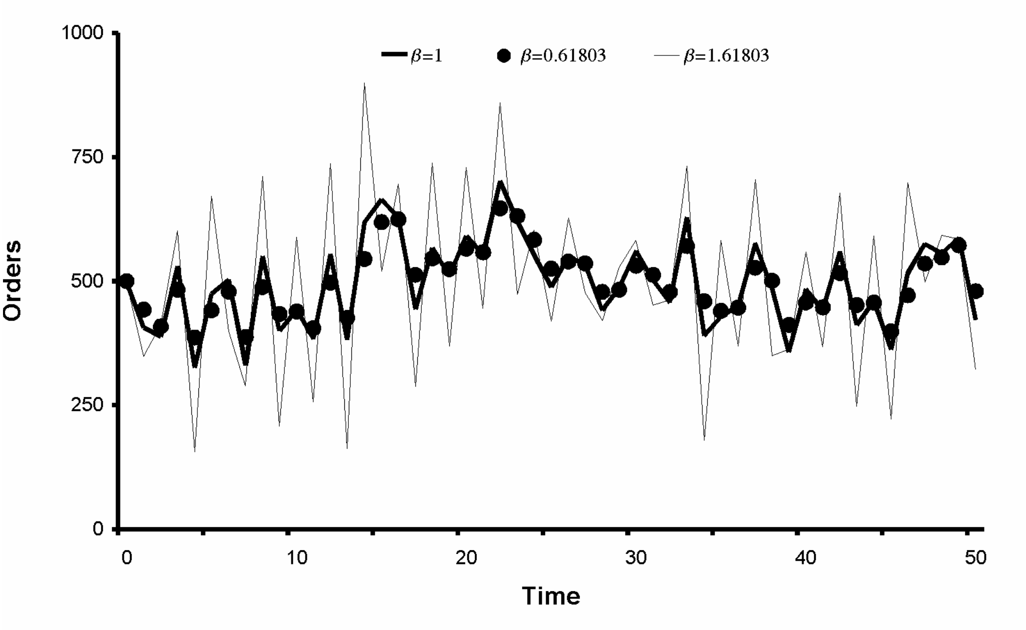

FIGURE 3

Bullwhip with stationary demand and matched feedback controllers

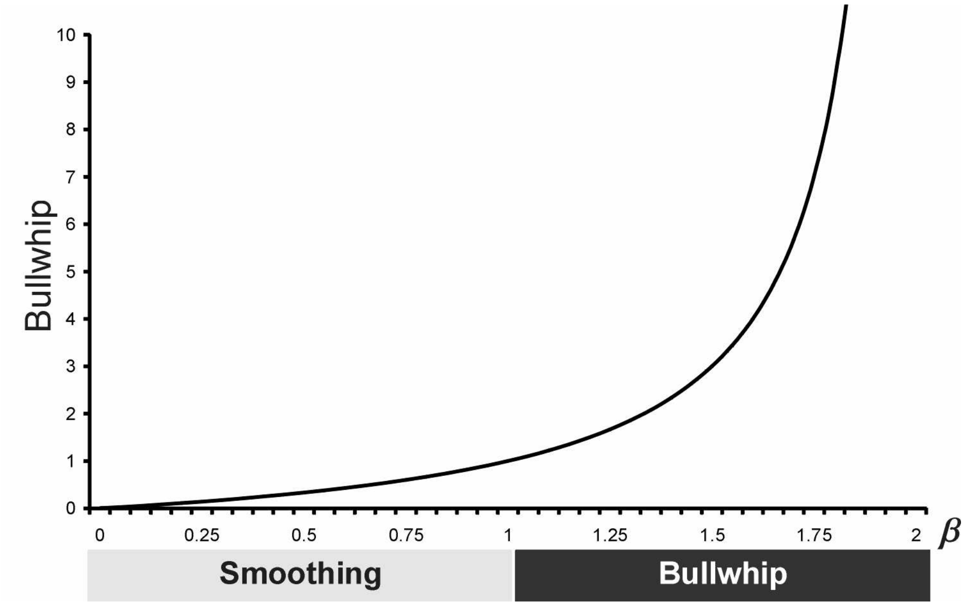

FIGURE 4

NSAmp as a function of $\beta = \gamma$ *; Tp = 2*

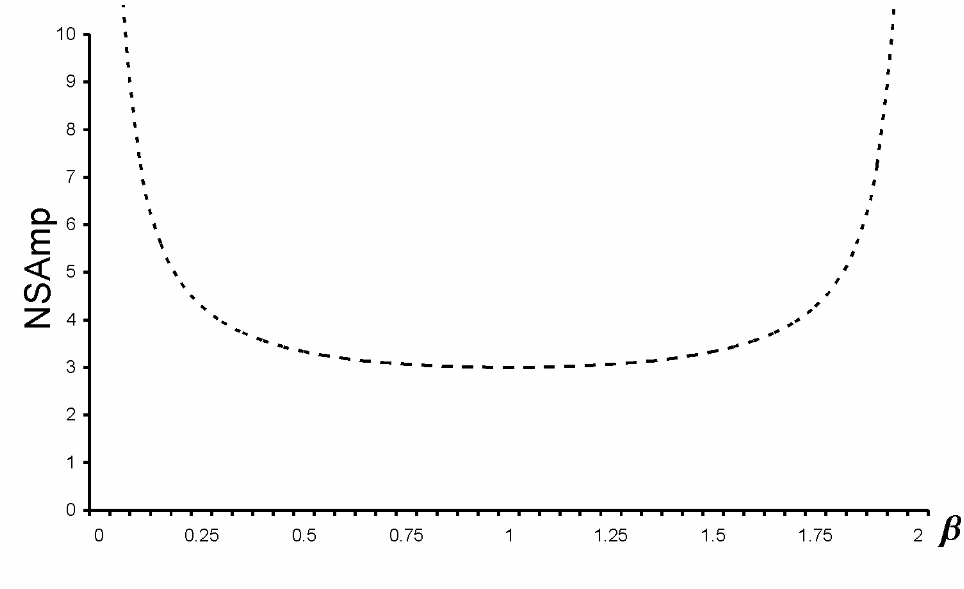

FIGURE 5

The variability trade-off (using the same data as Figures 3 and 4)