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Abstract

The traditional approach to discriminate amongst two competing hedging strategies is to compare the sample portfolio return variance implied by each strategy. This simple approach suffers from two drawbacks. First, it is an unconditional performance measure which is theoretically not coherent with a dynamic hedging strategy that minimizes the conditional portfolio return variance. Second, estimating unconditional performance over the entire period may not be sufficient since a strategy with a good unconditional hedging performance may not perform well at a particular point in time. In this paper, I use the Giacomini and White (2006), the Wald, and the Diebold and Mariano (1995) statistical tests in order to conditionally (and as a special case, unconditionally) compare the portfolio return variances implied by two competing hedging strategies. The attractive feature of the conditional perspective is that, in case of rejection of equal conditional hedging effectiveness among two initial strategies, it provides us with a new hedging strategy that selects at each date the initial strategy that will perform the best next period, conditional on current information. An application to several agricultural commodities illustrates the technique. For daily hedging horizons, it is found that most of the time Ederington's (1979) static strategy is superior to more elaborate dynamic strategies. This calls into question earlier results reported in the literature that were based on a much smaller database.

JEL classification: ....

Key words: hedging, GARCH

1 Introduction

Traditionally, the hedge ratio is determined by the slope of the OLS regression of the spot returns on the futures returns (and a constant) (Johnson, 1960; Stein, 1961; Ederington, 1979). The resulting hedge ratio is static, minimizes the unconditional variance of the portfolio and is often used as a benchmark. This approach, however, ignores the conditional heteroskedasticity in the returns. As a result, recent research focusses on the determination of time-varying optimal hedge ratios, minimizing the *conditional* variance of the portfolio return.

Numerous studies on hedging allow the joint distribution of spot and futures returns to vary over time (for short reviews, see Lien and Tse, 2002; or Chen, Lee and Shrestha, 2003). The bulk of the literature estimates dynamic hedge ratios using a conditional distribution associated with a bivariate GARCH (BGARCH) model. For instance, Baillie and Myers (1991), Myers (1991), Bera, Garcia and Roh (1997) and Haigh and Holt (2002) estimate time-varying hedge ratios in commodity markets; Kroner and Sultan (1993) in foreign exchange markets; Park and Switzer (1995) and Lafuente and Novales (2003) in stock markets; Lien, Tse and Tsui (2002) in all of the above markets; Gagnon and Lypny (1995) in fixed-income markets and Byström (2003) in electricity markets.

In order to rank two competing strategies, the traditional approach is to compute the ratio of the sample unconditional return variance of the first strategy to that of the second. However, this simple measure suffers from two major drawbacks. First, it is an unconditional measure. Evaluating a dynamic strategy resulting from the minimization of the *conditional* portfolio return variance by means of the implied *unconditional* portfolio return variance is not adequate. Secondly, it is an empirical fact that any return on a traded asset and, by extension, any portfolio return, exhibits conditional heteroskedasticity. Thus, at any particular moment in time, when comparing two minimum variance (conditional or unconditional) hedging strategies, the hedger should pay attention to the relative risk reduction in terms of the *conditional* portfolio return variances, instead of solely focussing on comparing unconditional portfolio return variances. Said differently, simply checking unconditional relative performance is not sufficient since a hedging strategy with a good unconditional relative hedging performance may nevertheless have a poor conditional relative hedging performance at any particular moment.

In this paper, I use the Giacomini and White (2006), the Wald, and the Diebold and Mariano (1995) statistical tests in order to compare the portfolio return variances implied by two competing hedging strategies conditionally (and as a special case, unconditionally). The attractive feature of the conditional perspective is that, in case of rejection of equal conditional hedging effectiveness among two initial strategies, it provides us with a new hedging strategy that selects at each date the initial strategy that will perform best next period, given current information. An application to several agricultural commodities illustrates the technique.

The remainder of this paper is structured as follows. Section 2 discusses some existing hedging strategies. Section 3 presents a methodology to evaluate conditional hedging effectiveness. Section 4 applies this methodology to agricultural commodity data and section 5 concludes.

2 Standard hedging strategies

I begin with a brief discussion on standard hedging strategies in order to establish ideas and fix notations.

Consider an agent with a one-period hedging horizon who wants to place a hedge on a long spot position. Let s_t denote the log of the spot price and f_t the log of the nearest-to-maturity futures price.[1] Assume that the agent has a portfolio with a long position of one unit in the spot market and a short position of h_{t-1} units in the futures contract. At time t, the return of the portfolio is

$$r_t(h_{t-1}) = \Delta s_t - h_{t-1}\Delta f_t. \tag{2.1}$$

The hedge ratio h_{t-1} has to be determined in some optimal way. On the one hand, Johnson's (1960), Stein's (1961) and Ederington's (1979) (henceforth ED) approach minimizes the unconditional variance of the portfolio return to derive the optimal static hedge ratio

$$h^* = \frac{\operatorname{cov}(\Delta s_t, \Delta f_t)}{\operatorname{var}(\Delta f_t)}. \tag{2.2}$$

Sercu and Wu (2000) extend this approach in order to pick up the lead-lag relationships between Δs_t and Δf_t. They derive the following static hedge ratio based on the Scholes-Williams estimator:

$$h^{**} = \frac{\operatorname{cov}(\Delta s_t, \Delta f_{t-1} + \Delta f_t + \Delta f_{t+1})}{\operatorname{cov}(\Delta f_t, \Delta f_{t-1} + \Delta f_t + \Delta f_{t+1})}.$$

On the other hand, a large literature initiated by Baillie and Myers (1991) allows the ED

[1] For notational convenience and since only the nearest-to-maturity futures contract is used, I use f_t instead of $f_{t,T}$, the log of the futures price maturing on date T.

minimum variance hedge ratio to vary over time. The dynamic version of h^* is given by:

$$h_{t-1}^* = \frac{\text{cov}\left(\Delta s_t, \Delta f_t | I_{t-1}\right)}{\text{var}\left(\Delta f_t | I_{t-1}\right)}, \tag{2.3}$$

where I_{t-1} is the information set available at time $t-1$. The major issue then becomes the choice of the BGARCH model to estimate the conditional (co)variances. Other more tractable approaches that take into account the time-varying nature of the hedge ratio have also been proposed. For instance, Miffre (2004) modifies ED's traditional OLS-based estimation method to incorporate conditional information and Bera, Garcia and Roh (1997) introduce a random coefficient autoregressive model to estimate the dynamic hedge ratio in (2.3).

As we have seen there are a lot of available strategies to choose from. A strategy should be preferred if it leads to greater reduction of risk. Risk measurement is the subject of the next section.

3 A method to compare conditional hedging effectiveness and a new dynamic strategy

I split the data sample in two parts, an in-sample and an out-of-sample part. The in-sample observations $t = 1, ..., T_{in}$ are used to estimate the optimal hedging strategies. The out-of-sample observations $t = T_{in} + 1, ..., T$ are used to compare hedging effectiveness. Let $T_{out} \equiv T - T_{in}$ be the out-of-sample size.

Assume we can choose between two competing hedging strategies, h^1 and h^2, prescribing a hedge ratio at each time, conditional on available information at that time. Let $r_t^2\left(h^1\right)$ and $r_t^2\left(h^2\right)$ be the squared demeaned out-of-sample portfolio return implied by h^1 and h^2, respectively. It is well known that these are unbiased estimates of the true conditional variances. See Andersen and Bollerslev (1998) or Diebold and Lopez (1996), for instance.

In order to measure conditional hedging effectiveness, the conditional portfolio return variances implied by each strategy is compared. Denote the difference in squared returns by $d_t \equiv r_t^2\left(h^1\right) - r_t^2\left(h^2\right)$. The null hypothesis of equal conditional hedging effectiveness is then formulated as

$$H_0 : E\left[d_t | I_{t-1}\right] = 0. \tag{3.4}$$

The motivation behind conditional testing is that it represents the real-time problem of a hedger in deciding which of the two strategies reduces the next period portfolio return variance most, conditional on current information. In order to achieve that objective, a set of unconditional moment conditions are derived from H_0. Suppose z_{t-1} is a $q \times 1$ vector included in the information set and let $Z_t \equiv z_{t-1} d_t$ be a $q \times 1$ vector. Then, by the law of total expectations, H_0 implies[2]

$$H_{0,z} : E\left[Z_t\right] = 0. \tag{3.5}$$

Giacomini and White (2006) (GW) constructed a test of $H_{0,z}$ against the two-sided alternative

$$H_{1,z} : E\left[Z_t\right]' E\left[Z_t\right] > 0.$$

The GW test is based on the Wald-type statistic

$$GW_z \equiv T_{out} \bar{Z}' \hat{\Sigma}^{-1} \bar{Z}, \tag{3.6}$$

where $\bar{Z} \equiv T_{out}^{-1} \sum_{t=T_{in}+1}^{T} Z_t$ is a $q \times 1$ vector and $\hat{\Sigma} \equiv T_{out}^{-1} \sum_{t=T_{in}+1}^{T} Z_t Z_t'$ is a $q \times q$ matrix estimating the variance of Z_t. Under $H_{0,z}$, $GW_z \xrightarrow{d} \chi^2_{(q)}$. The test will reject $H_{0,z}$, and therefore H_0, at the level α whenever $GW_z > \chi^2_{q,1-\alpha}$, where $\chi^2_{q,1-\alpha}$ is the $(1-\alpha)$-quantile of the $\chi^2_{(q)}$ distribution.

This test deserves some comments:

1. In the application, I choose the variables z_{t-1} from the information set that have potential explanatory power for predicting the difference in the squared returns. As it is often the case in finance, it is assumed that all existing information is included in the first lag and z_{t-1} is chosen to be[3]

$$z_{t-1} \equiv \left(1, \Delta s_{t-1}, \Delta f_{t-1}, \Delta s_{t-1} \Delta f_{t-1}, f_{t-1} - s_{t-1}\right)'. \tag{3.7}$$

2. There is a relation between the Diebold and Mariano (1995) (DM) unconditional test and the GW conditional test. The DM approach is concerned with testing the null hypothesis of equal unconditional hedging effectiveness, *i.e.*

$$H_{0,DM} : E\left[d_t\right] = 0. \tag{3.8}$$

   Under $H_{0,DM}$, the statistic

$$DM \equiv \frac{\sqrt{T_{out}}\bar{d}}{\sqrt{\widehat{LRV}\left(d_t\right)}} \xrightarrow{d} N\left(0,1\right), \tag{3.9}$$

[2] H_0 and $z_{t-1} \subset I_{t-1}$ imply that $E\left[z_{t-1} d_t | I_{t-1}\right] = 0$. Taking the unconditional expectation on both sides gives $H_{0,z}$.

[3] Alternative choices such as z_{t-1} being quadratic in returns or a function of past relative performance have been tried. However, all of them predicted d_t less accurately than (3.7).

where $\bar{d} \equiv T_{out}^{-1} \sum_{t=T_{in}+1}^{T} d_t$ and $\widehat{LRV}(d_t)$ is an estimate of the long-run variance of d_t. Note that H_0 implies $H_{0,DM}$. Furthermore, H_0 implies that d_t and $d_{t'}$ have zero covariance for $t \neq t'$, so the long-run variance of d_t equals the variance of $d_{t'}$. If the DM test is implemented with the latter restriction, it is a special case of the GW test, corresponding to the choice $z_{t-1} = 1$. Consequently, if $1 \in z_{t-1}$, $H_{0,z}$ cannot hold if $H_{0,DM}$ does not hold, so a rejection of $H_{0,DM}$ logically implies a rejection of $H_{0,z}$ (although there is a probability that the test outcomes violate the logical implication). Note also that the conditional hedging literature initiated by Baillie and Myers (1991) sticks to the null hypothesis (3.8) since the relative performance is traditionally measured by the implied out-of-sample return variance difference or ratio (but they don't, strictly speaking, test $H_{0,DM}$).

3. Both the GW statistic (3.6) and the DM statistic (3.9) can be computed using standard regression packages. Define $Z \equiv \begin{pmatrix} d_{T_{in}+1}z'_{T_{in}} & \cdots & d_T z'_{T-1} \end{pmatrix}'$, a $T_{out} \times q$ matrix and ι, a $T_{out} \times 1$ vector of ones. As it is discussed in Comment 4 of GW, the GW statistic can be rewritten as

$$\begin{aligned} GW_z &= T_{out} \frac{\iota' Z}{T_{out}} \left(\frac{Z'Z}{T_{out}} \right)^{-1} \frac{Z'\iota}{T_{out}} \\ &= T_{out} \frac{\iota' Z \left(Z'Z \right)^{-1} Z'\iota}{\iota'\iota} \\ &= T_{out} R^2, \end{aligned} \tag{3.10}$$

where R^2 is the uncentered square multiple correlation coefficient for the artificial regression $\iota = Z\gamma + \epsilon$. The DM statistic can be written as the t-statistic in a regression of d_t on a constant with Newey-West standard error.[4]

4. Rejection of equal conditional hedging effectiveness does not indicate *per se* which strategy reduces the risk best next period. But it indicates that z_{t-1} contains information about the expected relative performance of the two competing hedging strategies. In other words, rejecting $H_{0,z}$ implies that

$$E\left[d_t | z_{t-1}\right] \simeq \hat{\beta}' z_{t-1} \neq 0, \tag{3.11}$$

where $\hat{\beta}$ denotes the OLS regression estimate of d_t on z_{t-1}, over the out-of-sample period. The sign of $\hat{\beta}' z_{t-1}$, computed at time $t-1$, indicates the direction of the rejection

[4] In the application, the default truncation lag in EViews is used to compute the Newey-West standard error.

of the two-sided GW test: a negative (positive) sign indicates that strategy h^1 would conditionally outperform (will be outperformed by) strategy h^2 at time t. Remark also that the sign of

$$\bar{d} \equiv T_{out}^{-1} \sum_{t=T_{in}+1}^{T} d_t = T_{out}^{-1} \sum_{t=T_{in}+1}^{T} \hat{\beta}' z_{t-1}$$

indicates the direction of the rejection of the two-sided DM test.

5. There is a relation between the Wald test implemented using the White heteroskedasticity consistent covariance estimator and the GW test. The GW statistic in (3.10) can be rewritten as

$$GW_z = \iota' Z \left(\sum_{t=T_{in}+1}^{T} (d_t z_{t-1}) (d_t z_{t-1})' \right)^{-1} Z' \iota.$$

Consider the regression (3.11). That is, $d = X\beta + \epsilon$, where $X \equiv \begin{pmatrix} z_{T_{in}} & \cdots & z_{T-1} \end{pmatrix}'$ is a $T_{out} \times q$ matrix. The heteroskedasticity consistent Wald statistic testing the null hypothesis $\beta = 0$ is given by

$$\begin{aligned} W_z &\equiv \hat{\beta}' (X'X) \left(\sum_{t=T_{in}+1}^{T} (\hat{e}_t z_{t-1}) (\hat{e}_t z_{t-1})' \right)^{-1} (X'X) \hat{\beta} \\ &= d'X \left(\sum_{t=T_{in}+1}^{T} (\hat{e}_t z_{t-1}) (\hat{e}_t z_{t-1})' \right)^{-1} X'd \\ &= \iota' Z \left(\sum_{t=T_{in}+1}^{T} (\hat{e}_t z_{t-1}) (\hat{e}_t z_{t-1})' \right)^{-1} Z' \iota \end{aligned} \tag{3.12}$$

where $\hat{e}_t = d_t - \hat{\beta}' z_{t-1}$. It thus follows that W_z equals GW_z when the robust variance of $\hat{\beta}$ is computed under the null $H_{0,z}$, that is, when $\hat{e}_t$ is replaced by d_t in (3.12). In the application, both GW_z and W_z are reported.

6. Rejection of the null of equal conditional hedging effectiveness naturally leads to a third strategy, h^3, which selects one of the two initial strategies, h^1 and h^2, based on currently available information: h^3 at time $t-1$ chooses strategy h^2 if $\hat{\beta}' z_{t-1} > 0$ and h^1 otherwise. That is,

$$h^3 \left(h^1, h^2\right) = 1 \left\{ \hat{\beta} z_{t-1} \leq 0 \right\} h^1 + 1 \left\{ \hat{\beta}' z_{t-1} > 0 \right\} h^2, \tag{3.13}$$

where $1\{\cdot\}$ is the indicator function. Remark that rejection of $H_{0,z}$, *i.e.* rejection of equal conditional hedging performance between h^1 and h^2 over the out-of-sample period, implies *per se* the (in-sample) superiority of h^3 over h^1 or h^2 over the out-of-sample period. This is not, however, a genuine out-of-sample test of h^3. Therefore, I slightly modify h^3 by replacing the out-of-sample estimate $\hat{\beta}$ by the in-sample estimate $\tilde{\beta}$, giving

$$h^3\left(h^1,h^2\right)=1\left\{\tilde{\beta}'z_{t-1}\leq 0\right\}h^1+1\left\{\tilde{\beta}'z_{t-1}>0\right\}h^2, \tag{3.14}$$

instead of (3.13). The genuine out-of-sample performance of h^3 (both unconditionally and conditionally) can now be compared to that of h^1 or h^2 in exactly the same manner as comparing the performances of h^1 and h^2.

7. One may wonder if we can go one step further in considering a more general linear combination of the initial hedge ratios than in (3.13). Indeed h^3 restricts the weights to be zero or one. Consider the strategy

$$h^3_\alpha\left(h^1,h^2\right)=\alpha h^1+(1-\alpha)\,h^2, \tag{3.15}$$

which generalizes (3.14). Since $r_t\left(h^3_\alpha\right)=\alpha r_t\left(h^1\right)+(1-\alpha)\,r_t\left(h^2\right)$, this can be seen as holding two portfolios whose weights sum to unity: one portfolio hedged with strategy h^1 and one with h^2. The goal is now to find the optimal α such that $\operatorname{var}\left(r_t\left(h^3_\alpha\right)|I_{t-1}\right)$ is minimal. Rewriting the objective function in terms of spot and futures returns, setting $\frac{d\operatorname{var}\left(r_t\left(h^3_\alpha\right)|I_{t-1}\right)}{d\alpha}=0$ and solving for α, we get after some basic algebra

$$\alpha^*_{t-1}=\frac{h^2\operatorname{var}\left(\Delta f_t|I_{t-1}\right)-\operatorname{cov}\left(\Delta s_t,\Delta f_t|I_{t-1}\right)}{\left(h^2-h^1\right)\operatorname{var}\left(\Delta f_t|I_{t-1}\right)}. \tag{3.16}$$

Assume we have reliable estimates for $\widehat{\operatorname{cov}}\left(\Delta s_t,\Delta f_t|I_{t-1}\right)$ and $\widehat{\operatorname{var}}\left(\Delta f_t|I_{t-1}\right)$ thus yielding an estimate $\hat{\alpha}^*_{t-1}$. But if this were the case, in a minimum variance setting, we would use them to construct either h^1 or h^2. Without loss of generality, assume $h^2=\frac{\widehat{\operatorname{cov}}(\Delta s_t,\Delta f_t|I_{t-1})}{\widehat{\operatorname{var}}(\Delta f_t|I_{t-1})}$. But then $\hat{\alpha}^*_{t-1}$ equals zero and h^3_α reduces to h^2. Thus if one of the strategies, say h^2, incorporates reliable estimates of the conditional covariances, it cannot be further improved upon. It follows that the same must hold for (3.13): h^3 cannot improve upon h^2. Of course this reasoning is based on the premise that z_{t-1} is properly taken to be part of I_{t-1} when the conditional covariances are estimated. If the estimates are based on a model that incorrectly imposes certain exclusion restrictions, the GW_z and W_z statistics may reveal this and h^3 may outperform h^1 and h^2.

4 Application to agricultural commodities

I now turn to an empirical application of the methodology developed above. In this section the data are first described, then static and dynamic hedge ratios are estimated and the results and some problems encountered are discussed. Then, the relative ability of each strategy to reduce risk is compared. Finally, the relative performance of the new strategy h^3 is analyzed.

4.1 Data

The data consist of daily spot and nearest-to-maturity futures prices of corn, wheat, soybeans and oats. All futures closing prices were extracted from the Chicago Board of Trade tapes and cover the period January 1979 through December 2003 for corn, soybeans and oats, and the period January 1983 through December 2003 for wheat.[5] For the same period, the closing spot prices (in cents per bushel) were extracted from Datastream. Qualities (and exchanges) of the spot prices are the following: oats, No.2 (Milling Minneapolis); wheat, No.2, Soft Red (Chicago); soybeans, No.1, Yellow (Chicago); corn, No.2, Yellow (Chicago).

Following current practices in the literature, and in order to avoid expiration effects, a contract that expires in month m is replaced with the next expiring contract on the last day of month $m-1$. Specifically, on the last day of $m-1$, Δf_t is set equal to the return on the former contract, while on the first day of m, Δf_t is set equal to the return on the latter contract.

The hedging horizons considered in this paper are one day and one week.[6] Similar horizons are considered in the literature (see for instance Lien, Tse and Tsui (2002) or Byström (2003)). Minimum-variance hedge ratios are computed for each horizon, based on non-overlapping data with the same frequency as the hedging horizon. Thus, the returns were aggregated to yield weekly returns, computed from Friday closing prices. Table 1 gives the mean, standard deviation, skewness, kurtosis and autocorrelations on Δs_t and Δf_t for each commodity. The returns data are non-normal as evidenced by the high kurtosis and the significant Jarque-Bera statistics (not reported here).

4.2 Static hedge ratios

The sample is splitted in two parts. The in-sample period is from January 1979 (1983 for wheat) to December 1993 and the out-of-sample period is from January 1994 to December 2003. The in-sample observations are used to estimate GARCH and ED's optimal hedge

[5] There are different beginning dates for weekly data than for daily data, see Table 1. The reason is that the conditional comparison of the strategies was first investigated on the weekly horizon and that, by that time, all futures prices were not extracted from the CBOT tapes yet.

[6] Longer hedging horizons such as one month and six months were considered but I have not been able to estimate the parameters of the bivariate GARCH-type models.

Table 1: Descriptive statistics on spot and futures returns.

	mean	std. deviation	skewness	kurtosis	autocorrelation coef.		
					ρ_1	ρ_6	ρ_{12}
Corn, daily, 1/04/1979-12/31/2003, 6520 observations.							
Δs_t	5.8072×10^{-6}	0.0144	−0.4832	8.9742	0.0189	0.0218	−0.0049
Δf_t	-2.8836×10^{-4}	0.0122	−0.0060	5.6544	0.0497	−0.0063	0.0098
Corn, weekly, 1/26/1979-12/26/2003, 1301 observations.							
Δs_t	-2.0968×10^{-5}	0.0326	−0.4958	7.2140	0.0392	0.0276	−0.0362
Δf_t	−0.0014	0.0280	0.3005	6.8211	−0.0166	−0.0101	0.0003
Wheat, daily, 1/04/1983-12/31/2003, 5477 observations.							
Δs_t	-6.5684×10^{-6}	0.0172	−0.9733	22.1083	−0.0079	0.0037	0.0205
Δf_t	-2.0203×10^{-4}	0.0137	0.0811	5.5408	0.0243	0.0034	0.0195
Wheat, weekly, 1/13/1984-12/26/2003, 1042 observations.							
Δs_t	1.0417×10^{-5}	0.0373	−0.6618	10.9648	−0.0203	−0.0206	0.0140
Δf_t	-9.9113×10^{-4}	0.0301	0.4218	4.6755	0.0014	−0.0357	0.0208
Oats, daily, 1/04/1979-12/31/2003, 6520 observations.							
Δs_t	2.6532×10^{-5}	0.0193	−0.0794	22.9636	−0.0295	0.0108	−0.0008
Δf_t	-3.3979×10^{-4}	0.0178	−0.0572	5.1270	0.0566	−0.0102	−0.0047
Oats, weekly, 1/26/1979-12/26/2003, 1301 observations.							
Δs_t	4.9289×10^{-5}	0.0420	−0.0550	7.6995	−0.0720	−0.0335	−0.0278
Δf_t	−0.0017	0.0415	0.1307	7.0114	−0.0512	−0.0183	0.0428
Soybeans, daily, 1/04/1979-12/31/2003, 6520 observations.							
Δs_t	2.4267×10^{-5}	0.0135	−0.3723	6.5939	−0.0290	−0.0208	−0.0104
Δf_t	-1.6056×10^{-4}	0.0129	−0.1511	5.3061	−0.0174	−0.0221	0.0032
Soybeans, weekly, 1/19/1979-12/26/2003, 1302 observations.							
Δs_t	6.6689×10^{-5}	0.0307	−0.1426	6.0890	−0.0873	−0.0139	−0.0078
Δf_t	-8.2016×10^{-4}	0.0294	−0.1573	6.3095	−0.0543	−0.0014	−0.0248

ratios. The static hedge ratio h^* in equation (2.2) is estimated by the OLS regression slope of Δs_t on Δf_t (and a constant). Estimated parameters and Newey-West standard errors are reported in Table 2 for each hedging horizon. As expected, the optimal hedge ratios are close to unity, except for oats. The reason might be that the oats spot market is the Minneapolis exchange whereas the futures contract is traded on the CBOT.

4.3 Dynamic hedge ratios

Following the hedging literature initiated by Baillie and Myers (1991), the static hedge ratio is compared to more sophisticated time-varying ones resulting from a bivariate GARCH model. To date, many multivariate GARCH specifications have been proposed (see Bauwens, Laurent and Rombouts (2005) for a survey). The most general expression when the variances and covariances are linear functions of the squares and cross-product of the innovations is the VECH model of Engle and Kroner (1995). But this model involves many parameters (21 parameters in a bivariate context) and imposes many nonlinear cross-coefficient inequality restrictions to yield positive definite stationary covariance matrices. Engle and Kroner (1995) proposed a

Table 2: Static hedge ratio estimates, January 1994-December 2003.

	Daily	Weekly	Daily	Weekly
	Corn		Wheat	
a	0.000269	0.001496	-2.27×10^{-6}	-0.000522
	(0.000124)	(0.000700)	(0.000215)	(0.001160)
h^*	0.980	1.002	0.845	0.957
	(0.022)	(0.027)	(0.054)	(0.045063)
Loglik.	13280.17	2091.48	8613.12	1183.64
N Obs.	3914	781	2869	522
	Soybeans		Oats	
a	0.000290	0.00155	0.000231	0.001892
	(0.000067)	(0.00030)	(0.000214)	(0.000776)
h^*	0.918	0.985	0.378	0.687
	(0.017)	(0.013)	(0.024)	(0.039)
Loglik.	14054.65	2524.30	10997.86	1695.45
N Obs.	3913	782	3913	781

Note. Parameters estimates of $\Delta s_t = a + h^* \Delta f_t + \epsilon_t$ and Newey-West standard errors in parentheses.

fairly general multivariate alternative, the BEKK model (named after Baba, Engle, Kraft and Kroner), that is guaranteed to yield positive definite covariance matrices. Empirical studies on hedging that use the BEKK model include Baillie and Myers (1991), Gagnon and Lypny (1995), Bera Garcia and Roh (1997) and more recently Haigh and Holt (2002). However Lien, Tse and Tsui (2002) and Byström (2003) reported failure to get convergence in the estimation process of the BEKK model.

The GARCH models most commonly used in practice impose restrictions on the VECH and BEKK models (Ledoit, Santa-Clara and Wolf; 2003). They include the diagonal VECH of Bollerslev, Engle and Wooldridge (1988), the diagonal BEKK model and the constant conditional correlation GARCH (CCC) model of Bollerslev (1990). Amongst these models, the CCC model is the most popular in the hedging context and has been used by Kroner and Sultan (1993), Park and Switzer (1995), Bera, Garcia and Roh (1997), Lien, Tse and Tsui (2002) and Byström (2003), among others.

In this paper, I follow the hedging literature in considering the CCC and the BEKK. The CCC model is given by

$$\begin{aligned} \Delta \mathbf{y}_t &= \boldsymbol{\mu} + \boldsymbol{\epsilon}_t \qquad (4.17) \\ \boldsymbol{\epsilon}_t &\sim i.i.d.\ N(\mathbf{0}, \mathbf{H}_t) \\ h_{s,t} &= \omega_{11} + \alpha_{11}\epsilon_{s,t-1}^2 + \beta_{11} h_{s,t-1} \\ h_{f,t} &= \omega_{22} + \alpha_{22}\epsilon_{f,t-1}^2 + \beta_{22} h_{f,t-1} \\ h_{sf,t} &= \rho\sqrt{h_{s,t} h_{f,t}} \end{aligned}$$

where $\Delta \mathbf{y}_t \equiv [\Delta s_t, \Delta f_t]'$, $\boldsymbol{\mu} \equiv [\mu_s, \mu_f]'$, $\boldsymbol{\epsilon}_t \equiv [\epsilon_{s,t}, \epsilon_{f,t}]'$, $\mathbf{H}_t \equiv \begin{bmatrix} h_{s,t} & h_{sf,t} \\ h_{sf,t} & h_{f,t} \end{bmatrix}$. There are 7 parameters to estimate in the (co)variance equations. This specification has the benefit of parsimony of parameters and gives positive definite and stationary covariance matrices provided that ω_{11}, ω_{22}, α_{11}, α_{22} , β_{11}, and β_{22} are all non-negative satisfying $\alpha_{11}+\beta_{11} < 1$, $\alpha_{22}+\beta_{22} < 1$ and $-1 \leq \rho \leq 1$. The problem with the CCC model is the assumption of constant conditional correlation, which is not always supported by the data.

The BEKK model (or more precisely the student-t version of it; see below), which allows more flexibility than the CCC model, can be written as

$$\begin{aligned} \Delta \mathbf{y}_t &= \boldsymbol{\mu} + \boldsymbol{\epsilon}_t \\ \boldsymbol{\epsilon}_t &\sim i.i.d.\ (\mathbf{0}, \mathbf{H}_t) \\ \mathbf{H}_t &= \boldsymbol{\Omega} + \boldsymbol{\alpha}\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1}\boldsymbol{\alpha}' + \boldsymbol{\beta}\mathbf{H}_{t-1}\boldsymbol{\beta}', \end{aligned} \tag{4.18}$$

where $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$ are 2×2 matrices, and $\boldsymbol{\Omega}$ is symmetric and positive definite. The latter condition is imposed by

$$\boldsymbol{\Omega} \equiv \begin{bmatrix} \omega_{11}^2 & \omega_{11}\omega_{21} \\ \omega_{11}\omega_{21} & \omega_{21}^2 + \omega_{22}^2 \end{bmatrix}.$$

The model imposes positive definiteness restrictions over parameters across equations. The model is stationary if the eigenvalues of $\boldsymbol{\alpha} \otimes \boldsymbol{\alpha} + \boldsymbol{\beta} \otimes \boldsymbol{\beta}$ are smaller than 1 in modulus, with $\otimes$ denoting the Kronecker product of matrices. There are 11 parameters to estimate in the conditional variance equation.

For the CCC model, estimation is performed by maximizing the quasi-likelihood, assuming conditional normality of the innovations. This ensures consistency of the estimates even when the innovations are non-normal. For the second model, the likelihood is maximized assuming the innovations $\boldsymbol{\epsilon}_t$ are *i.i.d.* drawing from the bivariate student-t distribution.[7] The reason for using the student distribution, as opposed to the normal distribution, is that only in a few cases Gaussian BEKK model estimation converged whereas the student BEKK (t-BEKK)

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[7] This bivariate distribution can take a number of forms according to the specification of the dependence. I retain the form which is the most generally used (see Johnson and Kotz (1972), section 3, page 134, for a description of the joint density function). Let Θ be the unknown vector of parameters in the model, including the degree of freedom ν. The loglikelihood for the t-BEKK model is given by

$$\begin{aligned} L_T(\Theta) &= \sum_{t=T_{in}+1}^{T} \left\{ \log\left(\Gamma\left(\frac{\nu+2}{2}\right)\right) - \log\left(\pi\nu\Gamma\left(\frac{\nu}{2}\right)|\mathbf{H}_t|^{1/2}\right) \right. \\ &\quad \left. -\frac{\nu+2}{2}\log\left(1+\nu^{-1}\mathbf{y}'_t\mathbf{H}_t^{-1}\mathbf{y}_t\right) \right\}. \end{aligned} \tag{4.19}$$

where $\Gamma(\cdot)$ is the gamma function.

Table 3: Descriptive statistics of the time-varying hedge ratios, January 1994-December 2003.

	mean	median	max	min	std dev.	corr(CCC; t-BEKK)
			Corn, daily			
CCC	0.9354	0.9141	3.2184	0.5492	0.1439	0.0051
t-BEKK	0.9746	0.9836	1.4699	−0.7808	0.1203	
			Corn, weekly			
CCC	1.0278	0.9864	2.1993	0.6892	0.1814	0.5979
t-BEKK	0.9857	0.9734	1.5388	0.2803	0.1276	
			Wheat, daily			
CCC(constr.)	0.9649	0.9020	5.1135	0.6257	0.2818	−0.3257
t-BEKK	0.9682	0.9884	1.3026	−2.5190	0.1311	
			Wheat, weekly			
CCC	0.9743	0.9494	2.0373	0.6122	0.1765	−0.2110
t-BEKK	0.9753	0.9871	1.1314	0.7372	0.0546	
			Soya, daily			
CCC	0.9171	0.9041	1.6787	0.6732	0.0968	0.1898
t-BEKK	0.9568	0.9700	1.3225	−0.0624	0.0912	
			Soya, weekly			
CCC	0.9853	0.9679	1.7592	0.7162	0.0919	0.8800
t-BEKK	0.9807	0.9714	1.3993	0.6232	0.0736	
			Oats, daily			
CCC	0.4105	0.3590	2.2815	0.1380	0.1899	0.1679
t-BEKK	0.4029	0.4159	3.3082	−1.5567	0.3624	
			Oats, weekly			
CCC	0.6755	0.6506	2.5811	0.2597	0.1796	0.4019
t-dBEKK	0.6247	0.6396	1.0870	0.0417	0.1547	

model estimation converged for all commodities and frequencies, except for oats on a weekly horizon.[8] For that particular commodity on weekly horizon, I estimated instead the diagonal t-BEKK model (t-dBEKK) which is given by (4.18) where $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$ are 2×2 diagonal matrices. The estimations were carried out using the Marquardt algorithm in the Logl object in EViews 5.1. Parameter values from univariate GARCH were used to initialize the BGARCH estimation.

The parameter estimates for the CCC, the t-BEKK and the t-dBEKK models along with standard errors are presented in Tables 8 to 15 in the appendix, for corn, wheat, oats and soybeans on daily and weekly hedging horizons. Please note that in Table 10, for wheat with daily data, the CCC model estimates do not satisfy the stationarity condition since $\hat{\alpha}_{11} + \hat{\beta}_{11} > 1$. This issue is discussed in the next section.

At time $t-1$, for each horizon, the dynamic hedge ratio is then computed as the ratio of the out-of-sample conditional covariance between Δs_t and Δf_t to the out-of-sample conditional variance of Δf_t, *i.e.* as $h^*_{t-1} = \frac{\hat{h}_{sf,t}}{\hat{h}_{f,t}}$ with $t = T_{in} + 1, ..., T$. Table 3 shows the descriptive

[8] The convergence problem of the BEKK model is discussed in the following section.

statistics of the various dynamic hedge ratios. As the standard deviations indicate, the hedge ratios vary considerably across time. There is a non-negligible difference between the hedges ratios implied by the CCC and the t-BEKK models. Moreover, the correlations between the hedge ratios are surprisingly small on daily horizons. These are even negative on both horizon for wheat.

4.4 BGARCH estimation issues

Before comparing relative hedging performances, it is worth pointing out several problems I encountered in estimating BGARCH models.

First, I experienced problems in estimating the BEKK model for most commodities/frequencies and in estimating the t-BEKK model for oats with weekly data. The problem lies mainly in the positive definiteness constraint that I impose on Ω. Typically, one of the two eigenvalues and the determinant of $\hat{\Omega}$ are pretty close to zero suggesting that Ω is nearly singular[9] (while I have constrained for positive definiteness). The problem persists even when I consider alternative constrained specifications for Ω. Trying to resolve the puzzle, I estimated an unconstrained symmetric specification for Ω. The singularity of the unconstrained Ω is pointed out by the determinant and by the eigenvalues of its estimate.[10] The similarity between the likelihood values of the model with unconstrained and constrained Ω leads to the conclusion that the loglikelihood is maximal on or slightly outside the boundary of the parameters space.[11] In other words, the problem in the estimation procedure is due to the fact that the algorithm forces Ω to be singular, because the maximum of the likelihood is found where Ω is singular, while I constrain it to be positive definite. As discussed above, to circumvent the estimation problem, I estimated the BEKK model with student-t innovations.

Another problem has occurred when estimating the CCC model for wheat with daily data. As discussed above, the CCC model estimates do not satisfy the stationarity condition since $\hat{\alpha}_{11} + \hat{\beta}_{11} > 1$. The CCC model with the constraint[12] $\hat{\alpha}_{11} + \hat{\beta}_{11} \leq 0.999$ is also estimated.

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[9] For example, for oats with daily data, the eigenvalues of $\hat{\Omega}$ are 2.26×10^{-14} and 1.92×10^{-4} and the determinant equals 4.35×10^{-18}.

[10] For example, for oats with daily data, the eigenvalues the unconstrained $\hat{\Omega}$ are -1.38×10^{-4} and 1.82×10^{-4} and the determinant is equal to -2.51×10^{-8}.

[11] For example, for oats with daily data, the likelihood is 3193.154 for the BEKK model with the constrained Ω, and 3194.470 for the BEKK model with the unconstrained Ω.

[12] The algorithm achieved convergence for the constrained CCC model but did not report standard errors of the estimates. This is not so important because the latter are bounded by the standard errors from the

As can be seen from the last column of Table 10, the constraint $\hat{\alpha}_{11} + \hat{\beta}_{11} \leq 0.999$ is binding, meaning that the likelihood of the constrained CCC model is maximal on the boundary of the parameters space. I therefore report the constrained CCC model for that particular commodity on a daily hedging horizon.

4.5 Out-of-sample comparison of hedging effectiveness: GARCH strategies versus ED's strategy

In this section, the relative out-of-sample unconditional performance of the dynamic strategies (CCC, t-BEKK or t-dBEKK) against ED's static strategy is investigated. The difference in the squared returns is $d_t \equiv r_t^2 \left(h^{STATIC}\right) - r_t^2 \left(h^{DYNAMIC}\right)$ with $t = T_{in} + 1, ..., T$, where $h^{DYNAMIC}$ is the strategy implied by the CCC, t-BEKK or t-dBEKK model.

Table 4: Dynamic vs. static strategy: out-of-sample relative hedging performances, 1/1/1994-31/12/2003.

$d_t \equiv r_t^2 \left(h^{STATIC}\right) - r_t^2 \left(h^{DYNAMIC}\right)$					
Competing Strategies	Unconditional performance			Conditional performance	
Dyn. vs. static	$\bar{d}$	Winning	*DM* p-val.	GW_z p-val.	W_z p-val.
		Corn, daily			
CCC vs. ED	-5.38×10^{-6}	ED	0.0095	0.0006	0.0010
t-BEKK vs. ED	-4.16×10^{-6}	ED	0.0097	0.0392	0.0387
		Wheat, daily			
CCC (constr.) vs. ED	-1.79×10^{-5}	ED	0.0563	0.2154	0.2130
t-BEKK vs. ED	2.19×10^{-6}	t-BEKK	0.0981	0.2589	0.2780
		Soybeans, daily			
CCC vs. ED	-1.07×10^{-6}	ED	0.0020	0.0117	0.0068
t-BEKK vs. ED	-3.33×10^{-7}	ED	0.5324	0.0312	0.0211
		Oats, daily			
CCC vs. ED	-2.91×10^{-6}	ED	0.5169	0.2367	0.1953
t-BEKK vs. ED	6.16×10^{-6}	t-BEKK	0.5466	0.0578	0.0650
		Corn, weekly			
CCC vs. ED	-2.08×10^{-5}	ED	0.1919	0.4837	0.1814
t-BEKK vs. ED	-1.63×10^{-5}	ED	0.3305	0.9045	0.2018
		Wheat, weekly			
CCC vs. ED	-4.69×10^{-5}	ED	0.0004	0.0011	0.0008
t-BEKK vs. ED	5.21×10^{-6}	t-BEKK	0.4088	0.3057	0.2705
		Soybeans, weekly			
CCC vs. ED	-7.73×10^{-6}	ED	0.2562	0.5350	0.5425
t-BEKK vs. ED	-3.79×10^{-6}	ED	0.0902	0.5859	0.6039
		Oats, weekly			
CCC vs. ED	1.02×10^{-5}	CCC	0.8168	0.2097	0.1587
t-dBEKK vs. ED	-2.90×10^{-6}	ED	0.9407	0.8962	0.8975

In the third column of Table 4, the "potential" winning strategy for the unconditional two-sided tests is reported. The p-values of the DM test are reported in the fourth column.

unconstrained CCC model.

The following conclusions can be drawn from Table 4. First, the CCC model has the worst overall unconditional performance and should not be considered for hedging these agricultural commodities. Second, ED 's static strategy mostly outperforms the GARCH strategies unconditionally on a daily horizon. In fact, the dynamic strategies never significantly outperform ED's strategy in the unconditional sense. This is in contrast with the conclusions (based on different data, however) reached in Baillie and Myers (1991). Third, the difference in unconditional hedging performance is less pronounced on the weekly horizon than on the daily horizon. Fourth, significant differences in unconditional performances are also picked up by the conditional performance tests. In addition, the latter tests reveal in some cases differences in conditional performance while the unconditional performances are very similar.

4.6 Out-of-sample comparison of hedging effectiveness: new strategy versus initial strategies

The objective of this section is to investigate the relative performance of h^3. As discussed above, I proceed in two steps. First, the initial static hedge ratios in section 4.2 are re-used and the dynamic hedge ratios are computed over the period $t = 1, ..., T_{in}$ with the GARCH parameters estimated in section 4.3. Table 5 shows the GW_z and W_z p-values, *i.e.* the relative conditional performance of h^1 and h^2 over the same period. Compared to Table 4, rejections of $H_{0,z}$ are only marginal.[13] Then h^3 is computed from the rule (3.14).

Table 5: Dynamic vs. static strategy: in-sample relative hedging performances, 1/1/1979-31/12/1993.

$d_t \equiv r_t^2\left(h^{STATIC}\right) - r_t^2\left(h^{DYNAMIC}\right)$					
DAILY			WEEKLY		
Competing Strategies	Conditional performance		Competing Strategies	Conditional performance	
Dyn. vs. static	GW_z p-val.	W_z p-val.	Dyn. vs. static	GW_z p-val.	W_z p-val.
Corn			Corn		
CCC vs. ED	0.2031	0.1803	CCC vs. ED	0.4895	0.6073
t-BEKK vs. ED	0.8216	0.8041	t-BEKK vs. ED	0.4478	0.4764
Wheat			Wheat		
CCC (constr.) vs. ED	0.0155	0.0023	CCC vs. ED	0.0305	0.3432
t-BEKK vs. ED	0.2041	0.2449	t-BEKK vs. ED	0.7218	0.6717
Soybeans			Soybeans		
CCC vs. ED	0.9617	0.0392	CCC vs. ED	0.2041	0.1762
t-BEKK vs. ED	0.2650	0.1342	t-BEKK vs. ED	0.0511	0.0408
Oats			Oats		
CCC vs. ED	0.2989	0.3210	CCC vs. ED	0.5740	0.5812
t-BEKK vs. ED	0.5349	0.5171	t-dBEKK vs. ED	0.7853	0.4079

[13] In the same way, in-sample rejections of $H_{0,DM}$ (not reported here) are only marginal compared to Table 4, even though the "potential" winning strategies are mostly the same.

Table 6: New dynamic strategy vs. initial strategies: out-of-sample relative hedging performances on a daily horizon, 3/1/1994-31/12/2003.

$d_t \equiv r_t^2\left(h^3\right) - r_t^2\left(h^{INITIAL}\right)$					
Competing Strategies	Unconditional performance			Conditional performance	
New vs. Initial	$\overline{d}$	Winning	DM p-val	GW_z p-val	W_z p-val.
			Corn, daily		
h^3(CCC,ED) vs. ED	9.39×10^{-7}	ED	0.0138	0.1002	0.0574
h^3(CCC,ED) vs. CCC	-4.45×10^{-6}	h^3	0.0234	0.0122	0.0132
h^3(t-BEKK,ED) vs. ED	1.11×10^{-6}	ED	0.0547	0.2687	0.2929
h^3(t-BEKK,ED) vs. t-BEKK	-3.05×10^{-6}	h^3	0.0232	0.0321	0.0366
			Wheat, daily		
h^3(CCC,ED) vs. ED	8.77×10^{-6}	ED	0.1348	0.1308	0.0819
h^3(CCC,ED) vs. CCC	-9.15×10^{-6}	h^3	0.1326	0.5160	0.5041
h^3(t-BEKK,ED) vs. ED	-3.54×10^{-7}	h^3	0.6516	0.0198	0.0208
h^3(t-BEKK,ED) vs. t-BEKK	1.84×10^{-6}	t-BEKK	0.0617	0.5694	0.5777
			Soybeans, daily		
h^3(CCC,ED) vs. ED	8.08×10^{-7}	ED	0.0042	0.0108	0.0117
h^3(CCC,ED) vs. CCC	-2.58×10^{-7}	h^3	0.1985	0.0642	0.0518
h^3(t-BEKK,ED) vs. ED	8.52×10^{-7}	ED	0.0679	0.2671	0.2926
h^3(t-BEKK,ED) vs. t-BEKK	5.19×10^{-7}	t-BEKK	0.0747	0.0334	0.0241
			Oats, daily		
h^3(CCC,ED) vs. ED	-4.77×10^{-6}	h^3	0.0536	0.3945	0.4052
h^3(CCC,ED) vs. CCC	-7.68×10^{-6}	h^3	0.0547	0.3434	0.3147
h^3(t-BEKK,ED) vs. ED	1.36×10^{-7}	ED	0.9888	0.1600	0.2231
h^3(t-BEKK,ED) vs. t-BEKK	6.30×10^{-6}	t-BEKK	0.1071	0.0865	0.1461

Define the difference in the squared returns $d_t \equiv r_t^2\left(h^3\right) - r_t^2\left(h^{INITIAL}\right)$. The out-of-sample unconditional hedging performance of h^3 relative to the initial strategies, is indicated by the DM p-values and the "potential" winning strategy reported in the fourth and third columns of Tables 6 and 7. The following conclusions can be drawn. First, h^3 improves both initial dynamic strategies in the case of corn on a daily horizon and improves both CCC and ED for oats on a daily horizon, as well. In general, however, the new strategies h^3 do not outperform the static strategy. Second, the difference in hedging performance is less pronounced on weekly horizons than on daily horizons.

I also report the conditional (in-sample) tests through the GW_z and W_z p-values in the fifth and sixth columns of Tables 6 and 7: the rejection indicates that there is some information left to construct a new strategy, h^4, which selects amongst h^3 and $h^{INITIAL}$ the strategy that will perform the best next period, conditional on current information.

5 Conclusion

In this paper I have tried to answer the following two questions: among two initial competing hedging strategies (i) which strategy will reduce the portfolio risk more on an average? and (ii) which strategy will reduce the next period risk more, given current information? The former

Table 7: New dynamic strategy vs. initial strategies: out-of-sample relative hedging performances on a weekly horizon, 1/1/1994-31/12/2003.

$d_t \equiv r_t^2\left(h^3\right) - r_t^2\left(h^{INITIAL}\right)$

Competing Strategies	Unconditional performance			Conditional performance	
New vs. Initial	$\bar{d}$	Winning	DM p-val	GW_z p-val	W_z p-val.
			Corn, weekly		
h^3(CCC,ED) vs. ED	4.03×10^{-6}	ED	0.1026	0.5990	0.6013
h^3(CCC,ED) vs. CCC	-1.67×10^{-5}	h^3	0.3031	0.3843	0.1132
h^3(t-BEKK,ED) vs. ED	-1.07×10^{-8}	h^3	0.9966	0.6918	0.7340
h^3(t-BEKK,ED) vs. t-BEKK	-1.63×10^{-5}	h^3	0.3249	0.7850	0.0633
			Wheat, weekly		
h^3(CCC,ED) vs. ED	1.88×10^{-5}	ED	0.0120	0.0740	0.0365
h^3(CCC,ED) vs. CCC	-2.81×10^{-5}	h^3	0.0023	0.0176	0.0300
h^3(t-BEKK,ED) vs. ED	-2.39×10^{-6}	h^3	0.3339	0.5997	0.5613
h^3(t-BEKK,ED) vs. t-BEKK	2.82×10^{-6}	t-BEKK	0.6430	0.4690	0.5635
			Soybeans, weekly		
h^3(CCC,ED) vs. ED	7.44×10^{-6}	ED	0.2927	0.3672	0.3798
h^3(CCC,ED) vs. CCC	-2.86×10^{-7}	h^3	0.8729	0.1487	0.4034
h^3(t-BEKK,ED) vs. ED	2.76×10^{-7}	ED	0.8891	0.7202	0.6759
h^3(t-BEKK,ED) vs. t-BEKK	-3.51×10^{-7}	h^3	0.0062	0.0217	0.0210
			Oats, weekly		
h^3(CCC,ED) vs. ED	-1.86×10^{-5}	h^3	0.5024	0.7863	0.8868
h^3(CCC,ED) vs. CCC	-8.44×10^{-6}	h^3	0.8012	0.1877	0.1327
h^3(t-dBEKK,ED) vs. ED	-8.25×10^{-7}	h^3	0.9624	0.7301	0.7860
h^3(t-dBEKK,ED) vs. t-dBEKK	-3.73×10^{-6}	h^3	0.9214	0.4524	0.5259

question is addressed by making use of the Diebold and Mariano (1995) unconditional test. The latter can be answered by the Giacomini and White (2006) and the Wald conditional tests.

The theoretical relations between the three tests is discussed and the idea that rejection of the null of equal conditional hedging effectiveness naturally defines a third strategy is developed. This strategy selects the initial strategy that will perform best next period, conditional on available information. In addition, it is shown that, in a minimum variance framework, it was not possible to generalize that new strategy to a strategy that is a weighted average of two strategies.

One traditional static strategy and two dynamic strategies based on popular bivariate GARCH models are considered and the problems encountered in the estimation are outlined.

The tests are then applied to four agricultural commodities to compare those strategies. It is found that more elaborate GARCH strategies do not outperform the simple OLS regression hedge ratio on daily and weekly horizons. Moreover, the new strategy implied by the tests do not often reduce the risk significantly as compared to the static strategy. The empirical results are disappointing for the conditional approach to commodity hedging and call into question earlier results reported in the literature that were based on a much smaller database.

Appendix: Estimation results

Table 8: t-BEKK and CCC estimates for corn using daily data from 2/1/1979 to 31/12/1993

	t-BEKK		CCC	
μ_s	0.000380	(0.000145)	0.000354	(0.00016) 1
μ_f	−0.000141	(0.000133)	-2.74×10^{-5}	(0.000148)
ω_{11}	0.001762	(0.000175)	1.13×10^{-5}	(6.28×10^{-7})
ω_{12}	8.44×10^{-5}	(0.000184)		
ω_{22}	0.001048	(0.000140)	5.76×10^{-6}	(6.31×10^{-7})
α_{11}	0.365211	(0.023806)	0.097748	(0.003815)
α_{22}	0.235820	(0.022522)	0.078896	(0.005503)
α_{12}	−0.041977	(0.021115)		
α_{21}	−0.133040	(0.024072)		
β_{11}	0.770123	(0.022974)	0.833157	(0.006149)
β_{22}	0.932680	(0.021096)	0.869667	(0.009422)
β_{12}	0.029777	(0.020194)		
β_{21}	0.191540	(0.024165)		
ν	3.989207	(0.201341)		
ρ			0.804914	(0.003945)
Loglik.	29036.73		26020.68	
SIC^c	−14.81157		−13.28058	
N. obs.	3914			

Table 9: t-BEKK and CCC estimates for corn using weekly data from 19/1/1979 to 31/12/1993.

	t-BEKK		CCC	
μ_s	0.001972	(0.000761)	0.001372	(0.000934)
μ_f	−0.000739	(0.000706)	−0.000647	(0.000782)
ω_{11}	0.006565	(0.001168)	7.45×10^{-5}	(9.70×10^{-6})
ω_{12}	0.007438	(0.001332)		
ω_{22}	0.003730	(0.000519)	0.000102	(2.42×10^{-5})
α_{11}	0.158349	(0.054215)	0.171585	(0.012951)
α_{22}	0.390815	(0.071067)	0.128247	(0.017947)
α_{12}	−0.102882	(0.049838)		
α_{21}	0.126900	(0.073613)		
β_{11}	0.939962	(0.054331)	0.759041	(0.014941)
β_{22}	0.711744	(0.085589)	0.699629	(0.051237)
β_{12}	0.098289	(0.054539)		
β_{21}	−0.081546	(0.083698)		
ν	3.634522	(0.393009)		
ρ			0.841710	(0.007705)
Loglik.	4569.660		3973.957	
SIC^c	−11.59755		−10.11280	
N. obs.	780			

Table 10: t-BEKK, CCC and constrained CCC estimates for wheat using daily data from 4/1/1983 to 31/12/1993.

	t-BEKK		CCC[a]		constrained[b] CCC
μ_s	0.000414	(0.000203)	0.000491	(0.000174)	0.000492
μ_f	1.80×10^{-5}	(0.000185)	0.000160	(0.000172)	0.000161
ω_{11}	0.002320	(0.000575)	7.68×10^{-6}	(6.96×10^{-7})	7.92×10^{-6}
ω_{12}	−0.000417	(0.000670)			
ω_{22}	0.001548	(0.000354)	5.35×10^{-6}	(9.28×10^{-7})	5.40×10^{-6}
α_{11}	0.353453	(0.029816)	0.167634	(0.005447)	0.167416
α_{22}	0.171771	(0.029001)	0.104523	(0.007829)	0.104447
α_{12}	−0.033202	(0.023524)			
α_{21}	−0.259523	(0.033195)			
β_{11}	0.524220	(0.048086)	0.832918	(0.004456)	0.831584
β_{22}	0.946172	(0.041628)	0.863992	(0.011515)	0.863531
β_{12}	0.021466	(0.039044)			
β_{21}	0.486268	(0.050912)			
ν	3.353898	(0.153612)			
ρ			0.735086	(0.006696)	0.734573
Loglik.	20614.61		17829.23		17829.22
SIC[d]	−14.33673		−12.40823		−12.40822
N. obs.	2869				

a. Notice that the stationarity condition is not met because $\alpha_{11}+\beta_{11} > 1$.
b. I forced stationarity by imposing $\alpha_{11} + \beta_{11} \leq 1 - 10^{-3}$.

Table 11: t-BEKK and CCC estimates for wheat using weekly data from 6/1/1983 to 31/12/1993.

	t-BEKK		CCC	
μ_s	0.002074	(0.001161)	−0.000437	(0.001486)
μ_f	0.000327	(0.001002)	6.57×10^{-5}	(0.000972)
ω_{11}	0.008151	(0.011126)	0.000192	(6.07×10^{-5})
ω_{12}	0.002253	(0.015324)		
ω_{22}	−0.000314	(0.110942)	5.13×10^{-5}	(2.90×10^{-5})
α_{11}	0.085760	(0.038587)	0.290261	(0.039502)
α_{22}	0.125357	(0.080125)	0.097603	(0.027362)
α_{12}	0.087636	(0.039699)		
α_{21}	0.104308	(0.090631)		
β_{11}	1.379511	(0.327410)	0.631969	(0.057908)
β_{22}	0.302600	(0.301664)	0.826642	(0.057459)
β_{12}	0.597409	(0.282569)		
β_{21}	−0.669437	(0.325710)		
ν	3.669311	(0.381857)		
ρ			0.722465	(0.022275)
Loglik.	2875.060		2381.121	
SIC[c]	−10.86860		−9.032514	
N. obs.	522			

Table 12: t-BEKK and CCC estimates for soya using daily data from 3/1/1979 to 31/12/1993.

	t-BEKK		CCC	
μ_s	0.000200	(0.000149)	4.17×10^{-5}	(0.000148)
μ_f	−0.000173	(0.000149)	−0.000247	(0.000149)
ω_{11}	0.000637	(0.000442)	2.69×10^{-6}	(2.34×10^{-7})
ω_{12}	−0.000400	(0.000783)		
ω_{22}	−0.001190	(0.000362)	3.09×10^{-6}	(3.02×10^{-7})
α_{11}	0.403448	(0.030066)	0.078454	(0.003195)
α_{22}	0.222638	(0.028126)	0.077958	(0.003725)
α_{12}	−0.053120	(0.027462)		
α_{21}	−0.231053	(0.029191)		
β_{11}	0.690788	(0.033542)	0.907217	(0.003440)
β_{22}	0.893452	(0.033134)	0.903621	(0.004525)
β_{12}	0.073965	(0.031814)		
β_{21}	0.289634	(0.034786)		
ν	3.377528	(0.145856)		
ρ			0.877837	(0.002381)
Loglik.	29887.11		26354.16	
SIC^c	−15.25010		−13.45447	
N. obs.	3913			

Table 13: t-BEKK and CCC estimates for soya using weekly data from 12/1/1979 to 31/12/1993.

	t-BEKK		CCC	
μ_s	0.000188	(0.000806)	0.000104	(0.000866)
μ_f	−0.001792	(0.000803)	−0.001509	(0.000842)
ω_{11}	0.005114	(0.001063)	9.83×10^{-5}	(1.49×10^{-5})
ω_{12}	0.004888	(0.001248)		
ω_{22}	0.002865	(0.000330)	0.000103	(1.69×10^{-5})
α_{11}	0.434596	(0.093833)	0.147309	(0.015651)
α_{22}	0.228820	(0.094001)	0.135949	(0.014875)
α_{12}	0.058995	(0.089333)		
α_{21}	−0.149972	(0.094546)		
β_{11}	0.745988	(0.113503)	0.737419	(0.028891)
β_{22}	0.921258	(0.109572)	0.729466	(0.029412)
β_{12}	−0.018772	(0.103022)		
β_{21}	0.168100	(0.119153)		
ν	4.502604	(0.588959)		
ρ			0.938979	(0.003197)
Loglik.	4826.173		4228.206	
SIC^c	−12.23956		−10.75092	
N. obs.	782			

Table 14: t-BEKK and CCC estimates for oats using daily data from 3/1/1979 to 31/12/1993.

	t-BEKK		CCC	
μ_s	0.000180	(0.000157)	0.000156	(0.000216)
μ_f	−0.000242	(0.000191)	−0.000370	(0.000232)
ω_{11}	0.001195	(0.000111)	1.26×10^{-5}	(1.03×10^{-6})
ω_{12}	0.000174	(0.000209)		
ω_{22}	0.000643	(0.000246)	5.02×10^{-6}	(8.35×10^{-7})
α_{11}	0.188137	(0.008655)	0.064838	(0.004665)
α_{22}	0.171785	(0.011488)	0.081297	(0.007055)
α_{12}	0.036997	(0.011735)		
α_{21}	0.015497	(0.009167)		
β_{11}	0.940508	(0.004771)	0.884306	(0.007715)
β_{22}	0.974814	(0.004004)	0.902066	(0.008168)
β_{12}	−0.025995	(0.007079)		
β_{21}	0.002326	(0.003789)		
ν	2.834261	(0.142549)		
ρ			0.361231	(0.011511)
Loglik.	24632.95		21872.25	
SIC^c	−12.56393		−11.16310	
N. obs.	3913			

Table 15: t-dBEKK and CCC estimates for oats using weekly data from 19/1/1979 to 31/12/1993.

	t-dBEKK		CCC	
μ_s	0.000437	(0.001128)	-7.29×10^{-5}	(0.001146)
μ_f	−0.001882	(0.001162)	−0.002782	(0.001269)
ω_{11}	0.011975	(0.001922)	0.000374	(9.66×10^{-5})
ω_{12}	0.006125	(0.001044)		
ω_{22}	0.005937	(0.001354)	0.000216	(5.94×10^{-5})
α_{11}	0.266370	(0.035174)	0.145773	(0.032727)
α_{22}	0.228367	(0.027407)	0.121961	(0.015691)
α_{12}	−	−		
α_{21}	−	−		
β_{11}	0.852220	(0.040438)	0.580137	(0.093910)
β_{22}	0.917187	(0.020079)	0.728981	(0.046541)
β_{12}	−	−		
β_{21}	−	−		
ν	5.292789	(0.828016)		
ρ			0.664129	(0.016834)
Loglik.	3645.719		3171.466	
SIC^c	−9.262621		−8.055127	
N. obs.	781			

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