Transform Analysis and Asset Pricing for General Diffusion Processes: A Recursive Approach

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In most cases, the transition density function of an Itô stochastic differential equation is not available in closed-form. Considering the Laplace transform of this transition density with respect to the time variable, the problem is reduced to the solution of the determination of an ordinary differential equation, where the boundary conditions are such that for x_T (endpoint) approaches infinity, the density equals zero, whereas there is a jump when x_T crosses x_t for the first order derivative. Using Feynman-Kac integration containing a potential and in addition a δ-function perturbation, we construct an exact recursion scheme for the Laplace transform of the transition density. This allows a very accurate and nearly analytical treatment of a wide range of valuation and econometric problems for both short time horizon and long time horizon. Generalizations of our technique to functionals of Lévy processes are also briefly discussed.

Keywords: continuous-time Markov model, diffusion process, transition density function, Feynman-Kac integration, real Laplace inversion.